

Figure 11 shows a beam ready for testing. The load is applied by the weighing jack and is checked by the 50,000-lb. capacity proving ring. The strain gages on the side of the beam are used to check the effect of Poisson's ratio near the top fibers. The home-made polariscope is in place for photographing the light

retardations in the glass as the stresses develop due to beam flexure.

The work presented here is in the nature of pilot tests. A program for making a series of tests along these lines has been adopted by the Engineering Experiment Station of The Ohio State University.

DEFLECTIONS IN SLABS ON ELASTIC FOUNDATIONS

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SYNOPSIS

The fundamentals of a new method of slab analysis are presented in this paper. This new analysis, employing a simple extension of moment distribution principles, has furnished results that have been substantiated by the theories of elasticity and the methods of finite differences.

The technique employed in this analysis consists of a transformation of any given slab into a gridwork of beams, a systematic load deformation, and a subsequent distribution of bending moment and torsional effects on the grid. Analyses of slabs with grid systems are not new. But a uniqueness is realized in this analogy by assuming rigid and structurally indeterminate grid joints throughout the system.

Investigations of the deflections developed in slabs on elastic foundations are made in this paper. The theory is completely illustrated by the analysis of a slab on a soil foundation.

In 1926, Westergaard¹ presented a method for determining the deflections and resulting stresses in elastically supported slabs subjected to wheel loads. The results of his very elaborate mathematical analysis yielded, among other things, a succinct expression for vertical deflection in a slab loaded at a considerable distance from its edges, and supported by uniform elastic reactions. Concentric contour lines, expressing the deflection under concentrated loads, are determined by his formula:

$$Z_i = C \frac{P}{k l^2}$$

in which

Z_i = vertical deflections measured at various radial distances from the concentrated load.

¹ H. M. Westergaard, "Stresses in Concrete Pavements Computed by Theoretical Analysis," *Public Roads*, Vol. 7, No. 2, April 1926; also *Proceedings*, Highway Research Board, Vol. 5, Part 1, p. 90.

C = numerical coefficient varying with the radius.

P = concentrated load.

k = modulus of subgrade reaction.

l = radius of relative stiffness =

$$\sqrt[4]{\frac{Eh^3}{12(l-u^2)k}}$$

E = modulus of elasticity.

u = Poisson's ratio.

h = slab thickness.

In this paper a simple numerical method is used to determine deflection values that are in striking agreement with those proposed by Westergaard. The procedure is thought to have merit not only because it furnishes results in close correspondence with those of the theoretical analysis, but also because it makes use of physical concepts familiar to all structural engineers. Although further research is needed to determine the procedure's complete range of applicability, the analysis of slabs,

complicated by variable boundary and loading conditions, seems to be possible.

In brief, the analysis consists of a transformation of the slab into an imaginary grid system and a subsequent series of numerical operations on the established grid.

The analysis of the elastically supported slab is summarized in the following basic steps:

1. The division of the slab into orthogonal strips which are considered rigidly connected continuous beams of the imaginary gridwork.
2. The calculation of moment and torque distribution factors at each grid junction.

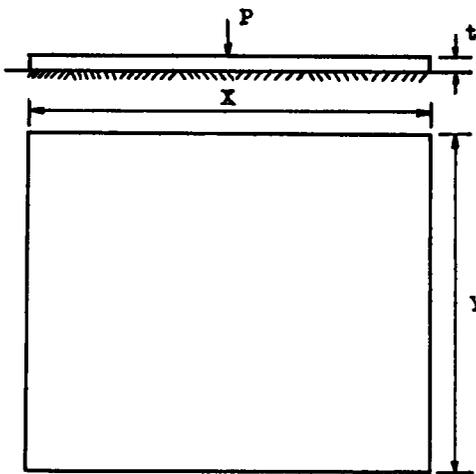


Figure 1

- $E = 1.5 \times 10^6$ lb. per sq. in.
- $u = 0.288$
- $t = \frac{1}{4}$ in.
- $P = 28$ lb.
- $k = 453$ lb. per sq. in. per in.
- $X, Y =$ indefinite slab divisions

3. The arbitrary vertical displacement of each singular joint in the system together with a simultaneous prohibition of vertical displacement at all other grid joints.

4. The consequent introduction of fixed end moments in the grid system.

5. The release of the fixed end moments and their transformation into bending and twisting moments by a moment and torque distribution process.

