DEPARTMENT OF DESIGN

Research on Highway-Bridge Floors

N. M. Newmark, *Research Professor of Structural Engineering*, and
C. P. Siess, *Research Associate Professor of Civil Engineering*,
University of Illinois

This paper reviews the research on highway bridge floors carried out at the University of Illinois during the period from 1936 to 1954 under the sponsorship of the Illinois Division of Highways and the Bureau of Public Roads.

The object of this research has been to determine the behavior of reinforced-concrete slabs subjected to vehicle wheel loads and to investigate the effects of the many variables encountered in different types of bridge-floor construction. The methods used have included a combination of mathematical analyses as well as extensive tests on scale-model bridges and on full-size elements of bridges.

Two types of bridges have been studied: the simple-span solid-slab bridge with integral curbs and the I-beam bridge. Tests have been made on both right and skew bridges of both types, and on both simple-span and continuous I-beam bridges. Extensive analytical and experimental studies of composite action in I-beam bridges have also been made. In addition to these studies of actual bridge types, there have been several analytical and experimental investigations of fundamental problems involved in the behavior of slabs subjected to concentrated loads.

The paper includes brief descriptions of each phase of the investigation with references to the several publications in which the research methods and results have been reported in more detail.

● A FEATURE common to nearly all highway bridges is the reinforced-concrete slab which serves as the bridge floor and carries directly the concentrated loads imposed by vehicle wheels. In some simple structures the slab may be the principal load-carrying member, transferring the wheel loads directly to the piers or abutments. This ideally simple case is seldom realized in practice, however, since the integrally-cast concrete curbs usually provided act as edge beams which stiffen the slab and themselves carry a portion of the load to the abutments. Even more complex in its behavior is the slab supported directly on beams, as in the I-beam or concrete-girder bridge, or in the floor systems of truss bridges.

The behavior of even a simple reinforced-concrete slab subjected to concentrated loads is fairly complex. If the slab is supported either wholly or in part by flexible beams with which it must coöperate in carrying the loads, the complexity of the problem is increased. And, finally, since in many cases the slabs are either skewed or continuous, or both, the problem of designing bridge floors is one requiring the utmost in knowledge and understanding of the behavior and strength of reinforced-concrete slabs carrying concentrated loads.

The bridge floor problem is twofold: First, the slab itself must be designed to support wheel loads safely and transmit them either to the piers or abutments or to the supporting beams or girders. And second, the loads transmitted by the slab to the supporting members must be determined so that they, in turn, may be designed.

In an attempt to answer some of the more-important questions concerning the design of highway bridge floors, a program of systematic and coördinated research was begun in September of 1936 at the University of Illinois in coöperation with the Illinois Division of

Highways and the Bureau of Public Roads. This research program was referred to as the "Concrete Slab Investigation."

OBJECTIVES AND SCOPE

The purpose of this research program, as set forth in the formal agreement between the coöperating organizations, was ". . . to make investigations and tests of reinforced concrete slabs to determine the behavior thereof under varying conditions and to develop information which will advance the art of concrete bridge building." Within the framework of this broadly stated directive, an extensive program of analytical and experimental research has been carried out during the past 18 years.

This research can be classified broadly into four categories: (1) fundamental studies and exploratory tests concerned with various aspects of the behavior of reinforced-concrete slabs; (2) investigations of the simple-span solid-slab bridge having integrally cast concrete curbs, both right and skew bridges being considered; (3) investigations of the slab-and-girder type of bridge, including both right and skew simple-span bridges and continuous right bridges; and (4) investigations of composite construction for I-beam bridges, with special attention to the function and behavior of shear connectors.

The results of these investigations were reported initially in 16 progress reports, 14 master theses, and 2 doctorate theses. However, complete and detailed reports covering practically all phases of the work have been made available in 16 bulletins of the University of Illinois Engineering Experiment Station; in addition, the results relating to particular aspects of the program have been reported or summarized in seven papers appearing in technical journals (see references).

Because of the scope of this research and in view of the many previous publications, it is neither desirable nor necessary to present a detailed report on this project in the present paper. Instead, it will be the purpose of this paper to review and discuss briefly each phase of the work, to outline the problems considered and the methods used to solve them, to mention the variables considered and the nature and limitations of the results, and to indicate where the interested reader may find more detailed discussions of the research techniques, the results obtained, and recommendations for design.

METHODS OF APPROACH

All of the problems studied in connection with the investigation were attacked by a combination of mathematical analyses and laboratory tests, correlated wherever possible with the results of field observations, and compared with current design practices as reflected by existing specifications or data furnished by design offices.

Mathematical analyses alone are not sufficient to describe correctly and completely the behavior of concrete slabs or highway bridges under the action of concentrated loads. They are limited by the assumptions on which they are based and by the fact that they are usually applicable only in the "elastic" range. Laboratory tests alone are also limited, since the results obtained cannot be considered to apply beyond the range of conditions actually encountered in the specimens tested. In the case of complex structures, such as those considered in this investigation, limitations of time, space, and money make it almost impossible to cover a sufficient range of variables by purely experimental means.

The solution to these problems involved a combination of analyses and tests. Usually, the analyses were made first in an attempt to determine the significant variables affecting the behavior of the structure. Since mathematical calculations can almost always be made more cheaply and more rapidly than laboratory tests, it was possible to study analytically a great many structures having a wide range of variables. On the basis of these analytical studies, experiments were then designed: variables were selected, specimens designed, and decisions made regarding the nature and quantity of the observations which should be made during the tests.

Laboratory tests were made on many types of specimens, but the most significant were those on small-scale or full-size models of highway-bridge structures or elements of such bridges. The tests on slab specimens or on bridge models were carried out to (1) determine the extent to which the theoretical analyses were applicable and establish, where necessary, the differences between the conditions of the test and those assumed in the analyses and (2) observe and measure the behavior and strength of the test structures in the range beyond yielding, for which the analyses were not applicable. If the results of the tests agreed with those obtained theoretically, the analyses could then be used to extend the applicability of the experimental findings over a wider range. If disagreements were found between tests and theory, they could usually be explained and the analyses revised or modified accordingly. This process of continually checking tests against analyses, and vice versa, was usually productive of results that could be applied with a fair degree of confidence to most structures in the range of practical designs, even though the scope of the tests was much more limited.

The method of attack may be summarized as follows: First, analyses were made to establish the variables and to aid in the planning of tests. Next, tests were made on laboratory specimens, usually scale models of highway bridges. And finally, recommendations for design were developed, based on the results of both the analytical and experimental studies. The design procedures proposed as a result of these investigations have usually been made as simple as possible. In some cases they represent merely simplifications of the original analyses, justified by the correlations between theory and tests. In other cases, the procedures have been based primarily on the experimental results but have been stated in a form derived from or rationally related to the theoretical analyses.

FUNDAMENTAL STUDIES

At the beginning of this project, and also from time to time during the course of the work, several analytical or experimental studies were made which were essentially fundamental in nature. By this it is meant that they have application to many problems in addition to those related specifically to highway bridges. Most of these studies, but especially those involving theoretical analyses, provided the basis and background for subsequent work on specific bridge types. In other cases, the studies in this category were made as corollary investigations to answer specific but fundamental questions which arose during the later stages of the work.

Analytical Studies

At the beginning of the investigation, during the period 1936–1937, two purely analytical studies were made of the moments in simply supported slabs loaded with concentrated as well as distributed loads. Both of these studies were based on the ordinary theory of flexure for medium-thick plates or slabs, as were all other analyses made in connection with this project. They differed from each other, however, both in the type of slab analyzed and in the manner of obtaining solutions for moments in the slab or in the supporting beams.

Bulletin 303. The analyses reported in Bulletin 303 (*1*) were made by the classical procedure of obtaining a solution of Lagrange's differential equation for the deflections of a slab. Algebraic expressions were derived for moments in the slab and in the supporting beams for a large number of cases involving slabs simply supported on two opposite edges. Solutions were obtained for the infinitely wide slab with a single rigid crossbeam, a single flexible crossbeam, or two flexible crossbeams, and loaded with a single concentrated load at various positions. Another case studied was the semi-infinite simply supported slab carrying a concentrated load and having the finite edge either simply supported, fixed, free, or supported by a flexible beam.

The rectangular slab simply supported on two edges and supported by flexible beams on the other two edges was treated at length because of its relation to the simple-span slab bridge with integrally cast curbs. Expressions were obtained for moments in the slab and in the beams for a single concentrated load at various points on the slab, for a uniform load over the entire slab, and for a line load along each beam. Solutions were also obtained for the rectangular slab having a center stringer as well as beams along each edge, for the edge-stiffened slab with the supporting beams continuous over interior supports, and for the three-stringer bridge with a continuous transverse diaphragm at midspan.

Although most of the solutions are given only in algebraic terms, numerical values were computed in certain cases, notably the infinitely wide slab with a rigid crossbeam, and the three-stringer bridge with a transverse diaphragm. In the latter case, the ratio of width to span and the relative stiffness of the beams and slab were the variables considered.

Bulletin 304. The work reported in Bulletin 304 (*2*) is concerned with a broadly applicable method of analysis rather than with particular solutions for various cases of loading or support. The method presented may be applied to any rectangular slab, simply supported on two opposite edges with any type of support on the other two edges, and continuous over any number and spacing of rigid or flexible simple beams extending transversely to the simply supported edges. The slab may have sections of different stiffnesses (owing to variations in thickness or modulus of elasticity) provided that the stiffness is constant over each rectangular section of the slab bounded by the two simply supported edges and transverse lines thereto. Under certain conditions, torsional restraint offered by the beams may also be taken into account.

In its essential features, the analysis is similar to and derived from the moment-distribution method of analysis developed by Hardy Cross for continuous beams and frames. Solutions are found in the form of terms in an infinite series, each term being obtained by numerical calculations involving fixed-end moments, stiffnesses, and carry-over factors applied to an analogous continuous beam. Any desired degree of accuracy can be obtained by taking a sufficient number of terms in the series.

Bulletin 304 contains tabulations of the required moment-distribution constants including stiffnesses, carry-over factors, and fixed-end moments for concentrated loads, uniform loads, and line loads. The moment-distribution procedure was used extensively in connection with the analyses of slab-and-girder highway bridges but may be used also for many other problems involving slabs supported on and continuous over flexible beams.

