DYNAMIC TESTING OF CONCRETE PAVEMENTS WITH THE SONISCOPE

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SYNOPSIS

THE SONISCOPE is an electronic instrument developed to measure pulse velocity of high-frequency sound waves through concrete. From this velocity the dynamic modulus of elasticity can be determined. Soniscopes have been used extensively by the Hydro-Electric Power Commission in Ontario, Canada, and by the Portland Cement Association in the United States for the past four years. Most of this work has been confined to the examination of dams, bridges, and other large concrete structures for areas of deterioration.

Since the summer of 1949, the Kansas Highway Commission has been conducting extensive investigations of concrete pavements and test specimens using the soniscope. Eight experimental pavements have been constructed, each of which is made up of a number of test sections of varying composition. One such pavement near McPherson, Kansas, which consists of 60 sections containing several pozzolanic additions and five different brands of cement, was constructed to study the reactive expansion of concrete. The sections of the other test roads, some of which were air-entrained, contain coarse and finely ground cement along with several types of aggregate. The purpose of these roads was to study the strength gaining qualities of the various types of concrete, and an effort was made to improve these qualities.

Sound velocity determinations have been made periodically with the soniscope on all test projects, and a sound velocity history is being maintained for the different sections. It is believed that changes in the modulus of elasticity, and hence in the sound velocity, are indicative of changes in quality of concrete. Such a history could therefore be used to check concrete for progressive improvement or deterioration.

A number of old pavements were investigated with the soniscope and an effort was made to correlate the velocity of sound with the degree of deterioration of the concrete. This report is a description of these investigations and of the equipment used, along with the results which have been obtained to date. No attempt has been made to draw conclusions or to establish values for such a method of concrete testing at this time.

● OVER THE PERIOD of years in which concrete has been in use, various means for controlling its quality have been devised. Tests are made on the cement and aggregates to see that they conform to specifications set up for certain types of concrete construction. During the placing of the concrete, consistency (or slump) tests and density (or yield) tests are made periodically to check the uniformity of the mix.

Tests made in the past on finished concrete have been made principally on test cylinders and beams cast at the same time the concrete was placed. If these test specimens are made carefully and receive the same curing treatment, they are presumed to be representative of the concrete in the finished product. After curing, the cylinders are crushed and the maximum compressive strength calculated; the test beams are simply supported and loaded to failure. Although these tests have become almost standard practice, they leave much to be desired.

There are several serious disadvantages to using test specimens as a measure of the quality of concrete. In the first place, it is difficult to make tests of this nature on the actual concrete used in construction. Another disadvantage is the destructive nature of the tests; once a specimen has been broken, it is no longer useful as a test instrument. For these reasons a nondestructive method of determining concrete quality would be advantageous.

One of the properties of concrete which only recently has been seriously investigated is the modulus of elasticity, commonly known as Young's modulus. The modulus of elasticity was known to vary with the type and quality of concrete, but not enough work had been done on the subject to give its value much significance.

During the past 20 years, however, the modulus of elasticity has assumed a more important role in the field of concrete testing.

Although the value of Young's modulus is not an actual measure of the strength of concrete, it has been found to be an indication of the quality of concrete (*1*). The modulus of elasticity, which varies considerably for concretes containing different types and quantities of the component materials, can not be used as a direct comparison of the quality of different types of concrete; the quality of concrete of a particular type or mix, however can be evaluated in this manner. In such cases, a high value for Young's modulus has been found to indicate concrete of good quality, and a low value, concrete which has undergone some type of deterioration. A testing program based on successive determinations of the modulus of elasticity could therefore be used to study the progressive changes in the quality of a given type of concrete or mix.

One advantage of using the modulus of elasticity as a measure of concrete quality is that it can be determined nondestructively. This method of testing leaves the test specimen intact for further treatment and subsequent testing. Until recently, Young's modulus for concrete has been evaluated statically by loading the test specimens in compression and reading the deformation with an Ames dial mechanism (*2*). The modulus of elasticity or the ratio of unit stress to unit strain can then be calculated. Although this method has been used with good results, the tests are only applicable to small test specimens and are tedious as well as time consuming.

DYNAMIC METHODS OF DETERMINING YOUNG'S MODULUS

In recent years two dynamic, nondestructive methods for determining the modulus of elasticity of concrete have been found. These are the resonant-frequency method and the pulse-velocity method, both of which have been used successfully.

The resonant-frequency method, which is commonly referred to as the sonic method, is only applicable to small sections of regular size and shape.

With this method of determination the specimen is simply supported and is vibrated flexurally to determine its fundamental resonant frequency. By varying the frequency of vibration until a crystal pickup shows a maximum amplitude of vibration, the resonant frequency can be ascertained. If the weight and the dimensions of the specimens are known, Young's modulus can be determined with the following equation (*3*).

$$E = CWN^2$$

Where: E = Young's modulus lb. per sq. in.
N = fundamental resonant frequency (flexural)
W = weight of specimen (pounds)
C = constant (takes into account size and shape of specimen)

With the pulse-velocity method of determining Young's modulus, hereafter referred to as the sound-velocity method, a sound wave or a small mechanical vibration is propagated through the material and its velocity calculated. The velocity of sound[1] through an elastic medium depends upon a number of factors. The following equation, which applies to mass concrete shows these relationships (*4*).

$$V = \sqrt{\frac{E(1-u)(144)\,32.2}{(1+u)(1-2u)d}}$$

In the equation:
V = Velocity of sound ft. per sec.
E = Young's Modulus lb. per sq. in.
d = density lb. per cu. ft.
u = Poisson's ratio

