Dynamics of Guardrail Systems

ROBERT S. AYRE and JOEL I. ABRAMS.
Structural Dynamics Laboratory, Johns Hopkins University

HIGHWAY-GUARDRAIL systems are being investigated by means of dynamical analysis and small-scale laboratory models to obtain a better understanding of the dynamical interaction of the vehicle and the guardrail structure and from this a more-complete basis for design. This paper is a first report on the progress of the investigation and includes: a general discussion of the variables; the differential equations of motion of the vehicle in contact with the guardrail; a derivation of the length, force, and time scales for the model; the construction of the first model and its relation to a definite full-scale vehicle and guardrail; and the laboratory methods, including the adaptation of multiple-flash still photography to the recording of the path of the vehicle and transverse displacements of the guardrail. It is possible to investigate a wide range of conditions of design and operation by the use of a controlled laboratory model.

● AS a result of the increasing speed and volume of highway traffic the adequate design of highway-guardrail systems is becoming of greater importance. The problem has usually been investigated on a full-scale basis by trial and by modification of various designs in actual use along the highway or, in some cases, by controlled field tests. When one deals with a structure containing as many variables and unevaluated constants as are to be found in the guardrail and vehicle there is, of course, no substitute for experience in the behavior of the full-scale structure under actual operating conditions. Even in this case, however, there is something to be said for the analytical approach and for an investigation by means of small-scale models. Full-scale testing and operational experience are costly, time consuming, and hazardous. The advantage of analysis and of small-scale modeling, used to supplement full-scale operational knowledge, is that a much-wider range of operating conditions and of types of designs can be investigated than is feasible in full-scale field tests.

The purposes of this investigation are to make a dynamical analysis of the vehicle and guardrail-impact problem, and to set up a small-scale laboratory model of the system in order to verify the analysis and to explore the effects of wide variations in the parameters of the system. While it is the ultimate aim to investigate guardrails of the beam type as well as of the cable type, the studies have been limited thus far to the cable type.

Even though the complexities of the problem are considerable, as will appear later, we hope that the investigation, when completed, will furnish information relative to post spacing, depth and nature of the post embedment, initial tension (assuming cable type), massiveness of end anchorages, stiffness of springs inserted in the cables, stiffness of post fittings, distance of the guard rail from the roadway, etc.

Earlier Work

The report of the Highway Research Board Committee on Highway Guard [Rails], G. A. Rahn, chairman (*1*), was published in 1941 to summarize the work that had been done in the development of highway guardrails from 1924 to 1941. It does not seem necessary to give a further historical summary here, except to mention that early work in the development of guardrails was reported by Pennsylvania, Georgia, Missouri, and Oregon, by the Bureau of Public Roads, by the American Road Builders' Association, and by several manufacturers. Unreported work undoubtedly has been performed by others. The most-comprehensive papers that we have been able to find, in addition to the committee report, are those published by Searcy B. Slack in 1934 (*2*) and Joseph Barnett in 1939 (*3*). Relatively little information has been made available since 1941.

Scope of this Report

This paper is mainly a progress report on the general methods of analysis and the experimental techniques. It is expected that more-detailed results will be presented in a second report.

Purpose of a Guardrail

Before we become involved with the technical aspects of the investigation, we should give some thought to the purpose of a highway guardrail. There seems to be a difference of opinion, among highway engineers as well as the public, as to what a guardrail should be able to do. On the one hand there is the point of view that the guardrail is merely a warning device and that under many conditions it may be best to dispense with it in favor of a roadside topography that would allow the driver to bring the vehicle to a controlled stop or to return it to the roadway. On the other hand, there is the view that the guardrail should be designed to deflect the path of a vehicle of a certain weight, speed, and direction so as to return the path to a direction more or less parallel to the roadway with a minimum of damage. The choice of point of view is partly a policy matter, involving cost, public reaction, and general topography. It is also partly a statistical problem, hinging on the following question: For a given type of site, is a guardrail apt to result in more or less damage to the vehicle and passengers than if the site were left "unguarded"? This question can be answered on the basis of statistics, assuming they are available. It might be possible, however, to investigate certain idealized cases in the laboratory.

