THE MECHANICS OF SOIL CONSOLIDATION BY ELECTRO-OSMOSIS

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SYNOPSIS

This paper discusses the mechanics of electro-osmosis and reviews the derivations which led to the quantitative expressions for hydrostatic pressure in terms of the physical, mechanical and electrical quantities. The electro-osmotic action when an EMF is applied to a fine grained clay is described and the physical limitations on its effectiveness are also discussed

A method for determining experimentally the actual pressure head for a particular soil sample subjected to an EMF is presented as well as the variation in this pressure head with decreasing voids ratio This experimental curve is compared with the theoretical equation for electro-osmotic pressure head.

The degree and rate of consolidation of a fine grained soil are shown to be both greatly increased by electro-osmosis It is further shown that the electro-osmotic pressure can be represented very simply as an increased consolidation load in the consolidation theory

Finally, consolidation test results under a relatively large EMF and again without an applied EMF are given and the log time-settlement curves for both are compared with the theoretical curve Load-settlement curves with and without the applied EMF are also compared.

A considerable portion of the material contained in this paper was extracted from a dissertation submitted by the writer to the Johns Hopkins University as partial fulfillment of the requirements for the degree of Doctor of Engineering.

The phenomenon of electro-osmosis is fundamentally a phenomenon of colloid and capillary chemistry. Its effect in soil consolidation will be influenced in no small measure by such factors as the crystal lattice of the predominating clay minerals and their hydration and adsorptive properties For a discussion of the chemistry of electro-osmosis, the reader is referred to references \mathcal{L} , \mathcal{G} , \mathcal{G} , 10, 11, 15 and \mathcal{D} at the end of this article. It is not the purpose of this paper to discuss these more basic concepts of electro-osmosis, but rather to try to correlate the overall mechanical effects with present theories of soil mechanics

GENERAL

The phenomenon known as electro-osmosis may best be demonstrated by means of the apparatus shown in Figure 1

A clay diaphragm A is inserted in the horizontal portion of a U tube containing water. Two electrodes are inserted on either side of the diaphragm and an electro-motive force applied from some external direct current source in the direction shown. Under these conditions the water will move through the diaphragm from the positive toward the negative electrode causing a differential pressure to be set up in the two arms of the tube. When the water in arm C reaches a certain height, depending on the size of the pores in the diaphragm and the applied EMF, a balance between the electrical pressure creating the movement and the hydrostatic head will be reached, and the movement of the liquid will cease.

Observations of electro-osmosis were made many years before any quantitative interpretation was given to the phenomenon. Helmholtz $(10)^1$ gave this first interpretation in 1879. Modifications in the theory were later made by Lamb, Smoluchowski, Perrin, and others but the quantitative results remained substantially unchanged.

The following explanation is based on Perrin's theory (6). Consider a capillary tube filled with water (Fig 2) in which two electrodes are inserted If the electrodes are connected to an external direct current source, causing a current to flow in the liquid, the inside of the capillary behaves as if it had an electrical double layer coating. Furthermore, the outer layer next to the wall of the tube has an excess of anions and the inner layer—

¹ Italicized figures in parentheses refer to the list of references at the end of the paper. in contrast with the free liquid in the tube —has an excess of cations. In the presence of an EMF the anions in the outer layer tend to migrate toward the positive pole and the cations in the inner layer toward the negative pole. The outer layer adheres to the wall of the tube, thus preventing movement of the liquid in this layer. The inner layer, however, is freer to move and the migration of the cations toward the negative electrode sets up a motion of the liquid in this layer which is transmitted by friction to the rest of the liquid in the tube.



Figure 1. Hydrostatic Pressure Produced by Electro-osmosis

Equation of flow —

The development of the following equations for quantity of flow and electro-osmotic pressure head, are based on the Helmholtz derivations as outlined by Freundlich (6).