6. The computation of reactions at the artificially supported grid joints in terms of the unknown vertical displacements.

7. The formation of linear equations by

equating vertical reactions at a joint to the externally applied load at the joint, and the simultaneous solution of these equations to determine the deflection pattern.

The object of this investigation is to determine the deflections in an elastically supported slab subjected to a concentrated load.

The slab, illustrated in Figure 1, is assumed to be supported on a homogeneous elastic foundation and loaded with a concentrated load at a considerable distance from the edges.

The physical characteristics of the slab and the magnitude of the applied load are those which might be applicable in a small scale experimental study. As defined by Wester-

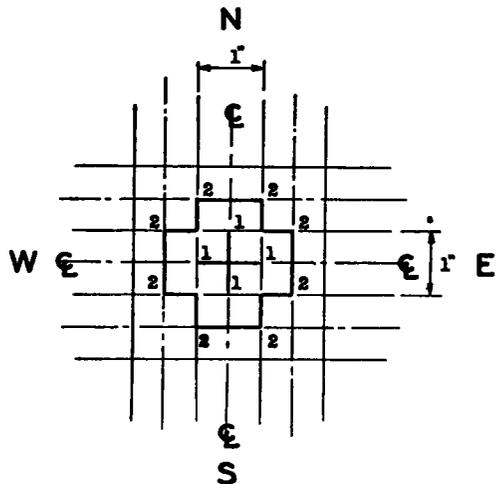


Figure 2

gaard, the modulus of subgrade reaction, k , is a measure of subgrade stiffness. The value of k , for any subgrade expressed in pounds per square inch per inch of deflection, is determined experimentally and is assumed to be independent of the resulting deflections.

THE GRID SYSTEM

If the $\frac{1}{4}$ -in. thick slab is divided into two sets of parallel orthogonal strips, each 1 in. wide, the resulting system can be likened to a series of rigidly connected continuous beams mutually supported along their center lines. As an example, the E-W and N-S grid strips in Figure 2 are considered continuous over supports 1-1 and 2-2 located at the center lines of the perpendicular strips. Furthermore, the beam segments 1-2-2-1, 1 by 1 by $\frac{1}{4}$ in.

are assumed free to rotate and to twist over the supports. Thus, the entire slab becomes a pseudo-grid consisting of a system of imaginary grid beams and joints.

In order to approximate slab action with this gridwork, composed of grid beams rigidly connected to all perpendicular grid beams, it is necessary to provide all beams with physical characteristics identical to those of the corresponding slab strips. This need is fulfilled by providing the beams with the same resistances to bending and to twist as those offered by the 1 by 1/4-in. slab strips.

Since the gridwork is to be deformed by vertical displacements at its joints, and the various elements subjected to bending and twisting moments, it is necessary to develop distribution factors at all of the joints in the system.

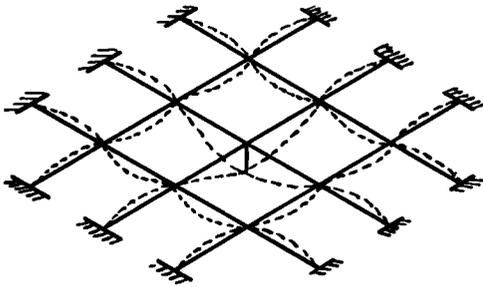


Figure 3

DISTRIBUTION FACTORS

When the schematically represented gridwork in Figure 3 is subjected to a vertical displacement at any one of its joints, the grid beams will simultaneously rotate and twist about the remaining joints. All joints but the one displaced are artificially restrained on a horizontal datum plane. In the system the difference in beam bending moment on opposite sides of a joint is resisted by torsion in the perpendicular beam.

A method similar to that developed by Cross² is used to distribute an unbalanced moment at a joint. However, the distribution of torsional resistance as well as bending resistance must be involved at the grid joints in order to simulate the desired slab action.

² Hardy Cross, "Analysis of Continuous Frames by Distributing Fixed End Moments," *Transactions A.S.C.E.*, 1932, XCVI, pp. 1-56.

Since all of the joints in the selected gridwork are formed by the intersection of identical beams, an unbalanced moment at any joint can be distributed with the same set of factors.