Orthogonally Nonisotropic Slabs. Analyses of slabs having different flexural stiffnesses in two directions at right angles to each other were made in 1938–1939 and reported in a thesis (*23*). Algebraic expressions in finite form were obtained for the principal moments in simply supported, infinitely wide slabs and in rectangular slabs simply supported on two opposite edges and either simply supported, free, or fixed at the other two edges. Both uniform and concentrated loads were considered in most cases. Numerical values of the moments were computed for various ratios of the stiffnesses in the two directions for the infinitely wide slab and for the rectangular slab simply supported on two edges and free on the other edges. Algebraic solutions were obtained also for certain cases of circular slabs with axially symmetrical loadings. These solutions are of fundamental interest in the theory of plates but also have applicability to concrete slabs with different amounts of reinforcement in the two directions and to certain extreme cases of slab-and-girder bridges in which the beams are closely spaced.

Experimental Studies

Plaster-Model Slabs. Analyses of slabs by means of the ordinary theory of flexure do not yield correct values of the stresses in regions near a concentrated load. Although special theories have been developed for computing these stresses, solutions for many important cases were not available and could not readily be obtained by analytical means. For this reason, tests of model slabs made of pottery plaster, a form of experimental stress analysis, were made in 1937–1938 and reported in Bulletin 313 (*3*). Such tests yielded the desired information regarding maximum stresses, since the type of plaster used possesses nearly linear stress-strain characteristics up to the point of failure, and failure seems to occur at a limiting tensile stress. For these reasons, the load causing failure of a plaster slab may be taken as a measure of the maximum tensile stress. The slabs tested were generally 1-inch thick and of 12-inch span. The variables studied were the shape of the slab, the manner of support, and especially the size and shape of the loaded area and its location relative to the edge of the slab or to a support. All loads were relatively concentrated as compared to the dimensions of the slab. Both circular and rectangular slabs were tested. Conclusions regarding the effects of size and shape of the loaded area, based on these tests and subsequently confirmed by tests on reinforced concrete slabs, were of considerable value in the planning of later tests on scale-model bridges.

Ultimate Strength of Beams. Since ultimate strength was considered to be as important as behavior at working loads, the problem of determining the maximum load-carrying capacity of reinforced-concrete slabs was encountered at almost all stages of these investigations. In the belief that a clear understanding of the behavior of beams must precede any attempt to explain the behavior of slabs, an extensive fundamental study of the ultimate strength of reinforced concrete beams was undertaken and the results reported in Bulletin 345 (*8a*) and elsewhere (*8b*). The results of numerous tests made by others were analyzed and a rational procedure was developed for predicting the ultimate moment-carrying capacity of beams reinforced in tension only. The expressions obtained for ultimate moments were rational in form and derivation but involved empirical factors, derived from tests on beams, to describe and account for the inelastic properties of concrete. Although other theories considering inelastic behavior had been presented previously, the procedures presented in Bulletin 345 were of particular value, because of their general nature and their ability to predict the behavior and strength of beams having widely varying properties.

Arrangement of Reinforcement in Slabs. In 1941–1942, tests were made on 15 reinforced-concrete slabs to determine the effectiveness of various arrangements of reinforcement in which the bars were not parallel to the direction of the principal moment. This problem arose primarily in connection with the tests of skew slab bridges, but the results are applicable to other problems in reinforced-concrete construction. The slabs tested were 5 inches thick and approximately 5 feet square, and were tested on a 5-foot span. They were simply supported on two opposite edges and loaded at the one-third points of the span on lines extending across the full width of the slab parallel to the supported edges. Each slab was thus subjected to a constant moment over the middle third of its span. Reinforcement consisted of two sets of parallel bars in the bottom of the slab. The bars in the two sets were oriented at various angles to each other and at various angles to the direction of the span and principal moment. All reinforcement was ⅜-inch deformed bars, but the spacing varied in the different slabs or in the two sets of bars in the same slab. The effectiveness of the various arrangements of reinforcement was evaluated in terms of both the ultimate moment capacities of the slabs and the observed relationships between moment and strains in the reinforcement or in the concrete. Rational expressions were developed for determining the moment-carrying capacity corresponding to any arrangement of reinforcement and for predicting the strains in the reinforcement and the concrete. The results of these tests and analyses have not yet been published but were reported in a thesis (*26*).

Continuous Slabs. In 1950–1952 a series of tests was made to study the effects of varying the ratio of positive to negative reinforcement on the manner and degree of cracking and the ultimate load-carrying capacity of slabs continuous over several supports. This problem arose chiefly because the degree of cracking at any location in a continuous slab is a function of the amount of reinforcement, as well as the moment at the location, while the magnitude of the moment depends on the relative stiffnesses of the various sections of the slab, which in turn is a function of the degree of cracking. These conditions had been encountered in the tests of I-beam bridges, and it was felt that additional fundamental studies were required in order to evaluate the many variables affecting the complex behavior involved. Tests were therefore made on reinforced-concrete slabs 3½ inches thick and 8 feet wide and continuous over five nondeflecting supports spaced 3 feet on centers. Four specimens were tested, the only variable being the amount and distribution of the positive and negative reinforcement. In three slabs the total amount of reinforcement was kept constant while the distribution between positive and negative moment regions was varied as follows: 1.33–0.67, 1.00–1.00, and 0.67–1.33. In the fourth specimen, only two thirds as much total reinforcement was provided, divided evenly between the positive and negative moment sections. All tests were made with concentrated loads applied at various locations on the slabs, and strains were measured in the reinforcement and on the concrete. Because of the complexity of the problem and the limited number of tests, no generally applicable conclusions can be presented. These studies, however, yielded much information contributing to a better understanding of the behavior of continuous slabs. The results of these tests have not been published but were reported in two theses (*28, 29*). The second thesis contains a complete report of the tests together with analyses of the results, comparisons with theory, and a limited set of conclusions.

EXPLORATORY TESTS

Simply-Supported Wide Slabs. The first laboratory tests of the concrete-slab investigation were made in 1937–1938 on two simply supported rectangular slabs having a width large compared to their span. This type of slab was chosen because it conformed closely to the infinitely wide slab for which extensive analyses had been made by H. M. Westergaard (*31*). The purpose of these tests was to establish the relationship between the theoretical and actual behavior of slabs subjected to concentrated loads, for a reasonably simple set of conditions. The test slabs were 6½ inches thick and 20 feet wide and were tested on a span of 6 feet 8 inches. They were loaded with a single concentrated load applied through a 6-inch circular disk at various locations, all of which were sufficiently far from a free edge to permit the analysis for the infinitely wide slab to be used.

The slabs were first tested in the uncracked state with loads small enough that cracking would not be produced. At this stage, extensive measurements were made of deflections and of strains in the concrete and the reinforcement, for comparison with those predicted by Westergaard's analysis. The slabs were then cracked by loading successively at various locations, and the cracking loads as well as the effects of cracking on the observed load-strain relationships were recorded. Next, a single concentrated load was applied at various points on the well-cracked slab, and strains and deflections were measured and compared with the results of the analysis. Finally, the slab was tested to failure under a single concentrated load. Since failure occurred locally by punching out a cone of concrete, this test was repeated at several locations on each slab.

The first slab tested was supported along one edge on a series of reaction rings, devices provided to measure reactions by the deflection of elastic rings. The results obtained from these measurements differed from the computed reactions by an amount which could be accounted for by the fact that the reaction-measuring devices, though quite stiff, were not entirely nondeflecting as required by the theory. No attempt was made to measure reactions on the second slab.

The procedures used and the results obtained in these tests have been reported in detail in Bulletin 314 (*4*), and also have been discussed in Reference 14a.

Square Slabs with Various Loaded Areas. Since the investigation was concerned primarily with the behavior of reinforced concrete slabs under the action of vehicle wheel loads, it was planned to make all of the tests with loads representing the concentrations produced by vehicle tires as far as possible. Because tires vary in size and shape and because dual tires also had to be considered, a question was raised as to just how much the strains in a reinforced-concrete slab would be affected by variations in the size and shape of the loaded area. Tests were therefore made in 1939 on 20 identical slabs, 6 inches thick, 5 feet wide, and simply supported on a span of 5 feet. The principal variable was the size and shape of the bearing block through which load was applied. The types used included: circular disks, 2, 6, 10 and 14 inches in diameter; circular rings, 1 inch wide and 6 or 14 inches in diameter; and pairs of circular disks each 2 or 6 inches in diameter. The 6-inch disk was used with a layer of either sponge rubber or plaster of paris between the disk and the slab, while the sponge rubber was used in all of the other tests.

The slabs were loaded at midspan and strains were measured in both the reinforcement and the concrete in the region immediately beneath and adjacent to the load. The measured strains in the reinforcement under the load were practically the same for loaded areas ranging from 2 inches to 10 inches in diameter and including the 6-inch ring as well as the pair of 2-inch disks on 3-inch centers. For the larger loaded areas, including the 14-inch disk and ring and a pair of 6-inch disks on 12-inch centers, the strains were generally smaller but again seemed to be independent of the actual shape of the loaded area. On the basis of these tests, it was decided to use a circular disk bedded on sponge rubber in subsequent tests. The diameter was to be taken at 15 inches for the full-size structure and reduced as required in the tests of scale models.

These tests have been described and the results reported in detail (*4*, *14a*).

Verification of Scale Models. The tests of wide slabs and of square slabs to determine the effect of the size of loaded area were made on full-size concrete slabs reinforced with standard-size deformed bars. Early in the test program, however, it had become evident that much of the experimental work would have to be done with small-scale models, and by 1939 the design of quarter-scale-model I-beam bridges had already begun. In order to establish the reliability of such models as a means of representing the behavior of full-size slabs it was decided to repeat the two series of tests just described, but using quarter-scale models. The dimensions of the model slabs were, in each case, exactly a fourth of those of the prototype structures. The materials used were a sand-cement mortar for the slab and reinforcing bars consisting of ⅛-inch-square, cold-rolled, SAE 1112 steel bars, pack annealed to lower their yield strength to about 45,000 psi., and rusted in order to produce a surface capable of providing good bond with the mortar. A total of 32 square slabs and four wide slabs were tested, duplicating the full-size tests in manner of support, size and shape of loaded area, location of loads, etc. Measurements included deflections, strains measured with a specially designed and constructed optical interferometer gage, yield loads, and punching loads. Because of occasional differences in the properties of the materials, direct comparisons could not always be made between the model and prototype slabs. However, a careful comparison of the results showed that the scale models were capable of reproducing the behavior and strength characteristics of the full-size slabs quite closely.

SIMPLE-SPAN SLAB BRIDGES WITH CURBS

The simple-span bridge consisting of a solid, reinforced-concrete slab with integrally cast, reinforced-concrete curbs along each side of the roadway was studied both analytically and experimentally from 1936 to 1942. Both right and skew bridges were studied. In both cases, analyses were made first to determine the variables which had the greatest theoretical effect on the behavior of the bridges. Tests were then made on scale models of bridges representative of typical highway structures. The scale factors used ranged from a fifth to a half. On the basis of the analyses and tests, design procedures were developed for the simple-span right bridge with curbs.

