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RATE OF COOLING OF LAVA FLOWS

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SYNOPSIS

The massive Mauna Loa lava flow of June 1950 buried about a mile of the main-belt highway on the Island of Hawaii to depths up to 60 ft. The initial temperature of the lava as it crossed the road was 950 C. Heat loss by thermal radiation from the surface was of importance only during the initial period of a few weeks. Thereafter the rate of cooling became dependent on the rate of heat conduction from the interior to the surface. Computations of temperatures at different depths for various periods of cooling from ½ yr. to 5 yr. were made by the theory of heat conduction. The study showed that at the depths of cut proposed in reconstructing the highway, the cooling rate was extremely slow, so that a prolonged period of waiting would probably be necessary. This conclusion seems supported, at least in a qualitative way, by comparative temperatures at depths of 18 in. or so within cracks in the lava taken 5 months (November 1950) and 15 months (September 1951) after the flow ceased. The measurements revealed only a slight drop in temperature in the colder cracks (250 F) and practically none in

the hot cracks (700 F). Rising hot gases tend to equalize the temperature within a crack to about the temperature of the lava at the depth to which the crack extends. On this basis the hotter cracks with the temperatures of 700 F are believed to extend down to a depth somewhere between $6\frac{1}{4}$ and $12\frac{1}{2}$ ft. In order to expedite the work of reconstruction, it seems best to depend primarily on borrow material for grading operations, limiting excavations to a maximum of approximately 6 ft. For such shallow cuts, the top half probably can be moved by blading with a bulldozer (because of the fragmental nature of the surface) and the balance loosened by mud-capping.

● For a number of years highway engineers, especially those engaged in the field of soil mechanics, have been familiar with the theory and practice of soil consolidation due to superimposed loads. From a mathematical point of view the soil-mechanics theory of consolidation is identical with the theory of heat conduction worked out in great detail by mathematicians and physicists during the nineteenth century. Thus it was possible to take the ready solutions that the latter theory offered and apply them to the analogous problems in soil mechanics.

Figure 1.

The theory of consolidation is well established. Although there are numerous engineers and specialists working on highway, airport, and other projects who are constantly dealing with problems of soil consolidation, probably very few, if any, will ever be called upon to deal with the theory of heat conduction on a massive scale.

The highway department of the Territory of Hawaii was confronted with just such a problem. On June 1, 1950, the volcano of Mauna Loa on the Island of Hawaii erupted lava on a gigantic scale, pouring out half a billion cubic yards in approximately three weeks. The lava ran down the southwest slopes of the mountain, crossed the round-the-island belt highway at three points for widths of 3,080, 1,350, and 1,085 ft., piling up to a height of 60 ft. or more in some places, thus completely blocking traffic (See Fig. 1). Fortunately there was no loss of life.

A few weeks after the flow ceased, equipment was moved in and a temporary road bulldozed across the flow. Drippings of oil and grease from the construction equipment would flash, indicating the flow was still quite hot. Evidently only a thin layer on the surface had crusted and cooled, but it was sufficient so that men and equipment could go over it. But no deep cuts were possible because of the heat. As of August 1951, or 14 months after the flow ceased, the surface felt warm to the touch, and looking over the flow one could see rising steam columns and atmospheric heat waves.

To make road cuts through the flow it is necessary to drill and blast. But because of the intense heat, blasting powder is out of the question, and it is probable that drill steel will not stand up.[1] The highway, of course, should be rebuilt as soon as possible. Thus some kind of an estimate as to the time required for the flow to cool to a safe temperature is desirable.

Conduction, Convection, and Radiation—There are three ways in which heat may travel from one point to another; viz, by conduction, convection, and radiation.