If an applied moment M_o in the plane of the beam AOB is released on the rigidly connected system shown in Figure 4, the tangent to the elastic curve at O will rotate through some angle θ . When the perpendicular beam DOC is fixed at its ends, it will twist through the same angle θ . Similarly a moment applied in the perpendicular plane will produce rotation of the tangent to the elastic curve in that plane and consequent twist in the beam AOB .

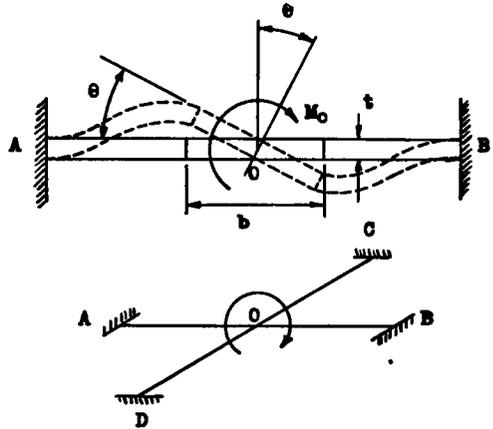


Figure 4

When the rotation through the angle θ has taken place,

$$M_o = M_{OA} + M_{OB} + M_{OC} + M_{OD}$$

Where M_{OA} and M_{OB} are bending resistances and M_{OC} and M_{OD} are torsional resistances.

For a rotation θ in the beam AOB

$$M_{OA} = \frac{4EI_1\theta}{l_1} \text{ and } M_{OB} = \frac{4EI_2\theta}{l_2}$$

For a twist θ in the beam DOC

$$M_{OD} = \frac{\beta_3 EI_3 \theta}{l_3} \text{ and } M_{OC} = \frac{\beta_4 EI_4 \theta}{l_4}$$

where $\beta = G/25E(1 - 0.63 t/b)$ for thin sections.³

³ A. Foppl, "Vorlesungen Über Technische Mechanik," 10 Aufl, Bd. 3, S 365.

- I = moment of inertia of grid beam cross-section about the horizontal axis.
- E = modulus of elasticity in tension or compression.
- G = modulus of elasticity in shear = $E/2(1 + \nu)$.
- l = length of grid beam.
- t = thickness of grid beam and slab.
- b = width of grid beam.

After the relationships between resisting moments and rotation angle are known, the distribution factors can be calculated as follows:

$$M_{OA} = \frac{\frac{4I_1}{l_1}}{\frac{4I_1}{l_1} + \frac{4I_2}{l_2} + \frac{\beta_3 I_3}{l_3} + \frac{\beta_4 I_4}{l_4}}$$

etc.

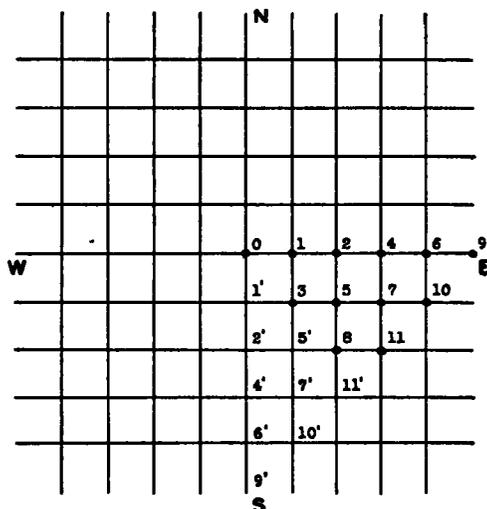


Figure 5

When the physical characteristics of the slab, and the constant length and cross-sectional dimensions of the grid beams are involved in expressions similar to that above, the following resisting moments will result:

$$M_{OA} = M_{OB} = 0.3683 M_O \text{ (bending)}$$

$$M_{OC} = M_{OD} = 0.1317 M_O \text{ (torsion)}$$

Therefore when an unbalanced moment M_O is applied on any joint of the selected grid-

⁴ L. C. Maugh—"Statically Indeterminate Structures," N. Y., 1946, p. 96.

work, two beams in bending resist 36.83 percent of the applied moment and two beams in torsion resist 13.17 percent.