If Poisson's ratio and the density are known, Young's modulus can then be calculated from the velocity of sound through the concrete. The equation takes on the following form for such calculations.

$$E = \frac{V^2(1+u)(1-2u)d}{(1-u)(32.2)144}$$

There are several electronic devices which can be used to determine the velocity of sound in concrete. The one used in the following experiments and described in this report is the soniscope (*5*). This instrument is used to measure the time that a compressional wave takes to traverse a given distance, and can be

[1] A sound wave, propagated in an elastic solid, is divided into several components which travel at different velocities. These components are: the compressional or longitudinal wave, the transverse wave, and the surface wave. The compressional wave has the highest velocity and is dependent upon Young's modulus for that velocity. The compressional wave is therefore the component which is used to determine Young's modulus. In this report the velocity of sound refers to this compressional wave and the other components are not considered.

used on small test specimens as well as large concrete sections.

The soniscope propagates a sound wave into the concrete specimen by means of a crystal-block transducer, commonly referred to as the driver. This crystal block is made up of Rochelle-salt crystals which warp or change volume when a voltage is applied to them. This change in volume is then transmitted through a castor oil medium to a rubber diaphragm. The diaphragm is placed in contact with the specimen and induces a small mechanical vibration or sound wave into the concrete each time that the crystal block is warped by an applied voltage. At a measured distance from the source, another crystal block transducer is placed with its diaphragm in contact with the specimen. This transducer is used as a receiver for the sound wave, and the output is fed to an amplifier.

The signal to the driver and the output of the receiver amplifier are applied to the vertical deflection plates of a cathode-ray oscilloscope, the beam of which is made to sweep linearly across the face of the tube. The time difference between the driving and received pulses, which is measured electronically in the equipment, is the transit time of the sound wave through the specimen, less a constant time delay referred to as "dead time".[2]

Soniscopes have been used extensively by the Hydro-Electric Power Commission of Ontario, and the Portland Cement Association in the United States for the past four years. Tests have been made on dams, bridges, road pavements and long time study field specimens. Based on these experiments, concrete velocities of 12,000 to 16,000 ft. per sec. indicate concrete of good quality. Velocities of 10,000 to 12,000 ft. per sec. indicate concrete of medium quality. Velocities of less than 10,000 ft. per sec. are found in concrete which has seriously deteriorated (*1*).

EARLY SONISCOPE INVESTIGATIONS

In the summer of 1949 the Kansas Highway Commission secured the loan of a soniscope for the investigation of concrete pavements. This instrument was one of the first models and had originally been the property of the Portland Cement Association. In the fall of 1949, soniscope investigations were begun on two experimental test roads then under construction.

One of these projects, a 5-mi. test road near McPherson, Kansas, included some 60 sections containing various pozzolanic additions and five different brands of cement. The purpose of this experimental project was to study the reactive expansion of concrete containing various pozzolanic additives in an attempt to reduce this expansion.

This expansion, which is accompanied by map cracking, is thought to be a chemical reaction between the cement and the aggregate, which can be reduced by the addition of proper constituents at the time the concrete is placed. Sound-velocity determinations were made for each of the sections at 28 days of age, and plans were made to repeat these readings every 6 months in an effort to correlate a change in the velocity of sound with the degree of this reactive expansion.

The second of these test projects consists of 8 mi. of concrete pavement south of Topeka, Kansas, on US 75. This project is made up of 36 test-sections which differ in the type of cement used and in the method of curing. Three types of cement were used, which were designated as Old Fashioned, Modern, and Modern Coarse. The purpose of this road was to study the strength gaining qualities of the various cement types, and an effort was made to improve these qualities. Sound-velocity determinations were made at 28 days of age and have been continued at 6-month intervals to study changes which may occur in the various test sections over a long period of time.

During the construction of the test road at Topeka, determinations of velocity were made for the different types of cement during the setting up period. Initial readings were made as soon as the concrete was firm enough for the propagation of a sound wave and were continued at regular intervals during the first 12 to 15 hr. after placing. The results were an interesting illustration of the setting phenomena for the different types of cement. The modern cements set quickly, hence the sound velocity increased rapidly. The old-fashioned cement set up more slowly and its velocity lagged behind that for the modern. Plots of sound velocity for the same type of cement but with different pouring temperatures

[2] This period of time is a fixed delay which occurs in the electronic circuits and is not a part of the transit time of the sound wave. The value for dead time is found by placing the diaphragms of the two transducers in contact and the delay time measured.

brought out the difference in set-up time due to prevailing temperature at the time the concrete was placed (*6*).

After using the soniscope through the summer and winter of 1949, the Kansas Highway Commission was interested in making further investigations involving the use of such an instrument. Plans were made to return the borrowed soniscope and to build another which would be the property of the Kansas Highway Commission.

RECENT SONISCOPE INVESTIGATIONS

New Equipment—Plans for an improved soniscope were procured from Cheesman and Leslie of the Hydro-Electric Power Commission of Ontario; these plans, with minor revisions, were used in the construction of a new soniscope. This new instrument was constructed, in the main, by J. G. Chubbuck, an electrical engineer from the staff at Kansas State College.