- If v = average velocity of flow in the capillary
 - $r_o =$ radius of the tube
 - V = volume of the liquid moved per unit time

 η = coefficient of viscosity of the liquid then:

$$v = \frac{V}{\pi \tau_0^2} \tag{1}$$

The liquid inside the double layer is moved with a velocity v. In the inner layer itself the gradient of velocity at any point is $\frac{dv}{dz}$ (where z is the distance from the inner face of the tube to the point) and since the thickness of the layer δ is very small $\frac{dv}{dz} = \frac{v}{\delta}$.

The value of δ for pure water as determined by Gouy (6) is 1010×10^{-6} mm corresponding to an interface charge between the layers of ten electrostatic units.

The friction force developed in a liquid by the motion of one layer over another is dependent on the viscosity of the liquid, the area of the surface, the acting velocity and the distance between layers.

The top plate in Figure 3 is moved with a velocity v relative to the bottom plate. The space between the two plates contains a liquid of viscosity η : if the distance between



Figure 2. Principle of Electro-osmotic flow

the two plates and the velocity v are relatively small the velocity gradient will be a straight line and the force F may be written as.

$$F = \frac{\eta A v}{\delta} \tag{2}$$

for unit surface area

$$F = \frac{\eta v}{\delta}$$

And for the case of a capillary tube

$$F = \frac{\eta V}{\pi r_0^2 \delta} \tag{3}$$

The force on unit surface area produced by the externally applied EMF, E, is the product of the charge e and the potential gradient H.

$$eH = \frac{\eta V}{\pi r_0^2 \delta}$$

$$e = \frac{\eta V}{\pi r_0^2 \delta H}$$
(4)

SOILS

Considering the double layer as a condenser of potential z with a dielectric between the two layers of constant D, then according to the law governing the potential across a condenser.

$$\begin{split} \varsigma &= \frac{4\pi\delta e}{D} \\ &= \frac{4\eta V}{Dr_0^2 H} \end{split} \tag{5}$$

from which

$$V = \frac{r_0^2 \zeta HD}{4\eta}$$

$$= \frac{r_0^2 \rho ED}{4\eta l}$$
(6)

where l = the distance between electrodes.



Figure 3. Velocity Distribution in Viscous Flow

From what has been said it is not apparent that the δ in Eq. 3 which was there defined as the thickness of the inner movable layer is the same as the distance δ in Eq. 5. Actually there is no clear dividing line between the inner and the outer layers and hence under these conditions there exists a "diffuse layer." Part of this layer lies in the liquid which adheres to the wall of the tube and part lies in the moving liquid. Thus there exists two differences of potential, namely, e between the solid surface and the moving liquid, and 5 between the liquid adhering to the wall and the moving liquid However, it can be shown that the same quantitative results are obtained if the diffuse layer is replaced by the Helmholtz layer of thickness δ The greater the charge and the ionic concentration, the smaller is the value of δ . Thus, δ may become comparatively large, as in the case of pure water where the concentration of ions is quite small.

Where the flow is through a group of capillaries as in the case of a clay diaphragm, the πr_0^2 is replaced by A_p —the cross sectional area of the pores. Equation 6 then becomes

$$V = \frac{A_p \zeta ED}{4\pi\eta l} \tag{7}$$

Since E = IR E = EMFand $R = \frac{1}{A_p \lambda}$ I = CurrentR = Resistance

 $\lambda =$ Specific Conductivity

$$V = \frac{\zeta ID}{4\pi\eta\lambda} \tag{8}$$

Equation 8 shows that the flow of liquid per unit time does not depend on either the length or area of the capillary.

ELECTRO-OSMOTIC PRESSURE HEAD

If the flow of liquid out of the tube is prevented as in Figure 1, then a pressure head will be built up which will be equal and opposite to the force producing the flow. To obtain an expression for this pressure head, Poiseuille's law is assumed to apply to the movement of the liquid in the capillary.

