Forecasting Distribution of Interzonal Vehicular Trips by Successive Approximations

THOMAS J. FRATAR, *Associate*
Knappen-Tippetts-Abbett-McCarthy, Engineers, New York

THE object of the successive-approximation method for forecasting interzonal vehicular trips is the determination of the most-logical distribution of trips expected to be made in a given area under definite conditions of regional development. The procedure is intended to provide a distribution compatible with the apparent relative attractiveness of each type of movement and with definite assumptions as to the total number of trips generated by each zone.

The procedure is not concerned with such techniques as regional planning and population forecasting, which must be performed regardless of the method used for estimating the trip distribution. The procedure does require that regional planning information be avilable for estimating the total traffic expected to enter and leave each traffic zone of the area under the new set of conditions for which the distribution is desired. The method also requires that origin-and-destination data be available for a given date and that a relationship be established between the regional development conditions as of the date of the origin-and-destination data and the date for which the new distribution is desired. With this basic information the successive-approximation method breaks down the anticipated total trips for each traffic zone into interzonal volumes which will fit the regional development plan and be compatible with the conditions assumed to exist.

The basic question to be answered is: How will the traffic of each zone be distributed in interzonal movements? For each movement the volume of traffic will reflect the relative attractiveness of that movement in competition with other possible movements. Changes in the relative attractiveness of any interzonal movement will result from changes in the traffic generated by the two zones involved relative to change in thė traffic generation of other zones.

To determine the interzonal volumes for a new set of conditions, the total new trips estimated for each zone are distributed to the interzonal movements involving the zone in proportion to the measured trips between it and each other zone and in proportion to the expected growth of each other zone. When the new traffic volumes into and out of all zones are distributed in this manner each interzonal trip has been assigned two tentative values, one as the result of the distribution for one of the zones involved and the other as the result of the distribution for the other zone involved. As a first approximation these pairs of tentative values are averaged.

The averaged values are added for each zone, adjusted to agree with the anticipated totals, and again distributed. The new distribution reflects the new tentative values of attractiveness of the various interzonal movements. The procedure is repeated until the desired degree of conformity is obtained between the anticipated totals and the assigned totals for each zone. Convergence is rapid, and with the use of high-speed calculators the distribution process is relatively economical.

● EACH vehicular trip on a highway network is prompted by a motive—to go to work, to go shopping, to go to school, to go home, or to engage in any of the numerous pursuits of community life. In any area and at any time the trips and purposes can be determined and the travel pattern can be established. With the aid of this measurable information and regional development trends, the highway-planning engineer must anticipate the travel pattern which will exist in the future in order to design highway networks which can efficiently accommodate the traffic loads imposed on them during their useful lives.

If the various parts of an area remain unchanged or if they change in a uniform way, the future traffic pattern will be the same as, or a uniformly expanded copy of, the existing pattern. That is rarely the case. More often portions of an area remain more or less stable while other portions expand or diminish. The future distribution of traffic will reflect those conditions—trips to and from the stable areas will remain unchanged, trips to and from the expanding areas will increase and trips to and from diminishing areas will decrease.

The method for predicting future interzonal volumes by successive approximations described in this article is based on the premise that if the character and growth conditions of traffic zones are known, or can be predicted, it is possible to estimate with equal dependability the total trips which will be made to and from each zone and the distribution of those trips in interzonal travel.

METHODS OF DISTRIBUTION PREVIOUSLY USED

Two general methods have been used for estimating the volumes and distribution of interzonal traffic for future conditions or for existing conditions when current origin-and-destination data are not available. By one of these methods a single expansion factor is applied uniformly to all interzonal movements. This procedure can lead to conclusions which are far from correct when significant changes take place in the concentrations of population, business and industry, or in the economic characteristics of the various zones. By the other method, each interzonal movement is expanded by multiplying it by the arithmetic or geometric mean of the factors representing the probable traffic growth of the two zones involved in the movement. An obvious error in this method is the discrepancy between the summation of the expanded trips to and from any zone and the anticipated trips from that zone. In recognition of this deficiency, planners have sometimes made arbitrary adjustments of the estimated interzonal volumes in order to obtain results more nearly consistent with the anticipated future state of development.

