STUDIES OF COMPRESSIVE STRESS DISTRIBUTION IN SIMPLY REINFORCED CONCRETE NEAR THE POINT OF FAILURE

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SYNOPSIS

In the theoretical development of formulae for the design of reinforced concrete by the plastic theory certain assumptions are made as a result of studies of standard concrete cylinder tests. The conditions of restraint in a standard cylinder being tested to failure in a testing machine are obviously quite different from the condition of restraint in a simply reinforced concrete beam or slab when tested to failure in flexure. The ideal information upon which to base such a theory would give the point to point stress distribution in the compressive area.

The theory of elasticity shows that stress in a normal rectangular beam made of isotropic material, subjected to pure bending, is correctly given by the formula $S = Mc/I$. Since it has long been realized that concrete is not isotropic and that concrete stress is not proportional to concrete strain, this formula is approximate for concrete and does not, in the opinions of many, give really satisfactory results. The fact that the strains in the compressive region of simply reinforced concrete in pure flexure do vary directly with the distance from the neutral axes does not mean that the compressive stress varies in the same manner.

Glass is an isotropic material, when properly manufactured, and is elastic in all ranges to its point of failure. It follows that, for glass, stress will be proportional to strain, which suggests the possibility of making photoelastic studies of this problem.

Pilot studies have been made in the Engineering Experiment Station of the Ohio State University using a 6- by 6- by 0.66-in. piece of special optical glass in the compressive area of a 6 by 12 in. by 15-ft. long heavily reinforced concrete beam resting on supports 12 ft. apart and loaded at the third points. The glass is set in the compressive region of the middle of the length of the beam which is in the region of no shear. The pressure from the concrete to the glass causes strain in the glass proportional to the pressure. Photoelastic photographs give information from which may be determined the average compressive stress at any distance from the neutral axis. The stress distribution thus found varies with the moisture variation in the beam and with the variation in quality of the concrete at different depths in the beam. In general the stress distribution may, but seldom does, follow a straight line variation. The compressive stress in the extreme fiber is usually several hundred pounds per square inch less in intensity than the maximum compressive stress. Indications are that variations in moisture and humidity conditions at the different layers greatly affect the stress distribution.

Other phenomena are being observed. The one second only in interest to the compressive stress distribution is the character of Poisson's ratio in the various levels of the compressive region. There is also a possible interaction between the glass insert and the concrete which might result in tension in the glass normal to the direction of the beam and to the direction of polarized light rays.

Many engineers are not satisfied with the so-called elastic theory for the design of flexural members in reinforced concrete. They contend that this theory does not give a realistic design since the concrete is definitely not entirely an elastic material. Furthermore, any ordinary reinforced concrete flexural member, designed by the usual theories, will fail first when the steel reinforcement bars are passing their elastic limit. If steel reinforcement bars having a high yield point are used in the member and the design has been made in the usual manner, failure will still usually occur when the steel is passing the yield point. Now, if the steel ratio in the member is increased and the strength of the concrete is

reduced by proper amounts, the resulting flexural member will fail at the instant when the concrete is being crushed in the compressive area and the steel is passing the yield point. Obviously, such a flexural member, in order to be most economical in the use of steel, should be reinforced with steel having a yield point as high as possible. Also such a beam will generally require a concrete strength lower than that customarily demanded. Such a combination of materials to make a structure equally strong in every part was expressed in Oliver Wendell Holmes' poem about the deacon's wonderful one "hoss" shay.

In order to develop theories to attain true balanced design, some information is necessary about the compressive stress distribution in simply reinforced flexural members during the time that the concrete in the compressive region is just beginning to fail by crushing. The loading should not be such as to cause quick failure, but rather should be of such intensity that failure would occur sometime after the load is applied. Direct evidence of stress distribution under this condition would be extremely interesting.

Concrete in a beam at flexural failure due to concrete crushing is capable of carrying a considerably higher unit stress than concrete from the same batch tested in 6- by 12-in. test cylinders. This suggests that tests of standard cylinders to obtain data regarding design apparently do not yield really desirable data. Such a design would depend in a large measure on the plastic properties of the concrete: either of two names, ultimate design or plastic design, seems to describe the purpose to be accomplished.

A great deal of work has been done in the field of plastic flow in concrete. Professor J. R. Shank in his "The Plastic Flow of Concrete" reviews much of the work of researchers and adds a few ideas of his own. No visible evidence of compressive stress distribution in simply reinforced concrete flexural members seems to be available. Such evidence, seemingly, would certainly furnish a better foundation for plastic design theories than that already in existence.

Probably the foremost theories dealing with the plastic design of concrete are due to Jensen and to Whitney. The assumptions used in these theories are, for the most part, based on information gleaned from test data on standard test cylinders. From these assumptions a shape of the stress distribution pattern has been more or less arbitrarily assumed. Mr. Whitney assumed the shape of the pattern to be rectangular above the neutral axis and Mr. Jensen assumed the shape to be trapezoidal, having a value of zero at the neutral axis. The Jensen pattern was divided into the elastic range and the plastic range. The plastic range depended on the quality of the concrete as evidenced by the ultimate crushing strength when tested in the form of standard test cylinders. Because the sustained ultimate load is less than the standard cylinder test ultimate load, around 85 percent of the value of the standard cylinder test crushing strength was used by Jensen as the sustained ultimate load. This percentage value varies somewhat with the quality of the concrete.

