Effect of Average Speed and Volume on Motor-Vehicle Accidents on Two-Lane Tangents

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THE accident rate on any road may be considered to be a function of definable features of (1) the roadway and its environment and (2) the traffic. For given types of drivers and vehicles, it should ultimately be possible to assign to any stretch of road a certain accident potential, based upon analysis of roadway and environmental features and dependent upon traffic speed and volume.

A natural first step toward this goal is to analyze in detail the effects of speed and volume on the accident rate. This should probably be undertaken separately for tangent sections, curves, and intersections, and for roads of various numbers of lanes. The present study is limited to two-lane tangents, but the methods used should be applicable also to accident analysis under broader conditions.

In this study, accidents are divided into three types: single-car, rear-end, and head-on. For each type, a theory is developed relating accident occurrence to speed and volume of traffic. The theory is then compared with accident experience on a group of California roads selected for uniformity in important features of highway condition and design.

● THE California Division of Highways made available a tabulation of nonintersectional accidents occurring in 1950 during daylight and good weather on two-lane roads with single center stripe, at points which were tangent, level, of concrete or asphalt surface, free of structures or roadway defects, and in 55-mph. zones. There were selected from this tabulation only those accidents which occurred on sections predominantly straight and level throughout, and without heavy roadside development.

Accidents were grouped for analysis according to the estimated traffic volume in the hour of their occurrence. Thus accidents occurring during hours of light traffic on a road of heavy average daily traffic were grouped with those occurring during peak hours on a more lightly travelled road. The advantage of using such hourly volume grouping, rather than the customary grouping by roads of similar average daily traffic, is that the peculiarities of individual roads become more nearly randomized. Smoother curves result, and the accident data can be studied with some confidence that the chief variables involved are the traffic volume and speed.

Details of the development of the accident data are discussed in Appendix A.

Accident rates per vehicle-mile are given in Table 1 and in Figures 1 and 2. It should be noted that these rates are far below the rates of actual accident occurrence. This is due primarily to the fact that the accidents tabulated are limited to those reported by the California Highway Patrol. These include nearly all accidents involving death or personal injury but only a minority of other accidents. Furthermore, the accidents were those occurring on unintercepted tangent sections and on dry roads, while the figure used for vehicle-miles includes travel during all daylight hours, wet or dry, and includes travel through intersections and on curves.

The computed accident rates, although too low, should nevertheless show the right general relationship with traffic volume. The curves of Figures 1 and 2 should differ from the true curves primarily in a change of scale on the vertical axis. This expectation rests on the following considerations: (1) randomization is obtained by the use of hourly traffic volumes (a single road section may be represented in as many as seven of the total of nine hourly

TABLE 1

DAYLIGHT ACCIDENTS ON SELECTED TWO-LANE CALIFORNIA TANGENTS BY HOURLY TRAFFIC VOLUME, 1950

a. Volume per hour	0–100	100–200	200–300	300–400	400–500	500–600	600–700	700–800	800–900	Over 900	Total
b. Average hourly volume	50	150	250	350	450	550	650	750	850	—	—
c. Daylight hour-miles (thousands)	2229	1346	868	966	582	250	106	41.8	10.9	7.7	6409
d. Single-car accidents	9	23	22	37	28	13	5	1	0	0	138
e. Rear-end collisions	14	35	45	94	84	58	42	13	5	13	403
f. Head-on collisions	5	16	23	44	42	24	10	4	1	0	169
g. Total accidents	28	74	90	175	154	95	57	18	6	13	710
h. Total vehicles[a] in accidents	53	136	181	356	322	203	128	42	15	31	1467
i. Single-car accidents per 10^8 vehicle-miles[b]	8	11	10	11	11	9	7	3	0	—	—
j. Rear-end collisions per 10^8 vehicle-miles	13	17	21	28	32	42	61	41	54	—	—
k. Head-on collisions per 10^8 vehicle-miles	4	8	11	13	16	17	14	13	11	—	—
l. Total multi-car accidents per 10^8 vehicle-miles	17	25	31	41	48	60	75	54	65	—	—
m. Total accidents per 10^8 vehicle-miles	25	37	41	52	59	69	81	57	65	—	—
n. Total vehicles[a] in accidents per per 10^8 vehicle-miles	48	67	83	105	123	148	186	134	162	—	—

[a] The number of vehicles included is limited to a maximum of three per accident.

[b] *d* divided by *bc*.

traffic groups); (2) of the accidents involving only property damage, the proportion reported is believed to be reasonably uniform on the roads in question; and (3) the roads used probably do not vary greatly in the number of wet hours during the year (nor in the proportion of mileage on curves), since the coastal and mountainous roads most subject to rain were omitted as not being predominantly straight and level.

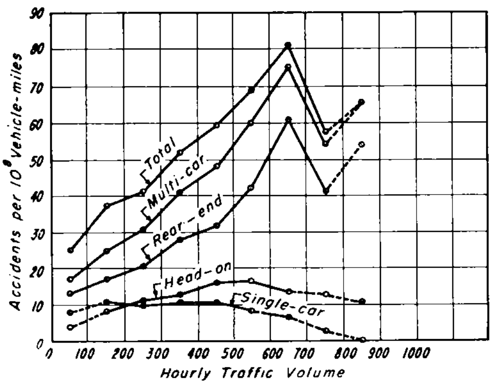

Figure 1. Daylight-accident rates on two-lane California tangents by hourly traffic volume, 1950.