- If P = pressure
 - l = an arbitrary length of tube
 - $r_0 = radius of tube$
 - $r = radius of any circle inside r_o$
 - u =velocity of flow at radius r

Equation (2) applied to a cylindrical thread of the liquid gives

$$\eta 2\pi r l \frac{du}{dr} = \pi r^2 P$$
$$u = \frac{P}{2\eta l} \int_{r_0}^r r \, dr = -\frac{P}{4\eta l} \left(r_0^2 - r^2 \right)$$

The volume of liquid V that crosses any section of the tube in unit time is obtained as follows.

$$V = \int_{0}^{r} u 2\pi r \, dr$$

= $\int_{0}^{r_{0}} \frac{P \pi r}{2\eta l} (r_{0}^{2} - r^{2}) \, dr$
$$V = -\frac{\pi r_{0}^{4} P}{8\eta l}$$
(9)

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Equating absolute values in Eqs. (9) and (6) there results

$$P = \frac{2\zeta ED}{\pi \tau_0^2} \tag{10}$$

Equation (10) was derived for a single capillary only. A similar equation may be obtained for a group of equal size capillaries by equating equations (7) and (9).

$$P = \frac{2(1+e_0)(ED)}{\pi^2 e r_0^4}$$
(11)

in which e_0 = initial voids ratio

e = voids ratio corresponding to P. If as in the case of a soil there existed considerable differences in the particle sizes or shapes, there would probably be void pockets of varying sizes. This would produce capillaries which were non-uniform in shape and size Equation (11) could not under these circumstances be expected to give exact quantitative results. However, assuming that the soil grains undergo no major disturbance during a reduction in voids ratio, the relation of pressure to voids ratio under constant EMF should follow a fairly uniform pattern provided there are no abrupt changes in ζ and D. If the terms in the numerator are constant in equation (11), the pressure varies inversely as some function of the voids ratio. This is demonstrated by the experimental curve (Fig. 5) obtained from actual test on a fine grained soil sample.

In order that the units in equations. (10) and (11) be consistent, all terms should be expressed in absolute units. Since one absolute electro-static unit of potential = 300 volts, E in volts must be divided by 300 before being put into the equations. η may then be expressed in gram. sec. per sq. cm. and P in grams. per sq. cm. ζ will have to be multiplied by 300 to convert it to volts

If the action in the capillary were reversed; i.e., if the water were forced through the capillary by a pressure P, then a difference eof potential would be produced between two electrodes placed in the stream. It is clear that this potential may be expressed as.

$$E = \frac{P_{\zeta}D}{4\pi\eta\lambda} \tag{12}$$

E in equation (12) is referred to as the "stream potential." Electro-osmosis is greatly

affected by the presence of dissolved substances in solution and the variation in the stream potential is one method of detecting the presence of these substances and measuring their concentration.

It would be possible to compute the value of P in equation (11) if the z potential, D and r_o could be measured. The z potential is always small and very difficult to measure. It would seem much more desirable to obtain P directly from a permeability test.

ELECTRO-OSMOSIS AND PERMEABILITY

A steady application of load to a fine grained soil sample will not produce any abrupt change in the pore spaces. It is reasonable to assume that the relative distribution of the shape and size of the pores in the soil will remain the same throughout the consolidation process, if the voids decrease uniformly as the soil consolidates and there is no abrupt change in ς or D in equation (11) Hence under a constant applied EMF the pressure-voids ratio should follow a uniform pattern Of course ς and Dwill not remain exactly constant but the change in these quantities will probably be small in comparison with effect of voids ratio change

Permeability tests were performed using a soil sample having the following characteristics:

Characterssisce	Value				
Particle size	passed	015	0-in	816VC	
Specific gravity of grains	2.85				
dex Sample size	11 2 5-in	dia	by () 72-in	height

The apparatus used for the test was a Casagrande type consolidometer with a permeability attachment as shown in Figure 4. The loading frame was fitted with an attachment so that the dial reading, i e. voids ratio, could be adjusted and held fixed.

