SOME LABORATORY STUDIES OF FACTORS PERTAINING TO THE BEARING CAPACITY OF SOILS

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SYNOPSIS

The paper briefly describes some small-scale bearing capacity studies, made in an attempt to evaluate some of the factors which control resistance of a mass of granular soil to failure under applied surface loads Techniques such as the displacement of buried glass beads, distortion of thin layers of white powder and X-ray examination of soil mass at various stages of loading, were used to obtain information on displacements within the soil mass, and on the shape and extent of the failure zone By means of the X-ray shadowgraphs, the shape of failure zones within a mass was clearly delineated.

From tests using model footings of various lengths, with length of footing almost equal to the width of the container, it was found that the friction between the soil and the wall of the container influences the shape of the failure surface for a distance from the sidewall equal to about two footing widths.

Very poor correlation was found between the bearing capacity found by the small scale tests and the results of calculation using existing methods of analysis. However, if account is taken of the effect of stresses generated by the applied load in inducing frictional resistance along the failure surface or zone, even the results of rough calculations were in reasonable agreement with the experimental results.

The purpose of this paper is to stimulate renewed thinking on the problem of the supporting capacity of soil masses under surface loads To that end the information presented herein is stated briefly, is largely qualitative, and the intent is primarily to highlight some observations of soil behavior, which may serve as leads to further research.

About two years ago in the Division of Civil Engineering at the University of California, some members of the staff and a group of graduate students, who were interested in soil mechanics, held a series of seminars in which the problem of supporting capacity of foundations was reviewed and critically discussed from various points of view. It was apparent to this group, as it is to all who have wrestled with the bearing capacity problem, that an adequate method of analysis, sufficiently comprehensive to be applicable to a variety of cases, has yet to be developed. Desiring to become mutually better informed, at first hand, concerning some of the factors which influence bearing capacity, some of the group undertook a series of experimental studies designed to substantiate or refute assumptions or hypotheses employed in some of the current analytic procedures. It is from these studies that the illustrations here cited are drawn

Concepts pertaining to the stability of earth masses developed by Coulomb, Rankine and Fellenius are basic and germane; and the contributions of Prandtl, Terzaghi, Krey and others to the specific problem of bearing capacity have led the way from the purely empirical to a more scientific approach. In fact, for predominantly cohesive soils, analyses such as those of Prandtl appear to give results which are in fairly good agreement with the results of experiments However, for those soils in which the resistance is predominantly due to internal friction, (called "granular" soils for convenience) the discrepancy between analytic results from existing methods of calculation and test results is variable and sometimes extremely large; in some cases and for some methods the discrepancy is so large as to cast grave doubt upon the validity of the methods, although one may also suspect that in many bearing capacity tests all variables are not controlled or that the criterion for failure does not, in effect, coincide with that envisioned by the analytic method with which a comparison is made.

In view of the circumstances to which allusion has just been made, it appears that attention may well be directed to the development of a comprehensive approach to the supporting capacity of soils in which internal friction is the predominant factor. That extrapolation of the results of bearing tests made with small sizes of bearing plate to full size footings can be made only on an empirical basis, attests the fact that our theories of bearing capacity are far from complete.

The experiments to which reference is made here were all conducted on sandy soils in the laboratory on a small scale. In most of the tests, the footing had lengths of 10 to 24 times the footing widths, and the widths were generally 1 or 1½ in. wide The soil masses on which

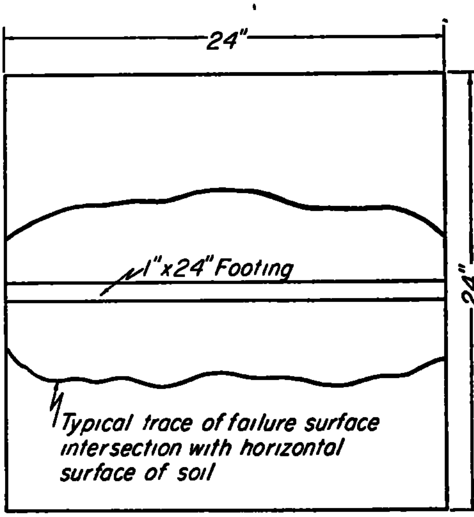


Figure 1. Typical Failure Surface Trace

the bearing tests were made had horizontal dimensions of from 1½ to 3 ft. and were of depth at least eight times the footing width. Experiments of bearing capacity on this small a scale are generally suspect, not only because correlation with large-scale behavior is uncertain, but also because correlation with results of existing theory is practically nil. Because of these objections, it was considered that small-scale tests should be where such studies should start.

Studies of Nature and Extent of Soil Displacements—Illustrations of the displacement of soils under model footings, made by observing or photographing particle movements through a glass-sided container, are familiar to students of the bearing capacity problem. It is recognized that friction between the glass and the soil during displacement influences the particle movements at this boundary. Such "side-wall" friction also may influence the results of bearing capacity tests made in containers. To obtain some idea of the influence upon displacements of such side-wall friction, experiments were made using containers of several widths with footings of length nearly equal to the width of the containers. The intersection of the surface of rupture with the top surface of the soil was observed as shown in Figure 1. It was concluded that the side-wall friction influences the mass of soil displaced at failure for a distance from the side-wall of the order of two to three times the footing width, in experiments on the scale here performed. While the number

Figure 2. Failure Surface Trace Utilizing Silica Layers

of these tests was too few to warrant precise conclusions, there was no consistent difference between the unit bearing loads at failure for footings having length-width ratios of 10 and of 24.