Assuming, for the purpose of this investigation, that a guardrail that is more than a mere warning device is desirable, there is still the question of what is meant by "minimum damage." If the purpose is to correct the path of the vehicle in such a way as to minimize the probability of other vehicles becoming involved, without regard for the extent of damage to the impacting vehicle and passengers, the problem is mainly one of kinematics, the strength of the rail, posts and connections, and the avoidance of structural details that will tend to snag the vehicle, stopping it abruptly, or throwing it out into the stream of traffic. On the other hand, if there is a dual purpose, namely, to minimize the involvement of other vehicles and also to minimize damage to the impacting vehicle and passengers, then in addition to the strength and the design details, the resilience and the energy dissipative properties of the entire system (automobile; guardrail, posts, and soil) become of considerable importance. We are basing our work on this last point of view.

General Statement of the Problem

Fundamentally the problem is one in dynamics. The vehicle is a dynamical system having many degrees of freedom, including three in rotation (rolling, pitching, and yawing), three in translation (two horizontal and one vertical), and at least three more introduced by the elastic connections between the wheels and the chassis. One has only to see some of the motion pictures of full-scale guardrail tests (*4*) to realize that rotational motions, as well as the translations, may be of appreciable significance. Still other degrees of freedom are involved if the possibility of motion of the load relative to the vehicle is taken into account. The springing of the vehicle and its mass and moments of inertia enter into the problem, as do also the frictional properties between the tires and the surface on which the tires roll or slide. To further complicate matters, the friction is affected by the wetness or dryness of the surface, by the degree of application of the brakes, and by the direction of motion of the vehicle. Additional factors are included in the deformational properties of the vehicle, the sliding friction developed between the vehicle and the guardrail, and the steering effect of the front wheels. Furthermore, there is the important human variable in the reaction of the driver that cannot be taken into account, either in theory or in the laboratory.

If in addition to the foregoing, we mention the parameters associated with the guardrail, we arrive at a discouragingly long list of variables, including the properties of the soil in which the posts are embedded, the elastic properties of the posts and their depth of embedment, and the elastic and geometric properties of the rail (including beam, cable, mesh, and plate types). The local topography and the grade and alignment of the guardrail are additional factors.

It is next to impossible to take all of these quantities into account, not only because of lack of time but also because of lack of knowledge of some of their magnitudes and

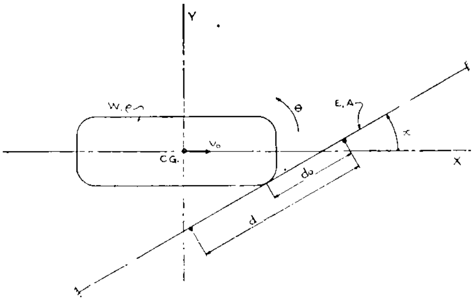

Figure 1. Coördinate system.

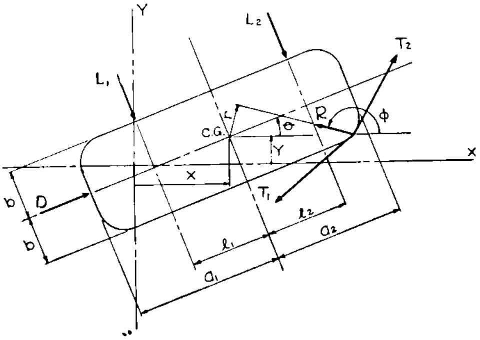

Figure 2. Displaced position of vehicle and force system.

physical relationships. It is necessary, therefore, to restrict the investigation, at first, to what appear to be the more-important variables, namely, the weight distribution of the vehicle, the velocity and the direction with which the vehicle strikes the guardrail, the friction properties, and the elastic properties of the guardrail. In a later phase of the investigation we hope to take into account the elastic suspension of the vehicle, the energy dissipative properties of the guardrail system, and the elastic coupling of the rail, posts, and soil.