In the cooling of a hot-lava flow, heat loss by radiation is of importance only in the initial stages. Actually the surface cools at such a rapid rate that in a matter of a few hours after the flow ceases it is possible to walk across it a short way, although one cannot remain very long over it. And in a matter of a few weeks the surface cools to within a few degrees of atmospheric temperature, although the interior remains hot for years. These facts show that cooling by radiation affects only the surface and is important only during the initial stages. Thereafter, the rate of cooling of the lava mass as a whole, as far as long-time effects are concerned, can, therefore, be calculated from the theory of heat conduction. The heat conducted from the interior to the surface is dissipated into the atmosphere as convection currents.

Theory of Heat Conduction—The time rate of change of temperature at any point within a solid due to the flow of heat within it is given in rectangular coordinates by the following partial differential equation (Fourier's equation),

$$\frac{\partial\theta}{\partial t} = \alpha\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2}\right) \qquad (1)$$

$$\alpha = \frac{K}{c\gamma} \qquad (2)$$

where θ = temperature
α = thermal diffusivity
K = thermal conductivity
c = specific heat
γ = density
t = time
and x, y, z = rectangular coordinates of the point where the temperature is measured.

[1] Special explosives that are not affected by high temperatures, for example, the kind used in blasting hot blast-furnace slag, are not available locally.

Equation 1 is one of the important partial differential equations of classical physics. Its derivation and methods of solution are given in books dealing with physical subjects and in treatises on differential equations, some of which are cited in the references given, (*1*, *2*, *3*, *4*). The equation has found wide application in fields other than heat conduction, e g., electricity and soil mechanics. Terzaghi (*2*) was the first to apply it in the latter field.

A lava flow is, in general, many miles long (coordinate x), many hundreds of feet wide (coordinate y) but only a few tens of feet thick (coordinate z). Hence, except for portions near the two edges, the flow of heat is vertically upward from the interior to the surface. The two partial derivatives in the x and y directions vanish and Equation 1 reduces to the following:

$$\frac{\partial\theta}{\partial t} = \alpha\frac{\partial^2\theta}{\partial z^2} \qquad (3)$$

Boundary Conditions—According to Dr. Gordon A. Macdonald of the Kilauea Volcano Observatory, U. S. Geological Survey, the temperature of the lava as it crossed the main highway was 950 C., as measured with an optical pyrometer. The mean daily temperature of the surrounding atmosphere will be taken as 20 C. We shall disregard the intense heat radiation during the short initial period of a few weeks when the surface temperature drops from its initial value of 950 C. to that of the mean daily temperature of 20 C. On the contrary, we shall assume that the surface temperature is immediately reduced to that of the mean daily temperature at time zero. The second boundary condition is that heat loss by conduction into the interior of the earth is negligible. This is done by specifying that the temperature gradient at the bottom of the flow is zero. (Without a temperature gradient there can be no flow of heat.) Because of the relatively low conductivity and specific heat of the weathered lava and soil forming the earth's surface here, this second boundary condition is probably approximately correct. The initial condition is that at time zero the whole mass was at a temperature of 950 C.

We shall measure the depth z vertically downward from the surface of the flow. Then if H be the thickness of flow we can state our boundary and initial conditions as follows:

$$\theta = 20 \text{ C. for } z = 0 \tag{4}$$

$$\frac{\partial \theta}{\partial z} = 0 \text{ for } z = H \tag{5}$$

$$\theta = T_0 = 950 \text{ C. for } t = 0 \tag{6}$$

The boundary condition (4) is awkward. We therefore introduce a new temperature scale, using the same width per degree as the centigrade scale except that referred to it the mean daily temperature is zero. Call this scale θ'. Then

$$\theta' + 20 = \theta$$

$$\frac{\partial \theta'}{\partial t} = \frac{\partial \theta}{\partial t}; \quad \frac{\partial \theta'}{\partial z} = \frac{\partial \theta}{\partial z}$$

and

$$\frac{\partial^2 \theta'}{\partial z^2} = \frac{\partial^2 \theta}{\partial z^2}$$

The differential equation (3) now becomes

$$\frac{\partial \theta'}{\partial t} = \alpha \frac{\partial^2 \theta'}{\partial z^2} \tag{3a}$$

The new boundary conditions are

$$\theta' = 0 \text{ for } z = 0 \tag{4a}$$

$$\frac{\partial \theta'}{\partial z} = 0 \text{ for } z = H \tag{5a}$$

$$\theta' = T'_0 = 930 \text{ deg. when } t = 0 \tag{6a}$$

The boundary conditions have been simplified but the form of the differential equation remains unchanged. After values of θ' have been found we can add 20 to them to return to the centigrade scale.