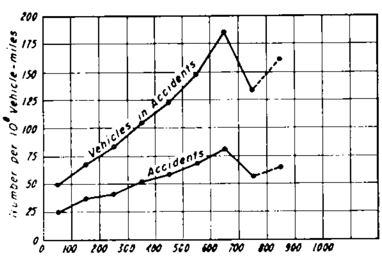

Figure 2. Rates of daylight accidents and of vehicles involved on two-lane California tangents by hourly traffic volume, 1950.

THEORIES OF ACCIDENT OCCURRENCE

The standard exposure theory of accidents (*1*) is that single-car accidents vary directly with traffic volume, while multi-car accidents vary with the square of the volume. In terms of accidents per vehicle-mile, the single-car rate is independent of volume, while the multi-car rate is directly proportional to volume. The total accident rate per vehicle-mile according to the exposure theory is of the form

$$A_{\text{vm}} = c_1 + c_2 n$$

where c_1 and c_2 are constants, and n is traffic volume per unit time.

This theory appears to be an adequate description of the total nonintersectional accident rate, up to the point where traffic becomes congested. The curve of total accident rates shown in Figure 1 is nearly linear, up to volumes of about 650 vehicles per hour, and so agrees well with this theory. So do the results of the comprehensive survey reported by Baldwin (*2*) for accidents on two-lane tangents, for volumes up to about 8,000 vehicles per day (see Fig. 3).

The theory is not, however, as successful in detail. Multi-car accidents per vehicle-mile appear to be more complicated in form than is supposed by the theory. The multi-car rates, as shown in Figure 1, do not approach zero at

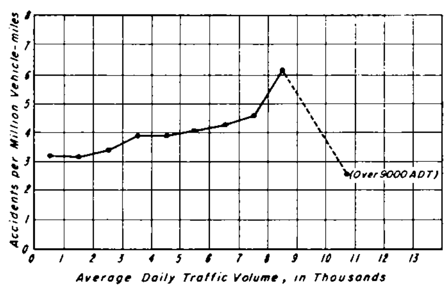

Figure 3. Accident rates on two-lane tangents (after Baldwin).

low volumes, and hence are not in accordance with the exposure theory.

The theories developed in the succeeding sections of this paper give a more detailed and possibly more adequate account of the relation of accidents to traffic conditions.

Summary of Theories

1. The number of single-car accidents per vehicle-mile varies with the average traffic speed, and is independent of traffic volume.

2. The number of head-on collisions per vehicle-mile increases directly with the traffic volume and with the average speed. The chance of collision between any pair of opposing vehicles is proportional to approximately the square of their approach speeds.

3. Rear-end collisions are of two types: 1) those occurring among vehicles travelling so close together that collision is inevitable should the leading vehicle suddenly stop, and 2) those

due to the failure to stop under circumstances where a safe stop could normally be made.

The first type of rear-end collision results in an accident rate per vehicle-mile which is similar in form to the head-on accident rate, increasing with traffic volume and with average speed. With the beginning of traffic congestion, all other forms of accidents begin to show declining rates (corresponding to a decline in average traffic speed) but this type of rear-end collision shows instead a rapid increase, falling off only at higher levels of congestion.

The second type of rear-end collision resembles the single-car accident; the vehicle collides with a slower vehicle instead of some other obstacle. The accident rate for this type of collision is similar in form to the single-car rate, and varies primarily with speed rather than volume.

4. The over-all accident rate per vehicle-mile for freely moving traffic is approximately

$$A_{\text{vm}} = c_1 v^{1.5} + c_2 n + c_3 nv$$

where c_1, c_2, and c_3 are positive constants, v is the average speed, and n is the volume per unit time.

At constant average speed, this expression is identical in form with that arising from the exposure theory. The term $c_1 v^{1.5}$ represents, however, not only single-car, but also a substantial number of rear-end collisions.

5. The number of vehicles involved in accidents is more important in many respects than is the number of accidents. As traffic volume increases, the average number of vehicles involved per accident also rises. In order to properly evaluate the risk of accident to a vehicle, under given conditions, the accident rate must be multiplied by the average number of vehicles involved per accident. The resultant rate of vehicles in accidents per vehicle mile is roughly

$$V_{\text{vm}} = v^2(k_1 + k_2 n + k_3 n^2)$$

The observed accident rates are compared in Figure 2 with the numbers of vehicles reported in accidents per vehicle-mile.

SINGLE-CAR ACCIDENTS

The probability, p, that a vehicle will suffer a single-car accident may be regarded as a function of its speed and of the distance travelled. As a first approximation, assume that

$$p = kdv_i^b$$

where b and k are constants, and v_i is the speed of the vehicle, assumed to be constant over a road of length d.

The average number of single-car accidents per hour on the road will be

$$A_{\text{hr}} = kd \sum_{i=1}^{n} v_i^b$$

where n is the average hourly traffic.

$$\text{If } b = 1, \qquad A_{\text{hr}} = kdnv$$

$$\text{If } b = 2, \qquad A_{\text{hr}} = kdn(\sigma^2 + v^2)$$

where v is the average speed of the traffic, and σ^2 is the variance of the speeds.[1]

Single-car accidents per vehicle-mile are accordingly:

$$\text{If } b = 1, \qquad A_{\text{vm}} = kv$$

$$\text{If } b = 2, \qquad A_{\text{vm}} = k(\sigma^2 + v^2)$$

The variance, σ^2, is a measure of the uniformity of the speeds, being minimum when the speeds are closely bunched around the average speeds. At low volumes, when speeds are high, σ^2 will be relatively high; under congested conditions, both average speed and σ^2 will be low. In all cases, however, σ^2 will be small compared to v^2, and may well be omitted in an approximate formula.