Right Bridges

Analyses. Extensive analyses for moments in the simple-span, right slab bridge were reported in Bulletin 315 (*5*). These analyses were based on the algebraic solutions given in Bulletin 303 for the rectangular slab simply supported on two opposite edges and supported on the other two edges by flexible beams. In the analytical treatment of this structure it was assumed that the supporting beams exert only vertical forces on the slab, i.e., interaction between the slab and beam resulting from horizontal shearing forces, and forces exerted on the slab as a result of the torsional resistance of the beams are both neglected. The nature of this assumption is to create a considerable difference between the behavior of the structure analyzed and the actual bridge, particularly in the neighborhood of the curbs. Numerical values of the moments at various locations in the slab and in the curb beam are given in Bulletin 315 for a wide range of structures having various values of width to span and various ratios of beam to slab stiffness. The moments are given for a uniformly distributed load over the entire area of the slab, representing the dead load of the slab itself; for a line load distributed uniformly along each edge beam, representing the dead load of the curb; and for concentrated loads at various locations on the slab, representing the forces exerted by truck wheels. Where truck loadings were considered, the span of the bridge was limited to 30 feet and the roadway width was limited to values corresponding to a two-lane bridge.

Laboratory Tests. Since the analysis involved certain assumptions and approximations regarding the interaction of the curb with the slab, it was considered necessary to make tests of structures corresponding more nearly to typical slab bridges. During 1939–1941, tests were made on seven quarter-scale models and two half-scale models of bridges designed on the basis of the data given in Bulletin 315. The principal variables studied were the span length and the relative stiffness of the curb and slab. In the prototype structures, the span varied from 10 to 30 feet. On one specimen, handrails were included to determine their effect on the stiffness of the edge beam. All of the test specimens represented two-lane bridges with a 24-feet width of roadway.

The quarter-scale models were divided into two groups. One group included span lengths, in terms of the prototype, of 10, 20, 25, and 30 feet, each slab having curbs of ordinary proportions. In the other group, the span was constant at 20 feet for the prototype, but the stiffness of the curb was varied over a fairly wide range from the slab with no curb to one with an ordinary curb stiffened by a reinforced-concrete handrail. The principal variable in the tests of the two half-scale models was the thickness of the slab, one slab being 2 inches thinner (4 inches thinner for the prototype) than the other. Both had spans of 12.5 feet corresponding to 25 feet in the prototype.

The specimens were first tested in their initial uncracked condition, since at this stage the slabs conform more nearly to the homogeneous and elastic conditions assumed in the theory. The slabs were then systematically cracked by applying groups of loads corresponding to the rear-wheel concentrations of two trucks at various locations on the bridge. In general, influence lines for moments at critical locations were determined experimentally from the measurement of strains produced by loads placed at various locations on the structures, both before and after cracking. The bridges were next tested under groups of concentrated loads simulating the effects of one or two heavy axles. These loads were placed at various locations to produce maximum moments or stresses. The loads were increased in increments and measurements made of strains at each increment in order to obtain information regarding the load-strain characteristics of the bridges. All bridges were eventually tested to failure under an added distributed load intended to make up the scale-model deficiency in dead load, together with a group of concentrated loads corresponding to the live load and placed so as to produce maximum moment at the center of the slab.

The strains measured in the tests were compared with those computed from the analysis. Strains measured at the center of the slab in the principal (longitudinal) reinforcement, after cracking, were usually somewhat smaller than the calculated values, but the agreement seemed to improve as the span increased. The measured strains in the transverse reinforcement, however, were always materially less than the calculated values. The measured compressive strains in the top of the curb at midspan were considerably larger than the calculated strains as a result of neglecting the interaction between the slab and the curb in the analyses. That such interaction was present in the test bridges was indicated by a significant lowering of the neutral axis in the curb at midspan.

The ultimate strength of these slabs, designed according to the elastic theory, was shown to be quite high. In most of the tests, the load producing first yielding of the longitudinal steel at midspan was equal to the dead load plus four to five live loads. In addition, the structures usually possessed a reserve strength, beyond yielding, of about 25 to 30 percent of the live load.

The results of these tests have been presented in Bulletin 346 (*9*).

Recommendations for Design. Comparisons of the results of the tests with those predicted by the analyses indicated that the elastic analysis did not provide a suitable basis for design. For one thing, the moments carried by the curbs were found to be appreciable, and any design procedure for the curbs which neglected their interaction with the slab would lead to cross-sections and reinforcement which would be unsatisfactory in service. Moreover, designs based on maximum localized slab moments yielded structures having load-carrying capacities in terms of live load considerably in excess of those usually desired for highway bridges. The capacities of the test bridges seemed to be more closely related to the average moment across the width of the slab than to the maximum moment.

On the basis of these findings from the tests, a modified method of design was evolved. This method is based partially on the analyses, since it makes use of three empirical equations which were derived from the numerical results given in Bulletin 315. The principal features of this simplified method are that the structure is designed for a total resisting moment determined mainly by statics, i.e. a form of limit design, and the reinforcement in the curb is determined for a composite section which includes a portion of the slab acting as the lower flange of the curb beam. Both of these features were justified by comparisons with the results of the tests. The scope of the simplified design method was extended beyond the range considered in the analyses and tests to include bridges with unlike curbs, bridges having a center longitudinal joint with inverted curbs, spans up to 45 feet, and truck-trailer loadings. This design procedure is described in detail in Bulletin 346 (*9*) which also contains comparisons between the results predicted by the simplified design procedure and those obtained in the tests of scale-model bridges.

Skew Bridges

Analyses. Analyses of skew-slab bridges with edge beams were made by means of difference equations. The difference-equation expressions for various networks of points suitable for use in connection with bridges having different angles of skew together with numerical solutions for moments in certain slabs, were presented in Bulletin 332 (*6*). The numerical solutions presented in this bulletin are for the uniformly loaded slab, simply supported on four edges and having skew angles of 0, 30, 45, and 60 deg.; the uniformly loaded slab, simply supported on two edges and having skew angles of 0, 30, 45, and 60 deg.; and the 45-deg. skew simple-span slab bridge with curbs. The latter structure was analyzed for a number of different loading conditions including both uniform and concentrated loads.

More-extensive analyses of simple-span skew-slab bridges with curbs were reported in Bulletin 369 (*12*). Detailed analyses for moments in the slab and curbs for various combinations of dead and live load were made for ten bridges having angles of skew varying from 30 to 60 deg., normal span lengths ranging from approximately 12 to 36 feet, and a clear width of roadway of 24 feet in all cases. The stiffness of the curb relative to that of the slab was also varied. The variation of dead and live load moments in both the slab and the curbs was studied as a function of the angle of skew, and extensive comparisons were made between moments in skew and right bridges.

Two assumptions made in the analyses of skew bridges were believed to require further study. First, the interaction between the curb and the slab was neglected, as was done also in the case of the right bridge; and second, it was assumed that the slab remained in contact with the supports at all points, whereas actually a tendency for the acute corners to lift from the supports would be expected in some cases. These effects were studied in 1940–1941 by tests on plaster-model skew slabs with curbs, similar in most respects to those described elsewhere in this paper (*3*). Since simple tests to fracture would not yield the desired information, strains were measured in the plaster slabs by means of a sensitive optical-interferometer strain gage. Influence values for strains at various locations on the slab were compared with the corresponding influence values for moments determined analytically. In general, the agreement was excellent, although some differences owing to interaction between the curb and slab or to lifting of the corners were observed. The test techniques have been described and the results presented in a thesis (*25*).

Laboratory Tests. Since the theoretical analyses of skew-slab bridges involved the same assumptions as the analyses of right bridges, tests were considered necessary in order to determine the extent to which the behavior of actual bridges corresponded to that predicted by the analyses. Tests were made on three half-scale models of 45-deg. skew bridges in 1941–1942, and on one fifth-scale model bridge having a skew of 60 deg. in 1941–1942. The results of these tests have been reported in Bulletin 386 (*15*). The three half-scale model bridges had identical overall dimensions and the same 45-deg. angle of skew. They differed only in the amount of reinforcement in the curbs and in the amount and direction of the spanwise reinforcement in the slab. In two of the bridges this slab reinforcement extended in the skew direction parallel to the curbs; in the other bridge, it was placed perpendicular to the supports. The fifth-scale 60-deg.-skew bridge was tested primarily to determine the effects of an extremely large angle of skew. All of the bridges were tested first in the uncracked condition and then were cracked systematically by applying at numerous locations a pair of loads representing the rear axle of a truck. Influence lines for critical strains were determined for both the cracked and uncracked condition, and subsequent tests were made with increasing increments of live loads representing one or two heavy truck axles. The final test in each case was a test to failure under an added distributed load plus a group of concentrated loads representing wheel-load concentrations. Tests to failure usually were made first with the concentrated loads placed to produce failure of the slab; a second test was then made with the loads placed to produce failure of a curb.

The results of the tests indicated that the theory presented in Bulletin 369 gives a

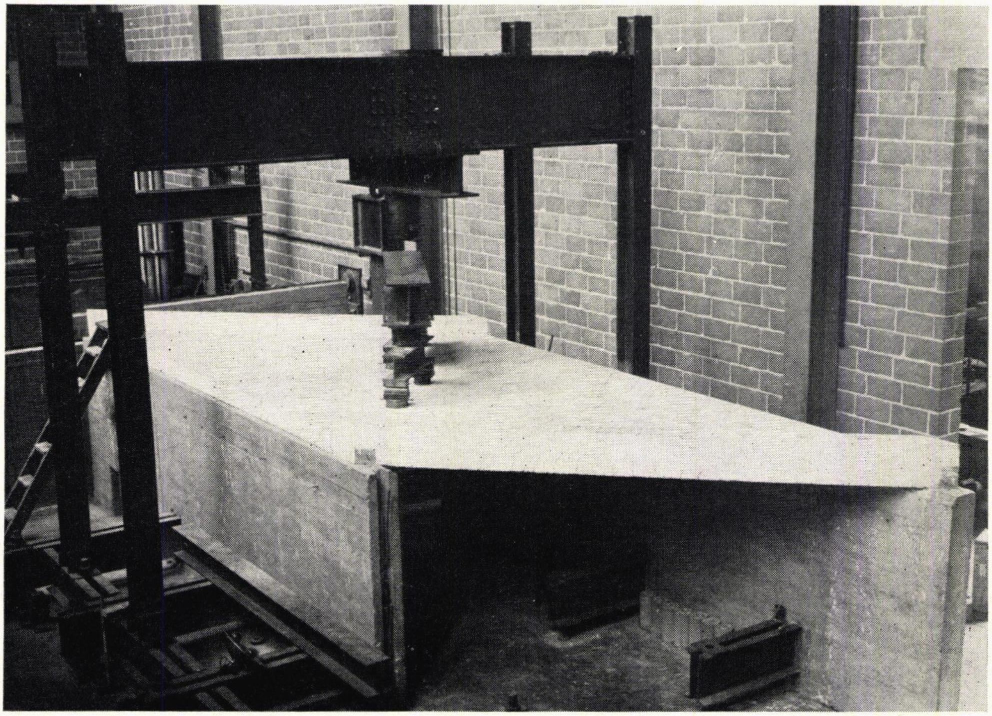

Figure 1. Test of a half-scale model of a 45-deg.-skew slab bridge with curbs. The model has a span of 9.75 feet and a clear roadway width of 13 feet. Four loads representing two heavy axles are being applied in the midspan region.

reasonably good qualitative picture of the behavior of a skew-slab bridge with curbs: The theory was found to predict with fair accuracy the location and direction of the maximum strains as well as the load positions for which these strains were critical. The relation between measured and computed strains in the skew bridges were generally comparable to those found in the tests of right bridges. Similarly, the ultimate strengths of bridges designed according to the elastic theory were found to be quite high, ranging from dead load plus 3.5 to 4.0 live loads for the 45-deg.-skew bridges to as much as 6.5 live loads for the 60-deg.-skew bridge.