AUXILIARY FORCE SYSTEMS FOR CONTROLLING DEFLECTIONS

Even in ordinary analyses, the moment distribution method cannot be applied directly to those structures that are acted upon by force causing translation of the joints. When the joints of continuous frames undergo translation as well as rotation, the successive approximation method cannot easily be applied because the convergence of carry-over moments is slow. The translation of a joint in a grid system of beams will create the same difficulty. Failure to obtain rapid convergence is due to the fact that a small translation of a joint has a measurable effect on the moments over many supports, whereas a rotation at a joint has only a localized influence on other joints.⁴ In order to control or prevent these joint translations, auxiliary force systems are introduced on the structure. These auxiliary forces are applied at each joint, and the translations they cause or prevent must be such as to make them compatible with the forces actually acting on the structure. The deflections at a finite number of points on the selected slab will be determined in this manner.

ARBITRARY DISPLACEMENT OF A TYPICAL JOINT

For all practical purposes, the gridwork of continuous beams, representing the elastically supported slab, can be imagined as infinite in extent. The infinite grid can be used only because the concentrated load is placed at a considerable distance from each boundary. Furthermore, all of the grid joints in the system are physically identical and make use of the same distribution factors. As a consequence, the vertical translation of any joint produces the same auxiliary force system over the grid. The vertical translation of joint O , shown in Figure 5 and assumed to be directly beneath the concentrated load P , will produce the necessary force system.

When the joint O is displaced an arbitrary vertical distance, fixed end moments equal to $\frac{6EI\Delta_0}{l^2}$ are developed at the joint O and at all

⁵ L. E. Grinter, "Theory of Modern Steel Structures," Vol. II, Macmillan Co., N. Y., 1937, p. 151.

joints immediately adjacent to it.⁵ The distribution of these fixed end moments throughout the grid system gives rise to a symmetric bending moment and torsion pattern. Because of this symmetry, the distribution process is confined to the southeast quadrant of the gridwork whose origin is at *O*.

THE DISTRIBUTION OF FIXED-END MOMENTS

The distribution of the east-west fixed end moments due to Δ_0 at point *O* is shown in

Joints are progressively released and moments are balanced by resistive bending moments and torques in the intersecting beams. In Table 1 for example, the fixed end moment -6000 at joint 1 is resisted in the balancing process by bending moments +2208 in the east-west grid beams and torsional moments +792 in the north-south beams. After all joints are balanced, a carry-over process is effected—torque moments carry over at full value but of the opposite sign, bending

TABLE 1

0					1					2					4					6					9
E	W	N	S	E	E	W	N	S	E	E	W	N	S	E	E	W	N	S	E	E	W	N	S	E	W
+6000	-6000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+2208	+792	+792	+2208	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+1104	0	0	0	0	+1104	0	0	0	0	-406	-146	-146	-406	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-203	0	0	0	0	0	0	0	0	0	0
0	+151	+54	+54	+151	0	0	0	0	0	+75	+27	+27	+75	0	+75	+27	+27	+75	0	0	0	0	0	0	0
+76	0	0	0	0	+76	+38	+38	+38	+76	0	0	0	0	0	0	0	0	0	0	+38	0	0	0	0	0
0	0	0	0	0	-70	-25	-25	-70	0	0	0	0	0	0	0	0	0	0	0	-14	-5	-5	-14	0	0
0	0	-16	-16	-35	0	0	0	0	0	-35	-11	-11	-7	0	0	0	0	0	0	0	0	0	0	0	-7
0	+26	+9	+9	+26	0	0	0	0	0	+24	+8	+8	+24	0	+24	+8	+8	+24	0	0	0	0	0	0	+3
+4820	+3616	+733	+733	+4820	+704	-133	-133	-438	0	-139	+24	+24	+92	0	+24	-5	-5	-14	0	0	0	0	0	0	-4

3					5					7					10
E	W	N	S	E	E	W	N	S	E	E	W	N	S	E	W
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-54	-14	-54	0	0	0	0	0	-54	-27	0	0	0	0
0	+45	+16	+16	+45	0	0	0	0	0	+30	+11	+11	+30	0	0
+22	0	0	0	0	+22	+25	+7	+15	0	0	0	0	0	0	+15
0	0	0	0	0	-25	-9	-9	-25	0	0	0	0	0	0	-7
+169	+337	+728	+106	+283	+35	-124	-40	-118	0	-24	-16	+11	+30	0	+6