DEVELOPMENT OF THE PROPOSED METHOD

In the course of planning a county-wide highway system for the Cleveland area, the writer and his associates were reluctant to accept the averaging method or to rely heavily on arbitrary adjustments. Intensive study was made of the possible ways of making the distribution. Out of these studies the procedure described herein was developed.

The study area in Cleveland is approximately 12 miles by 30 miles in extent and consists of approximately 256 traffic zones. These zones were selected so as to isolate areas with distinct characteristics and to limit the size of each zone so that its traffic volumes could be manipulated readily in the planning work. About 500,000 vehicles are registered in the area, and in 1952 almost 1¾ million trips were made each day. By 1975 much of the now-undeveloped portions of the suburbs will be taken over as sites for industrial centers, housing developments, shopping centers and the like. We estimated that as a result of anticipated increases in population and community activities there would be about 90 percent more trips made each day in 1975 than were made each day in 1952.

The problem was something like this. A definite number of trips was anticipated for 1975 because people will have to go to and from work, to and from shopping, and so forth, in about the same way as they do now. A difference will be that there will be more people, more shopping areas, and more places to work. What will be the new distribution of trips?

Certainly the volume of vehicles which moves between two zones at any time is an indication of the attractiveness of that movement at that time. Presumably, also the relative attractiveness will be affected by the growth of the two zones involved in the movement. What was needed was a system which would balance these relative-attractiveness factors.

We spent a great deal of time trying to use simultaneous equations. We found, as we should have known, that there are entirely too many unknowns. We then sought to bring the various factors of attractiveness into their proper relationship by successive approximations, and although the search led into many blind alleys, the methods described in this article were finally evolved and were found to work reasonably well.

METHOD OF SUCCESSIVE APPROXIMATIONS

Two basic compilations of data are essential tools in the successive-approximation method for forecasting interzonal vehicular trips: (1) a regional economic-development plan on the basis of which the growth of each traffic zone and changes in its characteristics can be estimated and (2) origin-and-destination data for a given date.

The regional development plan records the existing development of residential, commercial, industrial, and recreational zones and envisages the growth and changes in the characteristics of each part of the area. Its preparation requires an intimate knowledge of the area and of the controls and influences which tend to affect the characteristics of various parts of it. It is the framework within which the traffic pattern must be fitted.

With the regional plan as a guide and with the aid of pertinent trends, the development of each traffic zone of the area can be foreseen, and the total number of vehicular trips which will enter and leave each zone at some future date can be estimated within reasonable limits. Thus, for example, if a zone is entirely residential in character, the future trips to and from it will be estimated on the basis of the anticipated future population of the zone and the anticipated trips per capita; if a zone is entirely industrial in character, the future trips to and from it will be estimated on the basis of the anticipated future employment in the zone and the anticipated automobile usage. In the same manner, but with somewhat greater complexity for zones with mixed characteristics, estimates can be made of the total trips which can be expected to enter and leave each traffic zone of the area. In this process the proper relationship is maintained among population, worker trips, business and shopping trips, recreational trips, etc. The estimates must obviously represent possible conditions. For example, no one zone can have more trips into and out of it than enter and leave the other zones combined.

After arriving at logical estimates of the total trips which will enter and leave each zone, the next step is to employ a practicable means for distributing these trips so that the estimated totals for each zone remain unchanged and so that the distribution is logical. As mentioned previously, it was concluded that the attractiveness of any interzonal movement at a given time is indicated by the relative volume of that movement. At a future time, the attractiveness must be modified to reflect the intervening changes in the activity of the two zones involved and the relative changes in activity of these zones in relation to other zones of the area. For the latter purpose a comparison is made for each zone of the estimated future traffic and the present traffic as shown by origin-and-destination data. From these two volumes for each zone, growth factors are computed. These are merely ratios of the estimated future traffic volume to the present traffic volume for each zone. Thus, if it is anticipated that the population of a residential zone will be doubled and it is believed that the travel characteristics will remain in the future as they are at present, the trips into and out of that particular residential zone would also be doubled and the growth factor would be 2.0.

Because many computations are involved in the procedure some more or less automatic-calculating method must be used. Punched-card methods were used in the Cleveland area study. For each interzonal movement a punched card was prepared to show, at the outset, the known volume of trips made between the two zones as revealed by an origin-destination survey.