ANALYSIS

Assumptions. With the foregoing general statement in mind, we made the following assumptions: (1) the vehicle is a rigid body restricted to three degrees of freedom in horizontal plane motion: (2) the front wheels have no steering effect, it being assumed that they are fixed in direction parallel to the longitudinal axis of the vehicle; (3) the weight of the cable is negligible in comparison with the weight of the vehicle; (4) the posts are rigid; (5) the cable can slide without friction at the posts; (6) the only dissipation of energy is in the friction between the vehicle and the cable, between the vehicle tires and the ground, and in the braking of the vehicle.

Differential Equations of Motion. The coördinate system has been shown in Figure 1, where the origin of coördinates is at the center of gravity of the vehicle at the location of first contact with the guardrail, and the initial velocity of the vehicle has the magnitude v_0 and is directed along the x-axis. Assuming positive displacements and a force system as shown in Figure 2, the following system of differential equations of motion may be written.

$$\frac{W}{g} \cdot \frac{d^2x}{dt^2} = D \cos \theta - (L_1 + L_2) \cdot \sin \theta + R \cos \phi, \tag{1a}$$

$$\frac{W}{g} \cdot \frac{d^2y}{dt^2} = \sin \theta - (L_1 + L_2) \cdot \cos \theta + R \sin \phi, \tag{1b}$$

$$\frac{W}{g} \rho^2 \cdot \frac{d^2\theta}{dt^2} = -(L_1 l_1 + L_2 l_2) + Rr; \tag{1c}$$

where x, y, θ = coördinates of plane motion,
t = time, measured from instant of first contact of vehicle with guardrail;

where the starting conditions at $t = 0$ are $x = y = \theta = 0$; and

$$\frac{dy}{dt} = \frac{d\theta}{dt} = 0; \qquad \frac{dx}{dt} = v_0,$$

which is the initial velocity, i.e., the impact velocity;
and where

W = weight of vehicle,
g = acceleration of gravity,
ρ = radius of gyration of vehicle with respect to the vertical gravity axis,
$+D$ = driving force, parallel to longitudinal axis of vehicle,
$-D$ = braking force, parallel to longitudinal axis of vehicle,
$\pm L_1$, $\pm L_2$ = total transverse sliding friction force acting at rear wheels and at front wheels, respectively,
l_1, l_2 = distances from center of gravity to rear and to front axles, respectively,

R = resultant of instantaneous cable tensions T_1 and T_2, i.e., resultant force exerted by guard rail on vehicle;
r = length of moment arm of R relative to the center of gravity,
ϕ = angle of direction of R relative to the x-axis,
a_1 = distance from center of gravity to rear of vehicle,
a_2 = distance from center of gravity to front of vehicle,
b = half width of vehicle,
α = angle of direction of guardrail relative to x-axis,
d = distance between posts in guardrail,
d_0 = distance along guardrail from initial point of contact to first post ahead,
n = number of continuous spans in cable-type guardrail,
k = number of parallel cables,
E = effective modulus of elasticity of cable,
A = net cross-sectional area of one cable.

The instantaneous reaction R is a complicated function of the displacements x, y, and θ, and of the constants α, d, d_0, n, k, E and A of the cable. It should be noted that L_1 and L_2 are *passive* friction forces which exist only when there is translation transverse to the longitudinal axis of the vehicle or when there is rotation; they act in a direction to oppose the motion. L_1 and L_2 depend upon the weight distribution of the vehicle and upon the coefficients of sliding friction between the tires and the road or ground surface. The longitudinal force D may be positive, zero, or negative, and of variable magnitude, depending upon the reaction of the driver. In a more-complete analysis, L_1, L_2 and D must depend also upon any tendency of the vehicle to leave the assumed plane of motion.

It would be possible to solve the equations by step-by-step numerical methods for any specified values of the constants and of the initial velocity v_0. However, a more-fruitful approach is to use the general form of the equations for establishing the necessary relations between the prototype (the full-scale guardrail and vehicle) and the model and then to construct the model and to solve specific cases by experiment with it.

