CALCULATION OF STANDARD PROCTOR DENSITY AND OPTIMUM MOISTURE CONTENT FROM MECHANICAL ANALYSIS, SHRINKAGE FACTORS, AND PLASTICITY INDEX

DONALD T DAVIDSON, *Assistant Professor of Civil Engineering, Iowa State College*, AND WILLIAM P. GARDINER, *First Lieutenant, Corps of Engineers, United States Army*

SYNOPSIS

An empirical method of calculating standard Proctor density and optimum moisture content with the data obtained from the common soil indicator tests is described in this paper The calculation procedure offers the possibility of eliminating much of the time and labor involved in the standard Proctor laboratory method and appears to give results sufficiently accurate for many engineering purposes.

A new approach to the soil density problem was presented in 1948 by Professors Rowan and Graham[1] of Vanderbilt University, Tennessee. Their approach was a mathematical one for determining the standard Proctor density[2] and optimum moisture content. It is based on the premise that the Proctor test gives a density close to the density achieved when a wet soil is allowed to shrink under natural forces to the shrinkage limit. They calculated this density in pounds per cubic foot from the mechanical analysis and shrinkage test data of the soil. The density formula used is

$$\text{Calculated Density} = \frac{D}{1 + \frac{D - C}{62.5G}} \quad (1)$$

C = 62 5 R
R = Shrinkage ratio
$D = C \div \frac{B}{A}$
A = Percent of soil passing US No. 4 sieve
B = Percent of soil passing US No. 40 sieve
G = Approximate specific gravity as calculated from the shrinkage limit and the shrinkage ratio

When written in terms of the mechanical analysis and shrinkage test values, formula (1) becomes

$$\text{Calculated Density} = \frac{6250}{S\left(\frac{B}{A} - 1\right) + \frac{100}{R}} \quad (2)$$

S = Shrinkage limit

The optimum moisture content in percent by dry weight of soil was calculated from the mechanical analysis and shrinkage limit data by means of the formula

$$\text{Calculated Optimum Moisture} = S\left(\frac{B}{A}\right) \quad (3)$$

The results of experiments on ten soils indicated that the calculated values agreed fairly close with those secured by means of the Proctor laboratory tests. The greatest deviation in density between calculated and laboratory results was approximately 5 percent. This was considered within the limits of accuracy, and no correction was deemed necessary. However, the calculated percentage of moisture was usually 1 to 5 percent higher than the Proctor optimum moisture figure; so an arbitrary correction factor of 3 percent was subtracted from all calculated percentages.

The calculation procedure of Rowan and Graham offered the possibility of eliminating the time and labor involved in the laboratory method, especially for investigations of a routine nature. However, the amount of supporting data did not warrant unqualified acceptance of their procedure. Since only ten soils were tested, all of which were probably indigenous to the Southeastern part of the

[1] W H. Rowan and W W Graham, "Proper Compaction Eliminates Curing Period in Constructing Fills," *Civil Engineering*, Vol 18, pp. 450-451 (1948)

[2] The expression, standard Proctor density, is used to denote the maximum dry density in pounds per cubic foot obtained by the standard Proctor laboratory test

United States, a more comprehensive verification was considered necessary before full confidence could be placed in such a new development.

TEST DATA

The verification of the density and optimum moisture formulas was accomplished by comparing the calculated and laboratory values of 210 soils from widespread geographical localities in the United States. The test data for these soils were obtained from three major sources: personal laboratory tests, the files of

laboratory values as previously found by Rowan and Graham. As a whole the results were so inconsistent and often so much in error that the validity of the formulas was questioned. When the soil test data were segregated into the groups used in the Public Roads soil classification system, only the calculated densities and optimum moisture contents of the soils in the A-2 and A-4 groups compared favorably with the results of the standard Proctor control tests.