Two basic electrical changes were made in the new equipment which have improved its accuracy and increased its usefulness. One of these improvements has been a change in the type of electrical pulse fed to the Rochelle-salt crystals in the driving transducer. In the old equipment, this pulse was made up of a number of oscillations which tended to keep the crystals oscillating for too long a period of time, thereby causing interference with succeeding duty cycles. Only the very first part of the signal is usable; any energy propagated at a later time serves no useful purpose.

The new driving pulse is obtained from the sudden discharge of a gaseous electron tube which delivers a sharp pulse of voltage to the crystals. With this driving method, most of the energy is propagated near the beginning of the cycle, and a stronger, more-sharply defined signal is the result. The oscillations of the crystals are allowed to damp out between cycles, and the velocity measurements are more accurate and more easily obtained.

The second change of an electrical nature is the manner in which the transit time is measured. In the old equipment this time interval was determined by comparing the time difference between the driving pulse and the received signal, which were displayed on one trace of a two-trace cathode-ray tube with a calibrated square wave applied to the second trace.

Although this method gave usable values for transit time, some interpolation was still necessary to arrive at its value. The new equipment uses a calibrated marker pulse, or strobe, which is controlled by a potentiometer graduated directly in microseconds to measure transit time. This strobe is applied to the second trace of the cathode-ray tube and measures the transit time directly without interpolation.

The most obvious difference between the two soniscopes is the change in physical size and shape. Whereas the main control circuits of the old model were housed in three large units cabled together, the new control circuits are incorporated in one compact and easily transported unit. This difference is illustrated in Figure 1. The new soniscope has now been mounted semipermanently in a panel truck for use in the field.

Field Investigations—Since the establishment of the McPherson and Topeka test roads, which have been previously mentioned, six other test projects have been constructed. The concrete in the test sections in these projects, some of which are air-entrained, contain fine and coarse-ground cements. Velocity determinations are made each 6 mo. on all of these test projects and a history of sound velocity is maintained.

Sound velocity measurements for all test roads have been made with the diaphragms of both the driving and receiving transducers resting on the surface of the slab.[3]

The transducers are spaced 4 ft. apart, and a film of castor oil is applied to the concrete in the vicinity of the transducers to give maximum energy transfer from the rubber diaphragms to the concrete. Velocity determinations are made at regular intervals on each test section so that a number of readings are taken and a representative average can be found. The exact location for these readings are marked permanently in the slab so that similar conditions can be reëstablished for succeeding velocity measurements.

[3] For all tests made with the soniscope on concrete pavements, both of the transducers have been placed on the surface of the slab. Although this configuration is not conducive to the propagation of a compressional wave between the transducers, due to the nature of sound-wave travel through an elastic medium, such a wave is propagated. Becasue of the width of the rubber diaphragms of the transducers, it is difficult to ascertain the exact path length over which the transit time is being measured. However, investigations indicate that this distance is very nearly the distance from center to center of the diaphragms. All readings have been taken on the slab with the transducers 4 ft apart, thereby making all of the velocities relative in nature.