Model Analysis. If Equations 1a and 1b be divided by W, and Equation 1c by $W\rho$, the following dimensionless equations are obtained:

$$\frac{1}{g}\cdot\frac{d^2x}{dt^2} = \frac{D}{W}\cos\theta - \frac{L_1 + L_2}{W}\cdot\sin\theta + \frac{R}{W}\cos\phi \qquad (2a)$$

$$\frac{1}{g}\cdot\frac{d^2y}{dt^2} = \frac{D}{W}\sin\theta - \frac{L_1 + L_2}{W}\cdot\cos\theta + \frac{R}{W}\sin\phi \qquad (2b)$$

$$\frac{\rho}{g}\cdot\frac{d^2\theta}{dt^2} = -\frac{L_1 l_1 + L_2 l_2}{W\rho} + \frac{Rr}{W\rho} \qquad (2c)$$

Since Equations 2 apply either to the model or to the prototype, we may write the following equalities, where the subscript m refers to the model and p to the prototype:

$$\left(\frac{1}{g}\cdot\frac{d^2x}{dt^2}\right)_m = \left(\frac{1}{g}\cdot\frac{d^2x}{dt^2}\right)_p;$$

$$\left(\frac{D}{W}\right)_m = \left(\frac{D}{W}\right)_p; \quad \theta_m = \theta_p; \quad \text{etc.};\ldots \qquad (3)$$

$$\left(\frac{\rho}{g}\cdot\frac{d^2\theta}{dt^2}\right)_m = \left(\frac{\rho}{g}\cdot\frac{d^2\theta}{dt^2}\right)_p;$$

$$\left(\frac{L_1 l_1}{W\rho}\right)_m = \left(\frac{L_1 l_1}{W\rho}\right)_p; \qquad \text{etc.}$$

Furthermore, since $g_m = g_p$, we have

$$\left(\frac{d^2x}{dt^2}\right)_m = \left(\frac{d^2x}{dt^2}\right)_p;$$

$$\cdots\left(\rho\frac{d^2\theta}{dt^2}\right)_m = \left(\rho\frac{d^2\theta}{dt^2}\right)_p. \qquad (4)$$

Designating force, length and time dimensions for the model by f_m, l_m, t_m, and for the prototype by f_p, l_p, t_p, we obtain from Equations 4 the relationship,

$$\frac{l_m}{t_m^2} = \frac{l_p}{t_p^2},$$

and consequently arrive at the following well-known relation between the length scale and the time scale:

$$\frac{t_p}{t_m} = \sqrt{\frac{l_p}{l_m}}. \qquad (5)$$

From Equations 3 we see that the force scale is independent of the length scale and the time scale:

$$\frac{W_p}{W_m} = \frac{D_p}{D_m} = \cdots = \frac{f_p}{f_m};$$
$$\frac{W_p \rho_p}{W_m \rho_m} = \frac{L_{1p} l_{1p}}{L_{1m} l_{1m}} = \cdots = \frac{f_p}{f_m} \cdot \frac{l_p}{l_m} \qquad (6)$$

A scale for translational velocity may be derived as follows:

$$\frac{v_p}{v_m} = \frac{l_p}{l_m} \cdot \frac{t_m}{t_p} = \sqrt{\frac{l_p}{l_m}}. \qquad (7)$$

The elastic properties of the cable are best accounted for by modeling the product of A and E, which has the dimension of force, thus

$$\frac{(A \cdot E)_p}{(A \cdot E)_m} = \frac{f_p}{f_m}. \qquad (8)$$

This relationship may also be demonstrated on the basis of cable elongations, as follows: The cable elongation scale must be equal to the length scale. Representing cable elongation by δ we may write,

$$\frac{l_p}{l_m} = \frac{\delta_p}{\delta_m} = \frac{\left(\frac{Tl}{AE}\right)_p}{\left(\frac{Tl}{AE}\right)_m},$$

from which it is found that

$$\frac{(A \cdot E)_p}{(A \cdot E)_m} = \frac{T_p}{T_m} = \frac{f_p}{f_m},$$

In theory the motion has been limited to a horizontal plane, but this does not mean that the vertical dimensions can be disregarded. The height of the cable above the ground, the vertical spacing of the cables in the case of a multiple-cable system, the vertical location of the center of gravity of the vehicle, and the radii of gyration of the vehicle with respect to the horizontal gravity axes should all be modeled according to the length scale.