The value of θ' that satisfies the above differential equation (3a) and boundary conditions (4a) to (6a) is, (2)

$$\theta' = \frac{4T'_0}{\pi}\left[e^{-\alpha\pi^2 t/4H^2} \sin \frac{\pi z}{2H} + \tfrac{1}{3}e^{-9\alpha\pi^2 t/4H^2} \sin \frac{3\pi z}{2H} + \tfrac{1}{5}e^{-25\alpha\pi^2 t/4H^2} \cdot \sin \frac{5\pi z}{2H} + \text{etc.}\right] \tag{7}$$

where α = diffusivity
H = thickness of flow
T'_0 = initial temperature on the θ' scale = 930°
π = 3.1416
e = 2.71828, base of natural logarithms

It is seen that Solution 7 is exactly the same mathematically as that for the consolidation of a soft compressible soil layer sandwiched between an impervious bottom layer and a porous top layer and subject to a suddenly applied load which is uniform throughout its depth.

Equation 7 shows that the temperature at any time t and depth z depends on H the thickness of the flow, the thicker the flow the longer it takes to cool.

Because of the nature of a lava flow the surface is very uneven so that its thickness varies from point to point. But for a massive flow like this one, which is the greatest thus far in the present century, the thickness everywhere is bound to be substantial. In the computations to follow, two different thicknesses, 50 ft. and 25 ft., have been assumed.

In computing numerical values of θ, the following values, given by Daly, (5) have been assumed,

K = 0.00345 calories per sq. cm. per deg. C. temperature gradient
c = 0.270 calories per gram
γ = 2 70 gram per cu. cm.

Using the above values the diffusivity is by Equation 2,

$$\alpha = \frac{K}{c\gamma} = \frac{.00345}{.27 \times 2.70} = .00473$$

In Figures 2 and 3 the temperatures computed by Equation 7 using the above value of α and after conversion from the θ' scale to the θ (centigrade) scale have been plotted against the corresponding time in years. Figure 2 is for a thickness of flow of 25 ft. and Figure 3 for a thickness of 50 ft. Figure 2 corresponds to the thinner parts of the flow and Figure 3 to the thicker parts. For any known or given thickness intermediate between these values, estimates can be obtained by interpolation. The computations were made for time intervals of ½, 1, 2, 3, 4 and 5 years. For time intervals beyond 5 years (June 1955) the results will probably be affected by variations in the values of K with greatly decreased values of the temperature.

In the Handbook of Chemistry and Physics, Thirty-first Edition 1949, on page 1892, the

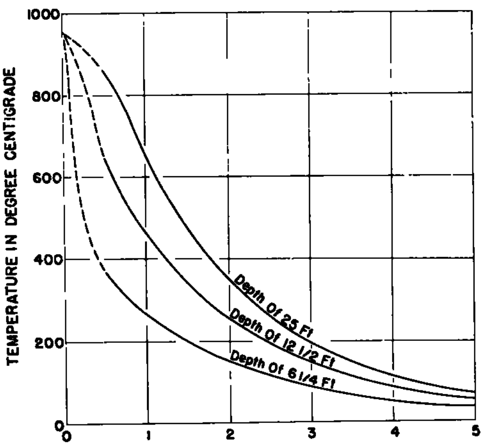

Figure 2.