It was further noted that the greatest errors in calculated density were obtained when the

TABLE 1
RESULTS OF PERSONAL LABORATORY EXPERIMENTS

Sample Location P R Classification Mechanical Analysis	1 Iowa A-4(8)	2 Iowa A-6(10)	3 Iowa A-7-6(18)	4 Iowa A-6(8)	5 Iowa A-7-6(20)	6 Iowa A-7-6(5)	7 Iowa A-6(10)	8 Virginia A-2-6(0)
Percent Passing								
No 4 Sieve			99 8	99 2	100 0	82 8		47 0
10			99 0	98 0	99 9	73 6	100 0	42 0
20		100 0	96 6	95 1	99 8	66 1	99 6	37 1
40		99 9	93 0	89 2	98 9	59 0	97 0	32 3
60		99 9	87 3	79 6	97 3	52 9	94 9	24 5
80		99 8	83 4	73 5	96 1	49 1	93 6	—
100		99 8	80 9	69 2	95 4	45 7	92 7	21 7
200	100 0	99 6	74 9	61 6	94 4	42 6	90 3	18 1
Soil Constants								
Liquid Limit, %	29 5	34 8	51 0	30 7	66 0	41 0	38 1	31 5
Plastic Limit, %	20 0	19 7	20 7	12 8	27 0	19 1	23 9	19 8
Plasticity Index, %	9 5	15 1	30 3	17 9	39 0	21 9	14 2	11 7
Shrinkage Limit, %	18 9	15 9	9 8	11 0	10 8	8 9	11 4	14 9
Shrinkage Ratio	1 76	1 82	2 02	2 02	2 11	2 18	2 00	1 80
Standard Proctor								
Density, pcf	107 6	110 0	106 6	120 0	98 2	124 1	117 2	119 8
Opt Moisture, %	16 5	16 5	16 8	11 7	22 0	10 5	14 0	12 0
Calculated								
Density, pcf	107 5	107 2	108 6	119 1	103 2	127 1	118 9	118 5
Opt Moisture, %	18 1	16 9	15 1	11 9	19 7	9 6	11 9	10 2

the Iowa State Highway Commission and the files of the US Bureau of Public Roads The personal tests were performed on seven Iowa soils and one Virginia soil (Table 1). Test data on 92 soils representing 28 Iowa counties were furnished by the Iowa State Highway Commission, and the Bureau of Public Roads provided test data on 110 soils sampled in ten different states. It is believed that all data used were obtained by means of the test procedures of the American Association of State Highway Officials.

DEVELOPMENT OF MODIFIED CALCULATION PROCEDURE

Application of the density and optimum moisture formulas to the three different groups of soil test data did not result in the same degree of correlation between calculated and

formula was applied to the highly plastic soils. To investigate this relationship the percentage error between the calculated and laboratory densities of each soil was determined as follows.

Percentage error

$$= \frac{\text{Calculated Density-Laboratory Density}}{\text{Calculated Density}} \times 100$$

These figures were plotted graphically against those for the plasticity index. Figure 1 shows the resulting scatter diagram.

The data of the scatter diagram appeared to have a straight line trend. This suggested that a curve might be fitted to the plotted points, and that such a curve could be used to apply a correction factor to the density formula of Rowan and Graham. By such a procedure the calculated density would be brought into closer agreement with the standard Proctor test density

It will be noted that it very nearly coincides with the curve of formula (4) through the plasticity index range 0 to 16.

The percentage error can be used to determine the density correction factor K_1 by