In order to set up the model, we choose an arbitrary length scale and from Expressions 5 and 7 determine the time scale and translational-velocity scale. Since one of the controlling factors in the construction of the model is the selection of a cable, we cannot be completely arbitrary in the choice of the force scale but must determine it from Expression 8 after a model cable has been selected and its AE value measured and compared with the AE of the prototype cable. Once the force scale has been determined, the weight of the model vehicle follows from Expressions 6.

The foregoing analysis results in a modeling of force and displacement and not of cable stress. This is not a serious limitation, because the model is not being set up for the purpose of a direct determination of cable stress and cable failure. However, some discretion must be used in the selection of a model cable. If the prototype cable has a linear stress-strain relation, then the model cable must also have a linear relation.

LABORATORY EXPERIMENTAL APPARATUS AND METHODS

General Apparatus

The experimental apparatus, shown schematically in Figure 3, consists of the model vehicle and guardrail, the "ground" surface, a device for accelerating the vehicle to the impact velocity, an electrical circuit for measuring the impact velocity, a still camera and stroboscopic light for use in recording the path of the vehicle by multiple-flash photography, electrical dynamometers and related equipment for recording the time-variation of force in the guardrail cable, and an electrical control circuit for coördinating the camera and the recording oscillograph with the approach of the vehicle.

Prototype

Vehicle I. The lengths of the full-scale vehicle are based on the "design vehicle" shown by E. R. Ricker in "The Traffic Design of Parking Garages" (*5*). The weight is assumed equal to 4,000 lb.

Guardrail A. The guardrail is assumed to consist of three ¾-inch, parallel steel cables with the center cable located 19 inches above ground and with the cables supported directly on rigid posts spaced 16 feet apart. Each cable has a total length of 160 feet and is anchored without springs to a rigid "dead man" at each end.

Model

Length, Time, and Velocity Scales. The length scale, which has been controlled to some

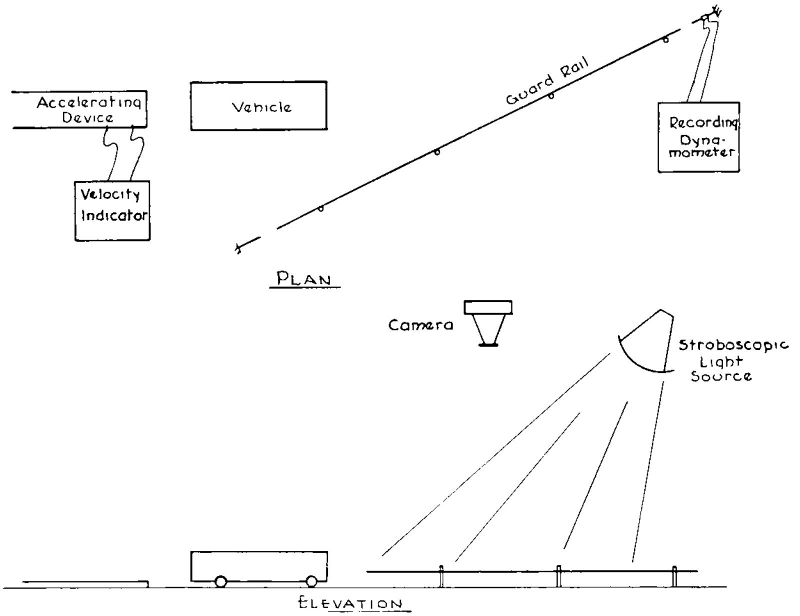

Figure 3. Schematic diagram of laboratory apparatus.

