Frictional Resistance under Concrete Pavements and Restraint Stresses in Long Reinforced Slabs

BENGT F. FRIBERG, *Consulting Engineer, St. Louis*

TEMPERATURE contraction of short pavement slabs is not greatly diminished by frictional-tension stress. But long pavement slabs, as used in continuously reinforced concrete pavements, actively contract and expand only for some hundreds of feet near each end. The length of actively moving ends and the movements are related to thermal change, to friction coefficient, and to the modulus of elasticity of the pavement. Simple mathematical relations are suggested.

The friction coefficient before sliding on the subgrade is not constant; in tests, large movements of long slabs have been observed without sliding at frictional resistance proportionate to the root of the movement. The overall modulus of elasticity of the reinforced pavement is much decreased below that for concrete alone due to closely spaced open cracks. Observed behavior of continuously reinforced pavements agrees well with that computed for frictional resistance increasing at a relatively low rate with movement.

Fully restrained central parts of continuously reinforced cracked pavements are subject to temperature restraint stresses in accordance with stress formulas which take into consideration the mechanics of concrete-steel bond. Applicable bond behavior of available reinforcing steel is suggested from tests. Steel stress increases with crack spacing and bond resistance near cracks. Concrete stress between cracks increases with increasing steel area and steel stress at crack. Cyclic crack widening is only partially prevented by the steel, but for typical close crack spacing, the opening at each crack is small.

Restraint stresses increase at a much-lower rate than temperature drop. Stresses for seasonal temperature change are only slightly greater than for daily temperature drop. Seasonal increase appears to be nonexistent in pavements up to 1,000 feet long, because of absence of frictional resistance to normally developing seasonal contraction in pavements of that length.

Restraint stresses in cracked pavements agree with those computed on the basis of relatively low frictional resistance for daily active slab ends. The average frictional resistance indicated under continuously reinforced pavements is much lower than the value commonly proposed for design of distributed reinforcement in conventional-length slabs.

● CONSTRUCTION of long reinforced pavements without joints has been explored in recent years. The oldest periodically observed project is 15 years old. Excellent data on movements and cracking behavior have been obtained. Movements for daily temperature changes extend in diminishing amount a few hundred feet from the ends, with parts further from the ends of unchanged overall length. Movements incident to seasonal temperature change on one project have been found to extend much further from the ends. Typical cracking for ⅓ percent or more of steel reinforcement consists of uncracked slab ends up to about 100 feet long and closely spaced cracks, from 3 to about 10 feet apart, in parts further away from the ends.

This research has inspired these notes to explore some rules for movements and stresses in long continuously reinforced pavements. It is particularly desired to find relationships between movements and frictional subgrade resistances and to suggest stress and strain conditions for full restraint in extensively cracked slabs, with the cracks free from obstructions and infiltrations. The two conditions, cause and effect, are compared. Simple formulas for frictional resistance and for restraint stress conditions have been used for interpretation of recorded observations, in spite of meager basic data. This is intended to aid critical review, discussion, and future research, rather than to suggest definite quantitative analysis, which should await further needed information.

It is hoped these notes may aid in developing design procedures for continuously reinforced concrete pavements. They give clues for future research and observations necessary for substantial interpretation. There is some promise of rational and constructive development of the type of construction.

The published field observations give the first recorded data on the effects of frictional restraints. Their interpretation is needed for predicting friction stresses in conventional length pavement slabs as well. These analytical deductions may help to establish realistic frictional coefficients, for use in design of distributed reinforcement in slabs of nominal length.

SLAB-END MOVEMENT AND FRICTIONAL RESISTANCE

Free Movement Limited by Frictional Resistance

A free concrete slab expands and contracts from its center with increase and decrease in temperature. At a distance x from the center, for thermal coefficient e, the movement corresponding to temperature change t is: $x\ e\ t$. Frictional forces between the slab and its subgrade, opposed to the movement, create a stress in the slab which increases in magnitude from the free end of the slab. For slab length L inches, concrete weight of $\frac{1}{12}$ lb. per cu. in., and average frictional coefficient F, the stress resisting the movement at point x, is: $\frac{(L/2 - x)\ F}{12}$ psi.

The concrete strain because of this frictional stress, for concrete with a modulus of elasticity or resistance of E_a, is: $\frac{F}{12\ E_a}\left(\frac{L}{2} - x.\right)$

Where the strain due to opposing stress equals the free rate of temperature length change $e\ t$, no movement takes place, and for stable conditions of temperature, friction, and elastic properties, no movement occurs at points further distant from the end of the slab. This point of zero movement, at distance x_0 from the center of the slab, or $(L/2 - x_0)$ from the end, then, is obtained from equation:

$$\frac{F}{12E_a}\left(\frac{L}{2} - x_0\right) = et; \tag{1}$$

The restraint of slab movements is pertinent to long monolithic concrete pavements. The "active length" A from the end, defined as $(L/2 - x_0)$ less than $L/2$, is given by Equation 1, in the simple form:

$$A = \frac{12E_a et}{F}. \tag{2}$$

The free temperature movement at the end for a distance A would be $A\ e\ t$. If the frictional coefficient is assumed constant, the stress would increase linearly from the end to A, and the average frictional stress for the active length A, would be $(F\ A/2)/12$, and change in length:

$$\frac{FA/2}{12E_d}\ A,$$

where E_d is the modulus of elasticity for the entire active length. The temperature movement D would be:

$$Aet - \frac{FA}{24E_d}\ A,$$

and by substitution of A from Equation 2, we obtain:

$$D = Aet\left(1 - \frac{E_a}{2E_d}\right); \tag{3}$$

or:

$$D = 6\ \frac{2E_d - E_a}{F}\ \frac{E_a}{E_d}\ (et)^2.$$

Equation 2 establishes the active length A with F, the average friction coefficient under the weight of the slab end, and E_a the modulus of stress at distance A, representing strains in concrete as well as in reinforcing steel across closely spaced cracks at that distance. The above expression for temperature movement in long monolithic concrete slabs is a simplification of actual conditions. Equation 3 is correct only for constant friction coefficient, independent of amount of movement on the subgrade of points along the active length. The modulus of strain E_d represents strains over the entire active length A, with stress increasing and crack spacing decreasing with increasing distance from the end.

Constant friction coefficient is conventionally associated with sliding between solids. This does not apply to pavements on a granular subgrade at least for small movements, in which the subgrade to some depth, may participate in decreasing degree with a gradually increasing resistance up to sliding. Such conditions could be expected particularly toward the inner end of the active length. The elastic modulus for stress E_a could be expected to be smaller than the elastic modulus for strain E_d because of cracks at A more closely spaced than the average for the active length, but E_d may approach E_a with increasing A, because of decreasing influence of the long uncracked slab ends. For preliminary considerations, the two can be assumed equal E, in which case

$$D = 6 \frac{Ee}{F} et^2 \qquad (4)$$

Figure 1 shows movements near ends of long pavements for 19- and 23-deg. temperature change, computed using Equation 4 for concrete with a thermal coefficient of 0.000005 and Ee/F of 10, 15, and 20. Values observed on the continuously reinforced pavement at Stilesville, Indiana, (Ref. 1, Fig. 5) are shown as well for quantitative comparison and to indicate the range of the ratios between unknown modulus and friction coefficient. For an indicated value of Ee/F of 15 for the Stilesville slab, with thermal coefficient close to 0.000005, corresponding values of E and F might be from 1,000,000 and 0.33, to 4,500,000 and 1.5.

To determine actual elastic and frictional values, the relations between stresses and strains in cracked slabs must be known. Variation in frictional resistance with movement, will be indicated below.