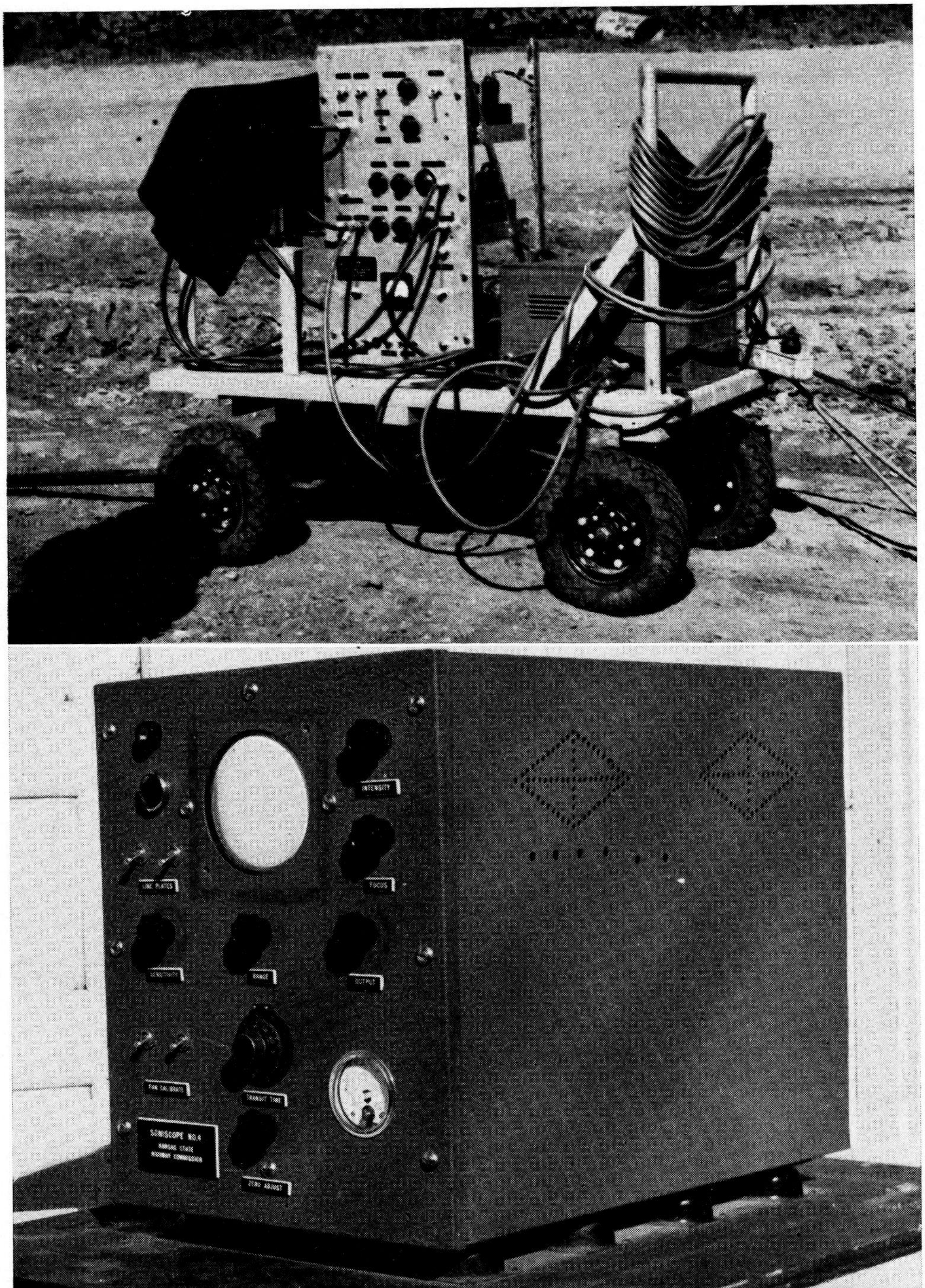

Figure 1. Shown (at top) is the original soniscope used in Kansas; a new soniscope is shown below.

The transducers were originally placed separately on the slab at a measured distance. Recently, however, they have been mounted on a metal carriage, shown in Figure 2. This method of spacing not only maintains a constant distance between the transducers, but greatly simplifies their placement.

Sound-velocity measurements have now been made periodically on the test projects for about 2 years. In Figure 3 sound velocities of four typical sections of the McPherson test-road are plotted against age. Considerable differences in velocity are found for the various admixtures. The average velocities for the various sections fall between 13,500 and 15,500 ft. per sec., but no definite trends in velocity have been established at this early date.

Sound velocities for the several types of cement used in the Topeka test road are plotted against age in Figure 4. The spread in velocity for the various types of cement is only about 500 ft. per sec. and the resulting curves are quite similar in nature. Enough work has not been done at this time with the soniscope to establish the magnitude of change in sound velocity which may indicate a significant change in the quality of concrete. Little significance can be attached to the velocity changes which have occurred to date, but the tests have been continued in an effort to correlate the changes in sound velocity with the changes in the quality of the concrete which will occur over a longer period of time.

During the past summer, a sound-velocity survey was made of a number of older concrete pavements containing three types of aggregate. Included in this group were 24 projects containing mixed aggregate, 19 projects containing coarse and fine aggregates, and 5 projects containing chat aggregate.[4] The pavements ranged in age from 1 to 29 years and were well distributed throughout this period.

Sound velocity determinations were made at regular intervals on each of these projects such that 20 to 40 measurements were made on each pavement. In order to get a representative cross-section of each project, the location for the individual readings were determined by distance measurement alone, and were not picked visually. At each of these positions the general condition of the slab was noted and recorded along with the transit time of the sound wave.

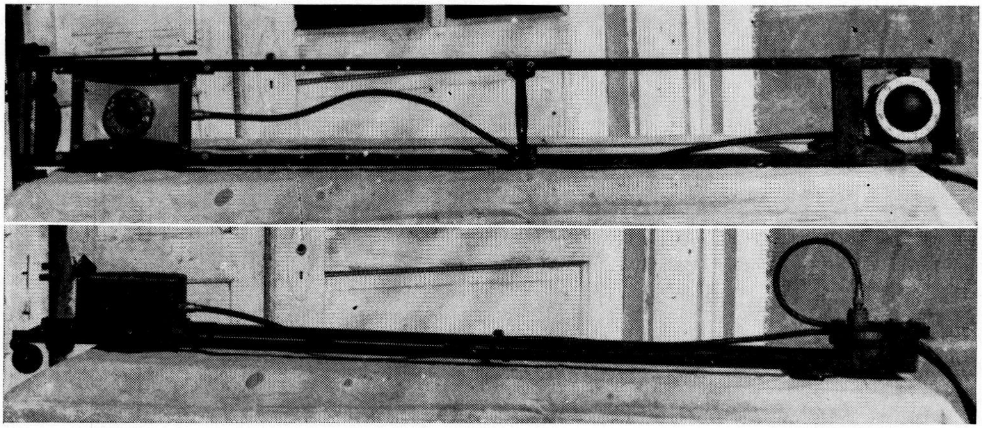

Figure 2. The underside of the transducer carriage is shown at the top: below it is shown in the normal position.

The individual velocity measurements were totaled and an average velocity determined for each of the projects. These averages are plotted in Figure 5 according to the age of the pavements. Although the individual velocity determinations varied from 13,000 to 16,200 ft. per sec., the section averages varied between 13,800 and 15,900 ft. per sec. The resulting graph shows little correlation between the velocity of sound and the age of the concrete pavements. The velocity of sound for the pavements containing chat aggregate tends to increase with age, no trend in veloc-

[4] The chat aggregate is a crushed flint which is a by-product of the lead and zinc mines and has a maximum size of ½ in

ity is apparent for the pavements containing coarse and fine aggregates, and the velocity of sound appears to decrease with age for the pavements containing mixed aggregates. The age of the concrete is undoubtedly a factor which affects the velocity of sound, but it is evidently not a predominate one.