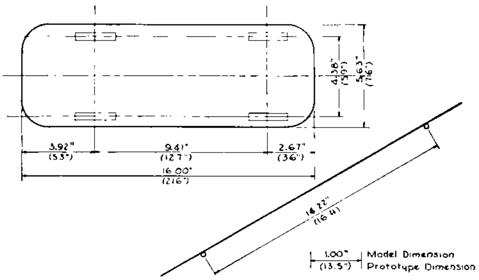

Figure 4. Plan view of vehicle and guardrail.

extent by the construction details of the accelerating device, is

$$\frac{l_m}{l_p} = \frac{1}{13.5}.$$

Figure 4 shows a plan view of the vehicle and guardrail, including comparative lengths of the model and the prototype. For example, an automobile 216 inches long overall, is represented by a model vehicle 16 inches long.

From Equation 5 it follows that the time scale is

$$\frac{t_m}{t_p} = \sqrt{\frac{1}{13.5}} = \frac{1}{3.67},$$

which is to say that the "model time" is roughly a fourth that of the "full-scale time." This reduction in time does not impose a serious limitation on the recording methods.

From Equation 7 we find that the velocity scale is

$$\frac{v_m}{v_p} = \frac{1}{3.67}.$$

If the prototype velocity is given in miles per hour, the model velocity in feet per second is

$$v_m = 0.40\, V_p.$$

For example, a full-scale velocity of 60 mph. is represented in the laboratory by a velocity of 24 ft. per sec.

Force Scale. The cabling of the model guardrail consists of one steel cable, of $\frac{1}{16}$-inch diameter and 6-by-7 stranding. Having selected the model and the prototype cables, the force scale may then be determined from Equation 8,

$$\frac{f_m}{f_p} = \frac{1}{430},$$

and the required weight of the model is found to be 9.3 lb. The model guardrail has been limited to one cable because, to date, we have not found a satisfactory cable small enough to allow each prototype cable to be represented by one model cable. However, this does not appear to place a serious limitation on the model, because the basic force-displacement relationships have been maintained.

Construction Details. The model vehicle has been constructed mainly of plywood and duralumin in a manner designed to allow easy adjustment of the weight distribution. The surface that rubs against the guardrail cable is covered with tin plate. Steel ball bearings are used for wheels, with the outer surface of the bearing acting as the tire.

The ground surface is a heavy kraft paper stretched over a metal-surfaced, rigid, plywood platform. The coefficient of transverse sliding friction between the steel tires and the paper has a nearly constant value of 0.33. If a stiff, cloth-base, rubber-gasket material is used for the ground surface, the coefficient is nearly constant at 0.56. These values can be raised or lowered by using other materials. They are well within the range of available information on actual transverse sliding coefficients.

The model is free running, except for the small amount of friction in the wheel bearings, so that there is no driving force and only negligible braking force. It conceivably is possible to modify the vehicle so as to include a driving force or a braking force, but we have no definite plans at the present for doing so.

Photographic Recording of the Path of the Vehicle

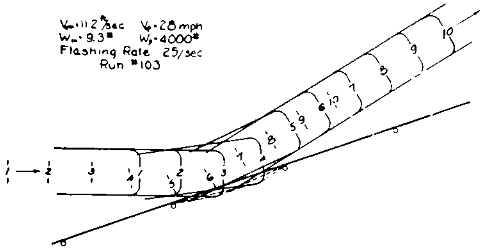

Figure 5. Tracing of stroboscopic record of impact; *Vop* = 28 mph.

A complete time record of the paths of the vehicle and of the cable has been obtained on a single photographic negative by the use of multiple-flash photography (*6*, *7*). The equipment consists of (1) a still camera mounted vertically above the ground surface and centered over the impact section of the guard rail, (2) an electrical device that opens the camera shutter as the vehicle leaves the accelerating track and closes the shutter after an interval of time long enough to include the approach of the vehicle, the impact, and the final path of the vehicle, and (3) a high-intensity stroboscopic lamp flashing at a constant rate that may be set at any value up to 100 flashes per second. The duration of each flash is of the order of 25 millionths of a second. Since the constant light intensity in the room is low in comparison with the flash intensity, the camera records the vehicle image only during each flash. For example, if the camera shutter is open for ½ second and if the lamp is flashing at the rate of 50 flashes per second the camera records 25 images of the vehicle at a time spacing of 1/50 second. Best results in the photography have been obtained by using a nonreflecting, dull black background and by outlining the vehicle and the guardrail in white.