In every case when this was done any residual load on the sample was released before the permeability test was started. A circular piece of copper wire mesh 1600 openings to the square inch was placed on the bottom of the sample and a similar piece on top. Porous stones A, B were placed outside the wire mesh (Fig. 4). The negative terminal of a D.C supply was connected to the cap C and the positive terminal to the base A The ring D

was insulating plastic thus insuring that the current flow through the sample

At a certain voids ratio, a permeability test was run by allowing the water in the stand pipe to flow through the sample upward, allowing only a drop of two or three centimeters from



Figure 4. Consolidometer with Permeability Attachment Used for Permeability and Consolidation Tests

the top of the stand pipe. The permeability coefficient k was then computed from the equation.

 $Q = k \imath A t$

Q =quantity of flow

i = average hydraulic gradient

 $= \frac{\text{Head (average)}}{\text{Length of sample } L}$

A = cross sectional area of sample

t = time of flow

$$k = \frac{Q}{\frac{H \text{ av}}{L} A t}$$
(13)

By running the test a second time at the same voids ratio but with an applied EMF a new time t was obtained for the same drop in head. The pressure of electro-osmosis could then be calculated from the equation.

$$P = \frac{Q \cdot L}{k \cdot a \cdot t} - H \text{ av.}$$
(14)

By repeating the process with different voids ratio the data shown by the solid curve in Figure 5 were obtained. The variation in permeability of the sample with decrease in voids ratio e is shown in Figure 6 The $e - \log k$ relationship is a straight line. Since this is known to be approximately true for fine grained soils in general it serves here to estab-



Figure 5. Relation between Voids Ratio and Electro-osmotiic Pressure Head

lish that no serious irregularities existed in the sample of apparatus before the EMF was applied The $e - \log P$ curve (Fig. 7) is also a straight line which suggests that this relationship, too, may expressed mathematically This solid curve in Figure 6 is representative of a number of such curves obtained from repeated tests on different samples of the same soil It can be readily seen that there is no direct correlation between equation (11) and the experimental curve It is not likely that the undetermined quantities or D change very much with change in voids ratio, so the experimental curve can be very closely approximated by the parabola

$$P = \frac{C}{e^8} \tag{15}$$

Equation (11) was derived on the basis of capillaries of uniform cross section No such regularity, of course, exists in soil capillaries. A given change in voids ratio may produce a small percentage reduction in the cross sectional area of the large pores and a very large percentage reduction in the area of the small pores According to equation (10) the small pores will be largely responsible for the electro-osmotic pressure head P. Hence the value of e is not directly a measure of the cross sectional area of the effective capillaries. It seems reasonable, however, that as the voids ratio e of a given soil is reduced the pressure Pmay change as some power of e. Equation (15) merely fits the experimental data for the par-



Figure 6. Relation between Voids Ratio and the Log of Coefficient of Permeability

ticular soil tested Another soil with a quite different particle size distribution curve and different Atterberg limits might yield a different equation. It seems, too, that the equation:

$$D = \frac{C}{e^n} \tag{16}$$

might apply for any soil in which electroosmosis was effective, the values of C and n, of course, depending on the soil properties.

The data shown in Figure 6 may be expressed by the 'equation:

$$e_0 - e = C_k \log \frac{k_e}{k_{e_0}} \tag{17}$$

Similarly the electro-osmotic pressure head P

vs. voids-ratio, (Fig. 7) may be expressed as:

$$e_0 - e = -C_p \log \frac{P_e}{P_{e_0}} \tag{18}$$

Where C_{s} is the slope of the line in Figure 6 and C_{p} is the slope of the line in Figure 7. P_{\bullet} and $P_{\bullet \bullet}$ are the electro-osmotic pressures corresponding to voids ratios e and e_{\bullet} .

