A THEORY ON LOADS ON NEGATIVE PROJECTING CONDUITS

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SYNOPSIS

Negative projecting conduits are defined as those which are placed in shallow ditches of such depth that the top of the conduit is below the adjacent natural ground surface which is covered by an embankment.

The theory for such conduits presented in the paper follows the general principles developed by Marston in the theory of loads on positive projecting conduits, which are those in which the conduit is placed in shallow bedding with the top of the conduit projecting above the natural ground surface, but under an embankment.

Since the load on a negative projecting conduit is less than that on a positive projecting conduit under the same fill, there is considerable advantage in the negative projection type of construction.

The load on conduit due to fill materials, lb. per ft. of length, $= W_c = C_n w B_d^2$, in which w is the unit weight of the fill material, B_d is the width of the ditch and C_n is the load coefficient for negative projecting conduits which varies with the ratio of height of fill above top of the conduit H to width of ditch B_d, settlement ratio r_{sd}, projection ratio p' and internal friction ratio $K\mu$.

Charts for determining values of C_n for various values of r_{sd}, p' and $K\mu$ are given in Figures 8, 9, 10 and 11.

This theory may be used to estimate loads on conduits installed by the im perfect ditch method. This method of increasing the supporting power of conduits consists in thoroughly compacting the soil on both sides and above the conduit for some distance above its top and then digging a ditch in this compacted fill down to the conduit. The ditch is refilled with loose compressible material, after which the embankment is finished normally.

One of the outstanding contributions of the Marston Theory of Loads on Underground Conduits is the demonstration by rational principles of mechanics that the load on a structure of this kind is greatly affected by certain environmental conditions of installation in addition to the height of fill over a conduit. The traditional view of many engineers is that the height of fill is the major variable which influences the load which a conduit must be designed to carry. Now it is known that there are several other factors which wield an important influence on the load. These factors are, for the most part, associated with the installation conditions which control the magnitude and direction of settlements of the prism of soil directly over the conduit relative to the settlements of the exterior prisms immediately adjacent to this central or interior prism. These relative settlements generate friction forces or shearing stresses which are added to or subtracted from the weight of the central prism to produce the resultant load on the conduit.

Because of the influence of these installation conditions and the importance of recognizing them when determining loads, underground conduits have been classified into several groups and sub-groups. This classification is shown diagramatically in Figure 1. There are two major groups; ditch conduits and projecting conduits. Projecting conduits are further subdivided into positive and negative subgroups, depending upon whether the top of the conduit, as installed, is above or below the adjacent ground surface. The essential features of these three classes of conduits are shown in Figure 2.

Ditch conduits are those which are installed in relatively narrow ditches dug in passive or undisturbed soil and then covered with earth backfill. Sewers, drains and water mains are typical examples of this class of conduits. Positive projecting conduits are those which are installed in shallow bedding with the top of the conduit projecting above the adjacent natural ground, and then covered with an embankment. Negative projecting conduits are those which are installed in shallow ditches of such depth that the top of the conduit is below the adjacent natural ground surface and then covered with an embankment which extends above this ground level. Highway and railway culverts are usually installed as projecting conduits, either positive or negative. than the flexible types in positive projecting conduit installations. Several State highway departments, notably those of California and Washington, are constructing highway culverts by methods which substantially conform