Recommendations for Design. Because of the large number of variables encountered in the skew bridges and the limited range of both the analyses and the tests, no generally applicable recommendations for design have been developed on the basis of these studies. It was concluded tentatively, however, that two-lane bridges having skews not greater than 45 deg. may safely be designed for moments in the slab and curbs corresponding to those in a comparable right bridge, i.e., a bridge which has a span length, roadway width, slab thickness, and curb detail identical to those of the skew bridge, the span length in both bridges being measured normal to the supports. Skew bridges designed in this manner may have ultimate load-carrying capacities or factors of safety somewhat higher than those of right bridges designed by the simplified procedure given in Bulletin 346. Although the 60-deg.-skew bridge, designed on the basis of the theoretical analyses, was found to have a factor of safety considerably in excess of that usually desired in highway bridges, it has not yet been possible to recommend modifications to the theoretical analysis because the complex behavior of structures with large angles of skew has not been fully explored, especially beyond the elastic range.

SLAB-AND-GIRDER BRIDGES

Basic Analytical Studies

For the purpose of analysis, the slab-and-girder bridge was assumed to consist of a rectangular slab simply supported on two edges and supported by a series of parallel flexible beams. All analyses for this type of structure were made by means of the moment distribution procedure described in Bulletin 304. For the three-girder bridge, the solution presented in Bulletin 303 may also be used but was not employed in any of the analyses made in connection with this investigation.

In actual highway bridges of the slab-and-girder type, the supporting beams may consist of steel I-beams acting independently, steel I-beams acting compositely with the slab, or concrete beams cast integrally and acting compositely with the slab. Since the analysis involves the assumption that only vertical forces act between the beams and the slab, no distinction was made analytically among these various types of beams; the differences in beam stiffness produced by composite action in I-beam bridges or by the use of concrete beams in place of steel beams were taken into account only by considering the increased stiffness of the beam. Torsional restraints offered by the beams or horizontal shearing forces between the beams and the slab were not considered in the basic analyses for moments in the slab and beams. The importance of these neglected factors was studied by means of the test program, which included only tests of I-beam bridges, however. The scope of the laboratory test program included both right and skew, simple-span, I-beam bridges and continuous, right I-beam bridges of two spans. In all cases, both composite and noncomposite bridges were investigated.

Simple-Span, Right, I-Beam Bridges

Analyses. The principal variables considered in the analyses of I-beam bridges were the number and spacing of the beams, the ratio of beam spacing to span, and the relative stiffnesses of the beams and the slab. In all of the analyses made in connection with this investigation, the beams were assumed to be equally spaced. Analyses for bridges with either three or five equally spaced beams were made in 1938–1939 and reported in a thesis (*24*). The analyses for the five-girder bridge were extended and refined, and reported in detail in Bulletin 336 (*7*). This bulletin contains tables giving influence values for moments in the slab and in the beams for 20 simple-span, right bridges having various ratios of beam spacing to span and various ratios of beam to slab stiffness. Influence values are given also for deflections of the beams. In addition, maximum live-load moments due to standard truck loadings were presented for 52 bridges having various span lengths, beam spacings, and ratios of beam to slab stiffness.

Laboratory Tests. Tests were made during 1939–1944 on 15 quarter-scale models of simple-span, right, I-beam bridges having five beams, and the results reported in Bulletin 363 (*11*). The principal variables were the span length, the amount of reinforcement in the slab, and the presence of shear connectors to provide interaction between the slab and the beams. The span lengths of the model bridges were 5 feet in nine cases and 15 feet in the other six, corresponding to prototype span lengths of 20 and 60 feet, respectively. The five supporting beams were spaced at 18 inches in all cases. The slab was made from sand-cement mortar reinforced with annealed ⅛-inch-square, cold-rolled bars and was 1¾ inches thick in all bridges. The beams of the short-span bridges consisted of 3-inch I-sections while those of the longer span structures were either 8- or 10-inch Junior Beams. Shear connectors, where used, consisted of short sections of 1-inch bar channel welded to the top flange of the steel beam and embedded in the concrete slab.

In general, the bridges were tested in pairs, i.e., two companion specimens embodying each set of variables were tested. This was done in order to determine the reliability of the experimental techniques as measured by the reproducibility of the test results.

The test specimens were not directly scaled down from full-size bridges but were themselves designed, using H-20 truck loadings reduced in accordance with the scale relations for the models. Moments used in design were based in part on the theoretical analyses and in part on standard designs then in use by the Illinois Division of Highways. Some bridges in the test series were not actually designed but were simply modifications of types previously tested. In the series of short-span bridges, one pair was tested with the natural bond between the beam flanges and the slab intentionally destroyed, another pair with this bond left in its natural state, and a third pair with shear connectors used to provide a positive mechanical bond and thus produce composite action between the slab and the beams. A similar pattern was followed in the case of the long-span bridges. Two were provided only with natural bond between the slab and the beams, another pair of bridges were identical with the first pair but with shear connectors added to provide a positive mechanical bond. The third pair of long-span bridges also had shear connectors, but in this case the size of the beams was reduced to take advantage of the resulting composite action. Three other short-span bridges were tested with the only variable being the amount of longitudinal reinforcement (parallel to the beams) provided in the slab. This varied from a relatively large percentage of such reinforcement to only a small amount.

The procedure for testing these bridges was similar in practically all respects to that described in connection with the tests of slab bridges. The first tests were made on the uncracked structure and usually consisted of the determination of influence lines for strains in the slab and in the beams for a single concentrated load placed at various locations on the structure. The slabs were then cracked by the systematic application of a pair of loads at various locations. After cracking, the influence line tests were usually repeated. Tests were then made with concentrated loads representing truck wheels placed at critical locations on the structure to produce maximum moments in either the beams or the slab. The load was increased in increments in these live-load tests, and strains and deflections were measured at each increment. The final test was a test to failure, usually with two loads placed to produce maximum moments in the slab. This test was carried until the slab failed by punching.

A principal object of the test was to determine whether the theoretical analysis, limited as it was by numerous assumptions, could be used to predict the behavior of the slab and beams in an I-beam bridge. The results were encouraging. Although measured and computed strains in the slab reinforcement were usually in relatively poor agreement, the discrepancies could be explained partly in terms of the usual differences between measured and computed strains in reinforced-concrete members and partly as the result of special conditions encountered in the tests. Outstanding among the latter was the redistribution of moment between cracked and uncracked sections of the continuous slab, a phenomenon which was investigated further in the tests described in another portion of this paper (*28, 29*). The distribution of moments to the several beams as determined from measured strains was in excellent agreement with the distribution predicted by the analysis. The effects of composite action on the behavior of the bridges was predicted quite closely by the theoretical analyses involving only simple assumptions regarding the increased stiffness of the beams.

Another objective of the tests was to obtain data on the ultimate strengths of typical I-beam bridges, an aspect of behavior which could not be predicted solely on the basis of analysis. In terms of applied live loads, the capacities of the slabs before yielding of the reinforcement ranged from 2.4 to 3.3 live loads for the short-span bridges without composite action and from 5.0 to 6.0 live loads for the long-span bridges and for the short-span bridges with composite action. Ultimate failure of the slabs was in all cases by punching, at loads averaging about 80 percent greater than those producing first yielding in the reinforcement. The capacities of the beams in bridges representing typical designs were equal to about 3.3 live loads as measured by the beginning of yielding.

Recommendations for Design. Recommendations for the design of simple-span, right, I-beam bridges were developed primarily on the basis of the theoretical analysis, since its general validity had been demonstrated by comparison with the test results and since a greater range of variables could be studied analytically than had been considered in the experimental program. Expressions were developed for the controlling moments in the slab and in the beams. These expressions involved originally all of the variables known to affect the behavior and strength of I-beam bridges. However, the number and range of variables was reduced and the design equations simplified by making designs for a large number of typical structures under various types of loading and determining the range of variables over which they needed to be considered. The resulting recommendations for design have been presented in two papers (*10*, *14b*). A discussion of the variables which affect the behavior of I-beam bridges and of the many factors which must be considered in design, particularly in connection with the distribution of loads to the girders, has been presented also in Reference 19.

Simple-Span, Skew, I-Beam Bridges

Analyses. No analyses of the skew I-beam bridge were made at the time this phase of the project was undertaken. Consideration was given to the use of difference equations in the manner utilized in connection with analyses of skew slab bridges with curbs, but the large number of solutions required and the number of simultaneous equations that would have to be solved in connection with each solution rendered such an attempt prohibitively costly in terms of both time and money. It was necessary, therefore, to plan and undertake a series of tests on skew I-beam bridges of such a nature that the results could be compared directly with those from the tests of right bridges and the effects of skew could thereby be evaluated.

Analyses were made in 1953 for moments in a number of skew I-beam bridges having five beams and having angles of skew of 30, 45, and 60 deg. Other variables included the ratio of beam spacing to span and the relative stiffness of the beams and slab. Influence coefficients were computed for moments in the beams at five locations at or near midspan, for transverse moments in the slab at four locations at midspan, and for deflections of the center beam at midspan. These solutions by means of difference equations were made possible by utilizing ILLIAC (the University of Illinois' electronic digital computer) for the solution of the simultaneous equations. The results of these analyses have been presented in a thesis (*30*).