ELECTRO-OSMOSIS AND CONSOLIDATION

It is assumed in the theory of consolidation that the time-consolidation curve for a given increment of load is not materially affected by the change in k during the consolidation process. This has been found to check the



Figure 7. Relation between Voids Ratio and the Log of Electro-osmotic Pressure

experimental time-consolidation curve very closely up to about 70 percent of the primary consolidation, (17) and (18) If this is so equations (17) and (18) suggest that the electro-osmotic pressure head P may also be assumed constant in the primary consolidation range. Since its effect is to increase the pressure of the moisture in the soil pores it acts in the same manner as the applied load and may in fact be considered as an added consolidation load. This is true for the time-consolidation relationship, but the effect of P on the total degree of consolidation makes it unlike ordinary increments of load as will be seen later.

For a confined sample of thickness H, drained through the top surface only, the pressure diagram under a consolidation load P_1 is as shown in Figure 8 (a), (17) and (18) With an applied EMF the pressure diagram may be represented by Figure 8 (b). P_{\bullet} is the electro-osmotic pressure which since it originates in the pore water will act in advance of P_1 in the consolidation process. The iso-rhoones t divide the pressure areas such that

in Figure 8 (a),
$$u = P_1 - \bar{P}$$
 (19)

in Figure 8 (b);
$$u = (P_1 + P_0) - \vec{P}$$
 (20)

where \overline{P} is the intergranular pressure and u is that part of the total consolidation pressure which at the time t and depth z has not yet been transferred to \overline{P} . As $t \to \infty$, u in each case $\to o$ throughout the depth H. The shape of the assumed isochrones (Fig. 8 (b)) indicates



Figure 8. Pressure Diagram Showing Isochrones for One-Dimensional Consolidation (a) Under Load Only

(b) Under Load and Electro-osmosis

that P_0 originates at the top, i.e., at the negative electrode and progresses downward.

Recent tests on colloidal soils by L. Casagrande (4) indicate that the electro-osmotic flow starts at the cathode. In his investigation on the movement of soil pore water by electroosmosis the water content at varying distances from the anode at specific time intervals was measured When no other water was provided except the initial water in the saturated soil it was found that the moisture content dropped most rapidly at the cathode until it reached a value of about 16 percent Then it began to drop at the anode and when the osmotic flow ceased the water content was the same from anode to cathode Thus, in consolidation the electro-osmotic pressure will progress from the top downward, it will be responsible for nearly all of the early consolidation and will continue to be effective until the consolidation has reached the point where the $\overline{P} = P_0$ The removal of moisture from the soil may of course continue beyond this point provided the moisture content of the soil is sufficient to sustain the osmotic flow. A supplementary effect of electro-osmosis is the creation of pore water tensions (4). This is exhibited by the abnormal increase in the degree of consolidation by the addition of an EMF. In the time-consolidation process this pressure, too, will be ultimately balanced by an equal intergranular compression.

Since the condition described above and shown in Figure 8 (b) is exactly analogous to



Figure 9. Illustration of One-Dimensional Consolidation Using Electro-osmosis

the ordinary consolidation process (Fig. 8 (a)), the differential equation of consolidation will be the same in both cases. In order to show that this is so, the development (18) of the equation is reviewed as follows.

Differential Equation of Consolidation-

Across a horizontal plane height z above the bottom impermeable layer, (Fig. 9) the velocity of flow is given by Darcy's law:

$$v = k \frac{dh}{dz} \tag{21}$$

Where $\frac{dh}{dz}$ is the loss in hydraulic head with

increase in z. If u = hydrostatic pressure in the pores due to the load plus the electroosmotic pressure and $\gamma_{*} =$ Specific weight of water

$$\frac{dh}{dz} = \frac{1}{\gamma_0} \frac{\partial u}{\partial z} \tag{22}$$

and the quantity of flow q is

$$q = k \frac{1}{\gamma_0} \frac{\partial u}{\partial z}$$
 (23)