Frictional Resistance for Increasing Slab Movement

Research at the Bureau of Public Roads (*2*) has established that frictional resistance to movement is not constant but increases with movement, rapidly at first and at a decreasing rate with increase in movement, closely approximated by parabolic relationship (*3*). The test data show decreasing coefficient of friction with increasing slab thickness and sliding commencing at a displacement of about 0.06 inch at a slab force of 14 to 9 psi. on 2- to 8-inch-thick and 4-foot-long slabs. The subgrade was found to participate in the movement in some degree even at 4½ inches of depth, but subgrade movement was clearly restricted for small

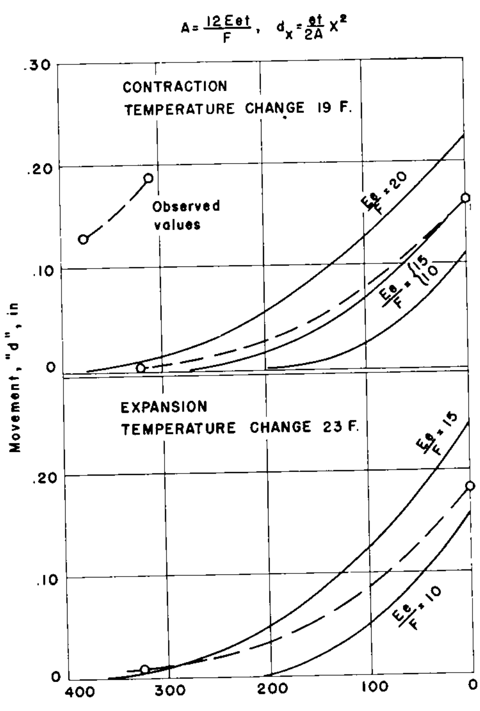

Figure 1. Slab movements observed at Stilesville for 19- and 23-F. temperature change in long slab and movements computed for constant friction modulus.

test slabs, due to soil resistance at the forward slab edge.

Force-displacement tests on a 100-foot-long pavement slab on subgrade paper, still in the curing stage, pushed back and forth at ¾-inch expansion joints at either end, are illustrated in Figure 2. These tests were made in conjunction with experimental stress curing in Missouri under the author's supervision. At first test sliding occurred at 0.08 inch average displacement and applied force of 150 psi., 1.5 coefficient of friction. For a return extended movement, however, the same frictional resistance was not reached until movement of ¼ inch beyond the original slab position. A repeat movement was resisted at the lower rate. The slab shortened due to compressive force, the far end movement, therefore, was slightly smaller than at the pressure end. On removal of force, lengthening occurred at the pressure end. Elastic rebound in the subgrade, indicated by some reversal of movement also at the forward end as the pressure was removed was less than 0.01 or 0.02 inch, about equal to that found in tests at the Bureau of Public Roads. This test shows a substantial decrease in subgrade resistance after initial disturbance and much greater movements before sliding than found for short slabs. Cyclic movements in excess of 0.01 to 0.02 inch appear to be positively resisted. Frictional coefficients for long slabs may be smaller than for short slabs, as they are also smaller for increasing thickness, and may develop at a much lower rate with movement, and to greater movement before sliding

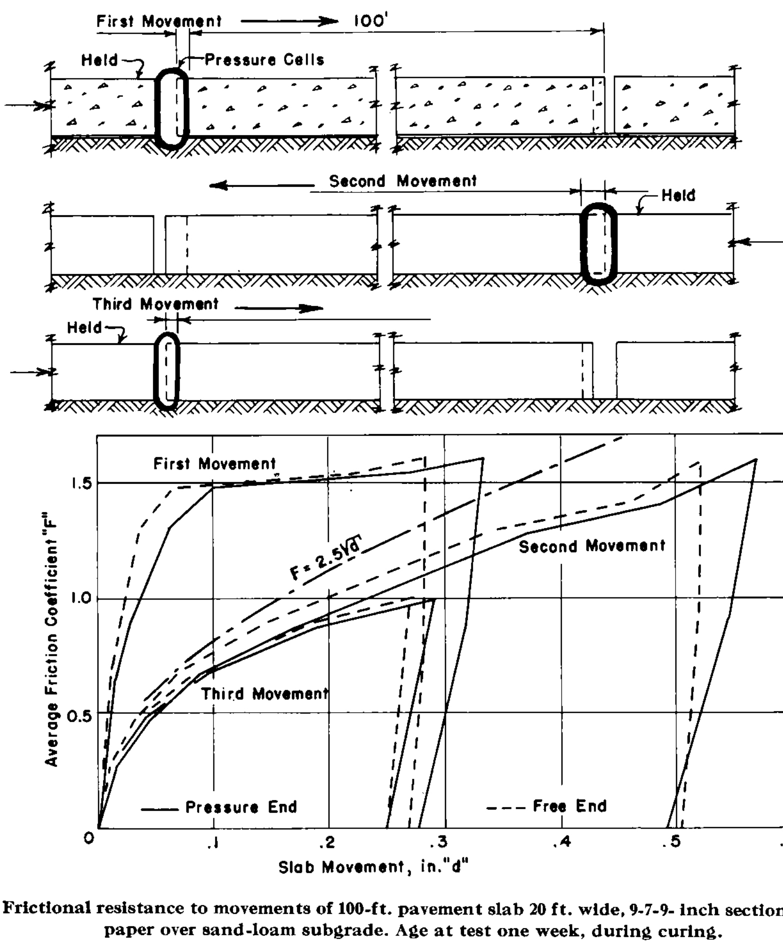

Figure 2. Frictional resistance to movements of 100-ft. pavement slab 20 ft. wide, 9-7-9- inch section, on subgrade paper over sand-loam subgrade. Age at test one week, during curing.

especially for contraction without end resistance to soil participation. Greater resistance to expansion than to contraction, is indicated by a comparison between observed movements (see Fig. 1).

The observed parabolic relationship between movement d at any point, and the frictional resistance or modulus v at that point, is represented by the formula (d in inches):

$$v = k\sqrt{d} \tag{5}$$

in which k is a constant, assumed to represent the physical conditions of the pavement and the subgrade. The maximum value of v is the constant sliding friction coefficient.

A displacement of 0.10 inch for sliding friction at $v = 1.5$ corresponds to k of 4.75; but a displacement of 0.5 inch for $v = 1.5$, as shown for repeat movements of the long slab in Figure 2, corresponds to k of 2.1. This range is believed to cover varying conditions of repeated temperature movements of concrete pavements on normal subgrades. Based on sliding friction at 1.5 for 0.06 in. displacement, as recommended by E. F. Kelley (*3*), the k value is 6.0; which may apply for very short slabs, and for long slabs at some initial "activation" of the subgrade.

The friction at the end of an active slab length A, which has not yet reached sliding at a movement of $0.5\ A\ e\ t$ inches (Equation 3, for $E_d = E_a$), would be:

$$V = k\sqrt{0.5\ A\ e\ t}$$

The generalized conditions of frictional restraints to temperature movements under long slabs are illustrated in Figure 3, showing frictional resistance, pavement strains, and movements, from a fixed origin at zero movement at distance A from a slab end. For constant pavement cross-section, the frictional stress is represented by the area under the friction diagram; at the origin the total frictional stress $FA/12$ equals $E_a et$; regardless of distribution, the full area under the friction diagram must equal the rectangular area for constant or average friction. At the origin, the total strain is zero, strains of temperature and stress must therefore be equal and opposed. The sum of total strains equals the movement at any point and is represented by areas between temperature unit strain and the frictional stress strain. For constant

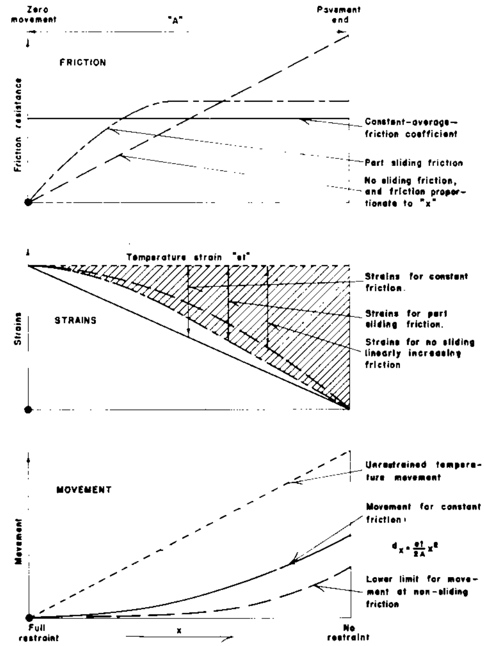

Figure 3. Related conditions of friction, strains, and movement, for temperature contraction and expansion of long pavement slabs.

friction coefficient, the evident triangular areas in the strain diagram, are represented by the parabola

$$d_x = \frac{et}{2A}x^2$$

shown in the movement diagram.