The actual manipulation of the cards by the accounting-machine methods may be done in several ways. The procedure used for the Cleveland area study was about as follows: For each zone, all the interzonal-trip cards involving that zone were selected and gang-punched to show the growth factor for the zone and the actual future volume of traffic estimated to enter and leave the zone in 1975. This operation was repeated for each zone.

The cards were then reselected by zones and for each movement the interzonal volume determined by the origin-destination survey was multiplied by the growth factor of the opposite zone on the card. Thus, when zone 101 was selected, the interzonal volume shown for each movement involving zone 101 was multiplied by the growth factor of the zone other than zone 101 on that card. For the group of cards representing a particular zone, the products of each volume and its respective opposite zone growth factor were then added and the sum was used as a common denominator in the initial distribution of future interzonal trips. For each interzonal movement involving the zone, this common denominator was divided into the product of the measured interzonal volume and the growth factor of the opposite zone and the result was multiplied by the total trips anticipated for the zone. By this means, the total trips anticipated for each zone were distributed to the various types of movements in proportion to the attractiveness of the movement as indicated by existing volumes and in proportion to the change in attractiveness as indicated by the growth of the zones which generate the traffic.

At the end of this operation all cards involving the movement to or from any zone would have tentative future volumes based on the most-probable distribution for each of the zones. This would be a logical place to stop, except for one important reason: Each interzonal movement now has two tentative values, one as the result of the distribution at one of the zones involved and the other as the result of the distribution at the other zone involved. Both of the values are punched into the card for the movement.

Some means must be introduced at this point to allow for the necessary balancing which will result in interzonal volumes compatible with the conditions for each zone. It appears, however, that if logical growths are anticipated for each zone the disagreement at this point is not startling. To resolve the disagreements, the pairs of interzonal volumes were averaged. This can be done automatically. The cards were then reselected by zones, the average volumes for each zone were added, and the sum for each zone compared with the desired volume for the zone. The ratio, for each zone, of this computed sum to the desired volume indicates how much the volume computed as above has to be changed to agree with the desired zonal volume. This ratio may be considered to be a new growth factor.

The process is then repeated, using the new growth factors and the new interzonal volumes. The distribution is made as before for each zone according to the apparent attractiveness of the interzonal movement. This time, however, the attractiveness is indicated by the tentative interzonal volumes just computed and by the new growth factors just determined. When the process is completed the second time each interzonal movement has two values, but this time the two volumes are closer together. These new pairs of volumes are averaged and new totals obtained for each zone. These new totals are also nearer the desired totals.

With these second approximations of interzonal volumes and second approximations of total volumes it is possible to proceed to the third approximation, fourth approximation, etc., if so desired.

It appears on the basis of our limited experience that two or three cycles are sufficient. For example, in Cleveland, at the end of the first approximation the maximum difference between the desired volume and the computed volume for any zone was 33 percent but the difference for more than three fourths of the zones was less than 5 percent. At the end of the second approximation the maximum difference was about 8 percent for any zone, and the difference for most of the zones was negligible.

Steps in the Method

1. Prepare dependable estimates of the total number of automobile trips which can be expected to enter and leave each traffic zone of the area under study at the future date for which the distribution is desired.
2. Distribute the total trips of each zone among the various movements in proportion to the attractiveness of each movement as indicated by existing interzonal volumes and by the anticipated growth of each of the other zones.
3. The distribution of trips for all zones will produce two tentative values for each interzonal movement. These pairs of tentative values are averaged to obtain the first approximation of the interzonal volumes.
4. For each zone, the sum of the first-approximation volumes is divided into the total volume desired for the zone to obtain a first approximation "growth factor" which will be used in the computations for the second approximation.
5. The originally estimated trips for each

zone are again distributed into interzonal movements, these new assignments being in proportion to the interzonal volumes and growth factors obtained by the first approximation. The pairs of tentative volumes obtained by this distribution are averaged as before, and the process is repeated until the desired conformity is obtained.

EXAMPLE OF PROPOSED METHOD

The following computations for a simple four-zone problem illustrate the proposed procedure. The situation is summarized below.