In considering this problem and how to get better data upon which to base plastic design theories, the possibility of using photo-elastic methods was first considered at The Ohio State University in conversations between Professors Large and Morris and Mr. Art Boase of the Portland Cement Association. The method involved passing polarized light through a glass insert which was to be cast in a concrete beam. The light was to pass through a thickness of glass equal to the width of the beam. The application of theories of photo-elastic principles in two dimensions to this problem was to be attempted, involving very careful work in mounting the glass. As a consequence of these discussions, work was started on the construction and testing of beams.

The first beams were 6 in. by 12 in. by 13 ft. long, supported on knife edges 12 ft. apart, and loaded at the third points. The portions of the beam subjected to shear were heavily reinforced with stirrups to prevent failure in diagonal tension. Figure 1 shows diagrammatically a glass insert and a stack of brass bars mounted in the middle third of the beam. The brass bars did not give the information which was being sought, so their use was discontinued. The main steel was high yield-point rail steel and had hooks as shown. As improvement in technique developed, the use of hi-bond bars was adopted and hooks were eliminated. The beams now being studied are reinforced with high-yield point hi-bond bars which conform to ASTM specifications. The

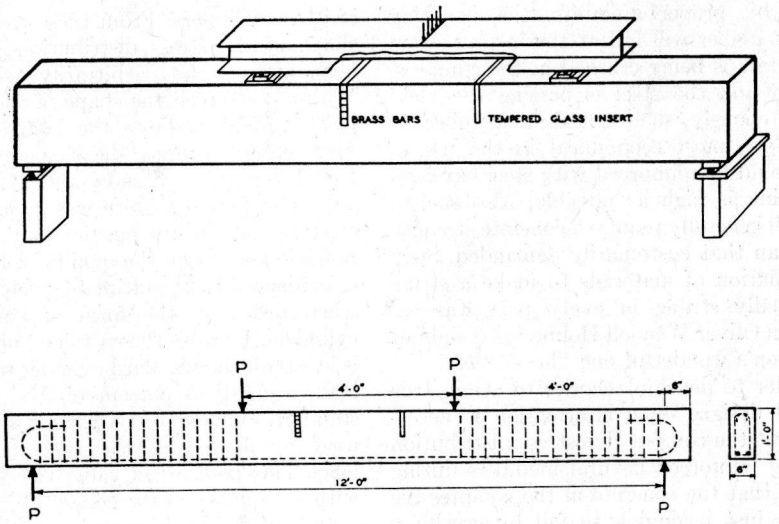


Figure 1. Loading Diagrams

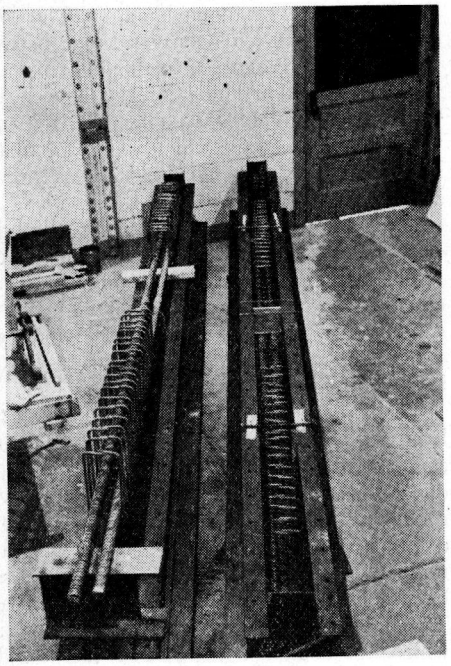


Figure 2. Beam Arrangement

glass insert is now being placed at the middle of the beam. Calibration of the glass showed that each additional photo-elastic fringe indicated an additional 193 psi. pressure from the concrete to the glass.

Figure 2 shows the arrangement of the reinforcement steel, the type of forms used, the manner of mounting the glass, and the waterproofed SR-4 gages at the middle of the bars. The lead wires from the gages are pulled through small holes in the bottom plate. A ground wire is fastened to the steel and pulled through a small hole in the form. This arrangement is used to test the gage-to-ground resistance. It was easily possible to obtain a resistance of 1,000,000 ohms between the gage and the steel to which it is mounted. A layer of cardboard is cemented to edges of the glass on all four of the edges. The two edges of the insert against the forms are thus protected from the pressure of the form. The bottom edge of the glass, embedded in the concrete, is protected by the cardboard as is also the top edge of the glass.

These two beams are designated 7-a and 7-b and are the subject of some discussion in connection with Figure 10. They are made of so-called 3000-lb. concrete. The test cylinders broke at 4000 psi. and the beam was carrying nearly a million inch-pounds bending moment when the steel began to yield. The bars are No. 11 billet steel, hi-bond type. The maximum concrete flexural compressive stress observed was in excess of 5500 psi. when the steel began to yield.

Figure 3 shows the appearance of the unmounted optical glass inserts as green circu-

larly polarized light is passed through the 6 in. of glass. The light center area shown in this edge face is in tension. The dark band denotes a stress-free region, while the light outside area represents compression. The extreme variation of stress intensity in the glass does not amount to more than 50 or 60 psi. This variation is small compared with the ultimate compressive concrete stress so no attempt was made to completely anneal the glass. This insert is a specially made striae-free annealed glass which is practically isotropic.

Figure 4 shows results of one of the early attempts to photograph evidence of the actual stress distribution using standard plate glass 0.66 in. thick. Note the position of the neutral axis in this beam, indicated by the zero fringe near the bottom of the fringe photographs. With the neutral axis in the picture, the photograph becomes a self-interpreting record of the stress distribution.