2'					5'					8'				
E	W	N	S	E	E	W	N	S	E	E	W	N	S	E
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	+19	0	0	0	+19	+38	0	0	0	-10	-11	0	0	0
0	0	0	0	0	-21	-7	-7	-21	0	+8	+2	+2	+8	0
0	0	-16	-2	-10	0	0	0	0	0	-2	-9	+2	+8	0
0	+10	+4	+4	+10	0	0	0	0	0	0	0	0	0	0
+19	+48	+132	+16	+38	-2	+31	-7	-21	0	0	0	0	0	0

4'					7'				
E	W	N	S	E	E	W	N	S	E
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	+2	0	0	0	+2	+7	0	0	0
0	0	0	0	0	-5	-1	0	0	0
+2	+2	-13	+2	+2	-1	+6	-1	-3	0

NOTE: LAST FIGURE IN EACH COLUMN IS THE FINAL MOMENT.

Table 1. Since the symmetry of the structure allows identical moment and force patterns to be developed from the distribution of north-south fixed end moments, these moments are not considered.

The fixed-end moments expressed at units of $EI\Delta_0/1000l^2$ are placed on the appropriate east-west grid beams in Table 1. When viewed from the south, the signs of these moments are in accordance with the moment distribution conventions.

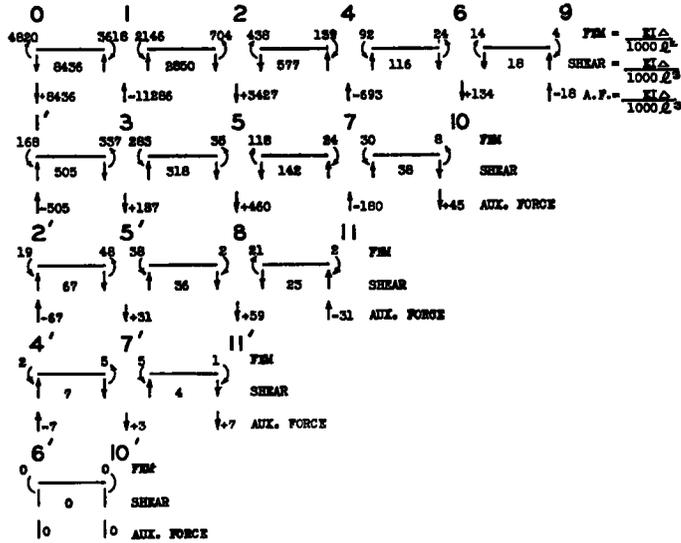
moments carry over at half value but of the same sign. A thorough dispersion of the original fixed end moments is realized after five cycles of distribution.

THE AUXILIARY FORCE SYSTEM DUE TO DISPLACEMENT Δ_0

The auxiliary forces shown in Table 2 are computed from the final distributed east-west moments in the southeast quadrant of the grid. End shears due to final bending moments

are determined on each grid beam segment 1, for example, can be determined by adding included between joints. The algebraic addition -11286 + 2(-505).

TABLE 2



THE COMPLETE FORCE SYSTEM

Neither the bending moments nor the auxiliary forces derived from them need be considered beyond the grid joints 9, 10, and 11 shown in Figure 6. When the force values at these boundary joints are compared with those near the displaced joint 0, the reason for this is evident.

The extent of the significant force pattern also defines the range of the significant deflections. Accordingly, deflections beyond the grid joints 9, 10, and 11 do not have to be determined. Furthermore, the deflections of grid joints 0 through 11 contained within one octant of the infinite grid, will define completely the elastic surface of the gridwork or slab.

In order to formulate twelve linear equations involving the twelve deflections Δ_0 through Δ_{11} as unknowns, auxiliary force patterns should be known for displacements at each of the twelve singular joints. However, no additional distributions are necessary in order to obtain the forces due to deflections. Since the selected gridwork is a completely homogeneous one, the auxiliary force pattern developed by a displacement at joint 3, for

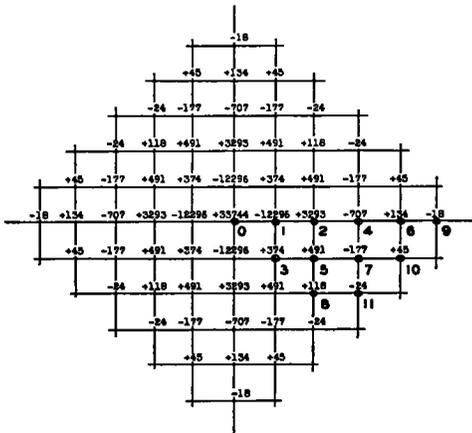


Figure 6

tion of the two end shear values at any joint produces the auxiliary force. Positive signs indicate downward forces.