To investigate the nature and extent of displacements within a soil mass three experimental methods were employed. In some of the tests mentioned in the preceding paragraph, small glass beads were buried in the soil before loading and their positions after failure were determined. The technique employed was as follows: the soil was compacted in one inch layers; on each layer along a line perpendicular to the footing at its mid length, beads were placed at one inch intervals by means of a template; after failure the soil was slowly saturated with water to provide artificial cohesion and then the soil mass carefully sliced and the displaced positions of the beads located. This procedure was tedious and lacking in precision; however it gave some indication of the extent, vertically and horizontally, to which appreciable displacement should be expected, and gave further qualitative indications confirmed by the other procedures to be described. A number of tests were made in which, in the top three inches of the soil mass, the upper surface of successive ½-in compacted spacing between the side walls of the container was 3 or 4½ in., the latter being the maximum thickness of soil which it was considered feasible to penetrate with the x-ray equipment available. Hence there was undoubtedly some effect of side-wall friction on the displacements observed. A diagram of the experimental set-up is shown in Figure 3. In some of the tests, sheets of x-ray film, 11 by 17 in., were

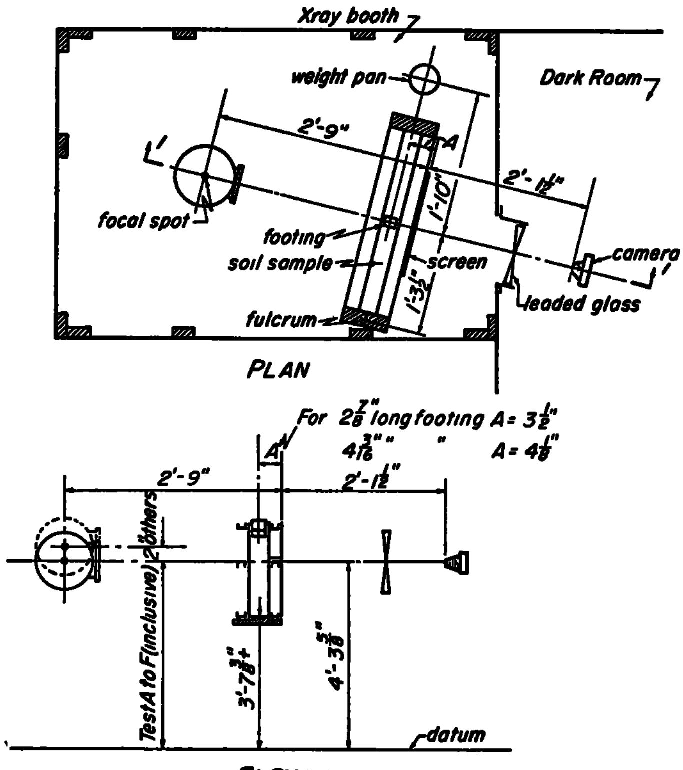


Figure 3. General Arrangement of Apparatus for X-Ray Study

layers were coated with finely ground white silica The soil in these tests was moistened to supply an effective cohesion during test and to facilitate slicing the soil mass after failure. A view of a sliced test specimen after loading to failure is shown in Figure 2 The surface of rupture may be traced by noting the breaks in the white lines marking the contacts between the original layers. A third series of tests was conducted in which an X-ray technique was used to trace the displacement of lead bird shot buried in the soil mass. In these tests, the placed behind the container; in others a fluorescent screen placed behind the container was photographed with a protected 35-mm. camera. Photographs were made after each of several successive increments of load until failure occurred The photographs were projected to enable plotting of points to a large scale. After the positions of the points were corrected for parallax, composite charts were made of particle displacements.

Samples of the x-ray photographs taken just after failure in three of the specimens are reproduced in Figures 4, 5 and 6. The dark spots are the shadows of the bird shot. Of special interest, however, are the traces of the failure zones extending outward from the bases of the footings, and of the densified areas immediately beneath the shadows of the footings. The soil was moderately-well compacted, to a density estimated to be above the critical density. In such a state, when appreciable shear displacement takes place, the mass expands as the grains ride over each other. The decrease in density which accompanied the shearing action along the zone of rupture was sufficiently great to affect the x-ray photographs.