The work is checked by comparing the internal or resisting moment (as determined from the fringe pattern or from the tensile. force in the steel) with the external moment. Recently a U. S. Bureau of Standards calibrated proving ring for measuring the applied loads was used. It was discovered that the weighing jack which had been used for measurement of the applied loads was becoming increasingly inaccurate for this purpose.

Figure 5 shows a series of fringe photographs taken at various fringe increments. This beam is made of Class C concrete as specified by the Ohio Department of Highways and was made with air-entraining ce-

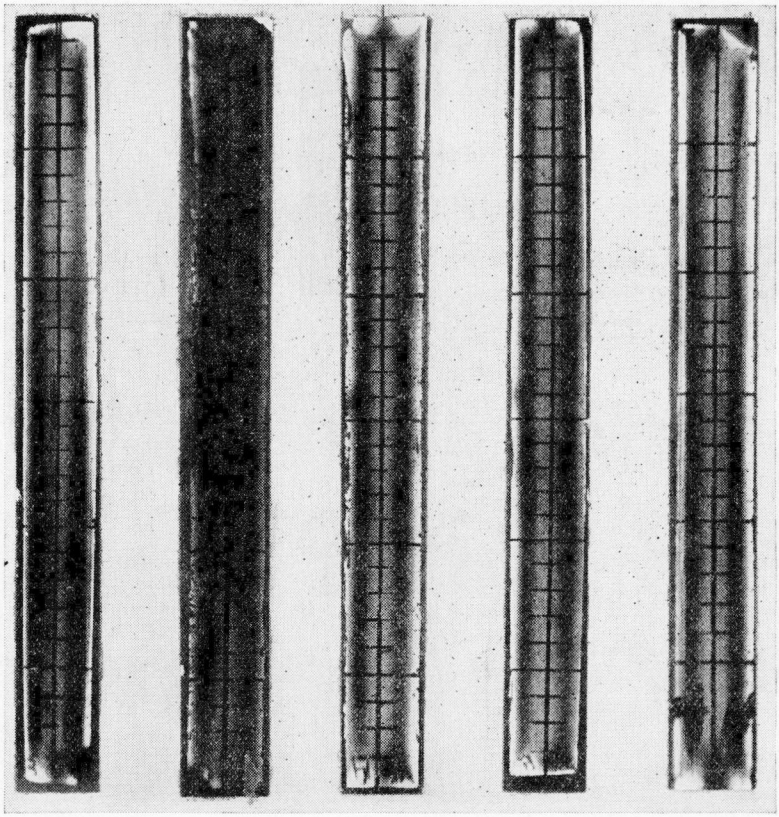


Figure 3. Five Unmounted Glass Inserts Taken in a Field of Circularly Polarized Light—The fringes in the picture indicate the type of initial stress in the glass. The etched lines are for the purpose of correcting lense abberations. The path of the light through the glass is 6 inches.

ment. The test cylinders broke at an average of 4500 psi. The steel used in this beam is the old-type rail steel bar. When 35 fringes had formed, the steel was yielding and flow in the concrete was progressing rapidly. The maximum flexural compressive stress in the concrete as shown is around 6500 psi.

There is some bending in the glass as indicated by the diagonal fringes in the lower fringe order pictures. When the bending moment became high, the effect of the bending was less noticeable.

Again the neutral axis is plainly shown by the zero fringe. Furthermore, the neutral axis moved up as the yield in the steel progressed. This is noticeable in the right three fringe photographs in the bottom row.

The discrepancy between T and C is probably due to the Poisson's ratio effect of setting up tension in the glass normal to the beam and to the direction of the polarized rays. The discrepancy is noticeable in much of the earlier work. Various types of coating on the faces of the glass in contact with the concrete have been tried in an attempt to eliminate this possibility.

Figure 6 shows the fringe value distribution drawn from the fringes at the two edges of the fringe photographs. Four load values are used here. A better way to interpret the fringe values is to take the fringe order along the vertical centerline of the fringe photograph. In this way, effects of bending in the glass are automatically eliminated.

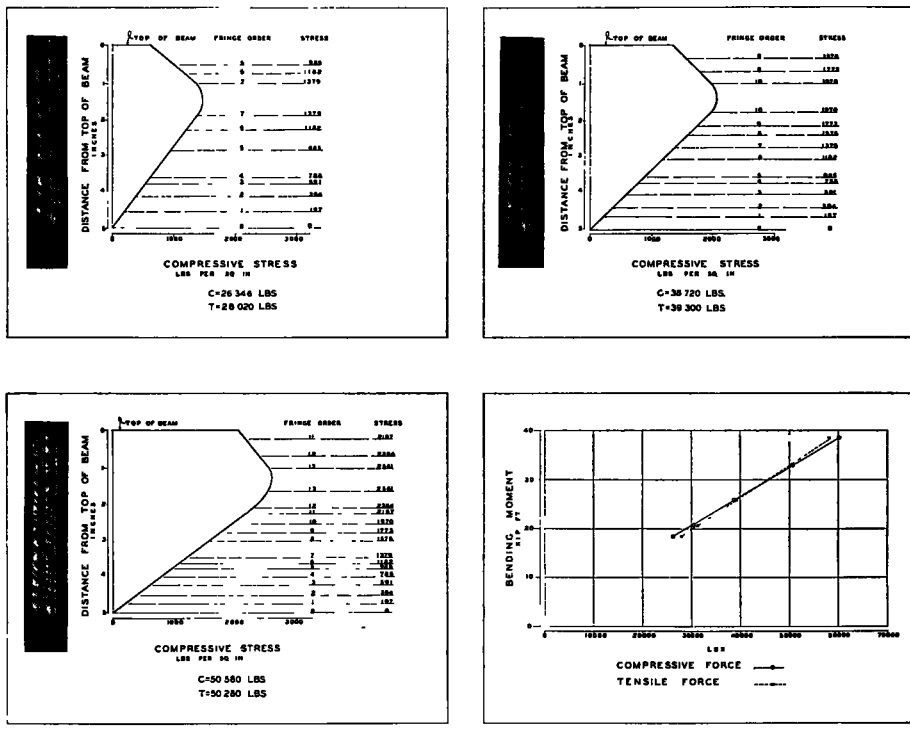


Figure 4

It is interesting that the distribution shape is nearly trapezoidal and agrees in this way with Jensen's assumptions. These photographs are four selected from the group shown in Figure 5.