Fortunately, an identical force pattern would develop in the N-S direction if north-south fixed end moments were distributed. The total forces, arising from the superposition of N-S and E-W force patterns, are shown in Figure 6. The auxiliary force -12296 at joint

example, is identical to that developed by the displacement of joint *O*. Obviously, the orientation of these forces over the gridwork will be such that symmetry is realized around joint 3 rather than around joint *O*.

Table 3 as units of $\frac{EI\Delta}{l^3}$. For example, at joint 0 the following forces exist:

$$+ 33.744 \frac{EI\Delta_0}{l^3} - 49.184 \frac{EI\Delta_1}{l^3} + 13.172 \frac{EI\Delta_2}{l^3} \text{ etc.}$$

TABLE 3
Auxiliary Force at Joint

	0	1	2	3	4	5	6	7	8	9	10	11
Deflection at Joint												
0	+33.744	-12.296	+3.293	+0.374	-0.707	+0.491	+0.134	-0.177	+0.118	-0.018	+0.045	-0.024
1	-49.184	+37.785	-12.021	-23.610	+3.073	+3.608	-0.635	-0.195	+0.920	+0.134	-0.043	-0.059
2	+13.172	-12.021	+34.114	+0.394	-12.362	-11.798	+3.293	+0.197	+6.856	-0.707	+0.536	-0.216
3	+1.496	-23.610	+0.394	+40.448	+1.0	-12.536	-0.354	+3.545	+0.020	+0.090	-0.749	+0.498
4	-2.828	+3.073	-12.362	+1.072	+33.744	+0.492	-12.296	-12.320	+0.982	+3.293	+0.374	+3.116
5	+3.928	+7.216	-23.996	-25.072	+0.984	+37.191	+0.906	-11.325	-25.916	-0.354	+3.234	+3.624
6	+0.536	-0.635	+3.293	-0.354	-12.296	+0.453	+33.744	+0.374	+0.236	-12.296	-12.296	+0.467
7	-1.416	-0.390	+0.394	+7.090	-24.640	-11.325	+0.748	+37.155	+0.394	+0.982	-11.829	-12.467
8	+0.472	+0.920	+6.856	+0.020	+0.982	-12.958	+0.236	+0.197	+34.012	-0.048	+0.453	-12.269
9	-0.072	+0.134	-0.707	+0.090	+3.293	-0.177	-12.296	-0.491	-0.048	+33.744	+0.374	+0.118
10	+0.360	-0.086	+1.072	-1.498	+0.748	+3.234	-24.592	-11.829	+0.906	+0.748	+37.037	+0.315
11	-0.192	-0.118	-0.432	+0.996	+6.232	+3.624	+0.934	-12.467	-24.538	+0.236	+0.315	+34.252

Note: All forces are multiples of $EI\Delta/l^3$.

TABLE 4
DISPLACEMENTS

	11	10	9	8	7	6	5	4	3	2	1	0
Summation of Forces at Joint												
11	+36.095	+1.315	+1.118	-12.269	-12.467	+467	+3.624	+3.116	+4.498	-216	-0.059	-0.024
10	+3.215	+38.890	+374	+453	-11.829	-12.296	+3.234	+0.374	-749	+536	-0.043	+0.045
9	+236	+748	+35.583	-0.048	+9.82	-12.296	-354	+3.293	+0.090	-707	+1.134	-0.018
8	-24.538	+906	-0.045	+35.855	+394	+236	+236	-25.916	+0.982	+0.020	+0.566	+0.920
7	-12.467	-11.829	+491	+197	+38.998	+374	-11.325	-12.320	+3.545	+0.197	-195	+1.177
6	+934	-24.592	-12.296	+236	+748	+35.587	+906	-12.296	-354	+3.293	-635	+1.134
5	+3.624	+3.234	-177	-12.958	-11.325	+453	+39.034	+492	-12.536	-11.798	+3.608	+491
4	+6.232	+748	+3.293	+982	-24.640	-12.296	+984	+35.587	+1.072	-12.362	+3.073	-707
3	+996	-1.498	+0.090	+0.020	+7.090	-354	-25.072	+1.072	+42.291	+394	-23.610	+374
2	-432	+1.072	-707	+6.858	+394	+3.293	-23.596	-12.362	+394	+35.957	-12.021	+3.293
1	-118	-0.086	+134	+920	-390	-635	+7.216	+3.073	-25.610	-12.021	+39.025	-12.296
0	-192	+360	-0.72	+472	-1.416	+536	+3.928	-2.828	+1.496	+13.172	-49.184	+35.587