The corresponding movement at the end is half of that for free expansion. In accord with tests the friction modulus increases gradually from zero at the origin of movement to a maximum for sliding movements; it must be higher than the average friction near the slab end to give unchanged maximum friction stress. Stress and its strain accordingly increase at greater than uniform rate near the end, the total strains are correspondingly decreased, as represented by the shaded area in the strain diagram. The movements are somewhat smaller but do not vary too greatly from the parabolic rate specific for the enclosing triangle in the strain diagram. The movements are approximately proportionate to distance x, squared, and as, according to tests, the friction can be assumed proportionate to the root of the movement, the friction modulus can be considered a linear function of x up to its value for sliding.

If no sliding should take place, the friction would then increase uniformly from zero at the origin to twice the average at the end, indicated in the friction diagram. The corresponding strain area would be the lower limit for movement at the end, about 35 percent of that for unrestrained temperature movement. This approximate movement curve has been indicated in the movement diagram. For simplicity it has been assumed that A is fixed in the graphical representation of Figure 3; actually, full restraint would occur farther from the end for slowly developing friction, because the total friction is related also to the area under the movement diagram. End movements can, therefore, be prognosticated between ½ and ⅓, probably close to 0.4, of that for unrestrained temperature, under assumptions derived from Figure 3.

The mathematical expressions may be derived by a differential procedure; however, our understanding of the relation between friction and movement, the elastic modulus of the cracked pavement, and crack distribution is approximate and mathematical refinement is unwarranted for the present purpose.

This analysis shows the approximate influence of the probably more nearly correct development of subgrade friction. Referring to Figure 1, actual end movements can be expected to be nearer 40 than 50 percent of temperature contraction computed for constant average friction per above. The observed contraction movement would correspond to values of $\frac{E}{F}\frac{e}{}$ between 15 and 20.

ELASTIC BEHAVIOR OF CRACKED REINFORCED SLABS

Concrete-Steel Bond and Strains at Cracks

It is generally assumed that in resisting loads, up to the point of cracking, concrete and encased steel in bond undergo identical strains with stresses proportionate to the modulus of elasticity for steel, and modulus of resistance for the concrete. The n factor gives the ratio between steel and concrete stress at unchanged temperature, or provided the steel and concrete have the same thermal coefficient.

Near an open crack the concrete is without stress, all of the stress on the section is carried by the steel. If the concrete and steel boundary on each side of the crack does not permit free slippage, bond in the conventional behavior of bond tests will take place, and the stress in the steel will be transferred back into the concrete, until the same strains exist in the concrete and the steel at some distance from the crack. Relative longitudinal deflections occur between the concrete and the more-highly strained steel, largest at the crack, assumed equal to slip at the loaded end of the conventional bond test for long embedments.

If bond-slip relations for the reinforcing steel are known, changes in crack width can be computed for cracked reinforced pavements, as a function total force or stress. No research data, directly applicable to the small pavement reinforcement percentages and large concrete sections per bar or wire, can be found in published bond information.

Figure 4, showing a concrete element fixed at one end and stressed by tensioned steel at the other end, illustrates the transfer of stress from steel at the crack to concrete and steel by bond. The steel stress is redistributed by bond with a differential deflection between concrete and steel at the crack equal to the

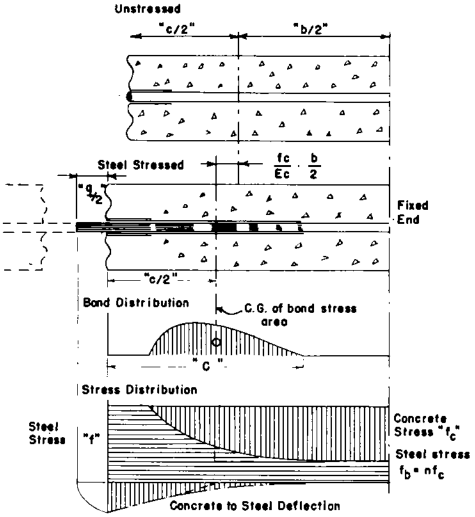

Figure 4. Mechanics of stress transfer from steel to concrete by active bond, and related deformations.

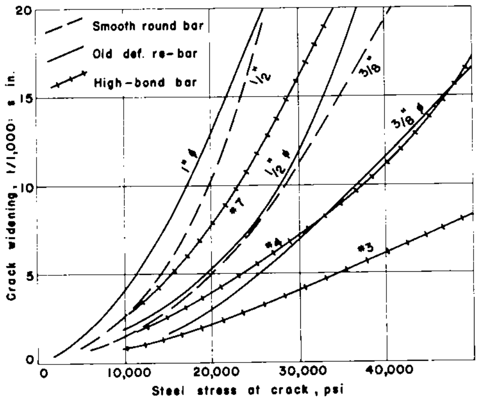

Figure 5. Crack widening, plotted as twice the slip at loaded end of conventional bond tests, suggested by data on tests with limited length of embedment.

stretch of the loaded steel over the length to the center of distribution of bond, as shown:

$$\frac{q}{2} = \frac{fc}{E_s 2}$$

where $q/2$ is slip at the loaded end, f the steel stress, and $c/2$ the distance to the center of bond. If the load is imposed through another mirror element, the crack opening q will consist of two loaded-end slips:

$$q = \frac{f}{E_s}\, c \tag{6}$$

Some idea of the steel stretch can be obtained from the few bond tests in the concrete literature, giving slips at loaded end, assuming that twice the end slip would be the crack widening (Fig. 5).

The crack openings suggested by conventional bond tests can be derived arithmetically, assuming the bond stress u on each side of a crack uniform over some active length c of the reinforcing member of size m diameter. A steel stress f at the crack is transferred to concrete and steel in combination, with stresses f_c and f_b, respectively, as assumed:

$$(f - f_b)\,\frac{\pi m^2}{4} = u\pi m c;$$

from which

$$c = \frac{m}{4u}\,(f - f_b) \sim \frac{f}{\frac{4u}{m}}$$

Entering this value in Equation 6, neglecting f_b, which for normal n values and small reinforcing percentages is small in comparison with f, gives:

$$q = \frac{m}{4E_s u}\, f^2. \tag{7}$$

In Table 1 are listed some bond values u computed per Equation 7 for different experimentally suggested crack widenings (Fig. 5) for different types and sizes of reinforcement, at different stresses.

The uniform bond stress computed for various observed bond-slip values, verifies the applicability of Equation 7, based on such an assumption. The data support the deduction that steel stress at a crack is transferred by active bond at high specific bond stress within a short distance on each side of the crack, irrespective of total greater length of embedment.