It is not easy to arrive at a precise value for b on theoretical grounds. Driver reaction time probably does not vary greatly with vehicle speeds. The distance a vehicle travels in an undesirable direction before correction is applied is therefore nearly directly proportional to its speed. Braking distance and maneuverability are more nearly proportional to the square of the speed. Hence a value of b somewhere between 1 and 2 is indicated.

We may assume as a first approximation that $b = 1.5$ and that single-car accidents per vehicle-mile are

$$A_{\text{vm}} = kv^{1.5}$$

[1] The relation $\Sigma^n v_i^2 = n(\sigma^2 + v^2)$ is perhaps more familiar in the form

$$\sigma^2 = \frac{\Sigma v_i^2}{n} - v^2,$$

σ^2 being the second moment around the mean, v.

The number of single-car accidents per vehicle-mile shown in Figure 1 is substantially constant for traffic volumes up to about 500 or 600 vehicles per hour, and falls off at higher volumes. This agrees qualitatively with the theory outlined above, since average speed declines at higher volumes, and is fairly constant, for two-lane roads, up to at least 400 vehicles per hour (*3*).

The number of single-car accidents in the higher volume groups, as may be seen in Table 1, is too low for any but the general conclusion that the accident rate declines. The data presented are thus insufficient to fix the appropriate value of b, even if the speeds at high volumes were accurately known. (Dotted lines in Fig. 1 indicate rates calculated for less than 10 accidents, and hence rates of low reliability.)

HEAD-ON COLLISIONS[2]

Assume that any pair of vehicles passing in opposite directions has a probability of collision, p, which depends only upon some power, m, of their approach speed:

$$p = k(v_1 + v_2)^m$$

where k is a constant, and v_1 and v_2 are the speeds of the respective vehicles.

If a vehicle traverses a road section, the average number of head-on collisions it will experience may be regarded as the sum of the individual probabilities of collision with each opposing car it meets during its trip. (The implausibility of a vehicle being involved in several head-on collisions in one trip does not seriously impair the analysis.) The opposing cars it will meet are: (1) those already on the section when it begins its trip (if t is the average time taken to traverse the section, and n_2 is the hourly volume of opposing vehicles, then at any time there will be an average of n_2t such vehicles on the section) and (2) those entering the section during its trip (if the vehicle travels at speed v_i, and the section is of length d, it will traverse the section in time d/v_i, and during this time an average of n_2d/v_i opposing vehicles will enter the section).

Hence the average number of head-on collisions the vehicle will experience will be

$$\sum_{j=1}^{n_2(d/v_i+t)} k(v_i + v_j)^m$$

[2] Accidents involving two or more vehicles, originating in contact between vehicles travelling in opposite directions.

If n_1 vehicles per hour traverse the section in one direction, and n_2 in the opposite direction, the average number of collisions per hour will be

$$C_{\text{hr}} = \sum_{i=1}^{n_1} \sum_{j=1}^{n_2(d/v_i+t)} k(v_i + v_j)^m$$

It may be illuminating to examine the assumptions that $m = 0$, 1, and 2.

Assuming $m = 0$ (probability of collision independent of speeds).

$$\begin{aligned} C_{\text{hr}} &= \sum_{i=1}^{n_1} \sum_{j=1}^{n_2(d/v_i+t)} k \\ &= kn_2 \sum_{i=1}^{n_1} (d/v_i + t) \\ &= kn_2(n_1 t + n_1 t) = 2kn_1 n_2 t \end{aligned}$$

assuming that t is the average time taken to traverse the section in either direction.

The average number of head-on collisions per vehicle-mile is accordingly

$$C_{\text{vm}} = \frac{2kn_1 n_2 t}{nd}$$

where n is the total hourly volume. The simplifying approximations $n_1 = n_2$, and $d/t =$ the average speed, v, produce

$$C_{\text{vm}} = \frac{kn}{2v}$$

This implies that the faster one travels, the smaller the chance of head-on collision (because fewer opposing cars are met). This argument is sometimes heard, but it does not seem to be supported by accident data. At high volumes and low speeds, the head-on accident rate falls far below that which would be suggested by the formula. Thus in Figure 1 the rates at over 500 vehicles per hour are far below the linear extension of the previous rates. Since speed falls off with volumes over about 500 per hour, the formula would predict accident rates above this linear extension.

The evidence would be more convincing if a larger number of accidents were included in the high-volume data, but the data are sufficient to show that the head-on accident rate did not rise sharply at high volumes. The assumption $m = 0$ is therefore to be rejected.

Assuming $m = 1$ (probability of collision directly proportional to the approach speed). Using the assumptions and approximations made above, the average number of collisions per hour is[3]

$$C_{hr} = \sum_{i=1}^{n_1} \sum_{j=1}^{n_2(d/v_i+t)} k(v_i + v_j) = kn^2d$$

and the average number of collisions per vehicle-mile is

$$C_{Vm} = kn$$

This implies that the accident rate increases linearly with traffic volume regardless of speed. It does not jibe with the observed reduction of accident rate at higher volumes. The assumption $m = 1$ must therefore also be rejected.