Final Deflections

	+0.0005	-0.0002	-0.0002	+0.0048	+0.0023	-0.0003	+0.00141	+0.00033	+0.00392	+0.00198	+0.00581	+0.0102
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Note: Deflection values expressed in inches.

The complete force pattern can be obtained by placing on a sheet of transparent paper a copy of the grid system that can be superimposed on the system shown in Figure 6. On this sheet only the 12 joints are numbered. If any deflected joint is placed over joint *O*, the auxiliary forces developed at each of the grid joints due to the deflection can be read through the tracing. All forces are expressed as multiples of $\frac{EI\Delta}{1000 l^3}$.

The twelve auxiliary forces accumulated at each of the singular joints are tabulated in

DEFLECTION EQUATIONS

The system of linear equations used to determine deflections is obtained by summing up the forces at each joint and equating them to the experimental constant *k*, times the deflection Δ which exists at the point where forces are totaled. The experimental constant *k* expresses the resistive force of the soil per unit deflection over a unit area and is equal to 453 lb. per sq. in. per in.

In the grid system, a reactive force developed over one square inch of foundation

area is applied at each grid joint. Therefore, $k\Delta$ or 453Δ is the reactive force at each joint.

The equations and their solutions are shown in Table 4. In each equation, the coefficient of the Δ term at the joint where the equation has been formulated, is different from that in Table 3 by an additional 453Δ . Since each term is defined in terms of EI units, the actual amount to be added to the Δ is not 453 but $453/EI$. The modulus of elasticity E is 1.5×10^6 lb. per sq. in. For the grid beam whose

A comparison of the results of this analysis and the theoretical displacements of Westergaard is shown in Figure 7. In order to plot the Westergaard values⁶ at the load and at varying distances from it, the radius of relative stiffness l must be calculated.

$$l = \sqrt[4]{\frac{Eh^3}{12(1-u^2)k}}$$

$$= \sqrt[4]{\frac{1.5 \times 10^6 \times (0.125)^3}{12(1-(0.288)^2)453}} = .874 \text{ in.}$$

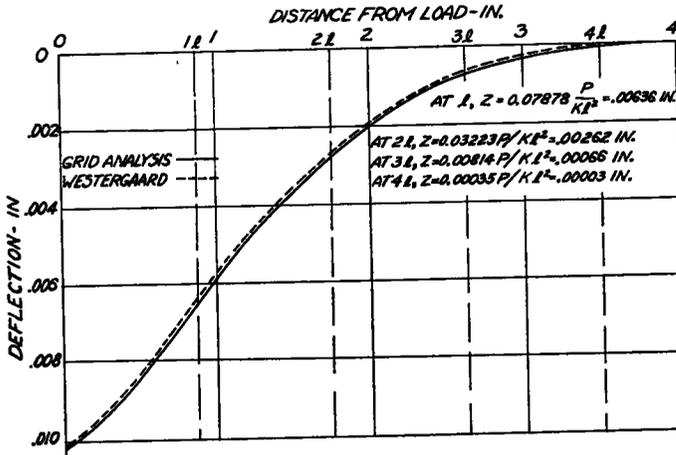


Figure 7—Deflections Produced by a Concentrated Load Acting at the Center of the Slab on Elastic Foundations.

cross section is one-eighth by one inch, the moment of inertia I is equal to:

$$I = \frac{1}{12}bt^3 = \frac{1}{12} \times 1 \times \left(\frac{1}{8}\right)^3 = 1.628 \times 10^{-4} \text{ in.}^4$$

therefore

$$EI = 1.5 \times 10^6 \times 1.628 \times 10^{-4} = 2.46 \times 10^2 \text{ lb. in.}^2$$

Thus the factor $\frac{453\Delta}{EI}$ becomes

$$\frac{453 \Delta}{2.46 \times 10^2} = 1.84 \Delta$$

The equation written at joint 0 is set equal to $(P/EI - 1.84\Delta_0)$ or $(0.1139 - 1.84\Delta_0)$. The first term is obtained by dividing the 28 pound applied load by the EI magnitude shown above. It should be noted that the load P is involved only in the equation written at joint 0.