Map cracking was evident on a number of the older concrete pavements containing mixed aggregate. In an effort to establish the relationship between the velocity of sound and the degree of map cracking, positions where map cracking was visible were classified as faint, medium, or severely map-cracked.

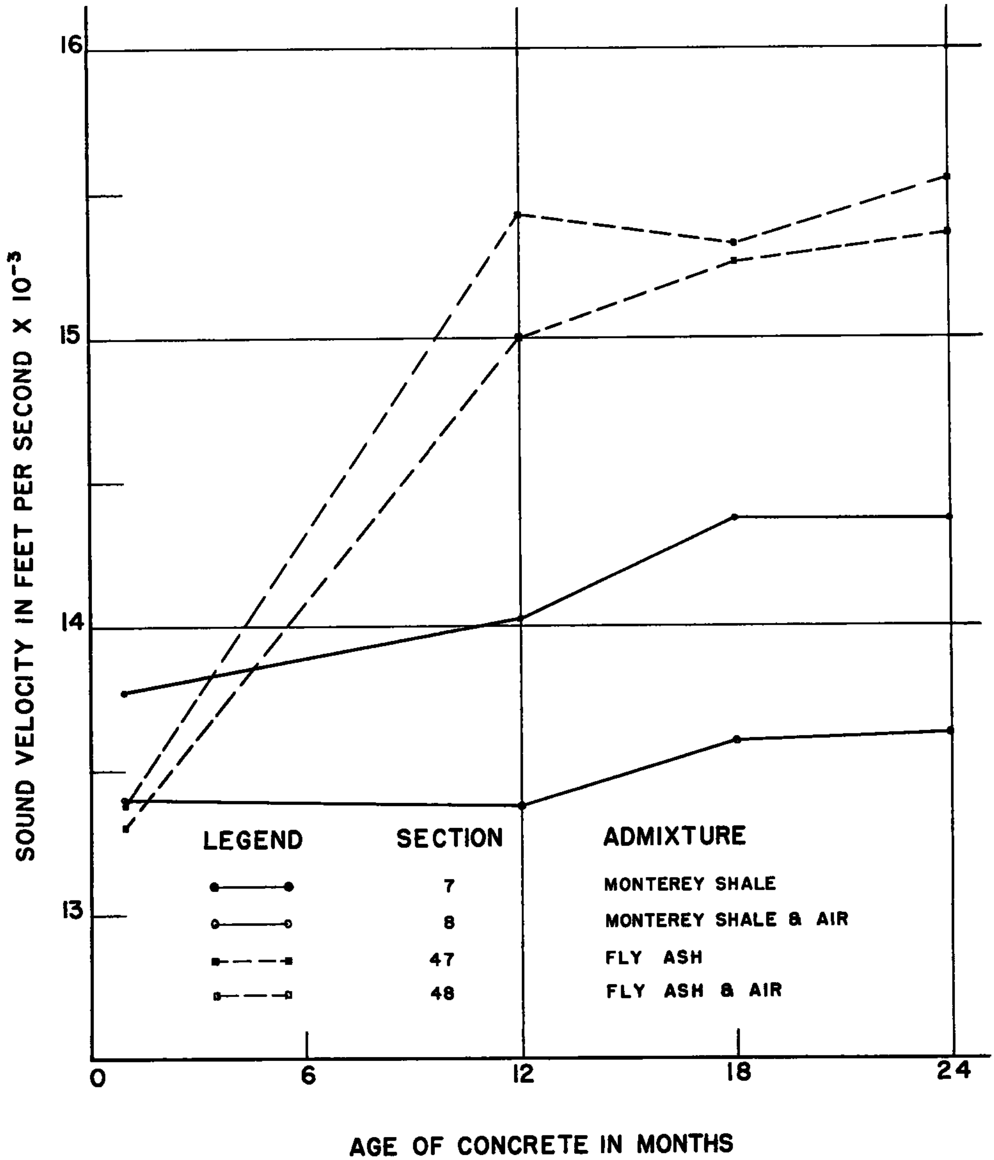

Figure 3. Sound velocity versus age of concrete: typical results of the McPherson Test Road.