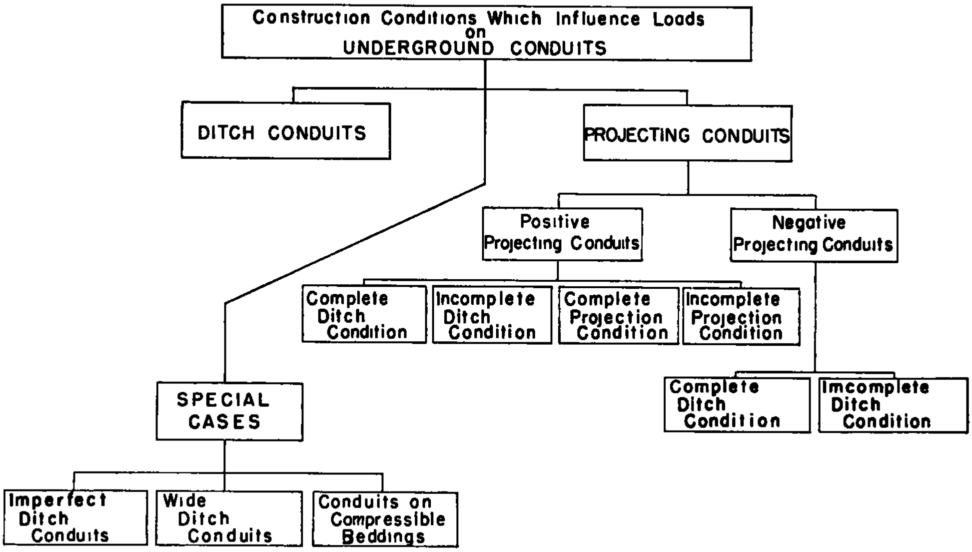

Figure 1

The theory of loads on ditch conduits was published by Marston (*1*)[1] in 1913. The load formulas resulting from the theory of loads on positive projecting conduits were given in 1930 (*2*) and a more extended discussion of the theory was published in 1950 (*3*). The purpose of this paper is to present a theory of loads on negative projecting conduits. This theory is based upon the same general principles which were employed by Marston in the two earlier developments mentioned.

There is a growing need for detailed understanding of the load-producing action of an embankment over conduits of this class, since it is favorable to the structure and permits the construction of higher fills without danger to the structure. For a given height of fill, the load on a negative projecting conduit is, generally speaking, considerably less than that on a positive projecting conduit. This is particularly true in the case of rigid structures, which normally are subjected to heavier loads

[1] Italicized figures in parentheses refer to the list of references at the end of the paper.

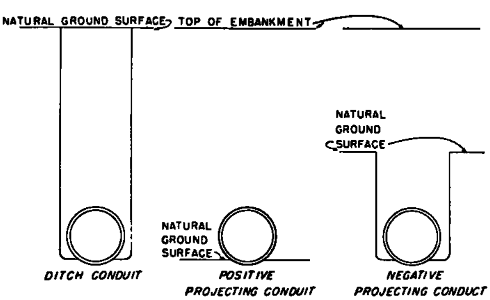

Figure 2. Classes of Conduits

with the methods prescribed for negative projecting conduits. Their results in terms of higher safe heights of fill have been satisfactory.

The following definitions apply in the case of negative projecting conduits. (See Fig. 3).

Critical Plane: The horizontal plane in the embankment material which was originally level with the natural ground surface before settlements occurred.

Plane of Equal Settlement: The horizontal plane in the embankment material at and above which the settlement of the interior prism of soil equals the settlement of the exterior prisms adjacent to it. Below the plane of equal settlement, relative vertical movements between the interior and exterior prisms induce shearing stresses which act upward along the sides of the interior prism and help to support its weight, thus reducing the load which reaches the conduit. Above the plane

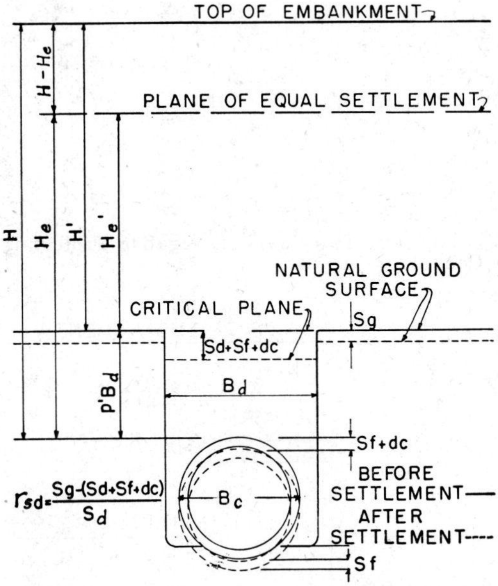

Figure 3. Elements of a Negative Projecting Conduit

of equal settlement no relative movements occur and no shearing stresses develop between this plane and the top of the embankment. An illustration of the fact of a plane of equal settlement is shown in Figure 4. Here a granulated rubber fill is placed over a negative projecting conduit in the laboratory. Note the greater settlements in the interior prism up to about the fourth horizontal string from the bottom.