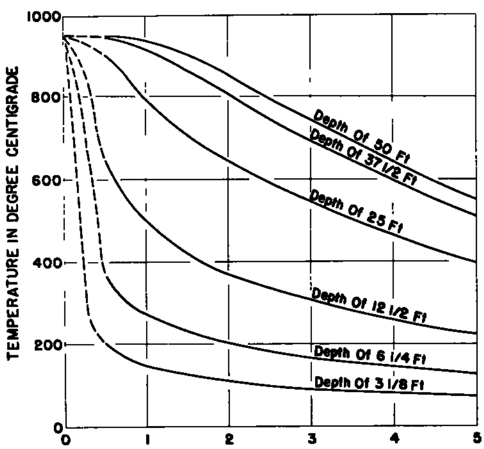

Figure 3.

conductivity of basalt (lava rock) at ordinary temperature is given as 0.0052 calories per sec. per sq. cm. per deg. C. temperature gradient per cm. The conductivity of lava rock varies with the temperature, being considerably less at high temperatures (*5*). Since the value of 0.0052 is for ordinary temperatures, it appears that during the early stages when temperatures are very high, the conductivity is closer to the previously quoted value of 0.00345.

Just as a matter of interest, the temperatures at a depth of 12½ ft. were recomputed using a value of conductivity of 0.0052 for the two thicknesses of flow previously assumed.

Ingersoll and Zobel (*1*), on page 87 of their treatise, also discuss the problem of cooling of lava flows. Their method of solution is by means of Fourier's integral. In the numerical computations they assumed a flow 20 meters thick (65 ft. approx.) and a value for the diffusivity of 0.0118 which is much higher than the value adopted here. Nevertheless, the final

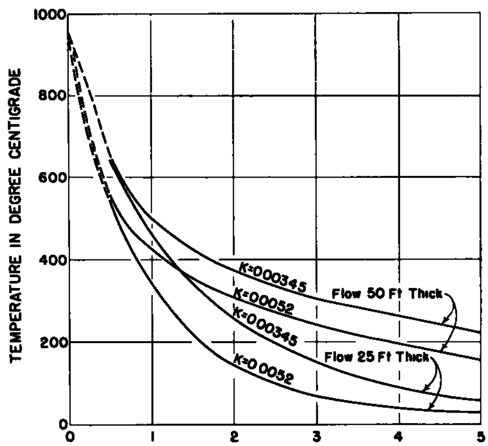

Figure 4.

results are comparable with those obtained here.

Experimental Verification—The most significant result revealed by a study of Figures 2 and 3 is the high temperatures, well over 300 C, still prevailing at the proposed depth of highway cut (15 ft.), 1½ years after the flow ceased. Figure 4 shows that even on the basis of a higher coefficient of conductivity and relatively shallow thickness of flow (25 ft.), the expected temperature at 1½ years is still somewhat in excess of 200 C. Because of possible variations in initial temperature, thermal conductivity, thickness of flow, etc., from point to point, the results achieved above can only be regarded in a very general way. It is interesting to compare the computed with the actual observed temperatures, due to Doctor

Macdonald (*6*). On November 18, 1950, (5 months after flow stopped), temperatures of 250 F to 700 F (121 C to 371 C) were observed about 18 in. below the surface through cracks in the lava. On September 9, 1951, ($1\frac{1}{4}$ yr.), the hottest cracks were essentially the same, namely 700 F. The cooler cracks, which apparently are less freely connected with the center of the flow or carry less rising gas, appeared a little cooler, but only by a few degrees. The corresponding computed temperatures at 5 months and $1\frac{1}{4}$ years are respectively 230 C and 130 C at a depth of $3\frac{1}{8}$ ft. for a thickness of flow of 50 ft. According to the charts a temperature of 700 F (371 C) at the $1\frac{1}{4}$ year period corresponds to a depth of somewhere between $6\frac{1}{4}$ and $12\frac{1}{2}$ ft. and it is quite possible that the hotter cracks extend down to this level. Both the computations and Doctor Macdonald's measurements indicate that a lava flow cools very slowly indeed, quite contrary to popular opinion.