Most of the fringe patterns obtained prior to the time that the photographs in Figure 7 were taken show the point of maximum intensity of stress in the compressive area to be 1 in. or more below the top of the beam. In the beam from which these pictures were obtained, the steel was placed in the top of the

forms and the beams were "cast upside down." The densest concrete was then on the compression side of the beam. It was thought that this distribution of concrete density would force the maximum fringe to come in at the top of the beam. The pictures show the maximum pressure to be only about one-half inch below the top of the beam. The photograph at the left in the top row was with zero moment. Notice that it compares well with the right hand photograph in the bottom row which is a picture of an unmounted glass.

Figure 8 shows pairs of pictures of the same fringe order, from the same beam. The left-hand picture of each pair was taken two days after the beam was removed from the forms. Note the increase in drop-off of fringe values in the pictures taken five days after removal from the forms. Note also the position of the neutral axis.

The additional three days of drying out of the top layers of the concrete explain this additional drop-off in fringe value and suggests a method of checking some of the effects of moisture in concrete.

The pictures in Figure 9 are the result of experiments to determine some of the types of stresses induced by wetting the top surface of the concrete. This beam is the companion beam to the one shown in Figure 8.

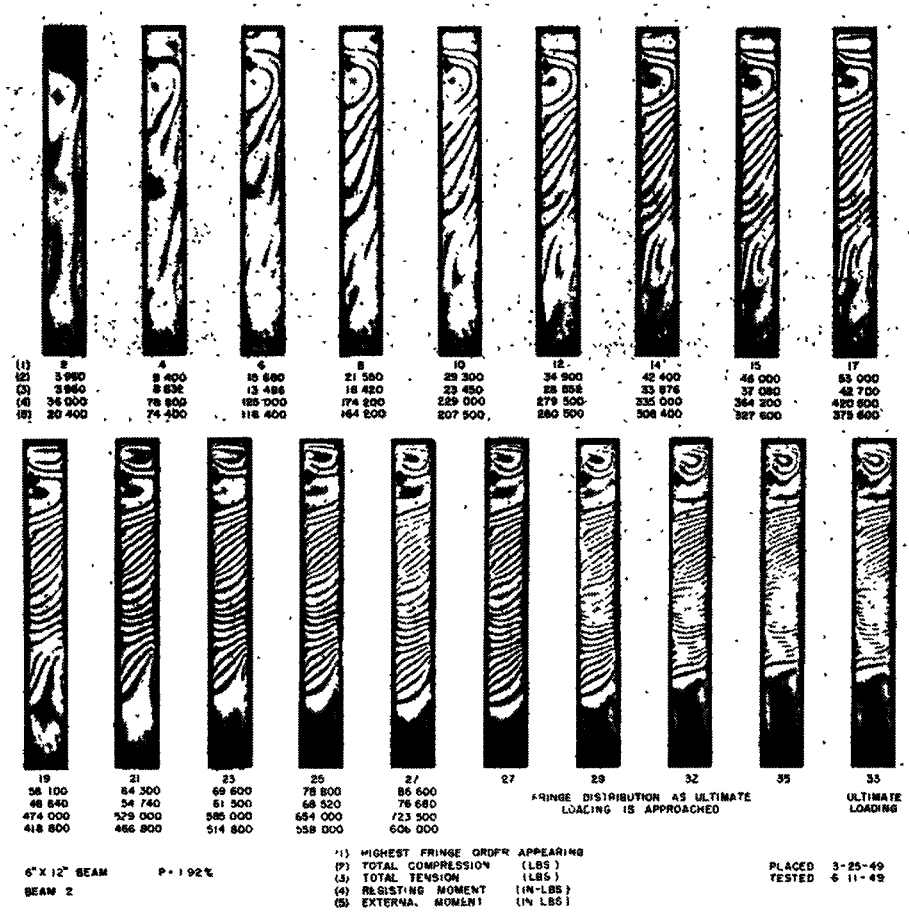
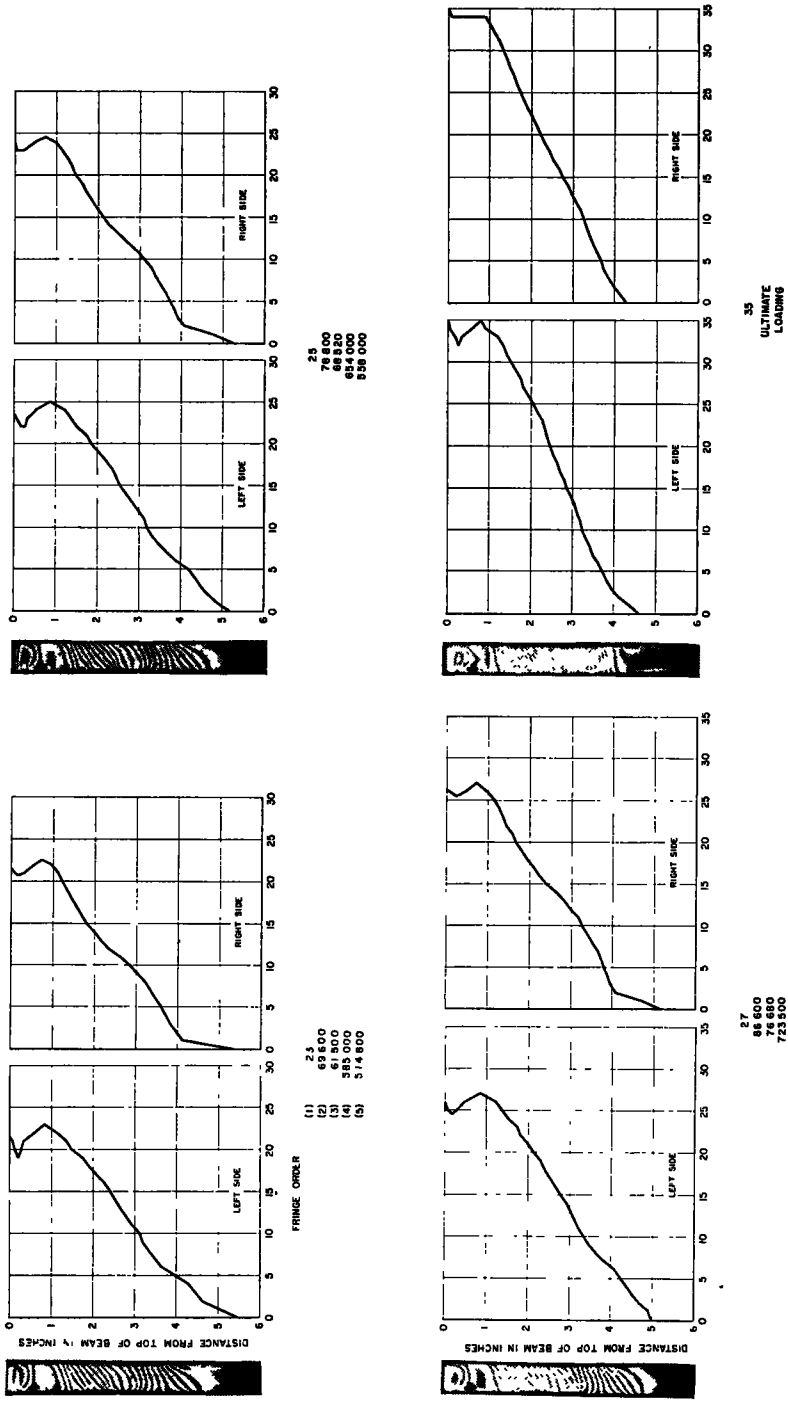