If a coördinate system is drawn on the ground so as to appear on the negative, it is possible, by projecting the negative to a large size, to measure the instantaneous linear and angular positions of the vehicle and also to compute the average linear and angular velocities for each time interval between flashes. It is also possible to determine the instantaneous lateral displacements of the guardrail cable from the same negative. Tracings of two of the negatives have been shown in Figures 5 and 7. The diagrams have been obtained by projecting the still film (a sample has been shown in Figure 6) to large size and tracing the resulting image. In these diagrams the sides and the rounded front corners of the vehicle have been outlined; the short transverse lines indicate the rear of the vehicle. The front outlines and the corresponding rear outlines of the vehicle have been numbered successively. The initial cable tension is 10 lb. in the model, which corresponds to 4,300 lb. (total) in the prototype. The cable was anchored directly to a rigid anchor at each end. It will be noted that the prototype velocity is 28 mph. in Figure 5 and 36 mph. in Figure 7 and that the vehicle leaves the guardrail with greater angle in Figure 7 than in Figure 5.

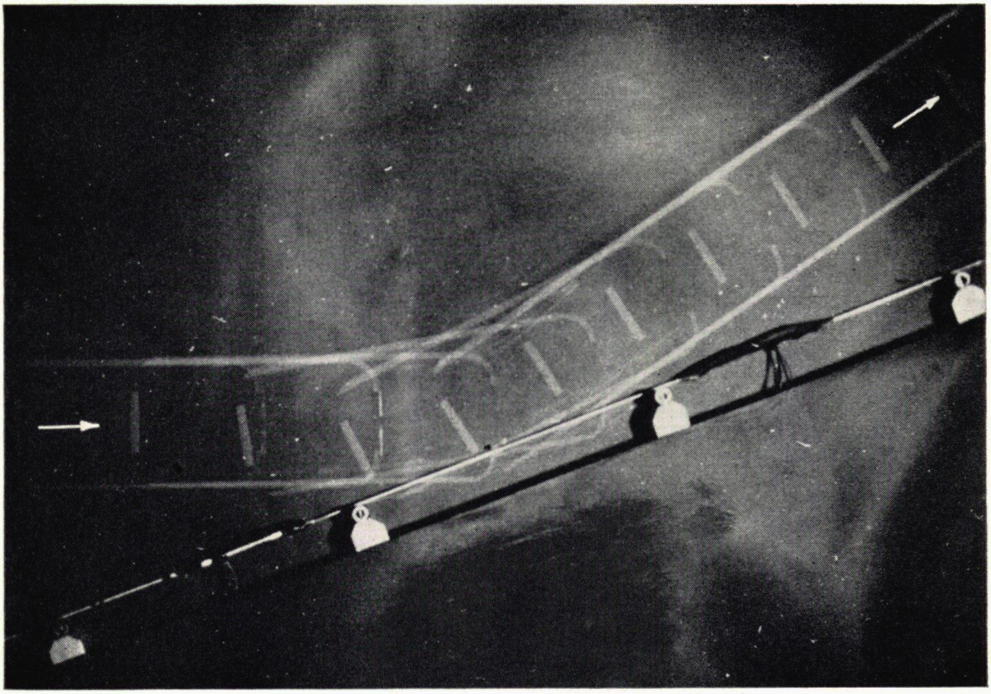

Figure 6. Stroboscopic record of impact. Print of negative from which tracing in Figure 5 was made.

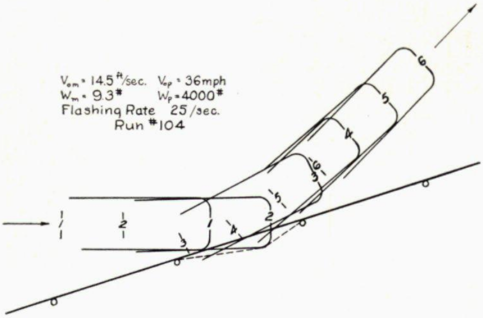

Figure 7. Tracing of stroboscopic record of impact; *Vop* = 36 mph.