It is proposed to install a recording thermometer at a convenient location, supplemented with individual readings at selected points. These records, when plotted, will give the actual changes in temperature with time for known depths below the surface. Such experimental curves can then be compared with the theoretical curves and the latter adjusted to conform with the experimental data. In this way the temperature changes at any other depth can be closely computed.

Plans for the highway crossing this lava flow called for cuts up to a maximum of approximately 15 ft. Because of the massive nature of the flow, huge blocks and ledges of solid dense rock will be encountered beginning at about 5 to 10 ft. below the surface, so drilling and blasting will be necessary. The charts show that as of January 1952 and for one or perhaps two years in the future, the temperature at that depth will probably be too high to permit the use of conventional explosives. One solution to the problem is to sacrifice line and grade, lowering the maximum cut to say 6 ft. or so. The top 3 or 4 ft. can then be fairly easily bulldozed and the balance loosened by mud-capping.

Cooling During Initial Stages—As previously mentioned, cooling during the initial stages of a few weeks is primarily by radiation.

Newton formulated a theory of cooling by combined radiation and convection according to which the amount of heat E given up per unit area of surface per second of time by a body at temperature θ_1, to its surroundings of temperature θ_2 is proportional to the difference in temperatures,

$$E = e(\theta_1 - \theta_2) \qquad (8)$$

where e is the emissivity of the surface. At the boundary between the surface of the lava flow and the atmosphere, after the surface temperature has dropped to a constant value, the rate at which heat is brought to the surface by conduction from the interior must equal the rate at which it is dissipated into the atmosphere by combined radiation and convection. Hence,

$$K \frac{\partial \theta}{\partial z} = e(\theta_1 - \theta_2) \qquad (9)$$

where z is the inward-drawn normal to the surface.

The Newtonian law of cooling as expressed by equation (8) has been found to hold true only for small differences in temperature. Thus, for a difference of 2 deg. in temperature, the heat loss is almost exactly twice that for a 1 deg. temperature difference. But if the difference is say 100 deg., the amount of heat radiated is vastly more than 100 times as great. Thus it is not applicable to lava flows, at least during the initial stages of cooling when temperature differences are great.

A quite different law is the Stefan-Boltzmann law deduced by Boltzmann in 1883 by the application of thermodynamic principles to radiation. According to this law the total emissive power or the total radiant energy for all wave lengths, emitted per unit of time per unit surface area of the radiating body is proportional to the fourth power of the absolute temperature of its surface. Thus,

$$E = \sigma T^4 \qquad (10)$$

where E = total emissive power
T = absolute temperature on the Kelvin thermodynamic scale
$\sigma = 5.735 \times 10^{-5}$ erg cm^{-2} sec^{-1} deg^{-4}.

The Stefan-Boltzmann law is strictly true only for a so-called black body or a thermodynamic enclosure. Since the hot-lava surface at temperature T is radiating to its surroundings which is at a lower temperature T_1, the net or differential emission is:

$$E = \sigma(T^4 - T_1{}^4) \qquad (11)$$

Expanding the factor in parenthesis we obtain:

$$E = \sigma(T - T_1)(T^3 + T^2T_1 + TT_1^2 + T_1^3) \quad (12)$$

If T does not differ much from T_1 the last factor on the right is very nearly equal to $4T^3$ and we have

$$E = 4\sigma T^3(T - T_1) \quad (13)$$

If T be considered constant, Equation 13 becomes the same as Equation 8. Thus the emissivity factor in Newton's law of cooling depends on the temperature.

From the previously given initial temperature of 950 C or 1,223 deg. absolute [2] and temperature of the surroundings of 20 C or 293 deg. absolute, we can compute the rate of heat loss by radiation by means of Equation 11 as,

$$E = \frac{5.735 \times (1223^4 - 293^4)}{10^5}$$

$$= 12{,}787 \times 10^4 = 12.79 \times 10^7 \text{ ergs per sq. cm. per sec.}$$

$$= 3.055 \text{ g. calories per sq. cm. per sec.}$$

The rate at which a hypothetical black body radiates thermal energy is the greatest that is theoretically possible.