Figure 5



VARIATION OF COMPRESSIVE STRESS-FRINGE DIAGRAMS WITH CHANGING BENDING MOMENT

(1) HIGHEST FRINGE ORDER APPEARING
(2) TOTAL COMPRESSION (LBS)
(3) TOTAL TENSION (LBS)
(4) RESISTING MOMENT (IN LBS)
(5) EXTERNAL MOMENT (IN LBS)

6" x 12" BEAM P-1 92%

FRINGE VALUE P-Q-196 PSI

PLACED 3-25-49
TESTED 6-11-49

BEAM 2

NOTE
CONCRETE PLACED IN BEAM 3-25-49
FORMS REMOVED 3-30-49
AIR DRIED UNTIL TESTED

Figure 6

The second picture from the left in the top row shows the fringe distribution pattern for a bending moment of 152,200 in. lb. At this intensity the maximum stress in the concrete is about 1600 psi. in compression. The load was removed and the top of the middle third of the beam was covered with wet sand. The fringe distribution pattern for dead load at the end of 17½ hr. is shown in the first photograph at the left in the bottom row. The fringe distribution for a bending moment of 152,200 in. lb. is shown in the second picture from the left in the bottom row. The distribution of fringes after 71 hr. is shown in the middle photograph in the bottom row. At the end of the 71-hr. period the wet sand was removed from the top of the beam and the fringe distribution 169 hr. later is shown in the right-hand photograph in the bottom row. Apparently, the high surface stress developed by the moisture on top of the beam caused plastic flow in those layers to such an extent that the final fringe pattern for the dry beam (for a bending moment of 152,200 in. lb.) is different from the initial fringe pattern when the beam was subjected to the same bending moment.