Present Number of Interzonal Trips

Zones	A	B	C	D
A	—	10	12	18
B	10	—	14	14
C	12	14	—	6
D	18	14	6	—
Present Totals	40	38	32	38
Estimated Future Totals	80	114	48	38
Growth Factors (Ratio of Present Totals to Future Totals)	2	3	1.5	1

For Zone A the future traffic volume of 80 trips would be distributed to the interzonal movements AB, AC, and AD in proportion to the attractiveness of those movements at A and for Zone B the future traffic volume of 114 trips would be similarly distributed to interzonal movements AB, BC, and BD according to the attractiveness of those trips at B. The volume of AB in each case would be:

$$\frac{\text{The future trips into and out of the zone considered (A or B)} \times \text{existing trips along AB} \times \text{growth factor of opposite zone}}{\text{Sum of products of existing trips ofthe zone considered (A or B)and the respective opposite growth factors}}$$

The distribution to AB at A would be:

$$\frac{80 \times 10 \times 3}{10 \times 3 + 12 \times 1.5 + 18 \times 1} = 36.4$$

and the distribution to AB at B would be:

$$\frac{114 \times 10 \times 2}{10 \times 2 + 14 \times 1.5 + 14 \times 1} = 41.5$$

The computations for the first approximation for each of the four zones in the example are summarized below. Line 1 for each summary shows the existing trips for the indicated interzonal movement. Line 2 shows for each zone the interzonal trips multiplied respectively by the growth factor of the other zone involved. These products are summarized for each zone to provide a common denominator for the distribution of the trips of that zone. This distribution is accomplished by dividing the common denominator into the total trips desired for the zone and multiplying the quotient by the products shown in Line 2. The new distribution, shown in Line 3, necessarily adds up to the total number of trips desired for the zone.

First Approximation

Zone	A	B	C	D	Sum of Products of Trips & Growth Factors	Desired New Total Trips	Ratio of Desired New Total Trips to Sum of Products
Growth Factor	2	3	1.5	1			
For Zone A							
(1)	—	10	12	18			
(2)	—	30	18	18	66	80	1.21
(3)	—	36.4	21.8	21.8	80		
For Zone B							
(1)	10	—	14	14			
(2)	20	—	21	14	55	114	2.07
(3)	41.5	—	43.5	29.0	114		
For Zone C							
(1)	12	14	—	6			
(2)	24	42	—	6	72	48	.667
(3)	16.0	28.0	—	4.0	48		
For Zone D							
(1)	18	14	6	—			
(2)	36	42	9	—	87	38	.437
(3)	15.8	18.3	3.9	—	38		

The pairs of interzonal volumes obtained by these computations are averaged as shown below to obtain the first approximation for interzonal trips.

	A-B	A-C	A-D	B-C	B-D	C-D
	36.4	21.8	21.8	43.5	29.0	3.9
	41.5	16.0	15.8	28.0	18.3	4.0
	77.9	37.8	37.6	71.5	47.3	7.9
First Approximations	39.0	18.9	18.8	35.7	23.6	4.0

The averages for the trips radiating from each zone are next summarized to determine new growth factors to be used in the second approximation, as shown below.

	A	B	C	D
	39.0 18.9 18.8	39.0 35.7 23.6	18.9 35.7 4.0	18.8 23.6 4.0
New Totals	76.7	98.3	58.6	46.4
Desired Totals	80.0	114.0	48.0	38.0
New Growth Factors	1.04	1.16	.82	.82

Additional cycles of approximations and corrections could be made, as shown below.

Second Approximation

Zone	A	B	C	D	Sum of Products of Trips & Growth Factors	Desired New Total Trips	Ratio of Desired New Total Trips to Sum of Products
New Growth Factors	1.04	1.16	.82	.82			
For Zone A							
(1)	—	39.0	18.9	18.8			
(2)	—	45.3	15.5	15.4	76.2	80	1.05
(3)	—	47.5	16.3	16.2	80		
For Zone B							
(1)	39.0	—	35.7	23.6			
(2)	40.5	—	29.3	19.7	89.5	114	1.275
(3)	51.6	—	37.3	25.1	114		
For Zone C							
(1)	18.9	35.7	—	4.0			
(2)	19.7	41.4	—	3.3	64.4	48	.746
(3)	14.7	30.8	—	2.5	48		
For Zone D							
(1)	18.8	23.6	4.0	—			
(2)	19.6	27.4	3.3	—	50.3	38	.755
(3)	14.7	20.7	2.6	—	38		