Figure 6 shows computed active bond lengths for any probable bond stress to bar size ratio. For practical purposes, with a given bonding surface and bar size, the length c can be considered proportional to the steel stress at the crack, although not entirely correct for new types of deformed bars. The value $4u/m$ represents the decrease in steel stress for each inch of active bond, and can be referred to as a bond modulus, U, dependent upon bar surface and bar size, and possibly on concrete quality. Its unit is pounds per square inch per inch, expressing

TABLE 1

Stress, psi.		20,000		30,000		40,000	
		Crack opening	Bond u	Crack opening	Bond u	Crack opening	Bond u
		in.	*psi.*	*in.*	*psi.*	*in.*	*psi.*
Smooth plain	½ in.	.01	170				
	¾ in.	.005	250	.011	250	.019	250
Old deformed	1 in.	.013	250				
	½ in.	.005	350	.012	300		
	¾ in.	.003	400	.007	400	.011	450
New deformed	#7	.008	350	.016	400		
	#4	.004	400	.007	550	.011	600
	#3	.002	600	.004	700	.006	900

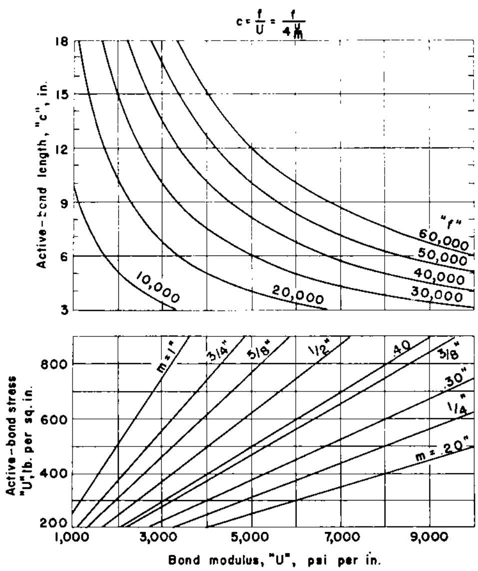

Figure 6. Relation between active bond stress, bar size, and bond modulus *U*, and corresponding active bond length for 10,000- to 60,000-psi. steel stress at a crack.

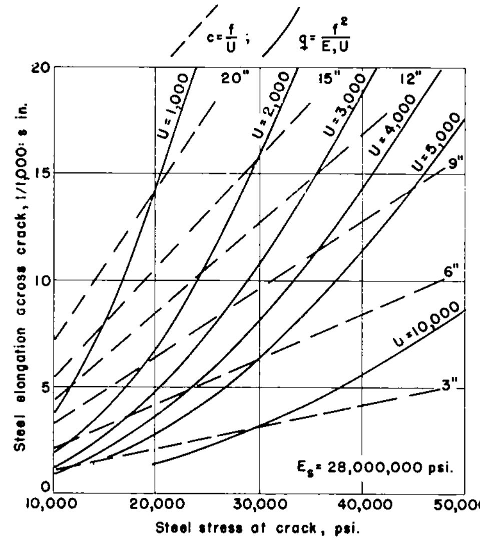

Figure 7. Theoretical crack widening for 10,000- to 50,000-psi. stress induced at a crack in bars with 1,000- to 10,000-psi. bond modulus. Active bond length on each side of a crack is shown.

the change in bar stress per inch of bond. The bond modulus defines the bond behavior of any reinforcing bar or wire for determination of relative deformation between bar and concrete and for determination of end deformation, or loaded-end slip. This slip may be concrete deformation as well as actual displacement at the concrete-bar boundary, with different degrees of elastic reversible or progressive movement. Research is not available on these issues.

Values of crack openings for different reinforcement stresses are shown in Figure 7 for some probable bond moduli. Values of c have been indicated as well. For a constant stretching length c between fixed anchorages, (possibly wire-fabric cross wires) or if bond can be considered active over a preactivated length, steel elongation also follows the linear rather than parabolic variation with stress. A combination of linear development within limits and parabolic relationship for higher stress is very possible.

Elastic Modulus for Cracked Slabs

Elastic strains in the cracked slabs are composed of concrete strains in fully bonded sections and strains in the steel, reinforcing percentage p, crossing the cracks. The total elongation consists of the sum of concrete lengthening and crack widening. These gross elastic movements take place in accordance with an elastic modulus, E_a different from that of uncracked concrete. With cracks closely spaced, its value is pertinent to overall pavement stresses and movements.

Subject to a concrete stress f_c and elastic modulus E_a, the elongation of a concrete element, a, including a crack, is as follows:

$$\frac{f_c}{E_a}\, a = \frac{f}{E_s}\, c + \frac{f_c}{E_c}\,(a - c)$$

Entering $$f = f_c\,\frac{1 - p + np}{p};$$

and $$c = \frac{1}{U}\,(f - f_b)$$

we can solve for E_a, and obtain

$$E_a = \frac{E_c}{1 + \dfrac{(1-p)^2}{Uanp^2}\, f_c} = \frac{E_c}{1 + \dfrac{f_c}{Z}} \qquad (8)$$

Values of E_a in terms of E_c are shown in Figure 8. As shown, the cracked-pavement

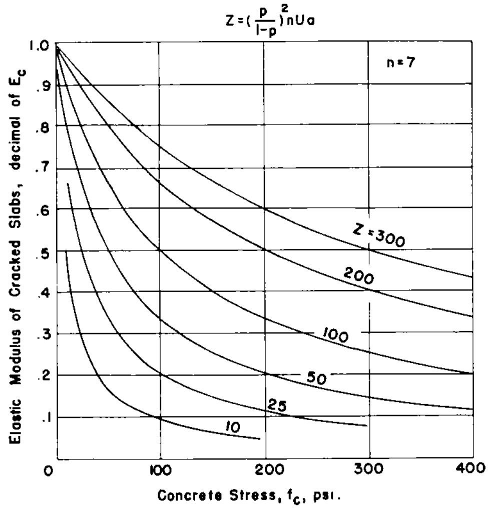

Figure 8. Reduction in Elastic Modulus of continuously reinforced concrete in tension, caused by closely spaced transverse cracks.

elastic modulus is not constant, but decreases with increasing pavement stress; it is higher for large crack distances a, for high reinforcing bond modulus U, and reinforcing percentage p. The curves in Figure 8 cover the probable range of conditions for continuously reinforced pavements. The effective elastic modulus for a continuously reinforced pavement with closely spaced cracks is obviously much smaller than that for uncracked concrete.

Friction Under Active Pavement Lengths

Using reduced elastic modulus values shown in Figure 8 and considering that the stress at the point of zero movement at distance A from the pavement end is determined by average friction coefficient, corresponding values of F and E_a can be determined for any known design condition. By matching appropriate value of $E\ e/F$ against values of E_a and frictional stress in Figure 8, the average friction coefficient is obtained.

The movements in Figure 1 for the Stilesville project were observed on a 1,310-foot slab with 1.82 percent reinforcement of old-style, deformed, 1-inch bars, assumed bond modulus 1,000 psi. The concrete modulus of elasticity was about 4½ million psi.; $n = 6$, and Z in Figure 8 accordingly is 2.05 a. Observed cracks numbered 5, 13, 20 and 26 for successive 100-foot distances from the end. Assuming full restraint at 300 feet from the end, crack spacing a of 50 inches and Z equal to 102, corresponding values of E_a and f_c, Figure 8, are 0.50 E_c at 100, 0.40 E_c at 150, and 0.33 E_c at 200 psi. stress. Restraint stresses of 100, 150, and 200 psi. correspond to an average friction modulus F of 0.33, 0.50, and 0.67, respectively, for a 300-foot active slab end. With e equal to 0.000005, corresponding computed values of E_ae/F are 33.8, 18.0, and 11.2. In line with Figure 1, and an indicated value of Ee/F between 15 and 20 for variable friction modulus, an average friction modulus of about 0.5 appears probable. Corresponding steel stress at a crack is 9,000 psi.

At an average friction modulus of 0.5, it is doubtful that sliding friction was reached at any point. Assuming friction varying with distance from 1.0 at the slab end to zero at 300 feet, we obtain numerically for the three 100-foot distances in order: average stresses are 50, 110, and 145 psi.; a equals 240, 93, and 60 inches; E_a is $0.9E_c$, $6E_c$, and $43E_c$; and total strains in each 1,200 inches, are 0.015, 0.047, and 0.086 inches. The unrestrained temperature movement for a 19-deg. temperature change would be 19 x 0.000005 x 3,600 = 0.34 in. The computed total movement, 0.34 − (.015 + .047 + .086) = 0.192 inch, compares with the observed 0.15-inch movement, Figure 1. A lower value of Z would quickly close the variation between computed and observed movement.

A friction modulus of 1.0 for a 0.15-inch movement indicates a k value in Equation 5 of 2.6. Both the average friction and the apparent maximum friction coefficient are much lower than have been assumed generally.