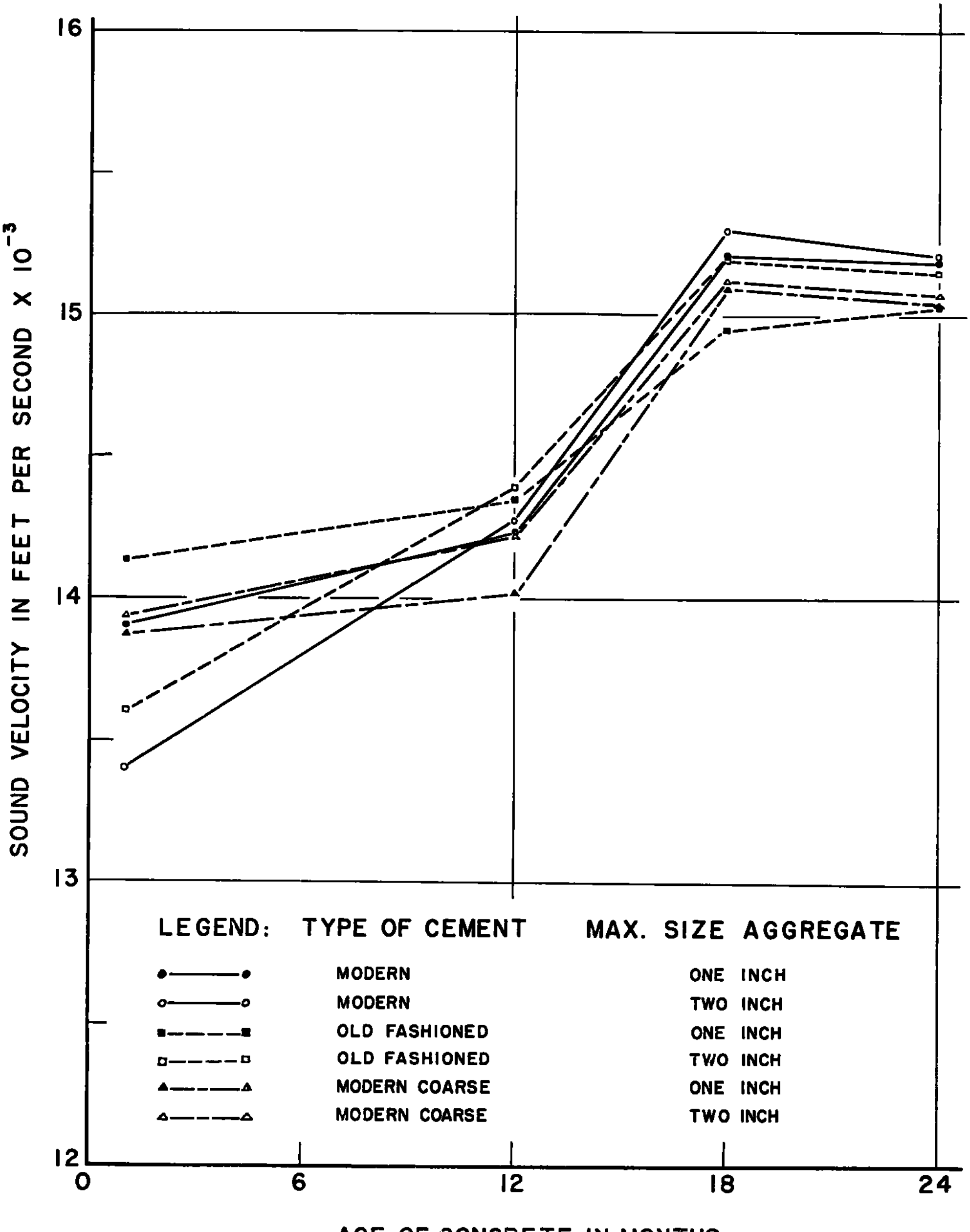

Figure 4. Sound velocity versus age of concrete, basic mixed aggregates, earth curing.

The velocity readings for each cracking category were totaled and the average velocity calculated. These averages are tabulated in Table 1 along with the number of positions which were included in each.

Although a decrease in sound velocity appears to accompany map cracking, the average decrease was only about 500 ft. per sec. between severely map-cracked areas and areas of no map cracking. A decrease of 500 ft. per

sec., which represents a change in sound velocity of only slightly over 3 percent, is thought at this time to be too small to have much significance. This apparent decrease in velocity may only be due to an increased path length for the sound wave because of the map cracking and may not indicate an actual change in the velocity of sound through the material.

One of the limitations placed upon the accuracy of the soniscope is the nonhomogeneous nature of concrete pavements. Not only does the velocity of sound vary from one pavement to another, but it also varies from one location to another in the same project. This condition is more pronounced on pavements which have deteriorated, but is also present in new concrete. For this reason it is important that all subsequent readings be taken at the same location if a change in the velocity of sound is to be recognized.

In an attempt to establish how the velocity of sound varies as an area of visible deterioration is approached, a number of special velocity investigations were made. Special emphasis was placed on areas and joints where considerable d-cracking was evident. Successive readings were taken in the vicinity of such a discontinuity so that a velocity pattern could be established. The velocity of sound was expected to decrease materially as the transducers were placed successively closer to such a deteriorated spot; this, however, was not the case. In a few instances there was a small decrease in velocity, 200 to 500 ft. per sec., as a discontinuity was approached, but no pronounced decrease in velocity was noted until the transducers were actually resting on a cracked or disintegrated area of concrete. If this deterioration was of a nature such that the concrete was no longer a continuous medium, a sound wave could not be propagated between the transducers and no velocity determinations could be made.