Laboratory Tests. In 1940–1941, tests were made on five I-beam bridges having angles of skew of 30 and 60 deg. and the results reported in Bulletin 375 (*13*). The structures tested were quarter-scale models of simple-span bridges and, except for the skew, were similar in all respects to the long-span, right, I-beam bridges described in Bulletin 363. One bridge with each angle of skew was constructed without composite action and corresponded to one of the right bridges similarly constructed. In the same manner, the 15-foot-span right bridge with composite action was represented in these tests by similar structures with 30-deg. and 60-deg. angles of skew. The fifth skew bridge was also provided with composite action, but the beam size used was the same as that used in the noncomposite bridges. This bridge was constructed only with a skew angle of 30 deg. and it too corresponded to one of the right bridges tested previously.

The tests made on these bridges were similar in most respects to those described in connection with the right I-beam bridges. However, instead of influence line tests in which a load was moved along a single line across the bridge, influence surfaces were obtained by applying a single load at a greater number of points extending in both directions from the point at which the influence was to be observed. This procedure was made necessary by the absence of a theoretical analysis. The influence surfaces thus obtained served as a guide to the proper location of the wheel loads in subsequent tests under simulated live loads and in the tests to failure.

The results obtained in these tests were compared in every case with the corresponding results from the tests on similar right bridges. In this way, the effect of skew over the range of 0 to 60 deg. could be evaluated. Strains in the beams were found to decrease as the angle of skew increased; the effect was quite small at an angle of 30 deg. and was on the order of 15 to 20 percent at an angle of 60 deg. The favorable effect of skew on the behavior of the beams is due primarily to a decrease in the total statical moment of the live loads resulting from a reduction in the right span length. This effect, however, is partially counteracted by the more-nonuniform distribution of moment to the beams in the skew bridges. Transverse strains in the slab reinforcement under a load at the middle of a panel between beams were increased as the angle of skew increased. These effects amounted to as much as 65 percent in the noncomposite bridges with a 60-deg. angle of skew. Transverse strains in the slab reinforcement over the beams, resulting from negative moments, were decreased as the angle of skew increased. This behavior was consistent with the increase in positive moment strain.

Strains in the slab reinforcement parallel to the beams increased with angle of skew for the noncomposite bridges and decreased appreciably for the composite bridges. The effect of skew on the ultimate strength of the bridge as measured by yielding of the beams could not be determined accurately, because of the presence of large residual stresses of unknown magnitudes in the beams. The ultimate capacities of the slabs as measured by the loads required to produce punching were practically unchanged from those obtained in the tests of right bridges.

Recommendations for Design. Because no analyses were made and because the tests were limited in scope, no generally applicable procedures for designing skew I-beam bridges could be developed on the basis of these investigations alone. However, a rational analysis for the effect of skew on the moments in the beams has been presented *(14b)*. Although the measured strains in the slab reinforcement were often appreciably greater in the skew bridges than in the corresponding right bridges, in no case were they greater than the strains computed on the basis of the theoretical analyses for the right bridges. For this reason, it was felt that the design moments recommended for the right bridge, and based almost entirely on the results of the analyses, could be used also for the skew bridges.

Continuous Right I-Beam Bridges

The studies of continuous right I-beam bridges were intended to: (1) compare the action of both the beams and the slab in the positive moment region of the continuous bridge with the behavior previously determined from the tests and analyses of simple-span bridges; (2) determine the behavior of the beams and the slab in the negative moment region of the bridge and compare the magnitudes and distribution of the strains in the beams and slab in this region with those in the region of positive moment; and (3) determine the effects of composite action on the behavior of continuous I-beam bridges and compare the behavior of composite bridges both with and without shear connectors provided in the region of negative moments.

Although the continuous, right, I-beam bridge could be analyzed by the methods described in Bulletin 304, the amount of time and effort required would have been several times greater than that required for the comparable analyses of simple-span bridges reported in Bulletin 336. It was decided, therefore, to rely primarily on laboratory tests in order to investigate the behavior of continuous bridges as compared with that of right bridges. The lack of theoretical data, except for the limited analyses mentioned subsequently, did not represent a serious deficiency, since the comparison between the action of the continuous bridge in the region of positive moment and the previously observed behavior of the simple-span bridge could be made more easily and more directly by experiment than by analyses. Moreover, the investigation of the effects of composite action and the presence or absence of shear connectors in the region of negative moment could be made only by means of tests.

These investigations were confined to two-span bridges. Although it was realized that most highway structures are of the three-span type, it was felt that the simpler, two-span structure involved all of the new variables resulting from the introduction of continuity and offered certain advantages in connection with the space required for the laboratory tests and the choice of model dimensions.

Analyses. As a guide in the planning of the tests, analyses were made for two bridges having five beams and continuous over two equal spans. The particular structures selected for analysis were chosen so as to permit the use of moment and deflection coefficients tabulated in Bulletin 336. Although the bridges analyzed were not representative of the structures tested, study of the results obtained permitted the development of a simplified method of analysis which gave results in good agreement with the more exact theory. This simplified method was used to make preliminary analyses of the test bridges. For the bridges with composite action, consideration had to be given to the fact that the moments of inertia of the beams were different in the regions of positive and negative moment. In the positive moment regions, the slab of the composite beam is placed in compression and can act, therefore, as a top flange, which appreciably increases the stiffness of the beam. In the negative-moment regions, the slab is placed in tension and contributes little to the stiffness of the structure after the slab has cracked. This variation in stiffness and its effect on the distribution of moments between positive and negative regions was taken into account approximately in the preliminary analyses of the bridges tested.

The theoretical analyses of both the exact and simplified method are described in the appendix to Bulletin 416 (*20*).

Laboratory Tests. During 1945–1948, three continuous I-beam bridges were tested and the results have been reported in Bulletin 416 (*20*). The test specimens were quarter-scale models of two-span continuous bridges; each span in the model was 15 feet. The other dimensions of the bridges, slab thickness, beam spacing, etc., were similar to those of the simple-span bridges discussed previously. The sizes of the beams, however, in the continuous bridges were determined from designs of prototype structures for H-20 lane loadings. The bridges differed primarily in provisions for composite action; one bridge was entirely noncomposite, one was provided with shear connectors throughout the entire length of the bridge, while the third was provided with shear connectors only in the positive-moment regions. In addition, slightly smaller beams were used and somewhat less transverse slab reinforcement was provided in the bridges with composite action. The test procedures were generally similar to those described in connection with the other tests of I-beam bridges. A noteworthy feature of these tests, however, was the extremely large number of strain measurements that were required in order to give a complete picture of the behavior of these bridges. Strains in both the beams and the slab reinforcement were measured by means of SR-4 electrical-resistance strain gages. A total of about 330 gages were used on each of the model bridges.

Strains in the slab reinforcement in the positive-moment regions of the continuous bridges were usually comparable to those observed in the tests of simple-span bridges. Strains meas-

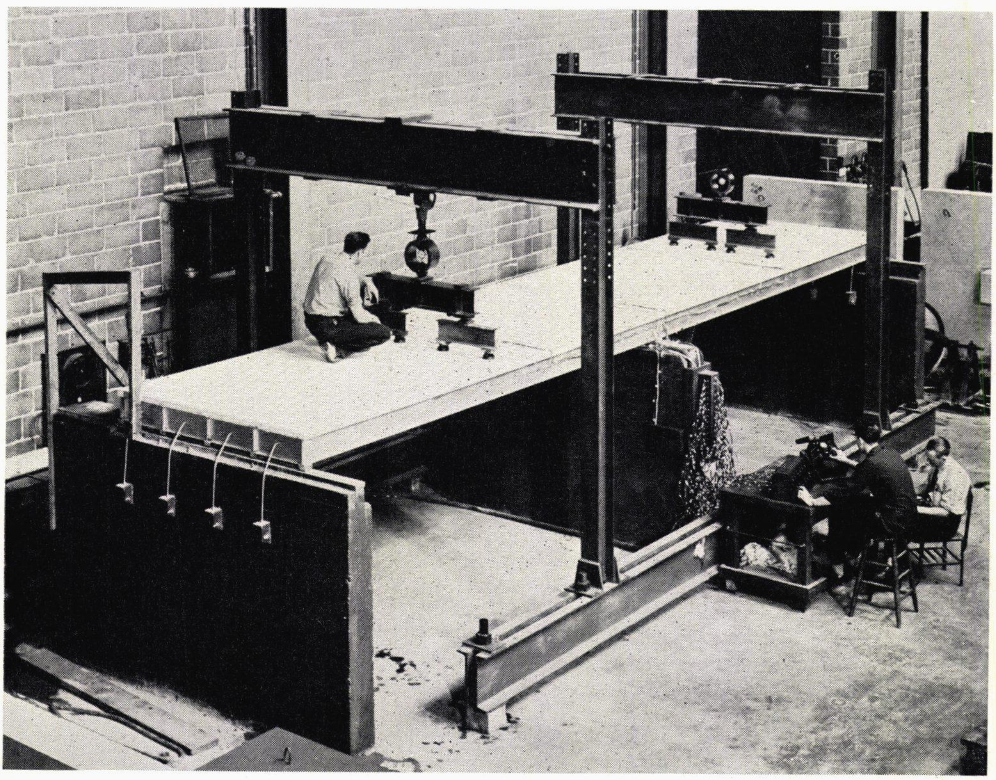

Figure 2. Test of a quarter-scale-model, continuous, I-beam bridge having two spans of 15 feet each. Loads representing heavy axles in both lanes are being applied near the middle of each span.

ured in the beams for various loading conditions indicated that the distribution of moment to the beams in the region of positive moment was essentially the same as that for a simple-span bridge having a span length equivalent to the span between points of contraflexure in the structures tested. The tests also demonstrated, in complete agreement with the results of preliminary analyses, that the distribution of moment to the beams in the region of negative moment over the center pier was, for all practical purposes, the same as the distribution of positive moments. In the region over and adjacent to the center pier, the measured strains in the top transverse slab reinforcement were usually significantly greater than the strains in the bottom or positive-moment reinforcement. This relationship between strains in the top and bottom of the slab was exactly opposite that obtained for the simple-span bridges and in the positive-moment regions of the continuous bridges. The difference is explained by the fact that the supporting beams were practically nondeflecting in the region adjacent to the pier. The longitudinal-slab reinforcement in the region over the center pier was subject to relatively high participation stresses in the bridges with composite action, but little difference was observed between the magnitudes of these strains in the bridges with and without shear connectors in the region of negative moment. The results of the tests bore out the assumption made in the analysis that the slab contributed little by way of composite action in the regions of negative moment. The presence or absence of shear connectors in this region had relatively little effect.