Projection Ratio: The ratio of the distance between the natural ground surface and the top of the conduit, to the width of the ditch in which the conduit is constructed.

Settlement Ratio: The ratio of the difference between the settlement of the natural ground surface and the settlement of the critical plane to the compression of the column of soil in the ditch within the distance between the top of the conduit and the natural ground surface. Since the soil in the ditch over the conduit is normally more compressible than the natural soil in which the ditch is dug, the critical plane tends to settle more than the natural ground surface as the fill is built above this surface. Therefore the numerical value of the settlement ratio will always be negative in this case.

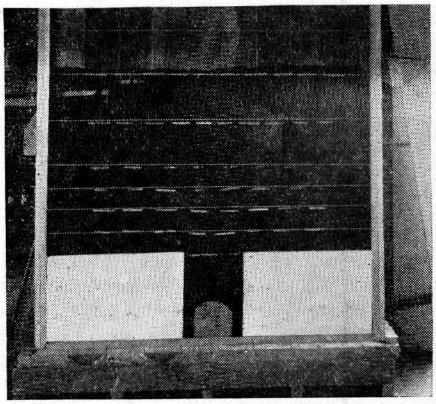

Figure 4. Laboratory Demonstration of a Plane of Equal Settlement

The following notation is employed in the mathematical development of the load formulas:

W_c = load on conduit due to fill material, lb. per ft. of length

w = unit weight of fill material, lb. per cu. ft.

V = vertical pressure on any horizontal plane in the interior prism of fill material, lb. per ft. of length

V' = vertical pressure on any horizontal plane in the exterior prisms of fill material, lb. per ft. of length

B_d = width of ditch in which the conduit is constructed, ft. (If the ditch has sloping sides, use the width at the top of the conduit.)

H = height of fill above the top of the conduit, ft.

H' = height of fill above the natural ground surface, ft.

H_e = height of the plane of equal settlement above the top of the conduit, ft.

H'_e = height of the plane of equal settlement above the natural ground surface, ft.

h = distance from the top of fill or from the plane of equal settlement down to any horizontal plane, ft.
C_n = load coefficient for negative projecting conduits
p' = the projection ratio
$p'B_d$ = distance from the natural ground surface down to the top of the conduit, ft.
d_c = the deflection of the conduit (i.e. the shortening of its vertical dimension), ft.
s_f = settlement of the conduit foundation, ft.
s_d = compression of the material in the ditch within the distance $p'B_d$, ft.
$s_d + s_f + d_c$ = the settlement of the critical plane, ft.
s_g = settlement of the natural ground surface, ft.
Note: d_c, s_f, s_d and s_g are the total settlements of these various elements due to the fill whose height is H
r_{sd} = the settlement ratio $= \frac{s_g - (s_d + s_f + d_c)}{s_d}$
λ = compression of the interior prism of fill material between the natural ground surface and the plane of equal settlement, due to the height of fill H'.
λ' = compression of the exterior prisms of fill material between the natural ground surface and the plane of equal settlement, due to the height of fill H'.
E = the modulus of compression of the fill material
μ = coefficient of internal friction of the fill material
e = base of natural logarithms
K = ratio of active horizontal earth pressure to vertical pressure
Use Rankine's formula

$$K = \frac{\sqrt{\mu^2 + 1} - \mu}{\sqrt{\mu^2 + 1} + \mu} \qquad (1)$$

There are two cases to consider in the development of load formulas for negative projecting conduits, the complete ditch condition and the incomplete ditch condition. The complete ditch condition prevails when the relative settlements between the interior and exterior prisms are such that the theoretical plane of equal settlenent is above the top of the embankment. Under these circumstances the upward shearing stresses on the interior prism act throughout the complete height of the fill, as shown in Figure 5 (a). The incomplete ditch condition prevails when the plane of equal settlement is at some elevation below the top of the embankment, as shown in Figure 5 (b). In this case the upward shearing stresses act only throughout the distance from the top of the conduit to the plane of equal settlement.