The ratio of the intensity of radiation emitted by a surface at any given wave-length and temperature to the intensity of radiation that would be emitted at the same wave length and temperature by a black body is the monochromatic emissivity of the surface. A "gray" surface is one whose emissivity e is the same for all wave-lengths and temperatures. For such a surface, the total thermal energy radiated per unit area per unit of time is

$$E = \epsilon\sigma(T^4 - T_1^4) \quad (14)$$

where T = absolute temperature of radiating body

and T_1 = absolute temperature of surroundings.

Daly (*5*) has computed the loss of heat by radiation from hot lava at Kilauea volcano, also on the Island of Hawaii, by extrapolation of data by Siegl. The results plotted on semi-log paper shows that at 950 C the radiation is 2.4 cal. per sec. per sq. cm. of surface. The emissivity of the lava, regarded as a gray surface is, therefore on the above basis, 2.4 ÷ 3.055 or .79 approximately.

Take a prism of 1-sq.-cm.-cross-sectional area and 762 cm. (25 ft.) high. Assume the density as 2.7 grams per cu. cm. and the specific heat as .27 cal. per gram per deg. C. Then the amount of heat given off when the above prism of lava is cooled from 950 C to 20 C is

$762 \times 2.7 \times 0.27 \times 930 = 516{,}600$ calories (neglecting latent heat of crystallization of minerals in the lava).

If the initial radiation rate can be kept constant throughout, the above amount of heat energy will be radiated in

$$\frac{516{,}600}{2.4} = 215{,}250 \text{ seconds}$$

$$= 2.49 \text{ days}$$

Hence, if the initial rate of radiation is maintained throughout, the whole mass of a lava flow, assumed to be 25 ft. thick, will cool in a matter of a few days and a thicker flow will cool in a time proportionally greater.

As the heat is radiated from the surface it must be constantly replaced by conduction from the interior, or else the surface cools.

The quantity of heat per unit of time that flows across a unit area of a substance is equal to the product of its thermal conductivity K, times the thermal gradient. Taking for the conductivity of lava the previously assumed value of 0.00345, the thermal gradient necessary to maintain the initial radiation rate is found to be 2.4 ÷ 0.00345 or 696 C. per cm. of depth. This means that for every centimeter of depth, the temperature must increase 696 C if the conduction rate is to equal the initial radiation rate. Since the maximum temperature was 950 C, such a high temperature gradient is quite impossible. As a matter of fact, the maximum temperature gradient at any time was probably less than one-tenth of the above. The gradient can also by calculated by differentiating Equation 7 with respect to z, and because of the exponential factor it decreases rapidly with time.

It is seen from the foregoing that the rate

[2] Here we are using absolute temperatures on the centigrade scale. Theoretically this scale is not identical with the Kelvin scale but the difference is small and will be neglected.

of heat conduction is very much less than the initial radiation rate The surface crust therefore cools very rapidly as already noted.

The fragmental nature of the surface of this flow also has a bearing on the rate of cooling It is probable that where the fragments are closely packed, this surface layer acts somewhat like an insulating blanket. Where the fragments are widely separated, resulting in cracks large enough to start convection currents, the rate of cooling will be somewhat hastened. Surface cracks also allow rainwater to penetrate deep below the surface. However as a whole, rainfall has only a minor effect on the rate of cooling, which is still measured in years.

CONCLUSIONS

Because of high temperatures, a lava flow cools during the initial stages by thermal radiation from its surface. After a few weeks the surface temperatures approach that of the atmosphere so that the rate of cooling becomes dependent on the rate at which heat is conducted from the interior to the surface, whence it is dissipated by convection currents into the atmosphere. Because of the relatively low conductivity of lava rock, the period of cooling is measured in years. Hence, any project calling for even moderate cuts of 15 ft. or so will probably have to be delayed several years. If immediate construction is desired, the project will have to be primarily a borrow job with cuts limited to 6 ft. or less.

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