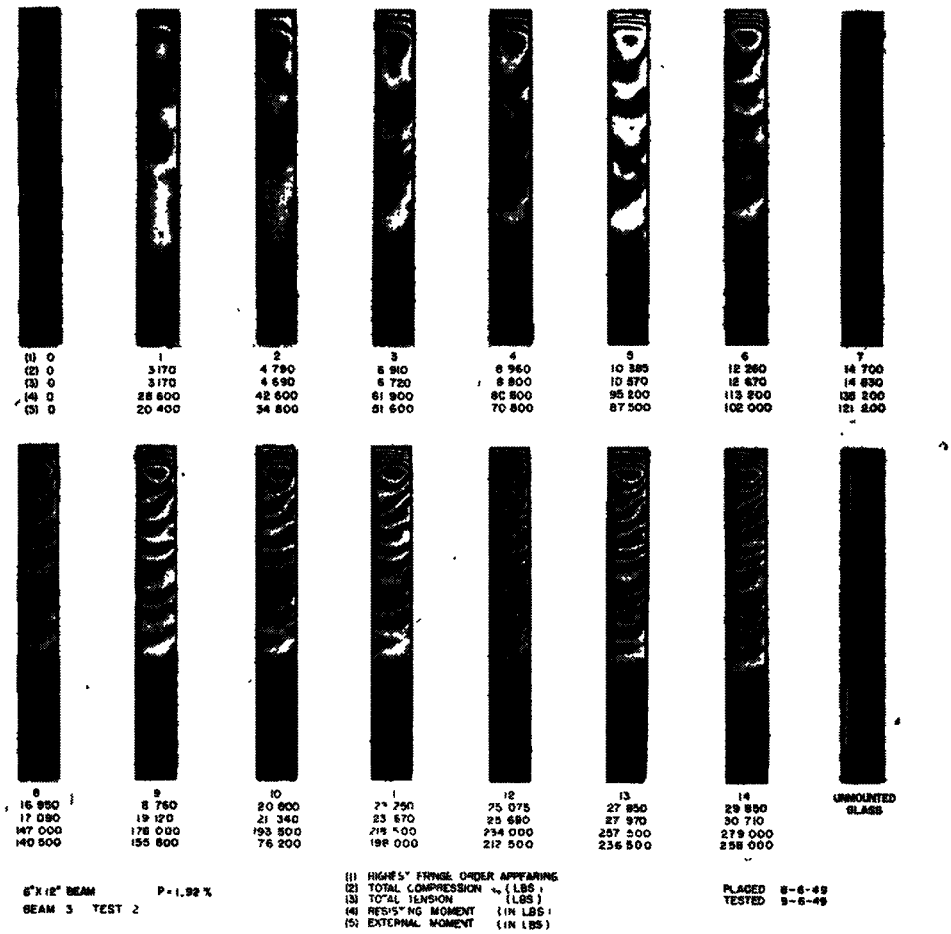


Figure 7

Figure 10 shows the result of tests made on one of the beams of the seventh pair which had