Recommendations for Design. The relation of the test results to design has been discussed in Bulletin 416, and more-detailed recommendations for the design of continuous I-beam bridges have been given (*14b*). The results of both the analyses and the tests indicate that the beams can be designed for the same proportion of a wheel load as that proposed for use in connection with simple-span bridges and that the slab in the regions of positive moment in the bridge may be similarly designed using the expressions for moment recommended for use with simple-span bridges. In both cases the span length of the simple-span bridge used in these calculations should correspond to the portion of the continuous bridge between points of contraflexure. The distribution of moment to the beams may be considered the same in regions of positive and negative moment, but in composite bridges the variation in stiffness of the beams must be considered in apportioning the moment to the positive- and negative-moment regions. It has been recommended further that the amount of transverse reinforcement in the top of the slab should be increased in regions over or adjacent to the piers. The tests revealed no significant differences in the behavior of continuous bridges with or without shear connectors in the region of negative moments. The question of composite construction in continuous I-beam bridges has been discussed also in Reference 21.

Concrete Girder Bridges

Bridges with concrete girders differ from those with steel I-beams principally in that the stiffness of the beams relative to the slab is much higher. No tests were made of this type of bridge in connection with the concrete-slab investigation. However, analytical studies similar to those reported for I-beam bridges in Bulletin 336 were made for a number of typical concrete-girder bridges. Beam stiffnesses were computed for a T-section including a portion of the slab having a width equal to the spacing between beams. On the basis of these studies, tentative recommendations for the design of the slabs and beams in concrete girder bridges have been presented (*10*). Certain aspects of the behavior of concrete girder bridges as compared to I-beam bridges have been discussed also in Reference 19.

Intermediate Transverse Diaphragms in Slab-and-Girder Bridges

Analyses. The effects of intermediate transverse diaphragms on the distribution of moment to the beams in slab-and-girder bridges was studied analytically in 1947–1951 and the results presented in a thesis (*27*). All of the analyses were made for bridges having five equally spaced and equally stiff beams, and only live loads represented by groups of concentrated loads were considered. The variables studied included the ratio of beam spacing to span, the relative stiffnesses of the beams and slab, the relative stiffnesses of the diaphragms and beams, and the number and location of the diaphragms in the structure. The results of these analyses showed clearly that the effect on load distribution of the slab and diaphragm acting together was not necessarily the same as, and might be appreciably less than, the sum of their effects acting separately. The additional load distribution provided by diaphragms was usually small, unless the diaphragms were very stiff as compared to the beams or the slab was initially incapable of providing much distribution as a result of its stiffness being small compared to the stiffness of the beams. This latter condition is likely to occur in the case of concrete-girder bridges and, to a lesser extent, in the case of composite I-beam bridges.

Laboratory Tests. The effects of diaphragms were investigated in the tests of both right and skew, simple-span, I-beam bridges (reported in Bulletins 363 and 375, respectively). The diaphragms used, however, were relatively light, having stiffnesses only 2 to 5 percent of the stiffness of the beams, and the results of tests made before and after the diaphragms were welded into place showed no detectable effect on the behavior of the bridge. These results are in agreement with the findings of the analyses.

Recommendations for Design. The analytical studies demonstrated that the behavior of slab-and-girder bridges with transverse diaphragms is exceedingly complex. Consequently, it has not been possible to formulate simple rules for the design of bridges with diaphragms or for the design of the diaphragms themselves. The effects of diaphragms on the distribution of loads to the girders has been qualitatively discussed (*19*). Generally, it can be stated that diaphragms to be most effective must be relatively stiff, as compared to the beams, and they will have their greatest effects in bridges for which the ratio of beam to slab stiffness is high, viz., in concrete-girder bridges. It has been pointed out further that the use of stiff diaphragms may not always prove beneficial insofar as their effect on maximum beam moments is concerned. For live loads placed on the bridge in such a manner that their center of gravity is located eccentrically with respect to the longitudinal center line of the bridge, the use of stiff diaphragms may increase appreciably the moment in the outside beam on the side of the bridge toward which the loads are shifted.

COMPOSITE CONSTRUCTION FOR I-BEAM BRIDGES

Introduction

A major portion of the concrete-slab investigation has been devoted to studies of I-beam bridges in which a shear connection has been provided between the slab and the beams in order to make these elements act compositely. The advantages of composite construction and the economies which may be brought about by its use have been discussed in Reference 14c.

The effects of composite action on the overall behavior of I-beam bridges was studied both analytically and experimentally in connection with the work on I-beam bridges described in an earlier portion of this paper. In the analyses it was assumed that the presence of composite action could be taken into account by considering only the increase in stiffness of the beams relative to the slab. The inclusion of bridges with composite action in the extensive test program on quarter-scale models provided a means for checking this assumption and furnished additional data regarding the effects of composite action on the ultimate load-carrying capacities of the bridges. Comparisons of the results of tests and analyses showed that the distribution of moment to the beams was more nonuniform for the bridges with composite action because of the greater relative stiffness of the beams. This finding was in excellent agreement with the results of analyses. Similarly, the effects of composite action on the moments in the slab were in general conformity with the analytical solutions. There was some indication, however, in the results of the tests, that greater restraints to rotation of the slab over the beams were present when the slab was attached to the beams by means of shear connectors. One effect of this added restraint was to decrease slightly the positive moments in the slab and to increase correspondingly the negative moments. A further consequence of these changes in slab moments was an increase in resistance of the slab to punching observed in the bridges with composite construction. The tests to failure also showed clearly that the toughness and reserve capacity of the I-beam bridges tested were increased markedly in the presence of composite action.

The studies of composite action in I-beam bridges, mentioned in the foregoing paragraph, provided adequate information for the design of such bridges, insofar as the moments in the slab and beams were concerned. However, additional studies were required to provide information needed in connection with the design and proportioning of the composite beams themselves. Investigations were made, therefore, to determine the strength and behavior of (1) the shear connectors themselves and (2) the isolated composite beam consisting of a single steel I beam and a narrow strip of concrete slab.

Shear Connectors for I-Beam Bridges

Laboratory Tests. The behavior of shear connectors was investigated during 1942–1950 by a relatively large number of static and fatigue tests on small-scale models and by a somewhat smaller number of static tests on full-size specimens. The small-scale tests have been described and the results presented (*14c*, *17*), while results of the full-size tests are included in Bulletin 405 (*18*).

Since the early tests were intended to be primarily exploratory in nature, small-scale models were used in order that a large number of variables could be studied with the most-economical expenditure of time and money. The first tests were made on 64 pushout-type specimens, each consisting of a short length of steel I beam and two cement-mortar slabs, one attached to each flange of the beam by means of a single shear connector. In the tests, the I beam was loaded so as to produce a shear on the two connectors, and slips between the beam and the slab were measured at each increment of load. Several types of shear connectors were included in the tests: channels, plates bent to a leaning Z shape, straight plates, angles, and tees. The variables included both the width and thickness of the connectors, as well as the strength of the mortar slab and the nature of the bond between the mortar and the beam flange or the shear connector itself.

The types of shear connectors tested can be divided into two categories: rigid connectors, consisting of channels, angles, or tees placed on end; and flexible connectors, consisting of plates or of channels with one flange welded to the beam. Although the load-slip characteristics of the rigid connectors were noticeably superior to those of the flexible type, the differences were much smaller than would be expected from the relatively great differences in stiffnesses of the two types. Furthermore, it was observed that moderately large variations in the thickness of the webs of the flexible connectors had only a small effect on their behavior. These observations led to the concept that flexible connectors acted much like a dowel embedded in an elastic medium and that the bearing pressures exerted by the mortar are nonuniformly distributed on the back of the connector, with the highest pressures at points close to the beam where a channel connector, for example, is stiffened by the flange and by the weld. This concept of dowel-like action led to two important conclusions: (1) The bending moments in the webs of the channel shear connectors were relatively small because of the low bearing pressures on the upper portions of the connector, and the magnitudes of these moments would be a function of the relative stiffnesses of the connector and the concrete. (2) The bearing pressures would be quite high on the lower portions of the connectors.

Further studies were made during 1944–48 to investigate the behavior of flexible channel shear connectors in fatigue. A need for studies of this nature was indicated in part by the fact that shear connectors in I-beam bridges acting compositely for live load only may be subjected to a great many cycles of stress ranging from zero to a maximum, or even to reversals of stress. Fatigue tests were also expected to determine whether the high concentration of bearing pressure at the bottom of the connector would lead to failure of the concrete in fatigue; in addition, the fatigue tests would provide some measure of the bending stresses in the channel web. These tests were made only with channel shear connectors of the flexible type, and again recourse was had to scale-model specimens. Since pushout specimens could not be tested conveniently in fatigue, the shear connectors to be tested were incorporated in a composite beam consisting of a 3-inch, steel I beam and a 1¾-by-18-inch mortar slab. These beams were loaded at the center of a 5-foot span. A total of 85 beams were tested, involving as principal variables the thickness of the channel web, the thickness of the flange welded to the beam, and the strength of the mortar slab. The beams were first loaded statically to the load to be used in the fatigue test in order to break the bond between the beam and the slab and measurements made of strains, deflections, and slip for each increment of applied load. They were then tested in fatigue under a loading varying from near zero to a maximum value, which was different for otherwise similar beams. Failure was considered to occur when a slip of 0.01 inch was observed at the end of the beam. Special studies indicated that a slip of this magnitude corresponded in all cases to a considerable decrease in composite action brought about by the failure of several shear connectors. At the conclusion of the fatigue tests, each beam was tested to failure under static load applied in increments, and strains, deflections, and slips were observed. In general, the tests were stopped after 2 million cycles, if failure had not occurred previously. However, a few tests were continued beyond this point, and failure was observed after as many as 9 million cycles.

Although the results of the fatigue tests from small-scale models were not entirely conclusive and could not be applied directly to predict the behavior of full-size structures under similar conditions, they yielded, nevertheless, a certain amount of quantitative data and provided considerable insight into the behavior of channel shear connectors. In general, the results of the fatigue tests confirmed qualitatively the concept of dowel-like action of flexible shear connectors. Although this concept would indicate the presence of exceptionally high compressive stresses in the mortar slab adjacent to the bottom of the connector, no failures of the slab in compression were observed, except in a few cases where the strength of the mortar was quite low. In all other cases, the channel shear connectors failed by a fracture of the web on a line through the fillet between the web and the flange closest to the beam. An increase in thickness of the channel web produced an increase in the fatigue strength, but the magnitude of this increase was much less than the change in section modulus of the web. Similarly, an increase in compressive strength of the mortar produced a corresponding increase in fatigue strength of the test beams, but this effect was smaller for high strengths than for low strengths. Although the principal emphasis in these tests was placed on the strength and behavior of the shear connectors themselves, some information regarding the behavior of composite beams was obtained. The fatigue failure of the shear connection in these beams was progressive, beginning at the ends of the beam and progressing toward the load point. A significant decrease in the degree of interaction between the slab and the beam took place only after several of the shear connectors had failed.