STRESSES AND CRACKS IN FULLY RESTRAINED PAVEMENTS

Continuously reinforced long pavements with 0.5 percent or more steel, normally crack from 3 to 6 feet apart in central portions. These central portions are fully restrained against daily temperature movements. It is of interest to determine restraint conditions, as a check on frictional resistances at active ends, as an indication of seasonal restraint stresses; and to understand crack size and stress relations.

Steel and Concrete Temperature Stress in Reinforced Fully Restrained Pavement for Decreasing Temperature

It is assumed that stress across cracks is carried by steel only without face to face or infiltration contact across the crack, that the cracks are spaced sufficiently far apart for bond to be fully effective, and that stress conditions are reversible. A fully restrained slab undergoes no change in length; temperature strains are balanced by concrete and steel strains between cracks plus the steel stress and strain across the crack.

The conditions are illustrated in Figure 9, showing the elongations necessary to overcome contracton because of a temperature drop t. With known crack spacing a, and fully reversible steel-bond slip conditions expressed as stretching length c across the crack, the remainder of each slab $a - c$, termed b can be assumed as a combined concrete-steel element. Thermal coefficients for concrete and steel are e_c and e_s. Three equations can be written for the three unknown, steel stress at crack f, and concrete and steel stresses between cracks, f_c and f_b, respectively, defined by the following conditions:

Equation 9, Equilibrium of forces at crack and half-way between cracks, for net reinforcing percentage p; (concrete area 1, rather than $1 - p$.).

Equation 10, The lengths of concrete and steel in the fully bonded section b must remain equal, regardless of temperature change t and common length change;

Equation 11, No overall length change takes place for the length a with temperature.

$$f_c + p \cdot f_b = f \cdot p \qquad (9)$$

$$\frac{f_c}{E_c} b - e_c tb = \frac{f_b}{E_s} b - e_s tb \qquad (10)$$

$$\frac{f_b}{E_s} \cdot b + \frac{f}{E_s} \cdot c - ate_s = 0 \qquad (11)$$

The solutions for the three unknown stresses are:

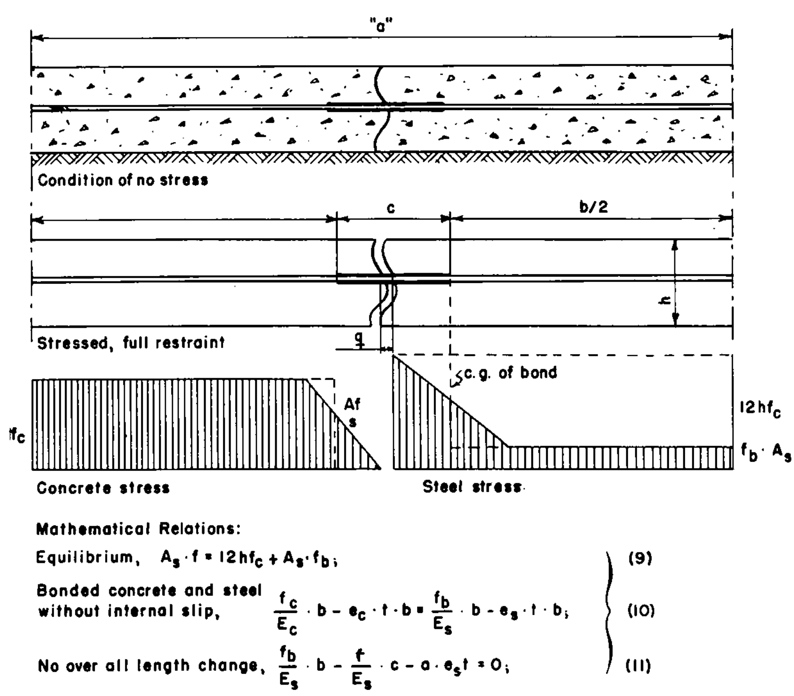

Figure 9. Stresses and strains in fully restrained, cracked reinforced concrete for decreasing temperature.

$$f_c = \frac{atpE_s e_c}{npa + c} \qquad (12)$$

$$f_b = E_s \cdot e_s \cdot t - \frac{atE_s e_c}{npa + c} \qquad (13)$$

$$f = E_s \cdot e_s \cdot t + \frac{btE_s e_c}{npa + c} \qquad (14)$$

The change in crack width, the crack opening q, including concrete contraction of the unbonded length c, is:

for $\qquad e_c = e_s$: $\qquad q = \frac{f}{E_s} \cdot c \qquad (15)$

and general:

$$q = ae_c t - \frac{f_c}{E_c} \cdot b = ae_c t \frac{npc + c}{npa + c} \qquad (16)$$

This change in crack width, crack opening as percent of free contraction, is shown in Figure 10.

The above formulas give general relations for the temperature restraint stresses. The term $E_s\ e_s\ t$ is temperature stress in fully restrained steel, which therefore is the upper limit for stress f_b between cracks, and the lower limit for stress f across cracks. For normal steel that stress is 180 psi. per deg. temperature change; the streel stress between cracks accordingly could not exceed between 5,000 and 7,000 psi. for 25- to 40-deg. daily temperature drop. The stress at cracks decreases as the length in bond b decreases, with decrease in concrete stress as well. Crack openings increase with increasing crack spacing and with increasing active-bond length. It might be possible to adjust bar size, bar surface, and steel quantity so as to obtain optimum combinations of steel stress, crack spacing, and width of cracks. On the other hand, if c is zero (for a bar with very effective bond surface), and provided the bar strength is high enough to prevent steel failure, the maximum concrete stress f_c becomes equal to full restraint stress, with the upper limit for steel stress at the crack $E_s e_s t + \frac{l}{p} E_c e_c t$; the stress equivalent to full restraint in both concrete and steel. There are accordingly clear upper limits to both amounts of steel and desirable reinforcement bond modulus.

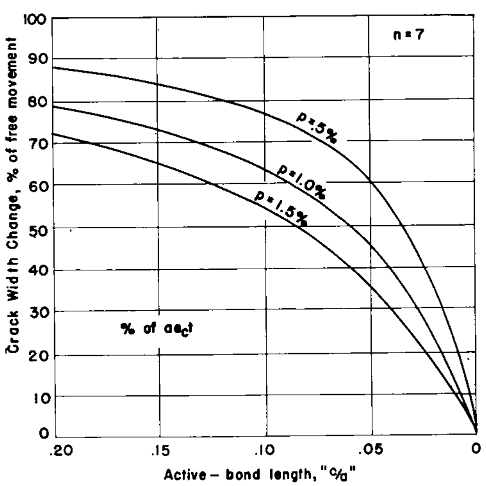

Figure 10. Change in crack width with change in temperature, as percent of change for freely contracting slab between cracks, for fully restrained reinforced pavement, with varying active-bond length on each side of cracks.

The range of maximum steel and concrete stresses for 0.5, 1.0, and 1.5 percent reinforcement per degree temperature drop is shown in Figure 11 for different ratios of actively bonded length c to total length between cracks and for typical concrete thermal coefficients. As seen, concrete stress increases with reinforcing percentage, steel stress decreases. Concrete stress over 10 psi. per deg. temperature drop occurs for active bond less than 5 to 10 percent of the distance between cracks. With steel stresses limited to between 1,000 and 1,500 psi. per deg. and reinforcement percentage less than 1 0, 5- to 10-percent active-bond length would be desirable minimum.