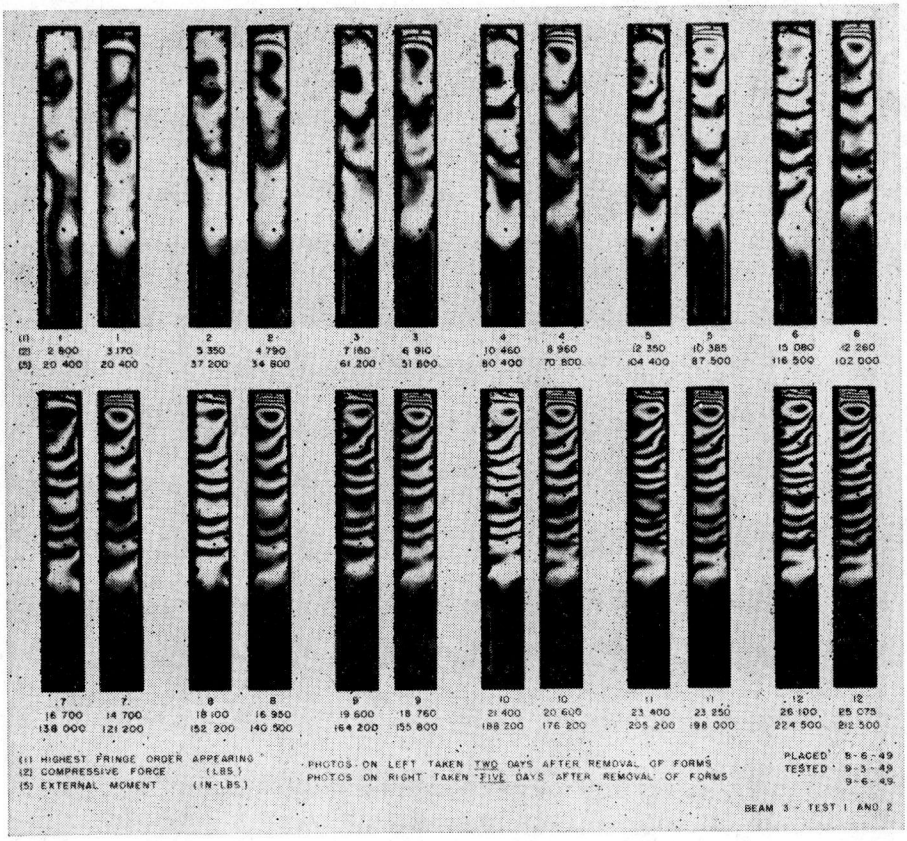


Figure 8. Effect of Natural Surface Drying

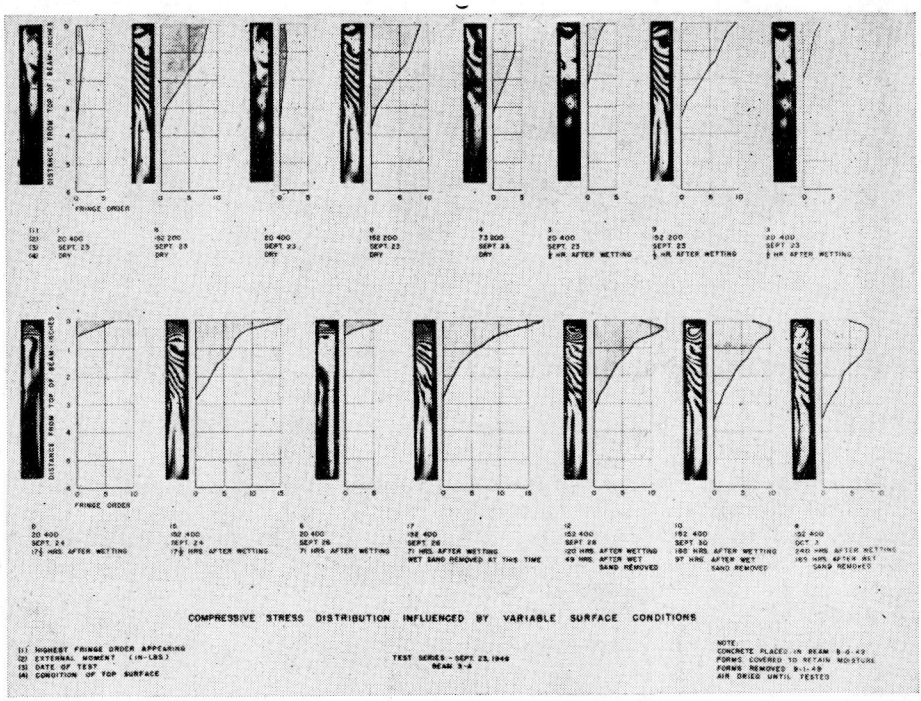
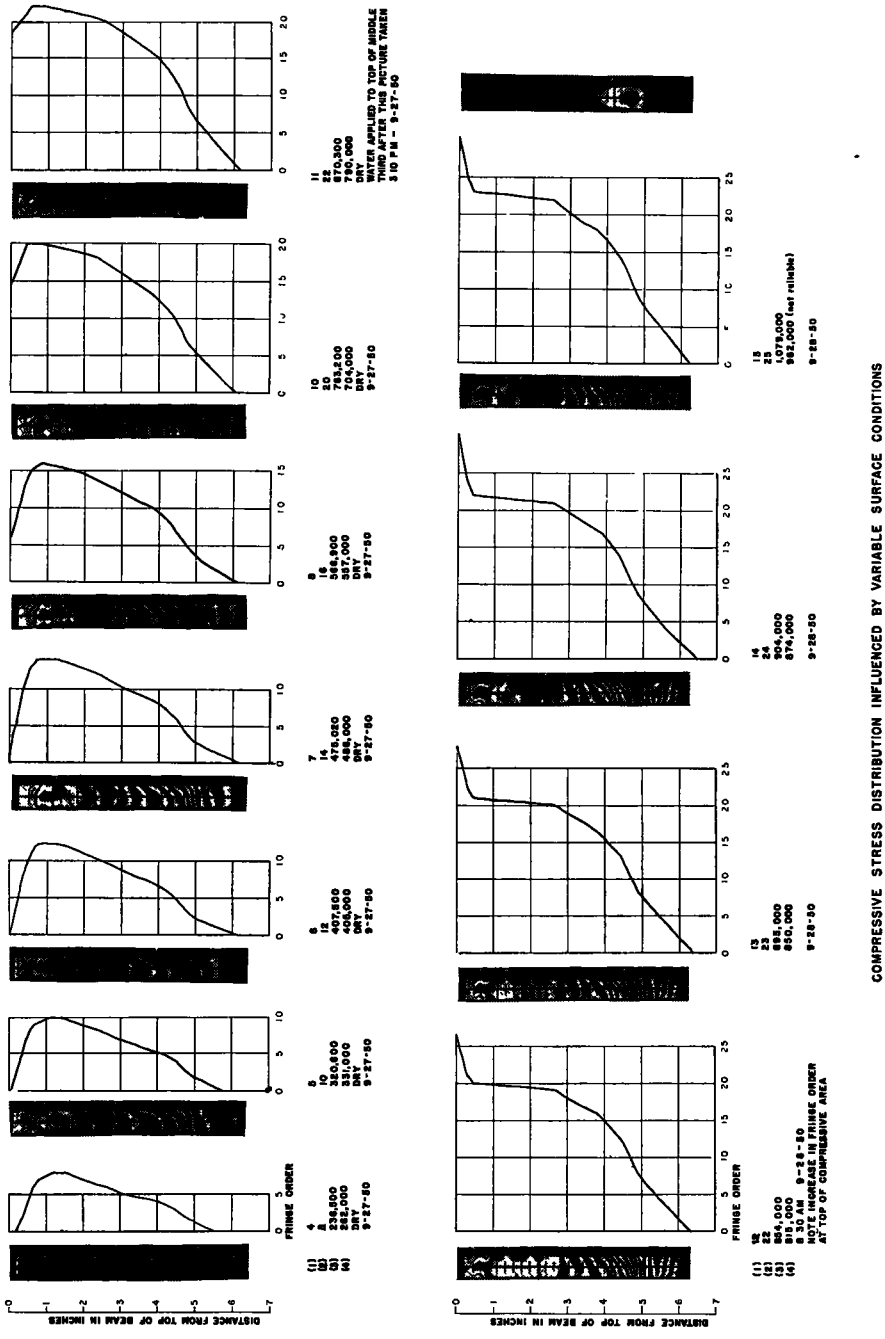


Figure 9



COMPRESSIVE STRESS DISTRIBUTION INFLUENCED BY VARIABLE SURFACE CONDITIONS

Figure 10

been made. The concrete used is Class C concrete, as specified by the Ohio Department of Highways, made with air-entraining cement and of 3-in. slump. After the concrete was placed in the forms August 21, 1950, it was covered with wet sacks and waterproof paper. The steel used is hi-bond type No. 11 billet steel bars. The SR-4 gages were mounted as previously described. On September 7, 1950, the beams were removed from the forms and the sides and ends were coated heavily with a compound approved by the Ohio Department of Highways Testing Laboratories, to prevent escape of moisture through the sides of the beam. The drying of the concrete in the beam was similar to the drying of a slab.