As is frequently the case in exploratory tests, somewhat more questions were raised than were answered by the tests on small-scale models. Further study of stresses in the channel webs and of the distribution of bearing pressure on the back of the channel was required before a complete picture of the behavior of channel shear connectors could be drawn. For these purposes, static tests were made in 1947–1950 on 43 full-size pushout specimens, similar in all respects to the small-scale specimens but approximately four times as large. In addition to slip measurements, strains on the webs of the channels were measured by means of strain gages attached to the web and protected from the concrete by means of a waterproof coating. The principal variables in these tests were the concrete strength and the dimensions of the channel shear connectors, including web thickness, flange thickness, height, and width. Both measured slips and web strains were used to evaluate the effects of these variables on the behavior and strength of the channel shear connectors. In addition, the distributions of strain along the height of the web were utilized to compute distributions of bearing pressure over the height of the connector.

The results of these tests confirmed those obtained from the tests with small-scale models, insofar as the effects of variations in web thickness or concrete strength were concerned. In addition, quantitative information was obtained regarding the effect of variations in the thickness of the channel flange attached to the beam. The effect of varying the width of the connector was linear, as would be expected, and the effect of varying the height of the channel above the surface of the beam was negligible over the range considered in these tests. This latter finding, as well as the observed strains and derived pressure distributions, confirmed fully the dowel concept. The quantitative data obtained from slip and strain measurements were utilized subsequently in the development of analytical procedures and design criteria as described below.

Analyses. A relatively crude analysis for channel shear connectors was presented in Reference 14c. In accordance with the dowel concept, the pressure distribution on the back of the channel was assumed to be a function of the relative stiffness of the connector and the concrete. Expressions were derived for stresses in the concrete and in the channel web. Subsequent tests on full-size channel shear connectors, however, showed this analysis to be unduly conservative, although qualitatively it served fairly well to account for the effects of some of the major variables.

The strain measurements on the full-size connectors and the pressure distributions derived from them provided the basis for a much-more-accurate (but also more-complex) analysis which was capable of predicting satisfactorily the behavior of channel shear connectors. This analysis was partly rational, in that it was based on the conventional expressions for a dowel embedded in an elastic medium, but it involved also a number of empirical coefficients evaluated on the basis of the available test data. For this reason, it is limited in applicability to channel shear connectors of the sizes and types tested. Expressions for computing the distribution of bearing pressure, the stress in the channel web, and the magnitude of the slip have been presented in Bulletin 405 and the results compared with those obtained in the tests. Since these expressions were rather complicated, simplified formulas for pressures, stresses, and slips were developed. These are given also in Bulletin 405 and compared with the test data and with the results obtained from the more exact theory.

Recommendations for Design. A rather complete discussion of the factors which must be considered in the design of channel shear connectors for composite I-beam bridges has been presented in Reference 21, and detailed recommendations for design, including the necessary formulas and an example illustrating their use, have been presented in Reference 22. Although the design formulas given are based on the theoretical analysis, the criteria for design were influenced to a considerable extent by observations of the behavior of channel shear connectors in the tests of full-size composite beams described subsequently. Design features covered in Reference 22 include: allowable loads, ultimate capacity, repeated loading, spacing, cover, and the design of welds.

Composite Beams

Analyses. The analysis of a composite steel and concrete beam by means of the usual transformed area method is satisfactory only if both materials remain elastic and if the interaction between them is perfect, that is, there is no slip. At the level of stress usually associated with working loads, the condition of elasticity will usually be satisfied, except for the possible presence of creep in the concrete if it is subjected to sustained stresses. Creep will occur if the composite beam is shored in order to carry dead load on the composite section, or if added dead load in the form of curbs, hand rails, or wearing surface is placed on the bridge after the slab has been cast. The effects of creep, however, are seldom important, except in the case of shored beams. As the beam approaches its ultimate capacity, inelastic action will be present in both the steel and the concrete, and the transformed-area method cannot be used. Unless the bond between the slab and the beam is still intact, some slip will occur at all levels of loading. The result is a beam with incomplete interaction, and the transformed-area method of analysis will be in error, unless the slip is quite small. Analyses have therefore been made of composite beams under two conditions for which the transformed area method is inapplicable: (1) for the beam with incomplete interaction and (2) for the ultimate capacity. The effects of creep have been discussed briefly (*14c, 21*).

A method of analysis for composite beams with incomplete interaction has been presented in a previous paper (*16*) and in the appendix to Bulletin 396. This analysis involves certain simplifying assumptions regarding the manner in which the shear connection is distributed along the length of the beam and requires a knowledge of the load-slip relationship for the shear connection. Expressions are derived in Bulletin 396 for stresses and deflection of the beam, loads on the shear connectors, and slip, in terms of the load-slip characteristics of the shear connectors. The theoretical analysis is compared with the results of tests, using load-slip relations for the shear connectors derived from the tests (*16*). An expression is given in Appendix B of Bulletin 405 for the load-slip characteristics of a channel shear connector in terms of its physical characteristics.

A simple method of computing the ultimate moment-carrying capacity of a composite steel and concrete beam with full interaction was developed and has been described (*21*). This analysis involves the following assumptions: (1) linear distribution of strain, (2) no tension in the concrete, (3) a rectangular compressive stress block in the concrete at failure, (4) failure of concrete at a limiting compressive strain, and (5) full yielding across the steel beam section at failure.

Laboratory Tests. Laboratory tests on three, quarter-scale model composite T-beams were made in 1943 and the results reported in Bulletin 396. More-extensive tests on four full-size beams were made during 1948–1950 and reported in Bulletin 405.

The quarter-scale model beams were identical with those used for the fatigue tests of shear connectors. Only two variables were considered: the compressive strength of the mortar slab and the stiffness of the shear connectors. A number of load tests were made on each beam, the last test being carried to failure. At each increment of load, measurements were made of strains in the I-beam and the slab, deflection of the beam, and slip between the slab and the beam at various points along the span. The results of these tests showed conclusively that some slip was present at even the smallest loads; however, the measured strains and deflections agreed well with those computed for a beam with incomplete interaction. It was observed also that the load-slip characteristics of the shear connectors in the beams compared satisfactorily with those observed in the pushout tests.

The full-size composite beams consisted of a 21- or 24-inch wide-flange beam and concrete slab 6 inches thick and 6 feet wide, tied together with shear connectors made from 4-inch channels. The beams were tested on a span of 37.5 feet. The first beam tested was made with a 24-inch wide-flange section and was constructed without shores, that is, the dead load of the beam and slab was resisted by the steel beam alone and composite action was effective only for added live load. The second beam was identical with the first, except that a shore was placed at midspan and not removed until the concrete had hardened; in this case, the moments resulting from the removal of the shore were carried by the composite section, and the stresses under dead load were smaller than those in the first beam. The third beam was also constructed with a shore at midspan, but a 21-inch wide-flange section was used in order to take advantage of the reduced dead load stresses. The fourth beam also used a 21-inch section but was constructed without shores and the shear connection provided was extremely weak, consisting of 4-inch lengths of 4-inch channels at a spacing of 3 feet. The bond between the steel beam and the slab was destroyed in all except the last beam where "natural" bond was permitted to develop. The beams were tested under a single concentrated load applied at midspan, at the quarter-point of the span, or near one support. Measurement was made of deflection, strains in the beam and slab, slip, and strains in the shear connector webs in a manner similar to that used in the pushout tests.

The results of these tests demonstrated clearly the great toughness and reserve of strength and deflection after yielding possessed by composite beams in which the shear connection was adequate to provide almost full composite action to failure. In the beam provided with a weak shear connection, the connectors yielded simultaneously with the steel beam, and the consequent decrease in interaction reduced appreciably the maximum load-carrying capacity of this beam. In all of the tests, the behavior of the beams, both at working load and at ultimate, was in good agreement with the predictions of the theoretical analysis. The behavior of the shear connectors in the composite beams compared closely with that observed in the pushout tests.

A major objective of these tests was to compare the behavior of beams constructed with and without shores. At working loads the stresses were in good agreement with those predicted by the transformed area method assuming complete interaction, that is, the dead-load stresses in the bottom flange of the steel beams were reduced and the loads causing yielding were slightly increased in the beam with shores as compared to a similar beam without shores. However, the ultimate strengths of these two beams were practically identical. On the other hand, comparisons of beams with and without shores, but having different size steel beams in order to produce equal stresses at working loads, showed that both the yield and the ultimate loads were appreciably less for the lighter beam designed for use with shores. These findings are in agreement with the analysis for ultimate strength which considers inelastic behavior after yielding and thus yields strengths which are independent of the differences in "elastic" stress distributions resulting from the use of shores. Even in the elastic range, however, the effect of using shores was found to be small as the yield load was approached, since only the dead-load stresses were affected by the method of construction, and the behavior of the beams under the added live load was the same in both cases.

Recommendations for Design. Some of the factors that must be considered in the design of composite beams for I-beam bridges have been discussed briefly (*14c*). Included were the effects of slip on the degree of interaction, the effective width of slab to be used as the T-beam flange, the value of the modular ratio to be used in computing properties of the transformed section, the effects of creep, and methods for proportioning beams utilizing unsymmetrical steel sections.

A more-complete discussion of design criteria, based on the results of the test programs as well as on the theoretical analyses, appears in Reference 21. These studies have shown that the transformed-area method may be used for proportioning composite beams if care is taken to provide an adequate shear connector. However, the tests have shown further that the great toughness of composite beams can be obtained only if the shear connection is strong enough to provide full interaction up to the maximum load-carrying capacity of the composite beam. In order to accomplish this, it is necessary to provide shear connectors that will not yield or otherwise permit large slips until the ultimate capacity of the beam is reached. Shear connectors having the same factor of safety as the beam will not prove satisfactory in this respect, since they will yield at the same time as the beam and the degree of interaction will decrease in the potentially large range between yield and maximum load. Other recommendations have been given for the effective width of slab to be considered as acting with the steel beam, and for procedures to be used in the design of beams constructed with shores.

It has been pointed out in several places (*14c*, *17*, *18*, *21*) that bond between the slab and the beam may provide an effective shear connection so long as it is present. However, the results of both laboratory tests and field experience have shown conclusively that bond cannot be relied on as a permanent shear connection under service conditions and that it will almost inevitably be broken before the ultimate load-carrying capacity of the beam is reached. For these reasons, it has been recommended most strongly that a positive mechanical-shear connection be provided in all beams designed for composite action.

ACKNOWLEDGMENTS

The investigation was carried out as a coöperative research project between the Engineering Experiment Station of the University of Illinois, the Illinois Division of Highways, and the Bureau of Public Roads. Since 1952, the project has been a part of the Illinois Coöperative Highway Research Program.