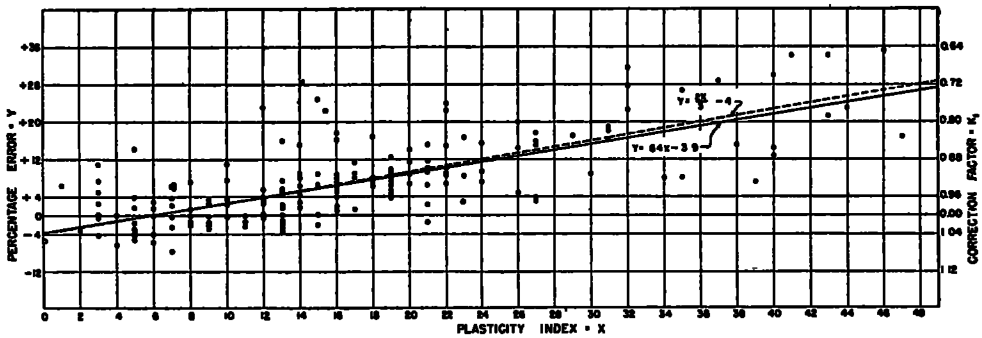

Figure 1. Correlation Between Calculated and Standard Proctor Densities

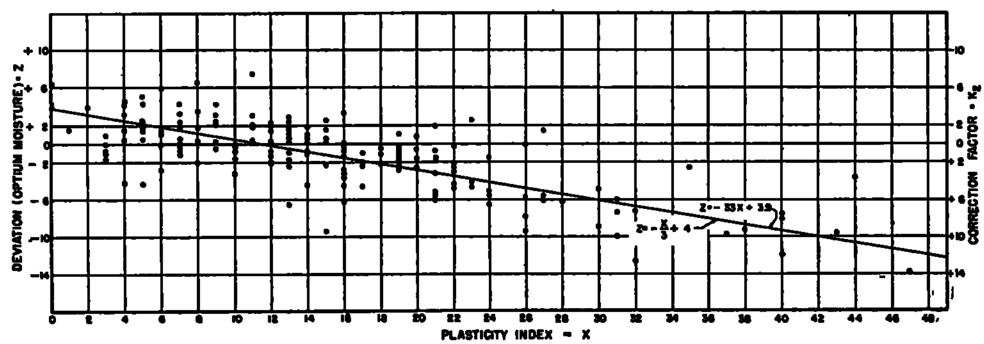

Figure 2. Correlation Between Calculated and Standard Proctor Optimum Moisture Contents

The curve (solid line in Figure 1) was fitted by the least squares method. Its formula is

$$y = .64x - 3.9 \qquad (4)$$

where y is the percentage error, and x is the plasticity index. The standard error of estimate was computed to be 6 percent. It is suggested that formula (4) be arbitrarily simplified to read

$$y = \frac{2x}{3} - 4 \qquad (5)$$

This simplified form is easier to use and is believed to be in keeping with the accuracy of the data of the scatter diagram from which it was derived. The curve representing formula (5) is shown by the dashed line in Figure 1.

means of the formula

$$K_1 = \frac{100 - y}{100} \qquad (6)$$

which, when expressed in terms of the plasticity index, becomes

$$K_1 = \frac{312 - 2x}{300} \qquad (7)$$

The value of K_1 may also be taken directly from the curve (dashed line) in Figure 1.

The modified density formula is obtained by multiplying formula (2) by the correction factor K_1. It may be written

$$\text{Calculated Density} = \frac{6250K_1}{S\left(\frac{B}{A} - 1\right) + \frac{100}{R}} \qquad (8)$$

An approach similar to that described above was employed to achieve closer agreement between the calculated and laboratory values of optimum moisture content. As in the case of the density computations, the greatest deviations from the Proctor test values occurred with the highly plastic soils. To establish a trend for this error, the deviation between the calculated and laboratory optimum moisture contents of each soil was determined as follows:

Deviation (Optimum Moisture) = Calculated − Laboratory

The figures were plotted on a graph against those for plasticity index. The resulting scatter diagram is shown in Figure 2.

The data of this diagram showed a linear relationship also, and a curve (solid line in Figure 2) was again fitted by the least squares method. The formula for this curve is

$$z = -.33x + 3.9 \qquad (9)$$

where z is the deviation in optimum moisture content, and x is the plasticity index. The standard error of estimate was computed to be 2.5 percent. Formula (9) may be arbitrarily simplified to read

$$z = -\frac{x}{3} + 4 \qquad (10)$$

The curve representing this formula practically coincides with the curve of formula (9).

The deviation in optimum moisture content can be used to determine the optimum moisture correction factor K_2 by means of the formula

$$K_2 = -z \qquad (11)$$

which, when expressed in terms of the plasticity index, becomes

$$K_2 = \frac{x}{3} - 4 \qquad (12)$$

If desired, the value of K_2 may be taken directly from the curve in Figure 2.