On September 27, 1950, 20 days after the top and bottom of the test beam were exposed to a natural drying action, a series of tests was made. The series of photographs shown in Figure 10 indicates the compressive stress distribution. The results clearly reveal that there is considerable moisture in the interior of the beam, as indicated by the bulging effect in the fringe order curve below the position of maximum fringe order. Toward the top surface of the beam drying has been more complete, as indicated by the rapid drop-off in fringe order between the maximum fringe value and the fringe value at the top of the beam. In the future more complete drying of the beams will be permitted before the testing is done.

At 3:10 p.m., (September 27) immediately after picture No. 11 was taken, as shown in Fig. 10, the top surface of the beam was wet in the middle third section. At 8:30 the next morning picture No. 12 was made. Surface moisture caused the formation of the maximum fringe order at the top surface of the beam rather than at the point shown in picture No. 11. This moisture effect is shown clearly in picture No. 12.

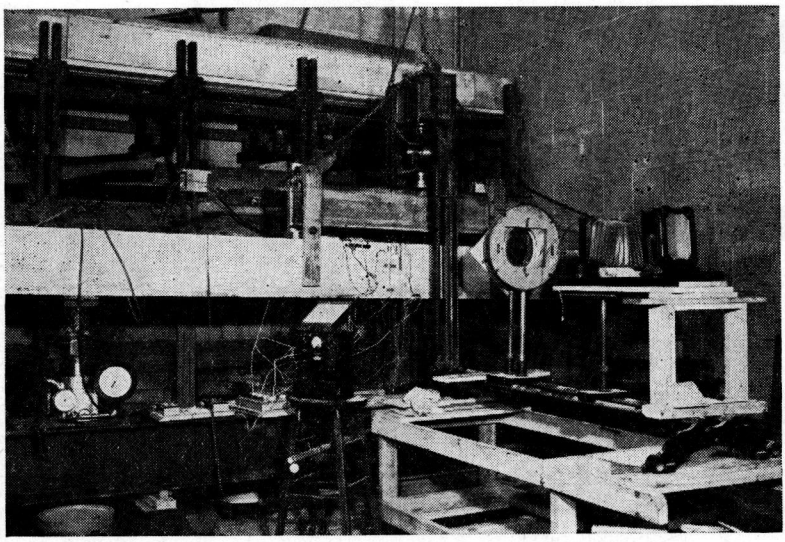


Figure 11

In Figure 10 the neutral axis does not appear in the fringe photograph. The steel ratio in this beam is so high that the neutral axis is about 6½ in. from the top of the beam, thus fringes 1 and 2 and the zero fringe are not shown in the picture.

The University has been furnished 20 of the all-important glass inserts through the generosity of Dr. S. Frank Cox of the Pittsburg Plate Glass Company Research Laboratories. These new inserts will extend 7 in. into the concrete and will collect all of the fringes between the neutral axis and the maximum fringe. Thus the photograph of the fringes will be a complete record of the fringe distribution.

Figure 11 shows a beam ready for testing. The load is applied by the weighing jack and is checked by the 50,000-lb. capacity proving ring. The strain gages on the side of the beam are used to check the effect of Poisson's ratio near the top fibers. The home-made polariscope is in place for photographing the light retardations in the glass as the stresses develop due to beam flexure.

The work presented here is in the nature of pilot tests. A program for making a series of tests along these lines has been adopted by the Engineering Experiment Station of The Ohio State University.

DEFLECTIONS IN SLABS ON ELASTIC FOUNDATIONS

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SYNOPSIS

The fundamentals of a new method of slab analysis are presented in this paper. This new analysis, employing a simple extension of moment distribution principles, has furnished results that have been substantiated by the theories of elasticity and the methods of finite differences.

The technique employed in this analysis consists of a transformation of any given slab into a gridwork of beams, a systematic load deformation, and a subsequent distribution of bending moment and torsional effects on the grid. Analyses of slabs with grid systems are not new. But a uniqueness is realized in this analogy by assuming rigid and structurally indeterminate grid joints throughout the system.

Investigations of the deflections developed in slabs on elastic foundations are made in this paper. The theory is completely illustrated by the analysis of a slab on a soil foundation.

In 1926, Westergaard[1] presented a method for determining the deflections and resulting stresses in elastically supported slabs subjected to wheel loads. The results of his very elaborate mathematical analysis yielded, among other things, a succinct expression for vertical deflection in a slab loaded at a considerable distance from its edges, and supported by uniform elastic reactions. Concentric contour lines, expressing the deflection under concentrated loads, are determined by his formula:

$$Z_i = C \frac{P}{kl^2}$$

in which

Z_i = vertical deflections measured at various radial distances from the concentrated load.

C = numerical coefficient varying with the radius.

P = concentrated load.

k = modulus of subgrade reaction.

l = radius of relative stiffness =

$$\sqrt[4]{\frac{Eh^3}{12(l-u^2)k}}$$

E = modulus of elasticity.

u = Poisson's ratio.

h = slab thickness.

In this paper a simple numerical method is used to determine deflection values that are in striking agreement with those proposed by Westergaard. The procedure is thought to have merit not only because it furnishes results in close correspondence with those of the theoretical analysis, but also because it makes use of physical concepts familiar to all structural engineers. Although further research is needed to determine the procedure's complete range of applicability, the analysis of slabs,

[1] H. M. Westergaard, "Stresses in Concrete Pavements Computed by Theoretical Analysis," *Public Roads*, Vol. 7, No. 2, April 1926; also *Proceedings*, Highway Research Board, Vol. 5, Part 1, p. 90.