Assuming $m = 2$ (probability of collision varying with the square of the approach speed).

The average number of collisons per hour is[3]

$$C_{hr} = \sum_{i=1}^{n_1} \sum_{j=1}^{n_2(d/v_i+t)} k(v_i + v_j)^2 = cn^2d\left(v + \frac{3\sigma^2}{8v}\right)$$

and the average number of collisions per vehicle-mile is

$$C_{Vm} = cn\left(v + \frac{3\sigma^2}{8v}\right)$$

For simplicity, the relatively small term involving the variance may be omitted. Collisions per hour then have the form

$$C_{hr} = cn^2dv$$

and collisions per vehicle-mile

$$C_{Vm} = cnv$$

This agrees moderately well with the accident data shown in Figure 1. Up to about 400 vehicles per hour, the average speed is probably nearly constant; formula and observation both show the accident rate varying nearly directly with traffic volume. At higher volumes speed declines; the accident rate falls below the trend established at lower volumes, in qualitative accordance with the formula.

Conclusion

A fair representation of head-on collisions results from the assumption that the chance of collision between any pair of opposing vehicles is proportional to the square of their approach speeds.

REAR-END COLLISIONS[4]

The probability of collision between a pair of consecutive vehicles may be regarded as a function of their speeds and of the distance between them. There are many possible formulations of this probability; the one described herein appears to be a reasonable first approximation.

Two types of rear-end collisions may be usefully distinguished: The first depends upon dangerously narrow vehicular spacing; sudden deceleration of one vehicle results in collision because there is not enough time for the following vehicle to avoid it. The second type occurs when there is originally ample time to prevent collision, but loss of control, poor judgment, or inattention causes a vehicle to run into a more slowly moving one.

Type 1

The average number of collisions of this type should be proportional to the average number of vehicles spaced dangerously close together. The critical distance is such that it is difficult to avoid collision should the preceding vehicle suddenly decelerate. For our purposes, it may be taken to be the "safe following distance" discussed by many authors. This is usually given as a second degree function of the velocity (*4*), a typical expression being

$$\text{SFD} = \text{safe following distance (mi.)} = (0.05\ v^2 + 1.5\ v + 15)/5280$$

where v is the speed in miles per hour.

In freely moving traffic, vehicles are known to have approximately random distribution (*5*), and the distribution of spacings between consecutive vehicles can be calculated (*3*, *6*). The average proportion of vehicles which are at less than the safe following distance from the preceding vehicle is approximately

$$1 - e^{-n_1 \text{SFD}/v}$$

where n_1 is the hourly volume in one direction.

The average number of Type 1 collisions per hour on a lane of length d is thus

$$C_{hr} = kdn_1(1 - e^{-n_1 \text{SFD}/v})$$

[3] Appendix B gives detailed derivations of the formulas for $m = 1$ and $m = 2$.

[4] Accidents involving two or more vehicles, originating in contact between vehicles travelling in the same direction.

and the average number of Type 1 collisions per vehicle-mile is, approximately,

$$C_{\text{Vm}} = k(1 - e^{-n_1 \text{SFD}/v})$$

Substituting the value of SFD suggested above,[5]

$$C_{\text{Vm}} = k[1 - e^{-n_1(.05v^2+1.5v+15)/5280v}]$$

For volumes and speeds typical of freely moving traffic, this expression is nearly directly proportional to the volume, n_1, and varies nearly linearly with the average speed, v. This suggests that for freely moving traffic, the number of Type 1 collisions per vehicle-mile may be well enough approximated by the simpler expression

$$C_{\text{Vm}} = c_1 n + c_2 nv$$

where c_1 and c_2 are constants, n is the total hourly traffic volume, and v the average speed.

Type 2

A collision of this type is due to failure to react to the presence of a dangerous object in the roadway (a more slowly moving vehicle) and is analogous to a kind of single-car accident. Spacing is not critical, since the driver is assumed not to react until it is too late to avoid an accident. As in the case of single-car accidents, the chance that a vehicle will cause such a collision may be regarded as primarily a function of its speed and of the length of road traversed.

The number of such collisions per vehicle-mile should accordingly follow a law of the form derived for single-car accidents:

$$C_{\text{Vm}} = c_3 v^b$$

where b has a value between 1 and 2, say 1.5.

Combining both types, the total number of rear-end collisions per vehicle-mile for freely flowing traffic is approximately

$$C_{\text{Vm}} = c_1 n + c_2 nv + c_3 v^{1.5}$$

For constant average speed, v, the graph of rear-end collisions per vehicle-mile versus total traffic volume should therefore be a straight line of positive slope, with a positive initial value at zero volume. This agrees with the graph of rear-end collisions shown in Figure 1 for volumes up to about 500 vehicles per hour.

For higher volumes, the basic assumption of freely flowing traffic is not justified. At the early stages of traffic congestion, clusters form of retarded vehicles very closely spaced (*3*), impatient for an opportunity to pass. The proportion travelling at less than the safe following distance increases markedly, with a corresponding increase in rear-end collisions. This may be seen for volumes between 500 to 700 vehicles per hour in Figure 1.

Beyond a certain degree of congestion, most drivers probably become resigned to an absence of opportunity to pass and so may drive less aggressively. This together with the lower speed may result in a lower accident rate (or at least in a lower rate of accidents serious enough to be reported). The number of rear-end collisions shown in Table 1 is too low to justify reliable conclusions for volumes over about 700 vehicles per hour.