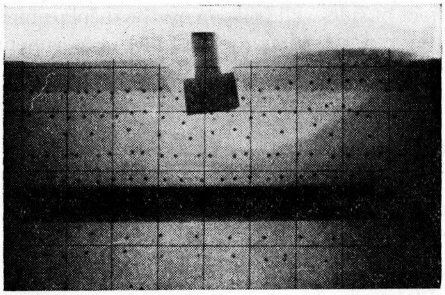


Figure 4. Density Differences Along Failure Surface

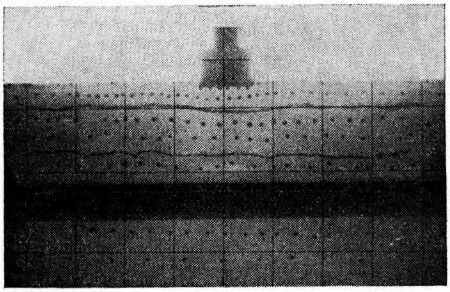


Figure 5. Soil Displacements by Means of Lead Shot—An Early Stage of Loading

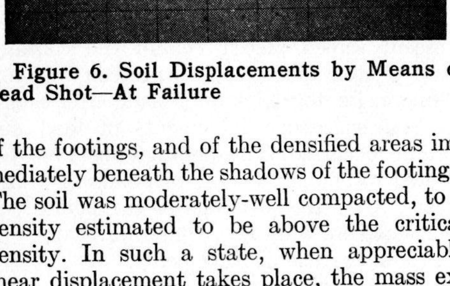


Figure 6. Soil Displacements by Means of Lead Shot—At Failure

In Figure 4, indication of failure zones on each side of the footing may be faintly discerned. These tests provide direct evidence of the following: (1) a dense or densified zone of triangular section may develop beneath a footing, which "wedge" appears to move, during the latter stages of loading, as if it were a part of the footing; (2) shearing failure may develop along a relatively thin zone of characteristic configuration on one or both sides of the footing.

Figure 7 is a typical plot of displacements, during the course of loading, from one of the x-ray studies. Figure 8 is a plot of contours of equal displacement, based on a plot such as that shown in Figure 7. Examination of the data from these and the previously mentioned tests indicates the following generalizations when no surcharge is present, at least for experiments on a small scale: (a) Displacements are inappreciable at depths below 3 footing widths; this should permit the use of shallower containers than those employed in these studies; (b) The failure zone, when it develops on one side of the footing, intersects the surface at a distance from the footing of not more than 5 footing widths, and lateral deformation is inappreciable beyond 6 footing widths; this indicates that the dimension of the container normal to the length of the footing should be at least 15 footing widths to eliminate interference with the free development of the failure zone.

From a study of the various evidence available on the shape of the failure zone, it was found that it may be closely approximated by a surface having a trace on a vertical plane of the shape of a logarithmic spiral.

One or Two Failure Zones—Over the several groups of experiments, considerable variability in load at failure was found. Generally the lowest values of bearing capacity were noted for those tests in which marked tipping of the footing occurred by the time maximum load was attained; in such cases, the failure zone development was markedly unsymmetrical. Here eccentricity of load, although unintentional and probably slight, appears to be an outstanding influence upon the resistance which can be developed. This is illustrated by the values of observed bearing capacity shown in Table 1, taken from the results of one of the series of tests.

In many tests, however, the footing settled evenly until after maximum load had been had been reached, and measurements relating to soil displacement made, such as surface upheaval in some tests, and x-ray photographs in others. While the evidence is not conclusive, it would appear that under concentric loading, the resistance to failure up to maximum load

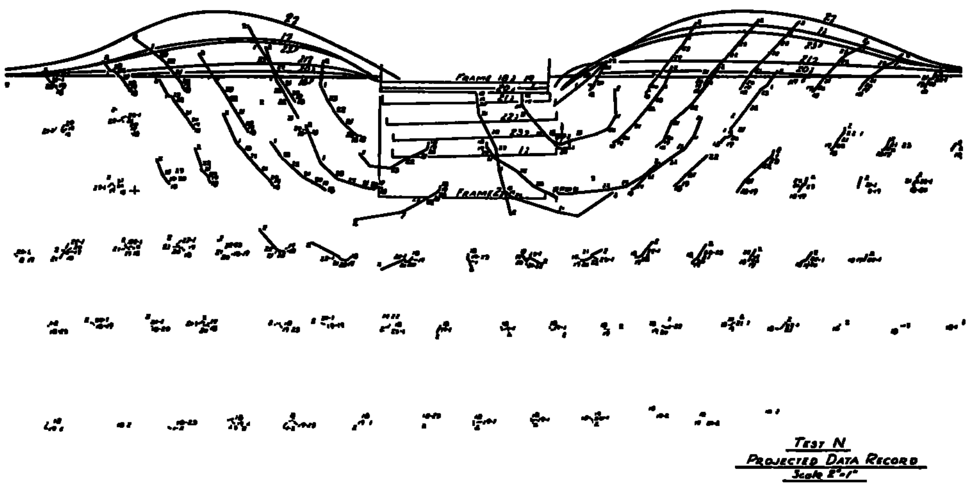


Figure 7. Trajectories of Lead Shot During Loading

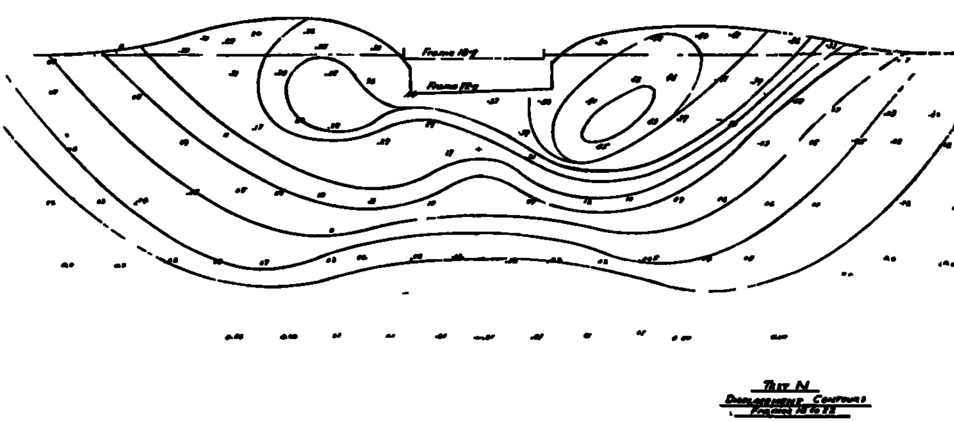


Figure 8. Contours Showing Extent of Soil Displacements During Loading

reached, although the zone of failure appeared finally to develop on only one side of the footing. In these cases, tipping of the footing became apparent after, but only after considerable settlement had taken place