Credit for initiating the program should go to Alfred Benesch, then engineer of grade separations for the Illinois Division of Highways, who pointed out the need for additional information on many aspects of highway bridge design and suggested that this research be undertaken. The continued and effective coöperation of the Bureau of Public Roads has been due in no small measure to the great interest shown in the project by E. F. Kelley, chief of the Physical Research Branch.

The work has been guided by an advisory committee composed of two or three representatives from the state, the bureau, and the university. This committee assisted and advised in the planning of the various projects and reviewed the results presented in progress reports or for publication. Its guidance, advice, and criticism contributed greatly to the success of the program. The membership of the committee has changed from time to time; throughout the course of the work the following persons have served: For the Illinois Division of Highways, A. Benesch, E. Lieberman, M. J. Fleming, G. F. Burch, L. E. Philbrook, W. J. Mackay, W. E. Baumann, W. E. Hanson, and W. E. Chastain, Sr. For the Bureau of Public Roads, E. F. Kelley, A. L. Gemeny, L.

A. Palmer, R. Archibald, and E. L. Erickson. For the University of Illinois, F. E. Richart, N. M. Newmark, V. P. Jensen, C. P. Siess, and E. J. Daily. Consultants to the advisory committee included T. C. Shedd, W. M. Wilson, and Hardy Cross, during the early stages of the project, and Fred Kellam, of the Bureau of Public Roads, who contributed much during the later stages.

Overall direction of the project was provided jointly by F. E. Richart, from 1936 until his retirement due to ill health in 1949, and by N. M. Newmark, from 1936 to 1954. Supervision of various phases of the work has been the responsibility at various times of V. P. Jensen (1936–43), R. W. Kluge (1936–41), C. P. Siess (1941–54), and I. M. Viest (1951–54). Other full-time members of the project staff were K. C. Lowe (1939–41), M. N. Tokay (1943–44), W. E. Johnson (1944–48), M. L. Gossard (1945–47), and C. B. Clarke (1947–48).

Most of the analytical or experimental investigations constituting the various phases of the program were conducted by research assistants who devoted half of their time to research on the concrete-slab investigation and half to graduate study. The following persons have served in this capacity: H. A. Lepper, Jr. (1936–38), W. A. Renner (1937–39), C. P. Siess (1937–39). R. R. Penman (1938–40), J. W. Allen (1939–41), W. M. Peckham (1939–41), C. B. Williams, Jr. (1939–41), J. C. Houbolt (1940–42), L. E. Goodman (1940–42), W. J. Austin (1941–42), E. J. Ward (1941–42, 1945–46), R. H. Westcott (1942–43), L. W. Jones (1946–47), J. H. Appleton (1947–49), A. D. Alexander (1947–49), I. M. Viest (1948–50), A. S. Veletsos (1949–50), B. C. F. Wei (1949–51), F. X. Blechinger (1949–51), J. R. Gaston (1950–51), and A. Laupa (1950–52). To these men must go much of the credit for the work of this investigation.

Acknowledgment must also be given to the laboratory mechanics, H. F. Baker, S. L. Smith, Sam Ellis, and W. C. Stitt, whose skill contributed a great deal to the success of the experimental work.

REFERENCES

Published Reports of the Concrete Slab Investigation

1. V. P. Jensen, "Solutions for Certain Rectangular Slabs Continuous over Flexible Supports," *Bulletin No. 303*, Univ. of Ill. Eng. Exp. Sta., (1938).
2. N. M. Newmark, "A Distribution Procedure for the Analysis of Slabs Continuous over Flexible Beams," *Bulletin No. 304*, Univ. of Ill. Eng. Exp. Sta., (1938).
3. N. M. Newmark and H. A. Lepper, "Tests of Plaster-Model Slabs Subjected to Concentrated Loads," *Bulletin No. 313*, Univ. of Ill. Eng. Exp. Sta., (1939).
4. F. E. Richart and R. W. Kluge, "Tests of Reinforced Concrete Slabs Subjected to Concentrated Loads," *Bulletin No. 314*, Univ. of Ill. Eng. Exp. Sta., (1939).
5. V. P. Jensen, "Moments in Simple Span Bridge Slabs with Stiffened Edges," *Bulletin No. 315*, Univ. of Ill. Eng. Exp. Sta., (1939).
6. V. P. Jensen, "Analyses of Skew Slabs," *Bulletin No. 332*, Univ. of Ill. Eng. Exp. Sta., (1941).
7. N. M. Newmark and C. P. Siess, "Moments in I-Beam Bridges," *Bulletin No. 336*, Univ. of Ill. Eng. Exp. Sta., (1942).

8a. V. P. Jensen, "Ultimate Strength of Reinforced Concrete Beams as Related to the Plasticity Ratio of Concrete," *Bulletin No. 345*, Univ. of Ill. Eng. Exp. Sta., (1943).

8b. V. P. Jensen, "The Plasticity Ratio of Concrete and Its Effect on the Ultimate Strength of Beams," *Proceedings*, American Concrete Institute, Vol. 39, pp. 565–584, (1943).

9. V. P. Jensen, R. W. Kluge, and C. B. Williams, Jr., "Highway Slab-Bridges with Curbs," *Bulletin No. 346*, Univ. of Ill. Eng. Exp. Sta., (1943).
10. N. M. Newmark and C. P. Siess, "Design of Slab and Stringer Highway Bridges," *Public Roads*, Vol. 23, No. 7, Jan.–Feb.–March, 1943, pp. 157–164.
11. N. M. Newmark, C. P. Siess, and R. R. Penman, "Studies of Slab and Beam Highway Bridges: Part I—Tests of Simple-Span Right I-Beam Bridges," *Bulletin No. 363*, Univ. of Ill. Eng. Exp. Sta., (1946).
12. V. P. Jensen and J. W. Allen, "Studies of Highway Skew Slab-Bridges with Curbs: Part I—Results of Analyses," *Bulletin No. 369*, Univ. of Ill. Eng. Exp. Sta., (1947).
13. N. M. Newmark, C. P. Siess, and W. M.

PECKHAM, "Studies of Slab and Beam Highway Bridges: Part II—Tests of Simple-Span Skew I-Beam Bridges," *Bulletin No. 375*, Univ. of Ill. Eng. Exp. Sta., (1948).

14. F. E. RICHART, N. M. NEWMARK, AND C. P. SIESS, "Highway Bridge Floors—A Symposium, *Transactions*, American Society of Civil Engineers, Vol. 114, pp. 979–1072, (1949). (Also *Reprint No. 45*, Univ. of Ill. Eng. Exp. Sta., (1949). (a) "Laboratory Research on Concrete Bridge Floors," by F. E. Richart, pp. 980–996. (b) "Design of I-Beam Bridges," By N. M. Newmark, pp. 997–1022. (c) "Composite Construction for I-Beam Bridges," by C. P. Siess, pp. 1023–1045.
15. M. L. GOSSARD, C. P. SIESS, N. M. NEWMARK, AND L. E. GOODMAN, "Studies of Highway Skew Slab-Bridges with Curbs: Part II—Laboratory Research," *Bulletin No. 386*, Univ. of Ill. Eng. Exp. Sta., (1950).
16. N. M. NEWMARK, C. P. SIESS, AND I. M. VIEST, "Tests and Analyses of Composite Beams with Incomplete Interaction," *Proceedings*, Society for Experimental Stress Analysis, Vol. IX, No. 1, pp. 75–92, (1951).
17. C. P. SIESS, I. M. VIEST, AND N. M. NEWMARK, "Studies of Slab and Beam Highway Bridges: Part III—Small Scale Tests of Shear Connectors and Composite T-Beams," *Bulletin No. 396*, Univ. of Ill. Eng. Exp. Sta., (1952).
18. I. M. VIEST, C. P. SIESS, J. H. APPLETON, AND N. M. NEWMARK, "Studies of Slab and Beam Highway Bridges: Part IV—Full-Scale Tests of Channel Shear Connectors and Composite T-Beams," *Bulletin No. 405*, Univ. of Ill. Eng. Exp. Sta., (1952).
19. C. P. SIESS AND A. S. VELETSOS, "Distribution of Loads to Girders in Slab and Girder Bridges: Theoretical Analyses and Their Relation to Field Tests," *Research Reports 14-B*, Highway Research Board, pp. 58–74, (1953).
20. C. P. SIESS AND I. M. VIEST, "Studies of Slab and Beam Highway Bridges, Part V—Tests of Continuous Right I-Beam Bridges," *Bulletin No. 416*, Univ. of Ill. Eng. Exp. Sta., (1953).
21. I. M. VIEST AND C. P. SIESS, "Composite Construction for I-Beam Bridges," *Proceedings*, Highway Research Board, Vol. 32, pp. 161–179, (1953).
22. I. M. VIEST AND C. P. SIESS, "Design of Channel Shear Connectors for Composite I-Beam Bridges," *Public Roads*, Vol. 28, No. 1, April 1954, pp. 9–16.

Unpublished Theses (Including only those theses containing material not published subsequently in the other references listed.)

23. W. A. RENNER, "The Bending of Orthogonally Non-Isotropic Slabs," M.S. Thesis, Dept. of Theo. and Appl. Mech., Univ. of Ill., (1939).
24. C. P. SIESS, "Moments in the Simple-Span Slab and Girder Bridge," M.S. Thesis, Dept. of Civ. Eng., Univ. of Ill., (1939).
25. J. W. ALLEN, "Analyses of Skew Slabs with Curbs," M.S. Thesis, Dept. of Theo. and Appl. Mech., Univ. of Ill., (1941).
26. J. C. HOUBOLT, "The Effectiveness of Various Arrangements of Reinforcement in Concrete Slabs," M.S. Thesis, Dept. of Civil Engineering, Univ. of Ill., (1942).
27. B. C. F. WEI, "Effects of Diaphragms in I-Beam Bridges," Ph.D. Thesis, Dept. of Civil Eng., Univ. of Ill., (1951).
28. F. X. BLECHINGER, "Tests of Continuous Reinforced Concrete Slabs under Concentrated Loads," M.S. Thesis, Dept. of Civil Eng., Univ. of Ill., (1951).
29. A. LAUPA, "Studies of Continuous Reinforced Concrete Slabs under Concentrated Loads," M.S. Thesis, Dept. of Civil Eng., Univ. of Ill., (1952).
30. T. Y. CHEN, "Numerical Determination of Influence Coefficients for Moments in Skew I-Beam Bridges," Ph.D. Thesis, Dept. of Civ. Eng., Univ. of Ill., (1954).

Other References

31. H. M. WESTERGAARD, "Computation of Stresses in Bridge Slabs due to Wheel Loads," *Public Roads*, Vol. II, No. 1, March, 1930, pp. 1–23.