Considering first the complete ditch condition (Fig. 6), in which $H < H_e$, (H_e is imaginary in this case) the vertical forces on any thin horizontal element of the interior prism may be equated as follows:

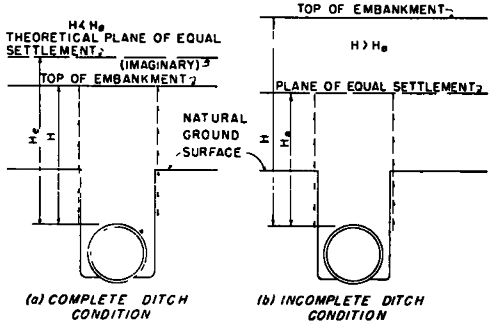

Figure 5. Two Cases of Negative Projecting Conduits

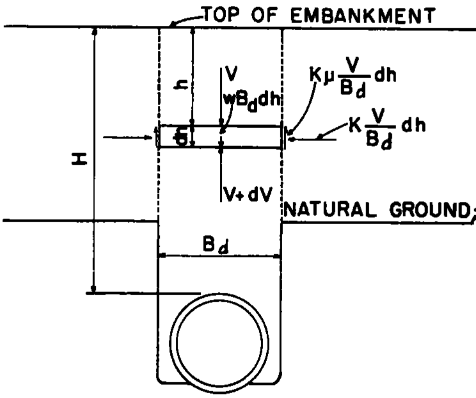

Figure 6. Force Diagram—complete Ditch Condition

$$V + dV = V + wB_d\,dh - 2K\mu \frac{V}{B_d}\,dh \qquad (2)$$

The solution of this equation is

$$V = wB_d^2 \frac{e^{-2K\mu(h/B_d)} - 1}{-2K\mu} \qquad (3)$$

At the top of the conduit, $V = W_c$ and $h = H$, whence

$$W_c = C_n w B_d^2 \qquad (4)$$

in which

$$C_n = \frac{e^{-2K\mu(H/B_d)} - 1}{-2K\mu} \qquad (5)$$

Next, consider the incomplete ditch condition (Fig. 7) in which $H > H_e$. Again equate

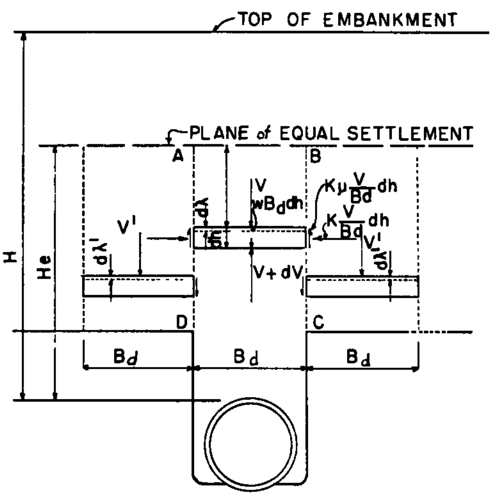

Figure 7. Force Diagram—Incomplete Ditch Condition

the vertical forces on a thin horizontal element at distance h below the plane of equal settlement.

$$V + dV = V + wB_d\,dh - 2K\mu\frac{V}{B_d}\,dh \qquad (6)$$

When $h = 0$, $V = (H - H_e)wB_d$, and the solution of equation 6 is

$$V = \frac{wB_d^2}{2K\mu} - \frac{wB_d^2}{2K\mu}e^{-2K\mu(h/B_d)} + (H - H_e)wBwe^{-2K\mu(h/B_d)} \qquad (7)$$

At the top of the conduit, $V = W_c$ and $h = H_e$, whence

$$W_c = C_n w B_d^2 \qquad (8)$$

in which

$$Cn = \frac{e^{-2K\mu(H_e/B_gd)} - 1}{-2K\mu} + \left(\frac{H}{B_d} - \frac{He}{B_d}\right)e^{-2K\mu(He/B_d)} \qquad (9)$$