The quantity of flow through unit area (Δz) above the plane is $q(z + \Delta z)$. Expanding $q(z + \Delta z)$ there results

$$q(z + \Delta z) = q(z) + (\Delta z) \frac{\partial q}{\partial z} + \frac{(\Delta z)^2}{2!} \frac{\partial^2 q}{\partial z^2} + \frac{(\Delta z)^3}{3!} \frac{\partial^3 q}{\partial z^3}$$
(24)

In the limit (Δz) becomes (dz). Representing q(z) by q and neglecting infinitesimals of a higher order than dz one gets

$$q(z + \Delta z) - q(z) = dq = \frac{\partial q}{\partial z} \cdot dz$$

from which

$$\frac{dq}{dz} = \frac{k}{\gamma_{\rm C}} \frac{\partial^2 u}{\partial z^2} \tag{25}$$

Within a unit volume the rate of change of q is equal to the rate of change of voids ratio e. The change in porosity (Δn) based on the original voids ratio e, becomes

$$\Delta n = \frac{\Delta e}{1 + e_0} \tag{26}$$

and

$$\frac{dn}{dt} = \frac{1}{1+e_0} \frac{\partial e}{\partial t}$$
(27)

dq is directly related to dn by the following

$$dq = \frac{dn}{dt} \cdot dz \tag{28}$$

Substituting equation (25) into equation (24) and again into equation (26) the following is obtained

$$\frac{1}{1+\epsilon_0}\frac{\partial e}{\partial t} = \frac{k}{\gamma_0}\frac{\partial^2 u}{\partial z^2}$$
(29)

or

$$\frac{1}{1+e_0}\frac{\partial e}{\partial u}\cdot\frac{\partial u}{\partial t}=\frac{k}{\gamma_0}\frac{\partial^3 u}{\partial z^2} \qquad (30)$$

ʻⁱn which

$$\frac{\partial u}{\partial t} = \frac{\partial \bar{p}}{\partial t}$$

 $\frac{\partial e}{\partial u}$ is assumed to be constant in the range of e. Equation (29) may now be written

$$\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2} \tag{31}$$

where

and

$$a = \frac{\partial e}{\partial u} = \text{constant}$$

 $C_v = \frac{k(1+e_0)}{a\gamma_0}$

A solution to equation (31) may be obtained by writing

$$n = Z(z)T(t)$$

Where Z(z) is a function of z only and T(t) is a function of t only (15). The boundary conditions will be the same as for ordinary consolidation under load except for u at the time t = 0. They are as follows:

$$t = 0; \ 0 \le z < H; \ u = P_1 + P_0$$

$$t = t; \ 0 \le t \le \infty, \ z = 0; \ \frac{\partial u}{\partial z} = 0$$

$$t = t; \ 0 \le t \le \infty, \ z = H, \ u = 0$$

$$t = \infty; \ 0 \le z \le H; \ u = 0$$

The solution of equation (31) is then

$$u = \frac{4(P_1 + P_0)}{\pi} \sum_{n=1}^{n=\infty} \frac{1}{(2n+1)} \cdot \left[\cos \frac{(2n+1)}{2H} \pi z \right] e^{\frac{-(2n+1)\pi^2 T}{4}}$$
(32)

where the time factor $T = \frac{C_{\rm s} t}{H^{\rm s}}$

The settlement δ is obtained by writing

$$d\delta = (\Delta n)dz \tag{33}$$

(Δn) may now be replaced by $\frac{a}{1+e}\Delta u$.

Noting that $\frac{\Delta u}{\Delta e} = \frac{\Delta \bar{p}}{\Delta e}$ and replacing $\Delta \bar{p}$ by $(P_1 + P_0) - u$ the expression for δ becomes

$$\delta = \frac{a}{1+e_0} \int_0^H (P_1 + P_0) dz - \int_0^H u dz \quad (34)$$

Substituting equation (32) into equation (34)

and integrating gives:

$$\delta = \frac{aH(P_1 + P_0)}{1 + e_0} \times \left[1 - \frac{8}{\pi^2} \sum_{n=1}^{n=\infty} \frac{1}{(2n+1)^2} e^{\frac{-(2n+1)^2 \pi^2 T}{4}} \right] (35)$$