In a few tests, in which special effort was made to avoid eccentric loading, the settlement of the footing was allowed to proceed by small increments, even though maximum load is developed along two incipient symmetrical failure surfaces, but that in the majority of cases, as settlement of the footing becomes large, sufficient eccentricity develops so the final gross failure occurs on one side only.

Estimates of Bearing Capacity—Some of the factors which may have an important influence upon the bearing capacity are: (1) the mode of failure, (2) shearing strength, and (3) degree to which shearing resistance can be mobilized in various parts of the mass.

In very loose soils and with small bearing areas, failure may occur by compression and internal displacement of the soil; an extreme example of this is the penetration of a rod into a loose sand. In this discussion attention is confined to failures in which lateral heaving and movement along a shear zone accompanies or governs failure. With this mode of failure, symmetrical or unsymmetrical development of the failure zones may characterize the failure, as mentioned in a preceding section.

TABLE 1
SUMMARY OF BEARING VALUES FROM LOADING TESTS—SERIES I

Material Cohesionless sand, density 102 pcf., angle of internal friction approximately 36 deg
Footing 1 in wide

Length of Footing	Surcharge	Test No	Load at Failure	Estimated Reliability of Test	Remarks
in	*in*		*psi*		
24	0	7	16 0	Fair	Load increment too large
		9	16 0	Excellent	Failure after load on for 5 min
		19	14 0	Good	
		20	18 0	Good	
	1	1	29 5	Fair	Considerable tipping
		2	24 5	Fair	Considerable tipping
		4	33 0	Good	Slight tipping
		5	32 0	Good	Slight tipping
		6	37 0	Excellent	Failure after load on for 30 min
10	0	9	22 0	Fair	Slight friction between end of bar and box
		10	20 0	Fair	
		16	14 0	Good	
		17	16 0	Good	
		18	22 0	Fair	Slight end friction
	1	11	34 0	Nil	Load increment too large
		13	36 0	Excellent	Failure after load on for 3 min
		14	26 0	Good	
		15	48 0	Nil	Considerable end friction
2 (Dia) Circular	0	21	17 0		Considerable tipping
		22	26 0		
		23	28 5		

The shearing strength may vary considerably in different parts of the soil mass depending upon density and confinement during shearing action. Accompanying the bearing capacity tests, several series of shear tests were made, both with the direct shear and triaxial compression devices. While with sandy soils of medium to high density and under moderate confining pressures the angle of internal friction was found to be of the order of 36 deg , on loose soil and very small confining pressure an angle of internal friction as low as 15 deg. was observed. In bearing tests on loose soils, where some densification must take place in the vicinity of the footing as load is built up, this variation in shear resistance is undoubtedly a very important factor to consider in attempting to estimate bearing capacity. Further, in small-scale testing, the control of uniform density of the test specimen, may be of much greater importance than previously suspected.

TABLE 2
COMPARISON OF CALCULATED BEARING CAPACITIES WITH RESULTS OF SMALL LOADING TESTS

Material Cohesionless sand, density 102 pcf , angle of internal friction approximately 36 deg
Footing 1 in wide, 10 in and 24 in long

	Bearing Capacity, *psi*	
	No Surcharge	1-in Surchage
Range in valid experimental values	14 to 20	26 to 37
Calculated by Krey method[a]	4 0	9 2
Calculated by Terzaghi Bearing Capacity Factors[b]	1 5	4 0
Calculated by Terzaghi-Hogentogler Equation[c]	0 6	2 1
Calculated by taking in account the effect of footing load in mobilizing resistance across failure surface	14 5	37 2

[a] Krynine, D. P , Soil Mechanics, 2nd ed, McGraw-Hill, 1947

[b] Terzaghi, Karl, Theoretical Soil Mechanics, Wiley, 1943

[c] Hogentogler, C A and Terzaghi, Karl, Analysis of Stability of Cohesive Earth under Strip Loading, *Public Roads*, Vol 10, May 1929, pp 51-52

The degree to which shearing resistance is mobilized in various parts of the soil mass is a factor which may be the clue to successful application of analytic procedures for estimating bearing capacity. Generally speaking, as the angle of internal friction of a material increases, an externally applied load becomes more predominant in comparison with the weight of the material itself in determining resistance to failure. It is not unreasonable to expect that in a material having a large angle of internal friction, the stresses across the potential surfaces of failure generated by the footing load may induce shearing resistance which may equal or exceed in importance the resistance that can be mobilized by the force of gravity actıng upon the mass of soıl which is subȷect to displacement as faılure occurs Utılızing thıs notıon, some rough calculatıons of supportıng capacity were made ın whıch the shearıng resıstance induced by the load were taken into account. For these calculations ıt was assumed the zone of failure followed a logarıthmic spıral path, and the states of stress along this path caused by the load were computed from the Boussinesq equatıons A number of crude approxımations were made ın the calculatıons, but the results when compared with the test results were most encouragıng. A summary of comparisons of various calculations wıth test results is given ın Table 2.