TOTAL ACCIDENT RATE ON TWO-LANE TANGENTS

For single-car, rear-end, and head-on accident rates, expressions were developed which seem to agree well with theory and observation. Agreement is best for freely moving traffic, but the formulas have the right tendency also for congested conditions.

In their most simplified forms, these expressions for the average number of accidents per vehicle-mile are

for single-car accidents: $k_1 v^{1.5}$

for rear-end colllsions: $k_2 n + k_3 nv + k_4 v^{1.5}$

for head-on collisions: $k_5 nv$.

Combining these expressions and collecting terms, the average number of accidents per vehicle-mile becomes

$$A_{\text{Vm}} = c_1 v^{1.5} + c_2 n + c_3 nv$$

where the c's are positive and constant for given driving conditions (other than speed or volume), v is the average speed of the traffic, and n is the total hourly volume.

The theoretical total accident rate may be compared with the observed rate shown in Figure 1, and also with the generally similar rate found in the survey reported by Baldwin (*2*), shown in Figure 3. The particular accident rates shown in Figure 1 reflect accident occurrence under highly selected conditions (daylight, good weather, 55-mph. zones, etc.), but the theoretical considerations are not so

[5] The symbol k or c is used as a formal constant, and need not represent the same value in the various sections of this paper.

limited, and should apply to all nonintersectional accidents on two-lane tangents. The value of the constants will depend upon roadway conditions, visibility, etc., but otherwise the shape of the accident rate versus traffic volume curves should be unchanged.

At low traffic volumes, average speeds on a given road are probably nearly constant. The theoretical and the observed total accident rates rise nearly linearly with volume.

At high volumes, congestion occurs and speeds are well below those of freely moving traffic. Theoretical and observed rates fall below the trend established at low volumes.

At intermediate volumes, corresponding to the early stages of congestion, there is a pronounced upward swing in the rate of rear-end collisions. (This is not predicted by the formulas given above, but is attributed to an abnormal increase in the number of vehicles travelling at less than the safe following distance.) Simultaneously, single-car and head-on accident rates fall off (with the lower speed) and substantially compensate for the rise in rear-end accidents. As a result, the trend of the total accident rate shows little change in Figure 1 up to a volume of about 700 vehicles per hour. Baldwin, however, found a sharp rise in accident rate just before the decline at high volumes (see Fig. 3), which may be characteristic of most roads.

NUMBER OF ACCIDENTS VERSUS NUMBER OF VEHICLES INVOLVED IN ACCIDENTS

One accident may involve many vehicles. Any primary accident, a single-car accident or simple rear-end or head-on collision, may occasion a number of additional, or secondary, collisions following in more or less rapid succession. The entire group of events is classified as a single accident. (In California state practice, collisions within 5 min. of a primary accident are considered part of the same accident.)

For many purposes the number of vehicles involved in accidents may be more significant than the number of accidents. The chance that a vehicle will suffer an accident is of course directly dependent upon the number of vehicles involved in accidents per vehicle-mile rather than upon the number of accidents per vehicle-mile.

An accident may be independent of other accidents, but the collisions involved in it are certainly not independent of each other. The theory so far developed in this paper is based on the assumption that accidents are independent events, that the occurrence of one accident does not affect the probability of other accidents. The average number of accidents per vehicle-mile may thus be expected to follow this theory, but the number of vehicles involved may follow a law of another form.

The mechanism of secondary collisions resembles that of primary rear-end collisions. The same two types are involved. In the first, a vehicle is so close at the time of an accident that collision is inevitable. In the second type, a vehicle is initially at a safe distance, but loss of control, poor judgment, or inattention causes it to add to the wreckage.

The probability of a secondary collision, when an accident has occurred, should therefore be of the same form as the probability of a primary rear-end collision, although with constants of different value.

The average number of primary accidents per vehicle-mile was shown to be approximately of the form

$$A_{\mathrm{Vm}} = c_1 v^{1.5} + c_2 n + c_3 n v,$$

as was also the rate of primary rear-end collisions.

Hence the average total number of vehicles involved in accidents per vehicle-mile should be approximately

$$V_{\mathrm{Vm}} = \sum_{i=0} (c_1 v^{1.5} + c_2 n + c_3 n v) \cdot (c_4 v^{1.5} + c_5 n + c_6 n v)^i$$

where the first term corresponds to the number of vehicles in primary accidents, the second term to the first additional vehicles involved in a secondary collision, the third term to the next vehicles to be involved, etc.

The terms decrease fairly rapidly for appropriate values of volume, n, and average speed, v, so that the first few terms represent the entire series sufficiently well.

Using the first two terms only, we have

$$\begin{aligned} V_{\mathrm{Vm}} &= c_1 v^{1.5} + c_2 n + c_3 n v + c_7 v^3 + c_8 n v^{1.5} \\ &\quad + c_9 n v^{2.5} + c_{10} n^2 + c_{11} n^2 v + c_{12} n^2 v^2 \\ &= (c_1 v^{1.5} + c_7 v^3) \\ &\quad + n(c_2 + c_3 v + c_8 v^{1.5} + c_9 v^{2.5}) \\ &\quad + n^2 (c_{10} + c_{11} v + c_{12} v^2) \end{aligned}$$

Ruthless simplification is called for; perhaps to

$$V_{\mathrm{Vm}} = k_1 v^2 + k_2 nv + k_3 n^2 v$$

This presumably is an approximation to the average number of vehicles per vehicle-mile which are involved in primary accidents or in the first secondary collisions. Additional terms in higher powers of n would take account of additional secondary collisions.