And the percent consolidation U is given by:

$$U\% = \frac{\delta}{\delta \max} \times 100$$

= 100 $\left[1 - \frac{8}{\pi^2} \times \sum_{n=1}^{n=0} \frac{1}{(2n+1)^2} e^{\frac{-(2n+1)^2 \pi^2 T}{4}} \right]$ (36)

to observe and measure. The load increments shown in Figures 10 and 11 were applied at the same time as the EMF. The temperature variation of the soil sample 1s also shown in all cases because the rate of consolidation is very considerably increased by an increase in temperature. The prolonged application of an EMF tends to heat up the sample considerably and the effect would be to make the consolidation produced by electro-osmosis appear greater than it actually was. It is believed. however, that the temperature rise for the tests reported did not influence the results appreciably. Figure 12 shows the effect of electro-osmosis on the degree of consolidation Curve No. 1 is the ordinary consolidation



Figure 10. Comparison of Theoretical Time-Consolidation Curve with Experimental Curve under Load Increment plus an Applied BMF

Equation (36) is the Terzaghi equation for consolidation under the effect of load only (17).

Experimental Consolidation Results-

The results shown in Figures 10, 11 and 12 were obtained using soil samples described for the permeability test The apparatus was that shown in Figure 4. The close correlation between the experimental time-consolidation curve and the theoretical curve in Figure 10 is evident. The voltage variation indicated on all curves was that measured across the sample itself, at the beginning and at the end of the consolidation period. The voltage supply used was approximately 110 On the size sample tested this gave an extremely high electric potential gradient. Thus the electro-osmotic effects were correspondingly large and easy curve using increments of load only. Curve No. 2 was obtained using increments of load plus an EMF on a different sample of the same soil. It is interesting to note that no effect of electro-osmosis was observed until a certain critical load was reached. Additional increments of load beyond this critical load produced very little further consolidation This is easily explained under the assumption that the electro-osmotic pressure P once started continues to act as long as there is sufficient moisture in the pores to keep it going. There is at the same time a corresponding decrease in voids ratio which makes the electro-osmosis effective even as the moisture content gets very small. Theoretically it would be possible to consolidate a soil completely in this manner even under a very small EMF. The smaller the EMF, however, the greater would be the time required. Actually the drainage effects of electro-osmosis decrease with time due to corrosion of the electrodes.

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compared with the theoretical pressure computed by the equation

$$P = \frac{k_e}{k}E \tag{37}$$









DISCUSSION OF RESULTS

The electro-osmotic pressure head may be measured by the more direct method of balancing the pressure against a column of mercury (3), (4), (7). Casagrande (3), (4) measured the pressure in this manner for a number of soils under varying potential gradients The initial voids ratio was assumed to remain constant in every case and the measured pressure was k is the ordinary coefficient of permeability, Eis the applied potential and k. is called the "electro-osmotic coefficient of permeability." Equation (6) gives the volume of water moved by electro-osmosis in unit time as

$$V = \frac{r_0^2 \zeta ED}{4\eta l} \tag{6}$$

This is written in the form

$$V = \frac{Dq\varsigma}{4\eta} \cdot \frac{E}{l} \cdot A \tag{38}$$

where q is related to the porosity, A is the cross-sectional area of the soil specimen, and the other quantities are as previously defined for equation (6). For a particular soil specimen subjected only to electro-osmosus, $\frac{Dqr}{4\eta}$ is assumed constant and is the k_c in equation (38), $\frac{E}{l}$ is *i*, the electric potential gradient. Equation (37) then becomes

$$V = k_e \iota_e A \tag{39}$$

By combining this equation with Darcy's law, namely,

$$V = k \iota_h A \tag{40}$$

where i_h = hydraulic gradient, the simplified form given in equation (37) is obtained.