In order to solve equation 9 it is necessary to know the value of H_e. An expression for determining H_e may be derived by equating the total settlement of the interior prism at the plane of equal settlement to the settlement of the exterior prisms at this plane. This equation is

$$\lambda + s_d + s_f + d_c = \lambda' + s_g \qquad (10)$$

or, since

$$r_{sd} = \frac{S_g - (s_d + s_f + dc)}{s_d}$$

$$\lambda = \lambda' + r_{sd}\cdot s_d \qquad (11)$$

To derive expressions for λ and λ' the following assumptions are employed:

1. That the internal friction in the fill materials distributes the infinitely small decrements of pressure from shear into the interior prism below the plane of equal settlement in such a manner that the effect on settlement is substantially the same as for uniform vertical pressure.

2. That the internal friction in the fill materials distributes the infinitely small increments of pressure from shear into each of the exterior prisms below the plane of equal settlement in such a manner that the effect on settlement is substantially the same as though the pressure were distributed uniformly over a width of prism equal to the width of the ditch, B_d.

Referring to Figure 7, the expression for λ the compression of prism ABCD is derived as follows:

$$d\lambda = \frac{V}{B_d E}\,dh \qquad (12)$$

in which

$$V = wB_d^2\left[\frac{e^{-2K\mu(h/B_d)} - 1}{-2K\mu} + \left(\frac{H'}{B_d} - \frac{H'_e}{B_d}\right)e^{-2K\mu(h/B_d)}\right] \qquad (13)$$

substituting equation 13 in equation 12

$$d\lambda = \frac{wB_d}{E}\left[\frac{e^{-2K\mu(h/B_d)} - 1}{-2K\mu} + \left(\frac{H'}{B_d} - \frac{H'_e}{B_d}\right) e^{-2K\mu(h/B_d)}\right] \quad (14)$$

Integrating between the limits $h = 0$ and $h = H'_e$

$$\lambda = \frac{wB_d^2}{E}\left(\frac{H' - H'_e}{B_d} - \frac{1}{2K\mu}\right)\frac{e^{-2K\mu(H'_e/B_d)} - 1}{-2K\mu} + \frac{wB_d^2}{E}\left(\frac{1}{2K\mu} \cdot \frac{H_e}{B_d}\right) \quad (15)$$

In a similar manner

$$\lambda' = \frac{3}{2}\frac{wB_d^2}{E}\frac{H_e}{B_d}\left(\frac{H' - H'_e}{B_d} + \frac{1}{2}\frac{H'_e}{B_d}\right) - \frac{wB_d^2}{2E} \cdot \frac{1}{2K\mu} \cdot \frac{H'_e}{B_d}$$

$$\frac{wB_d^2}{2E}\left(\frac{H' - H'_e}{B_d} - \frac{1}{2K\mu}\right)\frac{e^{-2K\mu(H'_e/B_d)} - 1}{-2K\mu} \quad (16)$$

The expression for s_d is

$$s_d = \frac{wp'B_d^2}{E}\left(\frac{e^{-2K\mu(H'_e/B_d)} - 1}{-2K\mu} + \frac{H' - H'_e}{B_d} e^{-2K\mu(H'_e/B_d)}\right) \quad (17)$$

Substituting equation 15, 16 and 17 in equation 11 gives

$$\left[\left(\frac{H'}{B_d} + \frac{H'_e}{B_d}\right) - \frac{1}{2K\mu}\right]\frac{e^{-2K\mu(H'_e/B_d)} - 1}{-2K\mu} - \frac{H'_e}{B_d}\left[\left(\frac{H'}{B_d} - \frac{H'_e}{B_d}\right) + \frac{1}{2}\frac{H'_e}{B_d} - \frac{1}{2K\mu}\right]$$

$$= \frac{2}{3}r_{sd}\,p'\left[\frac{e^{-2K\mu(H'_e/B_d)} - 1}{-2K\mu} + \left(\frac{H'}{B_d} - \frac{H'_e}{B_d}\right) e^{-2K\mu(H'_e/B_d)}\right] \quad (18)$$

From equation 18 values of $\frac{H'_e}{B_d}$ corresponding to $\frac{H'}{B_d}$ for various values of $r_{sd}p'$ may be obtained. Then since $H = H' + p'B_d$ and $H_e = H'_e + p'B_d$ (see Fig. 3), it is possible to determine values of C_n from eaquation 9. Substituting values of C_n in the load formula, equation 8, the loads on incomplete ditch condition negative projecting conduits may be obtained.