ACKNOWLEDGMENT

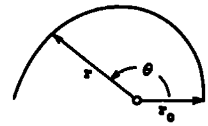
A number of the experıments reported hereın were performed by graduate students, Messrs. Frank Wormald, C. W McCormick, and R C Brıttaın, to whom acknowledgment is gratefully made.

DISCUSSION

D P. Krynine, *Unıversıty of California*—Professor Davıs, a co-author of thıs paper ıs a new member of the Proȷect Commıttee on Stress Dıstrıbution ın Earth Masses, and as the Chairman of that Commıttee, I welcome the presentatıon of this ınteresting paper From the methodologıcal point of vıew, the applıcation of x-rays to thıs kınd of research is a novel and an efficient tool that permıtted to see clearly the shape of the failure surface before it had actually developed The locatıon of the zones of decreased densıty ın sand, of course, suggests further thinkıng and perhaps further experimentatıon These zones are sıgns of the elasto-plastıc stage of equılıbrium that immedıately precedes the stage of plastic equılibrıum and subsequent failure.

In the opınıon of the wrıter, the most ımportant feature of this paper is the estimatıon of the bearıng capacıty of the earth mass The load applied at the surface of the mass is a source of two kınds of stresses (a) detrımental shearing stresses that tend to cut the mass along the eventual shearıng surface and (b) stabılizıng normal stresses or pressures that act on that surface and press the corresponding wedge to the rest of the mass thus opposıng the action of shearıng stresses. An analogous "play of forces" takes place ın the slope tendıng to slıde down with the essential difference, however, that ın the case of slopes both shearıng and normal stresses are produced by the weıght of the mass only, and not by the surface load The results of the authors' computatıons of the bearıng capacıty of a foundatıon on cohesıonless sand are the most reassuring for a desıgner, and the authors are to be commended for theır good work.

TABLE A
RADIUS OF LOGARITHMIC SPIRAL



$r = r_0 e^{\theta \tan \phi}$

θ	Angle of Internal Frıctıon, φ—*degrees*									
	0	5	10	15	20	25	30	35	40	45
	Relative Radıus, r/r_0									
degrees										
0	1 00	1 00	1 00	1 00	1 00	1.00	1 00	1 00	1 00	1 00
5	1 00	1 01	1 02	1 02	1 03	1 04	1 05	1 06	1 08	1 09
10	1 00	1 02	1 03	1 05	1 07	1 08	1 11	1 13	1 16	1 19
15	1 00	1 02	1 05	1 07	1 10	1 13	1 16	1 20	1 23	1 30
20	1 00	1 03	1 06	1 10	1 14	1 19	1 22	1 28	1 34	1 42
25	1 00	1 04	1 08	1 12	1 17	1 22	1 29	1 36	1 44	1 55
30	1 00	1 05	1 10	1 15	1 21	1 28	1 35	1 44	1 55	1 69
35	1 00	1 06	1 11	1 18	1 25	1 33	1 42	1 53	1 67	1 84
40	1 00	1 06	1 13	1 21	1 29	1 38	1 50	1 63	1 80	2 01
45	1 00	1 07	1 15	1 24	1 33	1 44	1 57	1 73	1 93	2 19
50	1 00	1 08	1 17	1 26	1 37	1 50	1 65	1 84	2 08	2 39
55	1 00	1 09	1 18	1 29	1 42	1 56	1 74	1 96	2 24	2 61
60	1 00	1 10	1 20	1 32	1 46	1 63	1 83	2 08	2 41	2 85
65	1 00	1 10	1 22	1 36	1 51	1 70	1 92	2 21	2 59	3 11
70	1 00	1 11	1 24	1 39	1 56	1 77	2 02	2 35	2 79	3 39
75	1 00	1 12	1 26	1 42	1 61	1 84	2 13	2 50	3 00	3 70
80	1 00	1.13	1 28	1 45	1 66	1 92	2 24	2 66	3 23	4 04
85	1 00	1 14	1 30	1 49	1 72	2 00	2 35	2 83	3 47	4 40
90	1 00	1 15	1 32	1 52	1 77	2 08	2 48	3 00	3 74	4 81
95	1 00	1.16	1 34	1 56	1 83	2 17	2 60	3.19	4 02	5.24
100	1.00	1.17	1 36	1 60	1.89	2 26	2 74	3 39	4 32	5.72
105	1 00	1 17	1 38	1 63	1 95	2 35	2 88	3 61	4.65	6.24
110	1 00	1 18	1 40	1 67	2 01	2 45	3 03	3 83	5.01	6.81
115	1 00	1 19	1 42	1 71	2 08	2 55	3 19	4 07	5.39	7.43
120	1 00	1 20	1 45	1 75	2 14	2 66	3 35	4 33	5.80	8 11
125	1 00	1 21	1 47	1 79	2 21	2 77	3 52	4 61	6 24	8 85
130	1 00	1 22	1 49	1 84	2 28	2 88	3 70	4 90	6 71	9.66
135	1 00	1 23	1 52	1 88	2 36	3 00	3 90	5 20	7 22	10 54
140	1 00	1 24	1 54	1 92	2 43	3 12	4 10	5 53	7 77	11.50
145	1 00	1 25	1 56	1 97	2 51	3 25	4 31	5 88	8 36	12.54
150	1 00	1 26	1 59	2 02	2 59	3 39	4 53	6 25	9 00	13.69
155	1 00	1 27	1 61	2 06	2 68	3 53	4 77	6 64	9 68	14.93
160	1 00	1 28	1 64	2 11	2 76	3 68	5 01	7 06	10 41	16 29
165	1 00	1.29	1 66	2 16	2 85	3 83	5 27	7 51	11 21	17 78
170	1 00	1.30	1 69	2 21	2 94	3 99	5 54	7 98	12 06	19 40
175	1 00	1 31	1 71	2 27	3 04	4 15	5 83	8 48	12 97	21.16
180	1 00	1 32	1 74	2 32	3 14	4 32	6 13	9 02	13 96	23 10