The electro-osmotic pressures computed in this manner by L. Casagrande were found to agree fairly well with the measured pressures when the EMF was first applied. The measured pressure for colloidal soils, however, fell off very rapidly after this initial period He attributed this to the opening up of small cracks in the soil under sustained electro-osmotic treatment. While the electro-osmotic pressure head was maintained it was observed that the soil underwent considerable consolidation even without an applied load. This internal consolidation was apparently the cause of the formation of the cracks and the subsequent reduction in the electro-osmotic pressure head.

Although equilibrium may be established between the hydrostatic pressure and the electro-osmotic pressure, the movement of the water through the soil probably continues Thus, in the small pores the water flows into the stand pipe and at the same time an opposite flow takes place through the large soil pores. This gives rise to a circulation rather than a static condition.

The data shown in Figure 5 are used there in an attempt to determine the manner in which the term qA is related to the electroosmotic pressure. The assumption is also made, of course, that D, ζ and η remain constant or approximately so, within a fairly wide range of q. Although no measurements of the pressure were made by the direct method described in references (3) and (4), it is behaved that the results obtained would not differ materially from those shown in Figure 5.

There was no abrupt falling off in electroosmotic pressure observed as the process continued This might very well have occurred in the permeability test, however, if the EMF had been applied for any considerable length of time. The small size of the sample and its rather high density probably prevented shrinkage and consequently the formation of any cracks. In the consolidation tests electroosmosis was always accompanied by a small consolidation load, so that as the pore water was reduced the soil consolidated under the load Under those conditions there was little or no opportunity for cracks to form From the experimental data available to date it would seem that the consolidation of colloidal soils by electro-osmosis can only be accomplished effectively when applied in conjunction with a certain minimum consolidation load.

CONCLUSIONS

In conclusion it may be stated.

1. The relationship between electro-osmotic pressure and voids ratio may be expressed mathematically for a given soil It is quite possible, too, that for relatively pure silts and fine sands, a mathematical relationship may be developed for electro-osmotic pressure in relation to the particle size Both represent conditions of constant applied voltage

2. Overall electro-osmotic pressure may be represented as an increased consolidation load in the consolidation theory This load, however, should be considered as acting in two ways, namely,

(a) by removing the moisture from the pores

(b) by creating pore water tensions which ultimately become balanced by intergranular pressures

3. Effective consolidation by electro-osmosis can only be fully achieved in very fine grained soils when used in conjunction with a minimum mechanical load.

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ELECTRICAL HARDENING OF CLAYS ADJACENT TO ALUMINUM FRICTION PILES

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SYNOPSIS

The paper reports the results of a small scale laboratory study directed toward the determination of certain facts relative to the L. Casagrande method of electro-chemical stabilization of soils in the vicinity of aluminum or aluminum sheathed friction piles The specific objectives of the tests were to determine the relationship between the clay content of the soils in which model piles were driven, the maximum increase in bearing capacity of the piles and the optimum amount of electrical treatment to produce maximum bearing capacity. An attempt was made to study the relationship between the type of predominate clay minerals in the soil and the above factors, but the results of this part of the work were inconclusive.

A series of ten pairs of model piles consisting of $\frac{1}{2}$ -in dia aluminum alloy rods were driven in various soils contained in waterproof boxes Each pair of piles was spaced 7 in c to c. and was driven 16 in. into the soil The soils were mixtures of various materials and ranged in two-micron clay content from 28 to 48 percent. Half the soils were montmorillonitic and half were kaolinitic. The soils were kept saturated throughout the tests by maintaining a layer of water about $\frac{1}{2}$ -in. deep over the soil.

After the piles were driven, initial bearing capacity of each of the piles was determined. Then a direct current of electricity was passed from one pile of a pair to the other and the amount of current and the voltage were measured. During the period of electrical treatment, measurements of the bearing capacities of the piles were made at frequent intervals It was found that the bearing capacity in all cases increased to a maximum value as treatment progressed