The tabulation provided by the California Division of Highways classifies accidents by number of vehicles, but shows only whether one, two, or more than two vehicles were involved. Hence the data show the primary accident and the first secondary collision, but not any additional collisions which may have occurred (except where a single-car accident started a chain).

These data are summarized in Table 1 and Figure 2 which show, per vehicle-mile, the number of vehicles involved in accidents (limited to a maximum of three per accident). Assuming constant average speed, v, this should correspond approximately to the quadratic form

$$V_{\mathrm{Vm}} = k_1 v^2 + k_2 nv + k_3 n^2 v$$

in contrast to the linear form which characterized the number of accidents per vehicle-mile.

This expectation is fairly well fulfilled by the curves shown in Figure 2.

The risk of accident to a given vehicle is seen to be at least a second degree function of traffic volume. It is more sharply dependent upon volume than is suggested by the exposure theory, or by the accident rate as usually computed. The difference would of course have been more clearly brought out had the data of Figure 2 included all vehicles involved in accidents, instead of being limited to a maximum of three vehicles per accident.

CONCLUSION

The foregoing theories seem to provide a plausible detailed account of the relation of speed and volume to various types of accidents. The theories involve some assumptions which are debatable. In particular, the direct relation between speed and accidents has often been challenged, and available accident data seem inadequate for a satisfactory general answer. The section on head-on collisions contains, however, what may be a convincing demonstration that head-on accidents, at least, cannot be independent of speed.

The theoretical methods used can be readily extended to the study of intersectional accidents and of accidents on multi-lane roads. The methods appear to be sufficiently promising to warrant such extension and a comparison of results with available accident data.

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APPENDIX A

Development of Accident Data

It was not feasible to analyze all the accidents on California two-lane tangents tabulated for 1950. Instead the following procedure was adopted:

Of the 1,100 two-lane road sections listed in the California State Highway System, 270 were chosen at random (using a table of random numbers). Of these, 102 were selected as being generally straight and level. These were chiefly roads of low traffic volume. In order to have an adequate number of higher volume, the list was augmented to include all physically suitable two-lane sections with average daily traffic of over 5,000 vehicles. In all, the analysis covers 172 sections with a total length of 1,420 mi. On these sections, 711 accidents occurred during 1950 of the type analyzed in this report.

In computing accidents per vehicle-mile, the total mileage of each section was used, without deduction for curves, reduced speed zones, etc. Traffic volume was based on the division of highways' estimate of 1950 average daily traffic. Hourly traffic volumes were estimated for each section using the factors shown below, approximations to the hourly volume distribution of typical California rural roads;

Hour beginning	% of ADT	Hour beginning	% of ADT
5 A.M.	1.71	1 P.M.	5.60
6	2.74	2	5.66
7	4.08	3	5.91
8	4.57	4	6.76
9	5.05	5	6.82
10	5.48	6	6.33
11	5.54	7	5.85
12 noon	5.54	8	5.05

The number of daylight hours in the year for each road was assumed to be:

Hour beginning	Daylight hours, 1950
4 A.M.	0
5	76
6	198
7 A.M. to 4 P.M.	365
5 P.M.	274
6	152
7	91
8	61
9	0

The number of daylight-hour–miles on the 172 sections was summed for each hourly traffic volume group. For example, a road of 5,000 average daily traffic was assumed to have a volume of 5,000 (0.0274) = 137 vehicles from 6 to 7 A.M. on each day of the year. If the section was 6 mi. long, it was assumed to have 6 (198) = 1,188 daylight-hour–miles annually between the hours of 6 and 7 A.M. These daylight-hour–miles were added to all others for which the estimated hourly traffic was between 100 and 200 vehicles. The total number of accidents (of the type being analyzed) occurring in traffic of 100 to 200 vehicles per hour was then divided by the corresponding total number of hour-miles and by the average hourly volume (150) to obtain the number of accidents per vehicle-mile.

For each hourly traffic volume group:

Accidents per vehicle-mile

$$= \frac{\Sigma \text{ accidents}}{(\text{av. hr. traffic})(\Sigma \text{ daylight hr.-mi.})}$$

APPENDIX B

Derivations of Formulas for Head-on Collisions

The average number of head-on collisions per hour is

$$C_{\text{hr}} = \sum_{i=1}^{n_1} \sum_{j=1}^{n_2(d/v_i+t)} k(v_i + v_j)^m$$

where k is a constant, v_i is the speed of any one of the n_1 vehicles per hour travelling in one direction, v_j is the speed of any one of the cars it meets travelling in the other direction, t is the average time taken by the traffic (in either direction) to traverse a section of length d.

The traffic volume per hour in the direction opposite to that of n_1 is n_2. A vehicle of speed v_i was shown to meet an average of $n_2(d/v_i + t)$ opposing vehicles while traversing d.