Restraint Stress and Bond Modulus

In the range of evenly distributed bond, as shown in Figure 9, the relation between bond modulus U and restraint stress can be computed. For any steel stress, the active bond length:

$$c = \frac{f}{4\frac{u}{m}} = \frac{f}{U}; \qquad (17)$$

can be entered into Equation 14, from which is then obtained the following relation be-

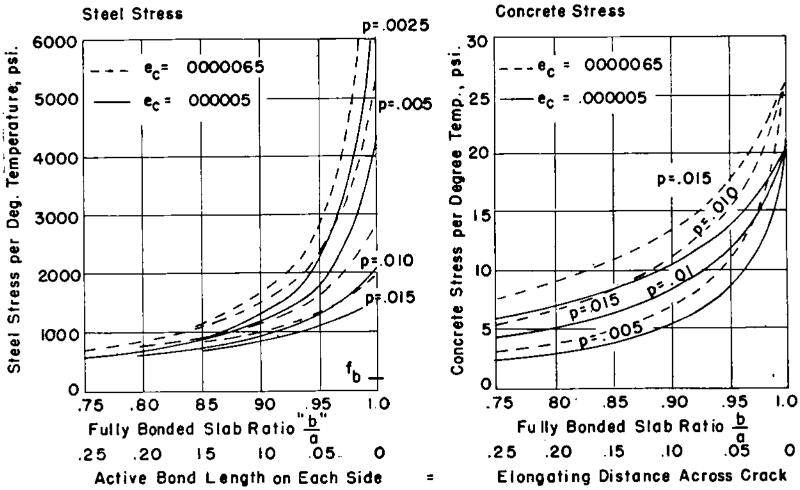

Figure 11. Steel and concrete temperature stresses in fully restrained cracked pavement for different amounts of reinforcement with varying active-bond length. E_s = 28 million psi.; n = 7; $E_s e_s$ = 180 psi. per deg. F.

tween steel stress at crack in fully restrained pavement and distance between cracks:

$$a = \frac{f}{U} \cdot \frac{f - E_s t(e_s - e_c)}{E_s e_c t - np(f - E_s e_s t)} \qquad (18)$$

This relation between crack spacing and restraint stress has been illustrated in Figure 12 for 20-, 40-, 60-, and 80-deg. temperature change, for bond moduli of 2,000, 3,000, and 4,000 psi. per inch, and for varying amounts of reinforcement and concrete stiffness as defined by np 0.025, 0.050, and 0.100. In determining steel restraint stress at crack, average crack spacing for some distance under restrained pavement length should be used, because variation in crack spacing could be equalized by slight movements on the subgrade without appreciable frictional resistance. Thermal coefficient of expansion of 0.000005 has been assumed for the concrete; for equal concrete and steel thermal coefficient restraint stress would be from 10 to 20 percent greater than shown in Figure 12. Length c is obtained directly from the steel stress by dividing by the bond modulus. Concrete restraint stress is easily obtained from steel stress by ratio $p/(l + np)$.

As seen in Figure 12, steel stress is relatively insensitive to changing amount of steel, and about equally dependent on crack spacing, bond modulus, and temperature change. The restraint stress is by no means proportionate to temperature drop; doubling the temperature change increases restraint stress only about 50 percent. This is about the measure of seasonal, compared to daily restraint stress.

After a certain bond length, c, has been activated, barring progressive failure, it may remain active. Restraint stress would then within that temperature limit be proportionate to temperature change (Figures 10 and 11, for a developed length relation). For example, a stress for a 20-deg. temperature drop, 20,000 psi. increasing to 44,000 psi. on further decrease to 80-deg. temperature drop, thereafter would be 11,000 psi. for the temperature equivalent to the 20-deg. temperature drop. Such elastic conditions seem probable; no applicable test information on repeated bond tests is known. The decrease in daily restraint stresses, after an initial high stress would be substantial.

Measurements and Computations for Continuously Reinforced Pavement Installations

The experimental projects of reinforced pavements (*1, 4, 5, 6*), have provided some information for comparison with the above deductions. Figure 13 shows actual crack spacing in the central portions of long continuously reinforced projects for reinforcement employed in the various jobs.

For the Stilesville pavement illustrated in Figure 1, for 19-deg. temperature drop, the

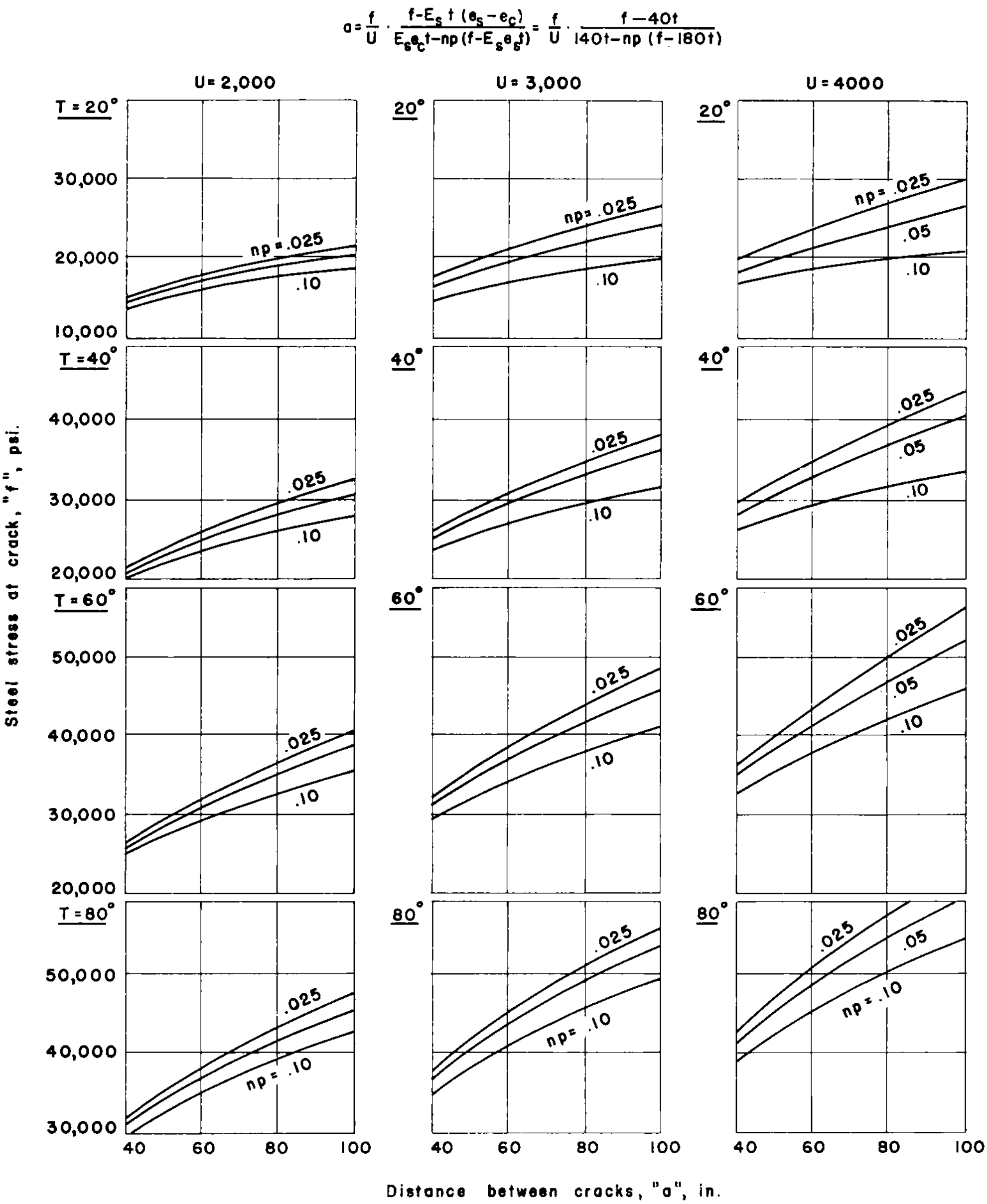

Figure 12. Steel stresses at cracks in fully restrained pavement with varying distance between cracks, for different amounts of reinforcement and concrete quality as shown for *np* of 0.025, 0.050, and 0.100, for reinforcement Bond Modulus of 2,000, 3,000, and 4,000 psi. per in., and for 20-, 40-, 60-, and 80-F. temperature drop.

steel-restraint stress computed in accordance with Equation 18, for a of 50 inches is 10,200 psi., which compares with the 9,000-psi. stress deduced as frictional stress. On the California project, (Ref. 4, Figure 7) at one month age and concrete strength about 4,500 psi., (E_c about 3½ million psi.), the stresses measured at cracks for about 30-deg. temperature drop were: 28,000 psi. for 0.62 percent steel (np 0.05) and some 60-inch crack spacing; and 27,000 psi. for 0.50 percent steel and some 50-inch crack spacing. The probable bond modulus for the old-style deformed ½-inch bars may be 2,500 psi. per inch. Stresses obtained from Figure 12 are 23,000 and 22,000 psi., respectively. On the Illinois project (*5*) for 70-deg. temperature drop in sections with 0.70 percent reinforcement of No. 5 high-bond bars, measured stresses were 62,400 and 42,000 psi. Crack spacings were not given so the specific stresses cannot be compared. For an average probable crack spacing of 10 feet at the time, and bond modulus 4,000 psi. per inch, for *np* of 0.05, a stress of 58,000 is extrapolated in Figure 12. The corresponding direct-tension concrete stress is 380 psi., a rather high value.