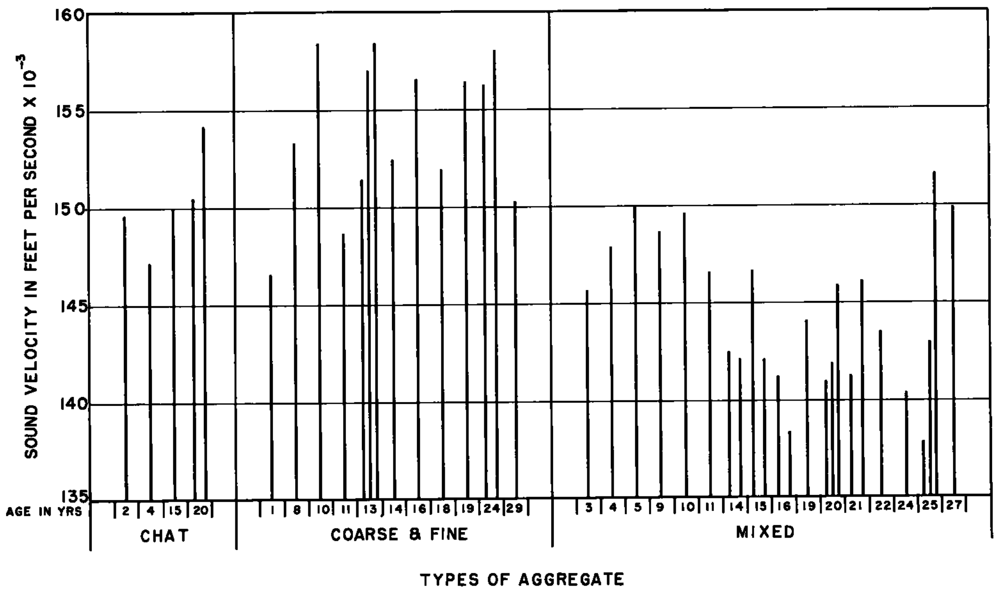

Figure 5. Sound velocity versus age of concrete pavements.

TABLE 1

Map cracking category	Number of Determinations	Average Velocity
		ft. per sec.
None	155	14,446
Faint	109	14,301
Medium	64	14,265
Severe	53	13,976

Laboratory Investigations—Sound velocities as found with the soniscope were originally used to calculate the modulus of elasticity for the various concretes, and comparisons were made on the basis of these values. For these calculations the value for Poisson's ratio was assumed to be 0.15, which had been the value

used for concrete in the past. The values for Young's modulus found in this manner were apparently higher than those found with the resonant-frequency method. For this reason, an investigation was made comparing the two dynamic methods of determining the modulus of elasticity.

A number of 3- by 4- by 16-in. and 6- by 6- by 36-in. concrete beams were procured and determinations of the modulus of elasticity were made with the soniscope and with the sonic machine at Kansas State College. Young's modulus for these beams as found with the soniscope were 10 to 20 percent higher than those found with the sonic. If Poisson's ratio were allowed to vary between the limits of 0.20 and 0.30, the two methods could have been brought into agreement Work done by J. R. Leslie and W. J. Cheesman indicates that Poisson's ratio as found by dynamic methods falls within these limits (*5*).

Due to the variation of Poisson's ratio with the type of aggregate, density, and water-cement ratio used in concrete, tests made recently have been confined to velocity determinations alone; no effort has been made to compare Young's modulus of the various concretes. It is felt that a change in the velocity of sound which can be more accurately determined will be as indicative of a change in quality as would a change in the modulus of elasticity.

In the past some speculation has been expressed on the effect of stress on the velocity of sound through concrete. For the purpose of investigating this effect, three 9- by 9- by 36-in. test beams containing coarse and fine aggregates were cast. After curing, these beams were loaded axially in increments of 20,000 lb. up to 180,000 lb. total load, or about 2,200 psi. During loading the transit time of a sound wave was measured over a 30-in. portion of the beam and was found to change slightly as the load was applied.

In going from a condition of zero stress to 2,200 psi., the transit time decreased about 3 percent. This would indicate an increase of about 3 percent in the velocity of the sound wave due to 2,200 psi. of compressive stress. This change was quite consistent for the three beams and also for repeated cycles of loading. No change in transit time was noted due to tensile stress up to the breaking point of the beams. The change in velocity was of such a small magnitude, that stresses which might exist in a concrete pavement would not perceptibly affect the velocity of sound as determined with the soniscope.

In an effort to find how the velocity of sound is affected by freezing and thawing, eleven 3- by 4- by 16-in. beams were subjected to 103 cycles of freezing and thawing. The beams contained several different types of aggregate and several types of cement. Ten beams of similar types were retained as pilots, and sound velocity measurements were made periodically on all specimens in both categories. After 103 cycles of freezing and thawing, some of the beams were beginning to crack badly, and the tests were discontinued. Final

TABLE 2
FREEZING AND THAWING DATA
FINAL RESULTS (AFTER 103 CYCLES)
3- by 4- by 16-in. Specimens

Type of Concrete	Control Beams			Active Beams		
	Sound velocity	Sonic modulus	Mod. of rupture	Sound velocity	Sonic modulus	Mod. of rupture
	ft. per sec.	psi. × 10^6	psi.	ft. per sec.	psi. × 10^6	psi.
Basic mixed aggregate[a]	15,500	6.50	997	15,690	6.45	932
	15,870	6.78	956	15,870	6.51	863
	15,870	6.52	997	15,690	6.60	909
	15,690	6 10	822	15,500	6.20	787
	15,500	6.21	892	15,330	6.02	781
Basic mixed aggregate with air[a]	15,690	6.19	1049	15,690	6.36	851
	14,650	5.57	863	14,490	4.96	676
	14,650	5.30	805	14,810	5.58	863
Coarse and fine aggregate[b]	15,330	6.44	874	11,200	2 73	192
				10,020	2 55	262
Coarse and fine aggregate with air[b]	15,500	5.97	985	12,820	4.15	478

[a] Opposite beams in these groups were of the same mix.
[b] Opposite beams in these groups were of the same type but of different mixes.

velocity measurements were made with the soniscope, and the beams were vibrated on the sonic machine. The beams were then simply supported and broken and the modulus of rupture calculated. The values for sonic modulus, sound velocity, and the modulus of rupture for all beams are tabulated in Table 2. The sound velocities for two beams which showed considerable deterioration, those containing coarse and fine aggregate with no air entrainment, along with a similar control beam are plotted in Figure 6.