If $m = 1$:

$$C_{\text{hr}} = k \sum_{i=1}^{n_1} \sum_{j=1}^{n_2(d/v_i+t)} (v_i + v_j)$$

$$= k \sum_{i=1}^{n_1} [n_2(d/v_i + t)v_i + n_2(d/v_i + t)v]$$

where v is the average speed of the traffic (in either direction). It is assumed here that the average speed of the opposing vehicles met by any car is equal to v. Expanding,

$$C_{\text{hr}} = kn_2 \sum_{i=1}^{n_1} \left(d + tv_i + \frac{d}{v_i} v + tv\right)$$

$$= kn_2 \sum_{i=1}^{n_1} (d + tv_i + t_i v + tv)$$

where t_i is the time taken to traverse d by a vehicle of speed v_i. Summing,

$$C_{\text{hr}} = kn_2(n_1 d + n_1 tv + n_1 tv + n_1 tv)$$

$$= kn_1 n_2(d + 3tv)$$

Using the approximations $n_1 = n_2 = \frac{n}{2}$, and $d = tv$,

$$C_{\text{hr}} = \frac{kn^2}{4}(d + 3d) = kn^2 d$$

If $m = 2$:

$$C_{\text{hr}} = k\sum_{i=1}^{n_1}\sum_{j=1}^{n_2(d/v_i+t)} (v_i^2 + 2v_iv_j + v_j^2)$$

$$= k \sum_{i=1}^{n_1} [n_2(d/v_i + t)v_i^2 + 2n_2(d/v_i + t)v_iv + n_2(d/v_i + t)(v^2 + \sigma^2)]$$

where σ^2 is the variance of the speeds (in either direction). It is assumed that σ^2 is also the variance of the speeds of the opposing vehicles met by a vehicle traversing d. Expanding,

$$C_{\text{hr}} = kn_2 \sum_{i=1}^{n_1} \left(dv_i + tv_i^2 + 2dv + 2tv_iv + \frac{d}{v_i} v^2 + tv^2 + \frac{d}{v_i}\sigma^2 + t\sigma^2\right)$$

$$= kn_2 n_1[dv + t(v^2 + \sigma^2) + 2dv + 2tv^2 + tv^2 + tv^2 + t\sigma^2 + t\sigma^2]$$

$$= kn_1 n_2(3dv + 5tv^2 + 3t\sigma^2)$$

Using the same approximations as before,

$$C_{\text{hr}} = \frac{kn^2}{4}\left(3dv + 5dv + \frac{3d}{v}\sigma^2\right)$$

$$= cn^2d\left(v + \frac{3}{8}\frac{\sigma^2}{v}\right)$$

where $c = 2k$.

DISCUSSION

T. W. FORBES,* *Technical Director, Committee on Highway Safety Research, National Research Council*—Belmont's paper is certainly interesting, stimulating, and of great value. The development from speed-probability assumptions alone of a series of mathematical functions which resemble empirical average curves of accident frequency rate is itself of interest. In our opinion, the author has made a very definite contribution in demonstrating the possibility of deriving functions from such assumptions which approximate certain portions of average accident-rate-volume curves. This demonstration should stimulate further attempts to include the many other factors which appear to affect the probability of accident occurrence.

* All opinions are those of the discussor personally and not necessarily those of the organizations with which he is connected.

The attempt to select stretches of highways in such a way as to eliminate certain undesired variables and to randomize other peculiarities of individual roads in the development of average accident-rate curves against which to test the theoretically derived curves is of importance. Such procedures should lead to better evaluation of theories, as should the division of accidents into various types. These may be viewed, therefore, as contributions to methodology.

However, there is considerable difference between developing such mathematical expressions and proving that the original assumptions with regard to individual vehicle speeds were correct. There seems to be some danger of misinterpretation of what the paper has shown from a casual reading of the synopsis, the statement of theories and the conclusion. It would be unfortunate if the paper were interpreted to have presented definite evidence that speed is the only factor—or even the major factor—in accidents, and we do not believe that the author meant to take such a position. Although statements in the synopsis and the theory statements might be interpreted in such fashion, his final conclusion indicates caution as to the validity of the theories.

It seems that the findings of the paper can be summarized as follows:

1. It was shown that the form of the exposure theory (volume) which was assumed did not fit the empirical average curves for accidents on two-lane tangents derived in the study. However, it should be pointed out that methods of analysis may sometimes determine whether curves such as those for multi-car accidents and rear-end collisions appear to approach zero at low volumes. Later in the paper, as shown below, the author does justify, in part and indirectly, relationships of the exposure type.

Furthermore, other types of exposure theory might be developed and these have not been ruled out.

2. The author has demonstrated that certain probability assumptions based on individual car speeds can lead to mathematical functions which show some resemblance to the empirical average curves obtained in his study.

However, as he notes in the paper, there are discrepancies which must be explained on the basis of various and sometimes questionable assumptions. For example, in connection with the final expression obtained for rear-end collisions, the author notes that the function would result in the necessary straight line relationship (to fit the first part of the empirical curve) if average velocity is assumed to be constant. Such an assumption means that in the initial part of the curve the expression has been converted to

$$C_{\mathrm{Vm}} = k_1 + k_2 n$$

which is his previously noted exposure type of function.

Moreover, as a volume of 500 vehicles per hour is reached, (which the author quotes as the point at which congestion is beginning), it would be expected that the average vehicle speed would decrease. This should result in a decrease in his power function whereas the actual curve of accidents per 10^8 vehicle-miles shows its greatest rise in this region, as he points out. On the basis of experience with speed-volume studies there seems to be no reason why a reduction in average speed should be delayed until a volume has been reached at which all vehicles are prevented from passing. Yet such a situation would seem to be required by the logic presented.