The above comparisons indicate that arithmetical deduction of stresses, based on bond modulus, is useable for determination of restraint in long reinforced-concrete slabs. If it helps to explain performance of continuously reinforced pavements, it may also help the useful development and intelligent design of such slabs.

Reinforcement, Early Temperature Restraint Stress, and Cracking

The above analysis indicates moderate steel stresses in continuously reinforced pavements with closely spaced cracks. For small reinforcing percentages, concrete stresses may not be sufficient to propagate cracking at close spacings even at early age. In that case, restraint stress for the greater seasonal temperature drops in pavements of sufficient length would be likely to result in bond or yield point failure of the reinforcement, rather than in rapid crack formation. An indication of this condition is given by the crack spacings for 0.3 percent of reinforcement on the Illinois project. Closer crack spacing might have been obtained for higher construction temperature on that project, with higher daily temperature change.

Observations on the projects built so far do not permit clear prognostications of minimum reinforcing amount to secure the characteristic closely spaced cracks which appear desirable for aggregate interlock across the cracks and, it is hoped, may not impair pavement serviceability. But the experience with about ½ percent of reinforcement (Fig. 13) indicates that 50- to 80-inch crack spacing,

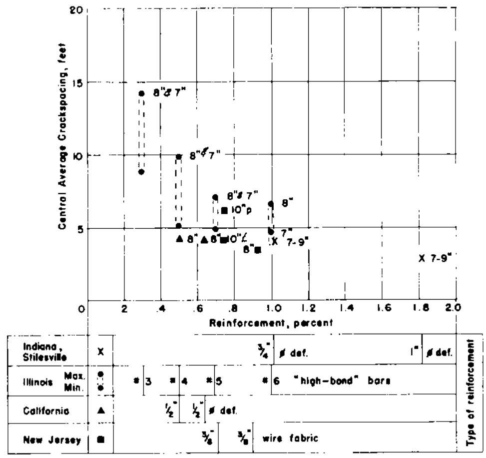

Figure 13. Average crack spacings observed in central parts of continuously reinforced pavement projects after 10 years on the Stilesville project and after 3 years on other projects. Reinforcement size and pavement thickness are shown. Closest crack spacing on the Illinois project about 200 feet from the ends, with appreciably wider crack spacing in central parts.

with a maximum steel stress at crack of some 40,000 psi. maximum seasonally, and 200-psi. maximum concrete tension, may be obtainable without special manipulation.

Seasonal Movements in Long Slabs

According to tests, frictional resistance is substantially inelastic, its directional characteristic should result in reversal of resistance with reversals of movement exceeding 0.01 to 0.02 in. Movement-direction changes would occur first near slab ends where frictional coefficient is greatest. The location of full cyclic restraint stress, therefore, would move further away from the ends at each reversal. On that supposition, gradual seasonal movements may be relatively unhindered for fairly long slabs; frictional resistance stress should not exceed that for daily cycle, and crack openings would not increase seasonally for such pavements of reasonable length.

Measurement of restraints to seasonal movements is complicated by unknown stress conditions in the pavement at the time of measurements. Seasonal displacements should be measured with the pavement in unstressed or equal-stress conditions for summer and winter temperature, but seasonal cumulative stress must be taken into account. Cumulative compression stresses may exist in central parts of long pavements at high temperature by reason of prior crack infiltration or bar slip, which would be seen as less than seasonal contraction, although not a restraint to such contraction. Decreasing concrete modulus of elasticity, relieving restraint, for long-time stress, and winter gain in concrete moisture content, apparent as a lower seasonal thermal coefficient, complicate the problem further.

Measurements at mean daily (morning) temperature during summer, 77 F., and during winter, 32 F., on the Stilesville project during its third year (*1*) for slabs of different length, have been plotted as displacements per degree (F.) temperature drop in Figure 14. For comparison, daily temperature movements have been plotted as well. The close agreement between unrestrained and observed seasonal displacements for at least 1,000-foot slabs is apparent. The data support the above supposition for slabs up to that length; however, the fairly high reinforcing percentages and low probable frictional steel stresses, must be considered. Other continuously reinforced pavements do not permit similar appraisal. The New Jersey project (*6*) includes detailed measurements of movement at ends and 200 and 500 feet from the ends, taken some 50 days or 1½ years apart, without correction for daily movements. Nevertheless, those measurements, compared

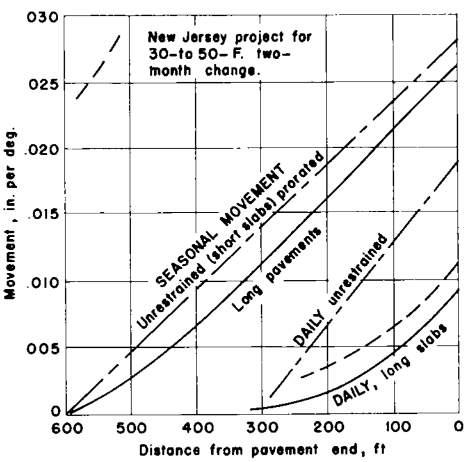

Figure 14. Seasonal and daily movements compared on the Stilesville project for up to 1,300-foot long slabs. End movement for 50-day temperature drop on a New Jersey project 1-mile slab shown as dashed line.

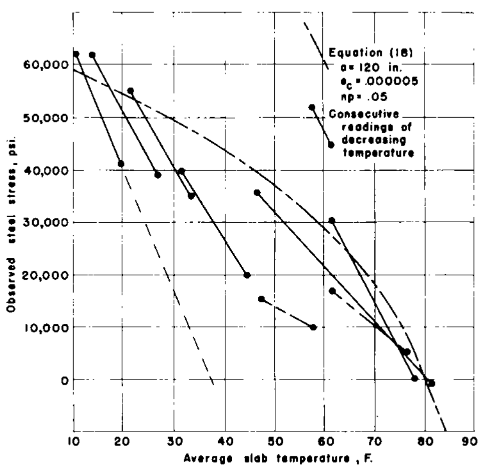

Figure 15. Observed steel stresses at different temperatures during first winter seasonal drop on the Illinois project, showing stress increase with each temperature drop near center of 3,500-foot slab.

for different seasons, show end movements per degree close to the daily movements observed at Stilesville. The reinforcement on that project is 0.7 and 0.9 percent of ⅜-inch cold-drawn wire and about 50-inch crack spacing.

Equal daily and seasonal end movements per degree, for the higher restraint stress of greater seasonal temperature drop, would be possible only if seasonal frictional restraint were cumulative in spite of substantial back-and-forth daily movements, which is contrary to test observations. Another explanation could be that restraint stresses, for some unknown reason, do not increase substantially over daily-cycle values for the amounts and type of reinforcement used in New Jersey. An increased active-bond length for the gradual seasonal restraint would have such effect.

Stress data from the Illinois project include some related information. Steel stress at cracks near the half-way point of the long pavements were obtained for daily cycles at seasonally decreasing temperature (Ref. 5, Figure 2), plotted in Figure 15, and individual stress increases are connected. Computed stresses (Fig. 12), fall close to observed maximum stress, but intermediate-temperature stresses before each temperature drop are lower than should be the case for elastic restraint condition. The difference cannot be explained by seasonal end movements, which in 4 months from time of construction, were less than 0.5 inch, 1,700 feet away for 55-deg. temperature change, and such influence should be noticeable for the maximum stress as well. Stress decreasing influences of some presently unexplained nature apparently act to decrease restraint stresses for intermediate temperatures.