Diagrams showing values of C_n versus $\frac{H}{B_d}$ for various values of r_{sd} have been drawn for values of $p' = .5$, 1.0, 1.5 and 2.0 and are shown in Figures 8, 9, 10 and 11. These

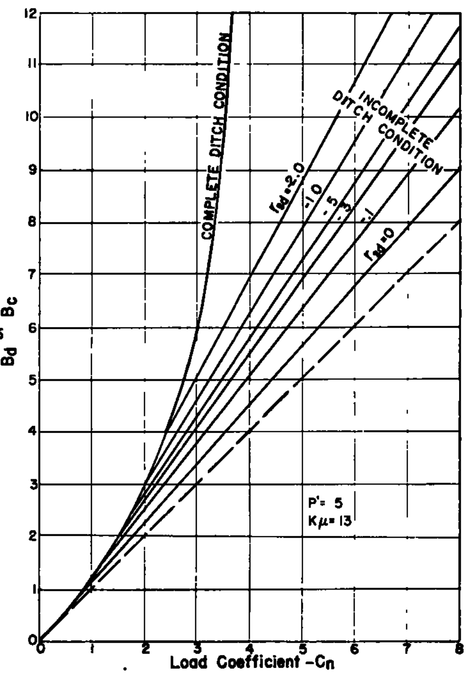

Figure 8

diagrams make it unnecessary to solve the cumbersome equation 18. For other values of p' between zero and 2.0, values of C_n may be obtained by interpolation with sufficient accuracy for design purposes. The coefficient of internal friction, μ, was assumed to be 0.2, ($K\mu = 0.13$) in the construction of the diagrams. This is believed to be a safe design value. If it is desired to determine loads for values of μ other than 0.2, these diagrams cannot be used.

In the early days of Marston's researches on conduit loads, he was impressed by the very high loads that may develop on positive projecting conduits when the conditions are such that large shearing forces are added to the weight of the interior prism of soil directly over the conduit. He worked hard to devise a method of construction that would reduce or eliminate these shearing forces, or would possibly reverse their direction so that they would act benevolently as in the case of ditch conduits. With this objective in mind he developed the imperfect ditch method of construction in which the soil on both sides and above the conduit for some distance above its top is thoroughly compacted by rolling and tamping. Then a ditch is dug in this compacted fill by removing the prism of material directly over the conduit. The ditch is refilled with very loose compressible material, after which the embankment is completed in a normal manner. The details of the imperfect ditch method are shown in Figure 12.

The purpose of this method of construction is accomplished by creating a condition wherein it is certain that the interior prism of material will settle more than the adjacent exterior prisms, and the ditch in the artificially compacted material must be deep enough and the backfilling material must be loose enough to insure this action. Straw or other highly compressible material may be used as part of the ditch backfill to increase the amount of settlement of the interior prism.

The foregoing theoretical analysis of loads on negative projecting conduits may be used to estimate loads on conduits installed by the imperfect ditch method. When used for this purpose, the top surface of the initially compacted material may be considered to be the natural ground surface. The width of ditch in the imperfect ditch method can be made much smaller than in the negative projecting case, and for the most favorable results, it should be made no wider than the outside width of the conduit, B_c.