The modified optimum moisture formula is obtained by adding the correction factor K_2 to formula (3). It may be written

Calculated Optimum Moisture

$$= S\left(\frac{B}{A}\right) + K_2 \qquad (13)$$

LIMITATIONS

On the basis of the limited experimental work done so far, the modified calculation procedure developed in this study is sufficiently accurate to justify its use for calculating standard Proctor density and optimum moisture content in situations where a high degree of accuracy is not necessary. Where highly accurate results are required, the standard Proctor laboratory method should be used. The data obtained from experiments on eight soils in the Soils Laboratory at Iowa State College are tabulated in Table 1. These data illustrate the results obtainable with the modified formulas.

One of the greatest limitations of the modified calculation procedure is that it cannot be used with accuracy for organic soils and for mineral soils having a high organic matter content. Organic matter is highly absorptive, and its presence in the soil makes it extremely difficult to obtain accurate determinations of the Atterburg shrinkage and plasticity limits.

Another limitation is that the calculation procedure is sensitive to small changes in the shrinkage limit and the shrinkage ratio. Because of this, the shrinkage test should be performed with particular care. In order to obtain the best results with the formulas, the shrinkage factors used should be the average values of several shrinkage tests and should be determined to the nearest one-hundredth. The plasticity index should also be carefully determined and perhaps should be the average of two or more tests.

While much more research is necessary before the true value of the modified formulas can be established, the great savings in time and labor obtainable by the use of them justify their further study and development.

DISCUSSION

W. H. Campen, *Omaha Testing Laboratories*—It is very evident from the data presented in this paper that the Standard Proctor Density cannot be calculated accurately from the gradation, shrinkage factors, and plasticity index. Therefore, the calculation method should not be used where rigid control or specification work is under consideration.

Donald T. Davidson, *Closure*—The extremely variable and complex nature of soils and the difficulties of moisture control make precise measurement of the maximum density impossible. Even with the widely used standard Proctor control test, it is generally impossible to obtain repeat values from a series of tests on the same soil The Proctor density does, however, approximate the compaction obtainable by field equipment and experience has proved the test to be an invaluable aid in the construction of highways, airfields, and rolled earth dams Extreme accuracy is not warranted for this determination because research has shown that there is no appreciable difference in watertightness or strength if the soil is compacted within two pounds of the Proctor density. For many engineering purposes not even this accuracy is needed since the majority of construction specifications require only that compaction be not less than 95 percent of the Proctor density

During the summer and fall of 1949, two senior Civil Engineering students[a] at Iowa State College made a statistical study under the supervision of Professor M G. Spangler to compare the accuracy of the laboratory and calculation procedures for determining standard Proctor density Only one soil, a sample of Mankato glacial till from Hamilton County, Iowa, was used in their study. It classified as A-6(8) by the revised BPR soil classification system.

In order to determine the degree of accuracy that could be obtained by the conventional standard Proctor laboratory test, the two students performed a series of 20 density tests on the single soil. Their results showed a maximum value of 116 1 pcf , a minimum value of 113 2 pcf , and an average value of 114 5 pcf The small sample theory of the method of least squares was used to compute the plus or minus variance of the data with respect to the true mean. The analysis indicated that the laboratory test can be performed with an accuracy of ±4 pcf. at least 99.7 percent of the time.

They next determined the number of shrinkage and plasticity index tests necessary to enable the standard Proctor density to be calculated by means of formula (8) with an accuracy of ±4 pcf. This was accomplished by performing the shrinkage limit, shrinkage ratio, and plasticity index tests 20 times and then using the small sample theory method to calculate the number of each of these tests necessary to give the variance of ±4 pcf 99 7 percent of the time.

On the basis of their study, they concluded that the calculation procedure can be as accurate as the laboratory method if two placticity index tests and five shrinkage tests are performed and the average values used in formula (8) Whether or not this conclusion is applicable to all soils will not be known until more studies of this nature are made on the various types of soil.

[a] Messrs. M. L. Calhoon and B R. Braymen.