Recording of Cable Tension

A tension dynamometer, consisting of a small metal strip with SR-4 wire-resistance strain gages attached to it, has been inserted in the cable in each section adjacent to the impact section. The dynamometers are used to indicate the initial tension in the cable as well as to record, through appropriate amplifiers and an oscillograph, the variation in the tensions during the impact.

CONCLUSIONS

1. A model can be made that reasonably well approaches the fullscale vehicle and guardrail as far as their characteristics are known. This includes friction affects.

2. Multiple-flash still photography provides a ready method of recording the displacements, including the lateral displacements of the guardrail, with respect to time. Velocities are easily calculated from the displacement-time data shown on the photographs. Cable tensions can be determined by a separate recording system making use of strain-gage dynamometers inserted in the cable. Multiple-flash photography has a number of interesting applications in the laboratory study of large displacements in dynamical systems.

3. A general criterion for the instantaneous direction of rotation of the vehicle in the horizontal plane can be set up on the basis of the differential equations of motion. It is hoped that more specific criteria will result from the laboratory studies.

4. Records of the preliminary experiments

show promise of the investigation leading to a useful overall understanding of the problem and to fairly definite information regarding some of the more-important variables.

5. It should be emphasized that we do not expect the analytical and the model studies to supplant the necessary full-scale work performed by various public agencies and by the manufacturers. We do feel, however, that much additional information can be obtained by means of dynamical analysis and controlled laboratory model studies based on this analysis. Furthermore, it is hoped that the laboratory studies will provide a guide to the design of full-scale field tests. This point of view is analogous to that taken in the aeronautical- and chemical-engineering fields where a design or process is developed from the laboratory stage, through the pilot model stage, and finally to full-scale tests.

WORK IN PROGRESS

A systematic set of experiments varying vehicle weight and velocity, impact direction, initial cable tension, total cable length, presence or absence of cable-anchor springs, post spacing, and friction between the vehicle and ground is in progress. A future set of experiments will vary the elasticity of the cable support fitting at the post and the elastic and plastic properties of the post and the post embedment. The specific information derived from the experiments includes the final direction of the vehicle relative to the guardrail, change in velocity and total energy dissipation, average and maximum linear and angular accelerations, maximum cable tensions ahead of and behind the impact section, and maximum transverse displacement of the guard cable.

ACKNOWLEDGMENT

This investigation is one of five on subjects relating to highway problems being carried out under a research contract sponsored by the Maryland State Roads Commission at the Civil Engineering Department of Johns Hopkins University. The contractual arrangements were made before the recent resignation of William F. Childs, Jr., as chief engineer of the commission and the subsequent appointment of his successor, Norman Pritchett. J. Trueman Thompson is chairman of the department and research contract director. The contract is administered by a Coördinating Research Committee composed of Walter C. Hopkins, deputy chief engineer of the commission; C. A. Goldeisen, A. L. Grubb, and Allan Lee, representing the commission; and J. Trueman Thompson, Robert S. Ayre, Walter C. Boyer, and Wen-Hsiung Li, representing the department. Joel I. Abrams is research staff assistant. The authors are indebted to the members of the Coördinating Research Committee for suggestions and to the research staff of the department for assistance in carrying out the investigation.

REFERENCES

1. "Report of the Committee on Highway Guard," G. A. Rahn, Chairman; *Proceedings* of the Twenty-First Annual Meeting of the Highway Research Board, 1941.
2. "Safe Design of Highway Guard Rails," by Searcy B. Slack, Civil Engineering, vol. 4, no. 8, p. 413; 1934.
3. "Design Loads for Guard Rails," by Joseph Barnett (with discussions), *Proceedings* of the Nineteenth Annual Meeting of the Highway Research Board, 1939.
4. Motion pictures of 1938 guard rail field tests performed by American Steel and Wire Co.
5. "The Traffic Design of Parking Garages," by E. R. Ricker, Yale University, Eno Foundation, 1948.
6. "Flash," by H. E. Edgerton and J. R. Killian: published by Hale, Cushman and Flint, Boston; 1939.
7. "High Speed Photography," by George A. Jones: John Wiley and Sons, New York: 1952.