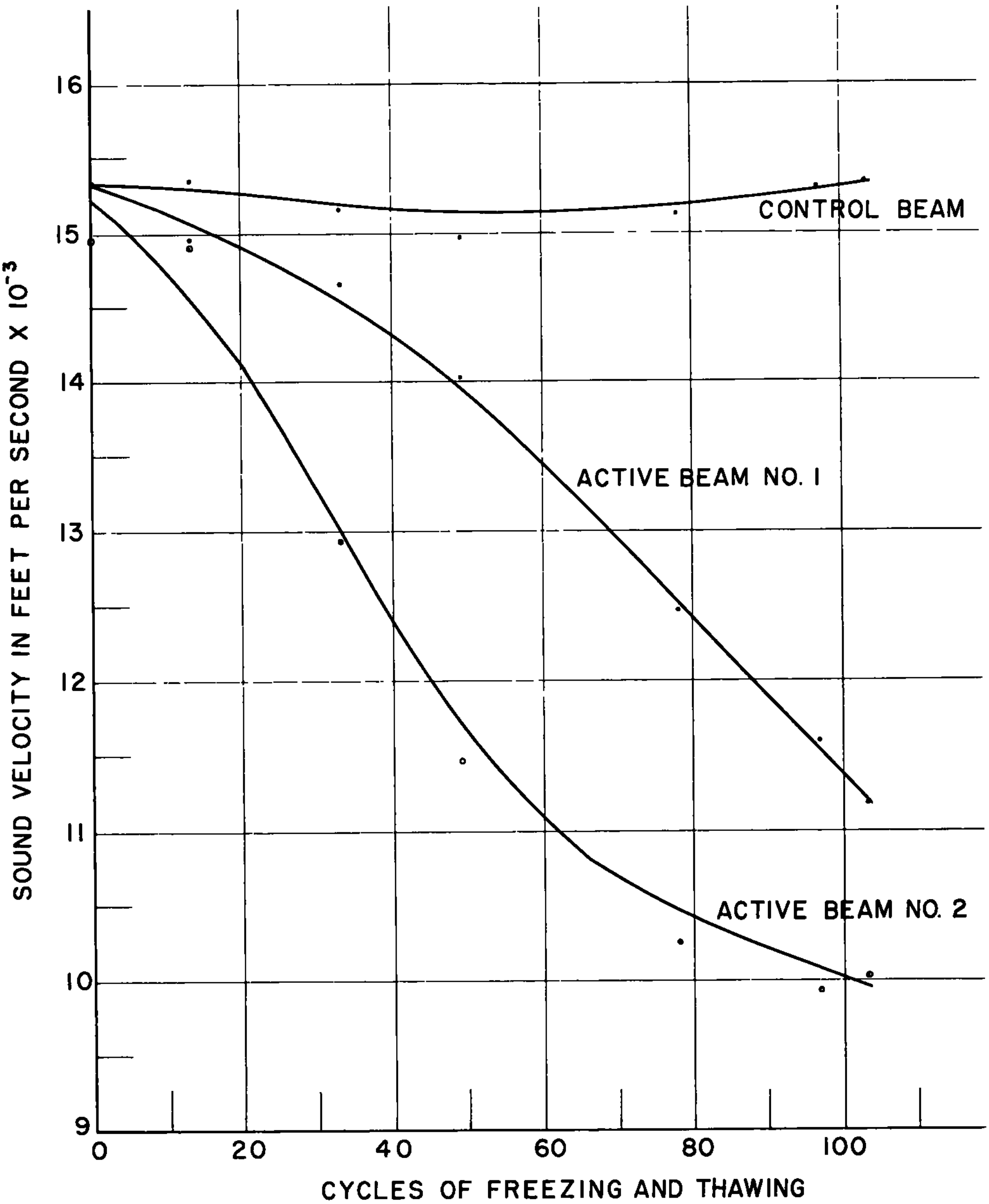

Figure 6. Sound velocity versus cycles of freezing and thawing of three 3- by 4- by 16-in. concrete beams containing coarse and fine aggregates.

For all of the beams investigated, active beam No. 2, plotted in Figure 6, appeared to be the most seriously deteriorated. The following changes in this beam were noted after 103 cycles of freezing and thawing. The sonic modulus decreased from about 6.4 to 2.5 million psi.—a decrease of about 60 percent. The sound velocity decreased from about 15,000 to 10,000 ft. per sec. Assuming that Poisson's ratio remained constant throughout the experiment, this decrease in velocity would indicate a decrease of about 55 percent in Young's modulus. The modulus of rupture decreased from about 870 to 260 psi.—a decrease of 70 percent.

These tests show a definite correlation between sound velocity and deterioration caused by freezing and thawing. These experiments afforded the lowest velocity found to date—about 10,000 ft. per sec. Concrete which has seriously deteriorated due to freezing and thawing may have sound velocities lower than the ones indicated in this experiment. However, the test beams with the low velocities had deteriorated to such an extent that the value of transit time was becoming indefinite because of cracks which had developed in the specimens.

SUMMARY

Although some differences in velocity have been found for various types of concrete, these differences have been confined to a rather-narrow velocity range. In no case has a velocity as low as 10,000 ft. per sec. been encountered on concrete pavements. Although many of the pavements tested were obviously deteriorated and in poor condition, the sound velocities have all fallen between 12,000 and 16,000 ft. per sec. which has been established by other groups as the "good-concrete" range. For this reason, the Kansas Highway Commission has not been able to fully establish the value of the soniscope as an instrument for testing concrete pavements for progressive deterioration.

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