It is believed that the theory of loads on negative projecting conduits provides a sound approach to the study of loads on this class of structures and that it points the way for the design and construction of conduits which

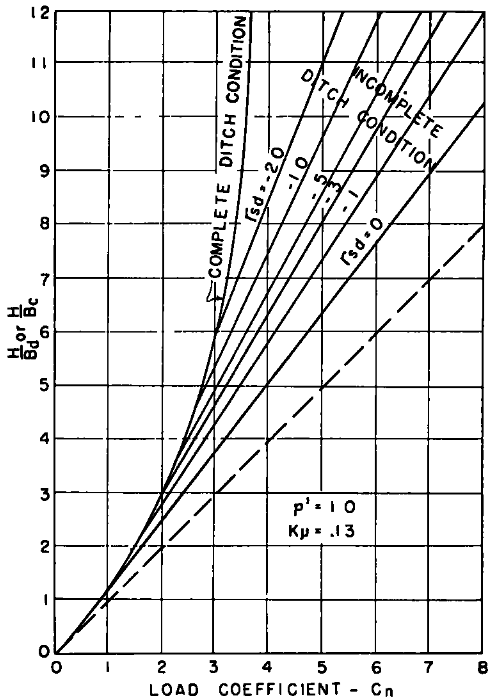

Figure 9

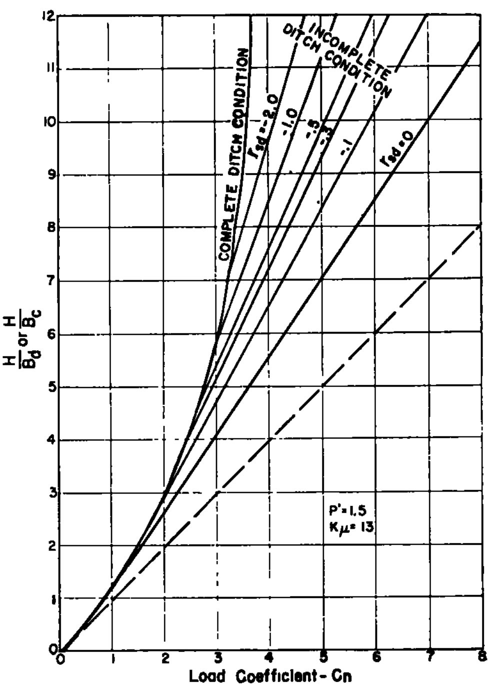

Figure 10

can safely withstand the loads produced by relatively high fills. However, there is no factual basis at the present time upon which design values of the settlement ratio can be recommended. Extensive studies of the settle-

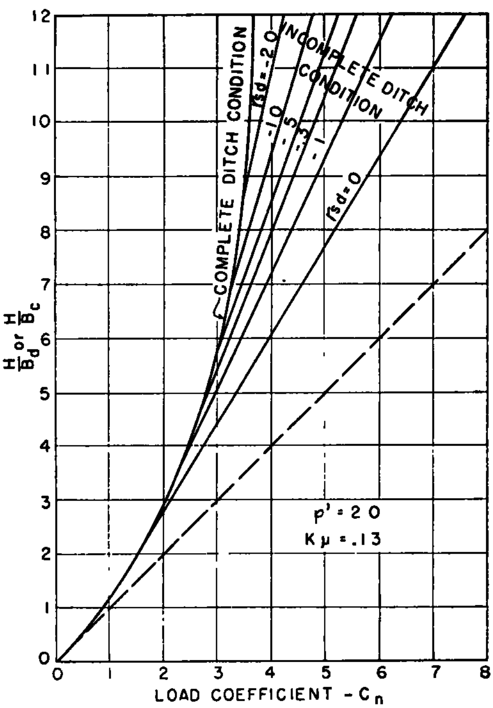

Figure 11

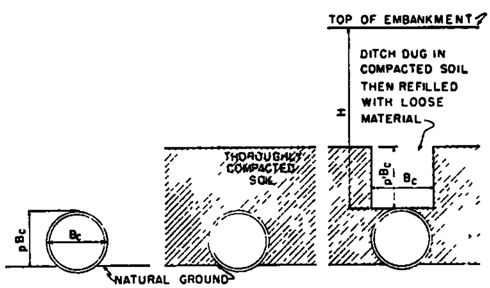

Figure 12

ment characteristics of actual conduits installed in this manner are needed to supply factual information relative to quantitative values of this important factor.

Although the main purpose of this paper has to do with the computation of loads on negative projecting conduits, it is appropriate to say a word or two about the supporting strength of rigid pipes when they are installed in this manner. Since negative projecting conduits are installed in ditches, the bedding conditions defined in connection with the field supporting strength of ditch conduits appear to be applicable. On the other hand, when rigid pipes are installed by the imperfect ditch method, the bedding conditions which influence the distribution of the reaction and the amount of lateral pressure on the pipe appear to be very similar to those which apply in the case of positive projecting conduits.