E S. Barber, *Unıversıty of Maryland*—The observatıons of bearing capacıty and movement of sand under small bearing areas presented in this paper are very interesting and stimulating

In line with the suggested use of logarithmic spirals in calculating bearing capacities, Tables A, B and C are useful. In using Table C

TABLE B
AREA OF SECTOR OF LOGARITHMIC SPIRAL

$$A = \frac{r_0^2}{4 \tan \phi}\left(e^{2\theta \tan \phi} - 1\right)$$

Moment of length of spiral = 2A

θ	Angle of Internal Friction, ϕ—degrees									
	0	5	10	15	20	25	30	35	40	45
	Relative Area of Sector, A/r_0^2									
degrees										
0	0.00	0 00	0 00	0 00	0 00	0 00	0.00	0 00	0 00	0 00
5	0.04	0 04	0 04	0 05	0 05	0 05	0 05	0 05	0 05	0 05
10	0 09	0 09	0 09	0 10	0.10	0 10	0 10	0.10	0.10	0 10
15	0.13	0 13	0 14	0 15	0 15	0 16	0 16	0 16	0 16	0 17
20	0 17	0 17	0 19	0 20	0 20	0 22	0 22	0 23	0 24	0 25
25	0 22	0 22	0 24	0 25	0 26	0 28	0.29	0 30	0.32	0 35
30	0 26	0 27	0 29	0 30	0 32	0 34	0 36	0 38	0.42	0 46
35	0 31	0 32	0 34	0 36	0 38	0 41	0 45	0 48	0 53	0 60
40	0 35	0 37	0 40	0 42	0 45	0 49	0 54	0 59	0 66	0 76
45	0 39	0 42	0 46	0 49	0 53	0 58	0 64	0 72	0 81	0 95
50	0 44	0 47	0 52	0 56	0 61	0 67	0 75	0 85	0 99	1 18
55	0 48	0 52	0 58	0 63	0 69	0 77	0 88	1 01	1 19	1 45
60	0 52	0 57	0 64	0 70	0 78	0.89	1 02	1 19	1 43	1.78
65	0 57	0 63	0 70	0 77	0 88	1 01	1 17	1 39	1 70	2 16
70	0 61	0 68	0 76	0 86	0 98	1 14	1 34	1 62	2 02	2 62
75	0 65	0 74	0 83	0 94	1 09	1 28	1 53	1 87	2 38	3.17
80	0 70	0 79	0 90	1 03	1 21	1 43	1.74	2 16	2 80	3.82
85	0 74	0 85	0 97	1 13	1 33	1 60	1.97	2 49	3 29	4 60
90	0 79	0 90	1 05	1 23	1 46	1 78	2 22	2 86	3 86	5.52
95	0 83	0 96	1 13	1 34	1 61	1 98	2 50	3 28	4 52	6 62
100	0 87	1 02	1 21	1 45	1 76	2 19	2 81	3.75	5 28	7 94
105	0 92	1 08	1 29	1 56	1 92	2 42	3 16	4 28	6 15	9 49
110	0 96	1 14	1 37	1 68	2 09	2 67	3 54	4 89	7 17	11 35
115	1 01	1 20	1 46	1 80	2 27	2 94	3 96	5 57	8 35	13 56
120	1 05	1 26	1 55	1 93	2 46	3 24	4 43	6 33	9 71	16 19
125	1 09	1 32	1 64	2 06	2 67	3 56	4 94	7 21	11 29	19 32
130	1 13	1 39	1 74	2 20	2 89	3 91	5 51	8 20	13 12	23 06
135	1 18	1 45	1 84	2 35	3 12	4 28	6 14	9 30	15 24	27 50
140	1 22	1 52	1 94	2 51	3 37	4 69	6 83	10 56	17 69	32 79
145	1 26	1 59	2 04	2.68	3 64	5 13	7 60	11 98	20 52	39 08
150	1 31	1 66	2 15	2 86	3 92	5 62	8 45	13 59	23 81	46 58
155	1 35	1 73	2 26	3.04	4 22	6 14	9 40	15 41	27 61	55 50
160	1 40	1 80	2 38	3 23	4 54	6 70	10.45	17 45	32 01	66.13
165	1 44	1 87	2 50	3 43	4 89	7 32	11.60	19 75	37.10	78.76
170	1 48	1 94	2 62	3 64	5 25	7 97	12 88	22 40	43.00	93.83
175	1 53	2 02	2 74	3 86	5.64	8 69	14 28	25 30	49.83	111.8
180	1 57	2 09	2 87	4 09	6.06	9 49	15 84	28 65	57 74	133.1

gravity acts perpendicular to r_0. Calculation of bearing capacity factors for one slip surface is illustrated in Figure A. The distribution of normal forces on the slip surface is immaterial since the resultant pressure when movement is imminent always passes through the center of moment. The minimum factors may be determined by trying several centers of rotation. The results of such calculation for two symmetrical surfaces as outlined by Terzaghi are shown in Table D. The fact that there is an immobile column under the center of the bearing area implies that the two halves of the load can act independently.