Again in the treatment of head-on collisions it appears that an exposure type of theory has gotten in "by the back door." The expression for head-on collisions is given as

$$C_{\mathrm{Vm}} = cnv$$

In order to make the derived function check with the corresponding empirical curve, it was necessary, first, to throw away a term involving variability of velocities between cars and, next, to assume that average velocity was a constant on the different road-volume samples up to 400 vehicles per hour. Again, this assumption results in the derived function becoming in effect

$$C_{\mathrm{Vm}} = kn$$

which is identical with the second term of the exposure theory which was discarded in the early paragraphs of the paper.

Beyond this point, in the volume groups for 600 vehicles and above, at least two alternate explanations of the drop in the accident rate curve are possible: (1) that less passing is possible and therefore less exposure, or (2) the lower average speed is responsible. Although in justifying the expression for rear-end collisions, the former explanation was used, in the head-on case the author chose the second explanation.

Aside from this inconsistency, it is rather interesting to speculate as to the proportional drop in average speed which would be necessary to account for the drop from 17 to 11 head-on accidents per 10^8 vehicle-miles corresponding to an increase in average hourly volume from 550 to 850 vehicles per hour (see Line k in Table 1). As the quantity n increases by a factor of 3/2 the factor v must decrease to 2/3 of its value, and then by an additional factor of 11/17 to produce the drop shown. Thus the average speed would have to drop by more than 50 percent. If an average speed of 60 mph. were taken as a reasonable value for these California highway tangents under low volume conditions, this would mean that the average speed would have to drop to about 30 mph. at volumes above 800 vehicles per hour. This would seem very doubtful for California conditions.

Finally, there is considerable evidence that numerous variables beside speed and volume are causally related to highway accident occurrence. As one example, the time driven on a particular trip when each vehicle reaches the sample highway location may well be involved. It also seems difficult to justify the elimination of speed variations between vehicles in formulating a logical theory. Since there is considerable evidence that many factors play a part in accident causation on the highway, it would be unfortunate if the paper should be misinterpreted as having demonstrated the correctness of a one-variable theory.

Nevertheless the approach suggested is a stimulating one, and should lead to multivariate studies. If used with mathematical evaluations of adequacy and goodness-of-fit against empirical data, it should be a useful technique. It should be used with considerable caution, however, since comparisons between average curves, in a field where interrelationships between variables is well-known, may involve some apparent relationships which are spurious.

D. M. BELMONT, *Closure*—Forbes quite correctly emphasizes that the theories presented in the paper are by no means definitely proven. They were intended as first approximations only, and no claim was made that they yielded good results in the region of traffic congestion. However, they do seem to yield better results than does the existing exposure theory. Forbes' calculation suggests that my theoretical head-on accident rate is too high for traffic of 850 vehicles per hour. But the standard exposure theory would indicate a rate about 50 percent higher still, if fitted to the lower volume data.

It is difficult to understand Forbes' remarks about a "one-variable theory" (especially in view of his efforts in behalf of the exposure theory). To examine the effects of speed and volume is hardly to deny the importance of cother fators. Speed may well be a major factor in determining accident rates, but we cannot yet measure its effect against that of, say, traffic signals or the emotional state of the driver.

In his concluding paragraph, Forbes advocates the application of statistical tests of goodness-of-fit to accident theories. This seems to me a rather premature suggestion. In the present rudimentary state of our theories, much progress must be made before one can reasonably hope for a close fit to actual accident occurrence. Furthermore, most of our accident records leave much to be desired, so that a negative statistical test might well result even for a perfectly sound theory. This weakness in the available data is a serious handicap to studies which would derive relationships from statistical analysis of the data. It may, therefore, be more profitable to work on the development of mathematical models from what seem to be reasonable a priori assumptions, and to be satisfied with rather modest agreement with the "facts."

Speed Characteristics on Vertical Curves

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THERE exist many miles of highways built 20, 30, and 40 years ago. These highways are obviously not designed for present day speeds. Critical points exist at vertical curves with restrictive stopping-sight distances. To determine the speed characteristics of today's passenger cars, at these points, the New York State Department of Public Works, in coöperation with the Bureau of Public Roads, made comprehensive speed observations at 20 such locations in New York State.

Conditions for site selection were: tangent length for at least 1,000 ft., each side of the vertical curve; full approach grade of a gradient to minimize its effect on average speed; no marginal influence; no speed zones; free-flowing traffic; traffic in lane adjacent to study lane at a minimum; good riding surface; and no-passing line markings where the sight distance was 500 ft. or less. Sight distance varied from 150 to 700 ft.

Field equipment included a constant-speed 20-pen recorder, electrically connected to 10 pneumatic detector units, 100 ft. apart. Seven detector units were laid on the approach to the vertical curve and three on the leaving grade. Speed for approximately 100 cars at each site was recorded. Three sight-distance criteria were studied: from 4½ ft. above the pavement to 4 in.; from 4½ ft. to 2 ft.; and from 4½ ft. to 4½ ft. above the pavement.

At each trap and for each site the following data was compiled and plotted: average speed; 85-percent speed; safe speed for the three sight distance criteria (according to AASHO policy for nonpassing minimum sight distances based on safe stopping distances); AASHO safe sight distance required for 85-percent speed; the percent of drivers exceeding the legal speed limit; and from the cumulative frequency distribu-