In order to illustrate the wide range of loads which may be produced on a conduit under the same height of fill, but installed under different environmental conditions, the following four examples are given for purposes of comparison. These comparisons are for one set of conditions only and should not be used as a basis for generalization.

Assume:

48″ reinforced concrete pipe with 5 in. walls. $B_c = 4.83''$

30 ft. fill $H = 30'$

$w = 120$ pcf.

Example 1

Pipe installed as a ditch conduit in a 6 ft. wide ditch $B_d = 6.0$

$\frac{H}{B_d} = 5, C_d = 2.80$ (*Ref. 4*)

$W_c = 2.80 \times 120 \times 36 = 12{,}100$ plf.

Example 2

Pipe installed as a negative projecting conduit with top of pipe 6 ft. below ground surface. $p' = 1.0$

$r_{sd} = -.5, B_d = 6$

$\frac{H}{B_d} = 5, C_n = 3.05$

$W_c = 3.05 \times 120 \times 36 = 13{,}200$ plf.

Example 3

Pipe installed as an imperfect ditch conduit. $p' = 1.0$

$r_{sd} = -.5$

$\frac{H}{B_c} = 6.21, C_n = 3.75$

$W_c = 3.75 \times 120 \times 23.3 = 10{,}500$ plf.

Example 4

Pipe installed as a positive projecting conduit. $p = .9$

$r_{sd} = +.5, r_{sd}p = .45$

$\frac{H}{B_c} = 6.21, C_c = 10.1$ (*Ref. 4*)

$W_c = 10.1 \times 120 \times 23.3 = 28{,}300$ plf.

These calculations show that a 30 ft. fill may produce loads on a 48 in. pipe ranging from 10,500 lb. per lin. ft. to 28,300 lb. per lin. ft. depending upon the nature of the installation conditions.

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BITUMINOUS PAVEMENT INVESTIGATION IN UTAH AND COLORADO

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SYNOPSIS

The failed spots in nine bituminous paving projects in Utah and fifteen in Colorado were examined to determine, if possible, the causes of failure. The method of study consisted of group observations and discussions supplemented by detailed information and laboratory tests on quality and thickness of base and subbase courses, character of subgrade soil, drainage and moisture conditions, quality and condition of bituminous surface mixtures. The effects of traffic and maintenance were considered.

It is stated that the extent of the investigation did not permit positive conclusions on the causes of the observed defects. However comparisons of the test data with specification requirements indicates that construction control was deficient on many of the projects.

Among the discrepancies are excess of fines, wide variance in bitumen content of samples from the same projects and lack of correlation between bitumen content and the proper amount for the aggregate in use.

Some distortion in the wheel paths indicates insufficiency of support due to inadequate design.

During early June 1949, nine typical bituminous paving projects in Utah and 15 in Colorado were examined in an attempt to obtain information on the causes of any failures observed. Most of the pavements in these States are of the low cost or intermediate bituminous type, and while most of them are in excellent condition and have given very satisfactory results, there have been enough local failures to warrant investigation. These projects varied in age from one to ten years and in most cases were old enough to show evidence of failure if such failures were to develop, yet they were not so old as to have been built under entirely inadequate design standards for the traffic. It was planned to make the investigation as early as possible in the season in order to observe the more adverse moisture and drainage conditions at that time.

Most bituminous surfaces in the two States are of the dense-graded road-mixed type about 2 in. thick, generally employing an MC-3 asphaltic material, although many of the older surfaces were mixed with an SC-3 type. In recent years, there has been a definite trend toward the hot plant-mixed type using paving asphalts, particularly on the more heavily traveled routes. The bituminous surfaces on the plant-mix projects are generally 2 to 3 in. thick using 85–300 penetration asphalt cements.

Traffic on the projects studied varied from 350 to 6800 vehicles per day with the commercial traffic percentages varying from 5 to 26 as shown by Table 1.