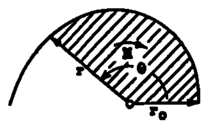
To check the relatively high bearing capacities obtained by the author several exploratory tests were made A strip 10 in. long and ½ in. wide on dry sand held 1.9 psi. Values calculated from Table D are 0 9 and 2.5 respectively for the ultimate (0.75) and maximum (0.9) coefficients of friction measured by direct shear. Dead weights were used to eliminate any restraint to tipping and one inch clearance was available at each end of the block. A disk 1.3 in. in diameter on dry sand in a 6-in diameter container held 3.1 psi. compared to 1.9 and 3.8 respectively for the ultimate and maximum coefficients of friction Bearing capacities for a circular loaded area were calculated from coefficients in Figure 11 page 35 in Vol. 26, *Proceedings*, Highway Research Board 1946. A test on a compacted stockpile of wet washed sand using the ½- by 10-in. bearing area gave a bearing capacity of 8 psi., compared to calculated values of 2½ neglecting cohesion but 16 including cohesion. Cohesion was determined as 0.12 psi. from the depth (18 in.) a vertical cut could be made around a prism of sand.

These few tests do not show the wide discrepancy between observations and calculations reported by the authors.

HARMER E DAVIS AND R. J. WOODWARD, *Closure*—Mr. Barber has presented some useful numerical tables to facilitate computations based on the assumption of a failure surface which conforms to the logarithmic spiral It is believed that computations based on such tables may serve usefully as first estimates and will tend to be conservative.

In the various series of tests conducted by the authors, more than 100 experiments were made on footings varying in width from one to four inches. A considerable range in values was encountered and, in general, the lower values were predominant where eccentricities of loading developed. In small-scale loading experiments, accidental eccentricities are very

TABLE C
MOMENT OF SECTOR OF LOGARITHMIC SPIRAL



$$M = \frac{r_0^3}{3 + 27 \tan^2\phi}\left[e^{3\theta \tan \phi}(\sin \theta + 3 \cos \theta \tan \phi) - 3 \tan \phi\right]$$

θ	Angle of Internal Friction, ϕ—*degrees*									
	0	5	10	15	20	25	30	35	40	45
	Relative Moment of Sector, M/r_0^3									
degrees										
0	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00
5	0 03	0 03	0 03	0 03	0 03	0 03	0 03	0 03	0 03	0 03
10	0 06	0 06	0 06	0 06	0 06	0 06	0 07	0 07	0 07	0 08
15	0 09	0 09	0 09	0 10	0 10	0 10	0 11	0 11	0 11	0 13
20	0 11	0 12	0 13	0 13	0 14	0 15	0 16	0 17	0 18	0 20
25	0 14	0 15	0 16	0 17	0 18	0 19	0 21	0 23	0 25	0 29
30	0 17	0 18	0 19	0 20	0 22	0 24	0 27	0 30	0 34	0 40
35	0 19	0 21	0 22	0 24	0 27	0 30	0 33	0 38	0.44	0 53
40	0 21	0 23	0 26	0 28	0 31	0 35	0 41	0 47	0 56	0 70
45	0 24	0 26	0 29	0 32	0 36	0 41	0 48	0 58	0 70	0 89
50	0 26	0 28	0 32	0 36	0 41	0 48	0 57	0 69	0 86	1 12
55	0 27	0 31	0 35	0 40	0 46	0 54	0 63	0 81	1 04	1 41
60	0 29	0 33	0 37	0 43	0 51	0 61	0 74	0 94	1 23	1 72
65	0 30	0 35	0 40	0 46	0 54	0 67	0 83	1 07	1 44	2 07
70	0 31	0 36	0 42	0 49	0 59	0 73	0 92	1 20	1 66	2 45
75	0 32	0 37	0 44	0 52	0 62	0 78	0 99	1 33	1 87	2 84
80	0 33	0 38	0 45	0 54	0 65	0 82	1 06	1 43	2 06	3 20
85	0 33	0 39	0 46	0 55	0 67	0 85	1 10	1 51	2 20	3 48
90	0 33	0 39	0 46	0 55	0 68	0 86	1 12	1 54	2 26	3 60
95	0 33	0 39	0 46	0 55	0 67	0 85	1 10	1 50	2 15	3 43
100	0 33	0 38	0 45	0 53	0 65	0 80	1 03	1 29	1 91	2 80
105	0 32	0 37	0 43	0 51	0 60	0 72	0 89	1 09	1 33	1 44
110	0 31	0 36	0 41	0 47	0 53	0 60	0 66	0 64	0 34	−1 01
115	0 30	0 34	0 38	0 41	0 44	0 43	0 32	−0 05	−1 24	−5 04
120	0 29	0 32	0 34	0 34	0 31	0 19	−0 14	−1 05	−3 58	−11 38
125	0 27	0 29	0 29	0 27	0 15	−0 12	−0 78	−2 45	−6 99	−20 93
130	0 26	0 26	0 23	0 15	−0 05	−0 51	−1 61	−4 35	−11 81	−34 97
135	0 24	0 22	0 17	0 03	−0 29	−1 01	−2 69	−6 88	−18.46	−55 3
140	0 21	0 18	0 09	−0 12	−0 59	−1 63	−4 07	−10 21	−27.51	−83 9
145	0 19	0 14	0 00	−0 29	−0 93	−2 38	−5 77	−14 49	−39.58	−124 0
150	0 17	0 09	−0 09	−0 49	−1 34	−3 27	−7 90	−19 96	−55.7	−179 3
155	0 14	0 04	−0 20	−0 71	−1 81	−4 34	−10 48	−26 89	−76.6	−254 9
160	0 11	−0 02	−0 31	−0 95	−2 35	−5 49	−13 63	−35 57	−103.9	−349 4
165	0 09	−0 08	−0 44	−1 22	−2 97	−7 06	−17 39	−46 33	−139 9	−494 0
170	0 06	−0 14	−0 57	−1 52	−3 66	−8 74	−21 85	−59 48	−183 8	−677 0
175	0 03	−0 20	−0 71	−1 85	−4 43	−10 69	−27 19	−75 63	−240 1	−917 0
180	0 00	−0 27	−0 86	−2 20	−5 29	−12 91	−33 39	−95 13	−311 1	−1232 0