It is suggested that maximum daily temperature drop can be the basis for stress prognostications in 1,000-foot or shorter pavements with closely spaced cracks, without regard to seasonal temperature change.

CONCLUSION

Closely spaced cracks are an essential element of continuously reinforced concrete pavements by which warping stresses are substantially relieved and a high degree of aggregate load transfer seems assured across individual cracks. The occurrence and progress of cracking in these pavements have not been considered within the scope of this paper. But the conditions which govern stresses in long, reinforced, cracked pavements other than conditions in common with short, uncracked slabs, have been suggested and analyzed, with results which appear to agree with observed behavior. The conditions existing in the mature pavements give indications for continued study of the intermediate early adjustments.

Stresses in the cracked slabs due to temperature changes can be computed. The relationships provide a guide for continued development and for interpretation of field observations. Refinements of computations have been left out, because much basic research information bearing on the subject is incomplete or lacking.

It seems evident that continuously reinforced concrete pavements may be subjected to some design analysis, that reinforcement and concrete stresses are closely linked with reinforcing-bond behavior, and that crack spacing and crack openings are related to reinforcement characteristics as much as to reinforcement amount. Reinforcement size and bond behavior may be adjusted to obtain the desired results with less steel and improved economy and, possibly, a predictable optimum crack spacing. Restraint stresses decrease with increasing number of cracks. Accordingly, they can be expected to be generally lower after most cracks have occurred, than during the period of adjustment.

Frictional resistance under active ends of pavements seems to increase at a relatively low rate with movement with substantial movements before friction modulus for sliding is reached. There is no reason to expect higher frictional resistance development under conventional length slabs on similar subgrade.

Realistic frictional resistance values should permit economies in less steel or longer slabs of conventional reinforced concrete pavements.

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DISCUSSION

Wayne R. Woolley—The author has suggested a means of calculating the friction between concrete and the subgrade which strongly indicates that this friction as it actually exists under the pavement is much less than generally assumed. He has also calculated that the maximum stresses in the steel in continuously reinforced pavement are less than they were previously thought to be. These calculations agree quite well with opinions I have formed from observations on the condition of reinforced concrete pavement which have been in service a number of years. It is particularly gratifying to me to know that these observations can be supported by analytical means. Friberg has made a valuable contribution toward a better understanding of reinforced concrete pavement design.

However, the stress in steel in pavements, which are the result of subgrade friction, are not the only stresses to which the steel is subjected. Axle loads moving over the pavement are known to cause deflection under the loads and this deflection is greater at transverse cracks than where there are no cracks. Deflection at cracks surely causes a strain in the steel that spans the crack, and this strain results in a stress which is in addition to that caused by subgrade friction. So far as I am aware, there are no published data to indicate the amount of this stress. Probably Friberg considered this to be outside the scope of his paper, but there is evidence that stresses in steel, due to axle loads, should be considered in the design.

The Stilesville Project referred to in Friberg's paper contained reinforcement ranging from 32 lb. per 100 sq. ft. of welded wire fabric to longitudinal bars one inch in diameter spaced 6 inches apart. All sections were designed on the assumption that the only stress in the steel was that due to movement of the concrete longitudinally over the subgrade, and that the coefficient of friction was 1.5. This is a considerably higher coefficient of friction than Friberg's calculations indicate to be correct, and therefore, on this basis all sections contain more longitudinal steel than necessary. It is of interest to observe that in spite of an over design of steel based solely on the subgrade friction theory, there are a number of open cracks indicating broken longitudinal steel in the sections containing 32-lb. mesh. Cashell and Benham, in their article published in the April 1950 issue of *Public Roads*, comment on this as follows: "In several cases the steel crossing the cracks that formed in the sections containing this light fabric broke, probably from shearing forces."

Numerous other instances of broken longitudinal steel were reported in a discussion I prepared on the report of the Stilesville Project published in the 1949 PROCEEDINGS of the Highway Research Board. These data tend to support the conclusion that stresses introduced by live loads may cause the steel to break, particularly in short length slabs. For longer slabs, designed for a subgrade friction of 1.5, the steel does not break. Very likely this is because the subgrade friction is considerably less than 1.5, so that the steel is actually adequate to take the combined stresses due to both subgrade friction and live loads.

Both observation of roads in service, and theory, indicate that pavement steel designed on the subgrade-friction theory alone results in the steel being underdesigned for short slabs and overdesigned for very long slabs. There is considerable evidence that steel for pavements should include a certain amount which is independent of slab length to resist live loads, plus an additional amount to overcome a subgrade friction somewhat less than 1.5 as proposed by Friberg. Neither existing theory nor observations give adequate information as to the amount of steel needed to resist live loads. The amount will probably vary according to the number and size of axle loads, the location of the steel in the pavement, the thickness of the pavement, and the firmness of the subgrade. It is believed that this principle of design will more nearly approximate the amount of steel needed than the use of the subgrade friction formula alone, as now in general use.

If it is assumed that subgrade friction is 0.75, allowable steel stress 40,000 psi., and that 0.07 sq. in. of longitudinal steel per foot of width of pavement is needed to resist live loads, an amount of steel is obtained which has been used satisfactorily for a number of years in several Midwestern states. These pavements were 8 to 10 inches thick, constructed on granular subbases, and subjected to heavy truck traffic, and contained contraction joints at intervals between 40 and 100 feet.

There is no theoretical basis for suggesting the use of 0.07 sq. in. of steel per foot of pavement width, but this amount, when used with the other assumptions shown above, does indicate the use of an amount of steel which has been satisfactory. This principle of design would be expected to be valid for all slab lengths up to the point where about 0.5 percent of the cross-sectional area of pavement is required. For lengths greater than this, the pavement is restrained against movement and subgrade friction does not apply. Experience indicates that 0.5 percent steel is adequate for continuously reinforced pavement.

BENGT F. FRIBERG, *Closure*—Woolley calls attention to stress-inducing conditions other than friction which affect pavement reinforcement. His suggestions deserve more attention than can be given within the scope of a discussion. Consideration of other stress increments would undoubtedly explain many observed steel failures and lead to improved material utilization. It is unfortunate that reinforcement analysis has been formulated with reference to frictional stress excusively, although of major importance in long slabs and controlling the behavior of continuously reinforced pavements.

Pavement behavior suggests three distinct stress conditions in pavement reinforcement at cracks: (1) frictional tension, within the scope of this paper; (2) bending of the steel as dowels across the crack under shear load, if the crack width is too great for effective aggregate interlock with a minimum of vertical deflection across the crack; and (3) tension as flexural reinforcement for the cracked section, if corresponding compression can develop near top or bottom of the slab. Without contact across the crack dowel stress Condition 2 would be greater than the steel flexing stress. Consideration of Conditions 2 and 3 are outside the scope of this paper; however the remarks which follow are occasioned by the discussion.

In small members deflecting as dowels across cracks bending stresses increase rapidly with decreasing member size and become formidable in comparison with the bearing stress between the round member and the concrete. The risk of steel dowel failure accordingly increases with decreasing steel size, but it may be minimized by limiting the stress so that wide cracks do not occur.

Flexural reinforcement tension is directly related to bond modulus, which would have to be exceptionally high to cause serious tension stress for pavement deflections on normal subgrades. Incident to large pavement deflections at joints on pumping subgrades, angular changes at adjacent cracks may be sufficient to cause serious stress in steel near the pavement surface. This may be minimized by lower steel placement near joints and steel with lower bond modulus.

These remarks illustrate the importance of steel characteristics, stress, and placement in pavement reinforcement design. The arbitrary addition of sectional steel area is no assurance against either of these failures and is no substitute for concerted investigation. The basic research can be pursued with available experimental technique. Prospects for directly applicable useful information are most promising.