In the theoretical analysis, the deflection under the center of the load is equal to

$$Z_1 = \frac{P}{8kl^2} = \frac{28}{8 \times 453 \times (0.874)^2} = .0101 \text{ in.}$$

As shown in Table 4, the value of deflection Δ_0 is .0102 in. Consequently, the two values for center point deflection differ by one percent.

Along Westergaard's curve, the displacements Z are given as multiples of P/kl^2 at distances away from the load expressed in terms of the radius of relative stiffness. If the theoretical deflections are to be compared with those furnished by the grid analysis, the

⁶ H. M. Westergaard, "Stresses in Concrete Pavements Computed by Theoretical Analysis," *Public Roads*, Vol. 7, No. 2, April 1926, p. 127; also *Proceedings*, Highway Research Board, Vol. 5, Part I, page 90.

magnitudes must be interpolated from the theoretical curve at one inch intervals. Had a grid beam width of 0.874 in. been selected, this interpolation could have been avoided. However, the selection of smaller grid beams would result in the calculation of more joint displacements, in order to define the elastically deformed surface of the slab over the same general area. The solution of more than twelve simultaneous equations by ordinary methods was not considered feasible.

In the previous analysis no modification was applied to the concentrated load in order to determine deflections and moments near the concentration. Westergaard has suggested the use of an equivalent diameter of a circular load to avoid the singularity existing in the ordinary slab equation at the point of application of the load. In a grid system, no such discontinuity exists and deflections can be obtained under concentrated loads in a man-

ner similar to that employed in ordinary beam theory.

CONCLUSIONS

The adaptation of the moment distribution method to the analysis of slabs is done with the realization that the physical characteristics of the grid system are different from those of the homogeneous and continuous slab. Nevertheless, the grid analysis procedure seems to compare favorably with other known methods of determining deflections. The accuracy of the analysis is necessarily dependent on the number of grid beams used to approximate the slab.

As indicated by the example in this paper, the investigation of slabs on elastic foundations does not present any technique not already understood by the average structural engineer. Conceivably, these same techniques can be employed in the analysis of elastically supported finite slabs with thickened edges.

REPORT OF COMMITTEE ON BRIDGE DESIGN

G. S. PAXSON, *Chairman, Bridge Engineer, Oregon State Highway Department*

The Bridge Committee of the Division of Design was formed this year (1949). The membership at the present time consists of Mr. Raymond Archibald of the Bureau of Public Roads, Mr. James P. Exum of the consulting firm of Howard, Needles, Tammen & Bergendoff, Mr. E. L. Erickson of the Bureau of Public Roads, Mr. T. R. Higgins of the American Institute of Steel Construction, Mr. R. Robinson Rowe of the California State Highway Department, Professor C. P. Siess of the University of Illinois, and Mr. G. S. Paxson of the Oregon State Highway Department as chairman. In forming this committee it was the intention to have representatives of State highway departments, the Federal Bureau of Public Roads, universities, consulting engineers, and commercial interests intimately connected with bridge projects.

The first meeting of the Committee which was held in Washington, D. C. on May 2, 1950 was largely concerned with formulating objectives for the work of the Committee and preliminary arrangements for the presentation of a program at the 30th annual meeting.

The broad general objectives of the Committee are:

1. To keep track of current structural research and to encourage publication of the results. A large amount of work is done each year by universities, various highway departments, and city and county organizations. Too often this research work is undertaken in connection with some specific project and the result having served its purpose is filed away and forgotten. Much of it has value and is well worth general distribution.

2. To act as a clearing house for suggestions for needed structural research and to help, in so far as possible, in arranging for sponsorship for worth-while projects. Many problems, on which information is needed, are not, in themselves, of sufficient magnitude to justify the expense of a major research program. These same problems, however, are common to many organizations and, by a combination of the resources of these organizations and a sharing in the cost of the work, much information could be obtained at a nominal cost to each of the interested parties.