difficult to control and it may be presumed, in the half-inch wide footings used by Mr. Barber, that accidental eccentricities would be even more difficult to prevent than they were in the case of the wider footings employed by the authors.

It was the intent of the authors to explore the behavior of small footings over the entire range of behavior that might be developed in the laboratory, and to attempt to determine

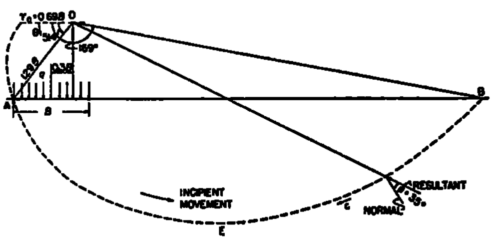


Figure A. Calculation of Resistance to Sliding on Log Spiral

Take moments about O (trial center)

Load moment = Weight moment (Sector OAEB − triangle OAB) + cohesion moment (arc AEB)

$$qB \times 0.3\,B = w\overline{0(.69B^3} \times 57\ 2 - B^3 \times 4.6) + c\overline{(0.69B^2} \times 41.8)$$

$$q = 47w\bar{B} + 67c$$

w = unit weight

c = cohesion

$\tan \phi = f$ = coefficient of friction

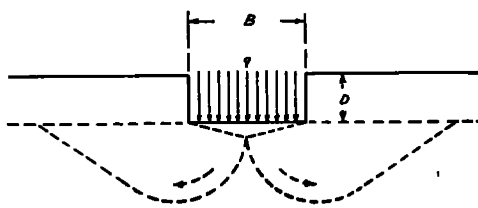
$r = r_1 e^{\theta \tan\phi}$

what analytical procedures would be necessary to account for the observed behavior. Hence, at this stage of the authors' study there was no attempt to establish minimum values, such as might be used for design

A factor which may be important in contributing to the higher bearing capacities found is the mobilization of resistance due to the stresses caused by the load itself. Thus, not only is frictional resistance developed due to the weight of the potential sliding segment but also due to frictional resistance developed as the result of the state of stress caused by the applied load, at points along the potential slip surface Furthermore, in the regions where the load stresses are high and, if the soil is to some extent compressible, there may occur some degree of densification with a resulting increase in the angle of internal friction. It may be, thus, that the angle of internal friction may be larger in the vicinity of the load than at points more distant therefrom It, thus, may further be expected that the failure

TABLE D

BEARING CAPACITY FACTORS FOR SHALLOW RECTANGULAR FOOTINGS



q = bearing capacity pressure
B = width
L = length
D = depth
w = unit weight
$s = c + fn$ = effective shear resistance

$$q = \left(1 + 0\ 3\frac{B}{L}\right) cF_c + wDF_D + \left(1 - 0\ 2\frac{B}{L}\right) wBF_B$$

Friction Coef , f	Bearing Capacity Factors		
	F_c	F_D	F_B
0	5 7	1 0	0 0
0 05	6 6	1 3	0 0
0 1	7 6	1 8	0 1
0 15	8 9	2 3	0 2
0 2	10 4	3 1	0 35
0 25	12 1	4 0	0 6
0 3	14	5 3	1 0
0 35	17	7 0	1 6
0 4	20	9 0	2 5
0 45	24	11 7	3 7
0 5	28	15 1	5 5
0 55	34	20	8
0 6	40	25	11
0 65	48	32	16
0 7	58	41	22
0 75	69	53	31
0 8	83	68	44
0 85	100	86	62
0 9	120	109	85
0 95	143	137	115
1 00	172	173	160

line is not a logarithmic spiral and that the resultant of the resisting stresses will not make a constant angle to the normal to the failure surface.

It is hoped that these suggestions may stimulate analysis of the foundation stability problem among those who have an interest in this field of study.