	A-B	A-C	A-D	B-C	B-D	C-D
	47.5 51.6	16.3 14.7	16.2 14.7	37.3 30.8	25.1 20.7	2.5 2.6
	99.1	31.0	30.9	68.1	45.8	5.1
Second Approximations	49.6	15.5	15.4	34.0	22.9	2.5

	A	B	C	D
	49.6 15.5 15.4	49.6 34.0 22.9	15.5 34.0 2.5	15.4 22.9 2.5
New Totals	80.5	106.5	52.0	40.8
Desired Totals	80.0	114.0	48.0	38.0
New Growth Factors	1.0	1.07	.92	.93

Third Approximation

Zone	A	B	C	D	Sum of Products of Trips & Growth Factors	Desired New Total Trips	Ratio of Desired New Total Trips to Sum of Products
New Growth Factors	1.0	1.07	.92	.93			
For Zone A							
(1)	—	49.6	15.5	15.4			
(2)	—	53.0	14.2	14.3	81.5	80	.982
(3)	—	52.0	13.9	14.1	80		
For Zone B							
(1)	49.6	—	34.0	22.9			
(2)	49.6	—	31.3	21.3	102.2	114	1.114
(3)	55.4	—	34.9	23.7	114		
For Zone C							
(1)	15.5	34.0	—	2.5			
(2)	15.5	36.4	—	2.3	54.2	48	.887
(3)	13.7	32.3	—	2.0	48		
For Zone D							
(1)	15.4	22.9	2.5	—			
(2)	15.4	24.5	2.3	—	42.2	38	.90
(3)	13.8	22.1	2.1	—	38		

	A-B	A-C	A-D	B-C	B-D	C-D
	52.0 55.4	13.9 13.7	14.1 13.8	34.9 32.3	23.7 22.1	2.0 2.1
	107.4	27.6	27.9	67.2	45.8	4.1
Third Approximations	53.7	13.8	14.0	33.6	22.9	2.0

	A	B	C	D
	53.7 13.8 14.0	53.7 33.6 22.9	13.8 33.6 2.0	14.0 22.9 2.0
New Totals	81.5	110.2	49.4	38.9
Desired Totals	80.0	114.0	48.0	38.0
New Growth Factors	.98	1.035	.973	.98

Fourth Approximation

Zone	A	B	C	D	Sum of Products of Trips & Growth Factors	Desired New Total Trips	Ratio of Desired New Total Trips to Sum of Products
New Growth Factors	.98	1.035	.973	.98			
For Zone A							
(1)	—	53.8	13.8	14.0			
(2)	—	55.7	13.4	13.7	82.8	80	.967
(3)	—	53.8	13.0	13.2	80		
For Zone B							
(1)	53.8	—	33.6	22.9			
(2)	52.7	—	32.7	22.5	107.9	114	1.055
(3)	55.7	—	34.5	23.8	114		
For Zone C							
(1)	13.8	33.6	—	2.0			
(2)	13.5	34.8	—	2.0	50.3	48	.953
(3)	12.9	33.1	—	2.0	48		
For Zone D							
(1)	14.0	22.9	2.0	—			
(2)	13.7	23.7	2.0	—	39.4	38	.965
(3)	13.2	22.8	2.0	—	38		

	A-B	A-C	A-D	B-C	B-D	C-D
	53.8 55.7	13.0 12.9	13.2 13.2	34.5 33.1	23.8 22.8	2.0 2.0
	109.5	25.9	26.4	67.6	46.6	4.0
Fourth Approximations	54.8	13.0	13.2	33.8	23.3	2.0

	A	B	C	D
	54.8 13.0 13.2	54.8 33.8 23.3	13.0 33.8 2.0	13.2 23.3 2.0
New Totals	81.0	111.9	48.8	38.5
Desired Totals	80.0	114.0	48.0	38.0
New Growth Factors	.99	1.02	.985	.985

For this simple four-zone problem, the maximum difference for any zone between the desired total number of trips and the adjusted total was about 3.5 percent at the end of the third cycle and about 2 percent at the end of the fourth cycle. Each successive cycle reduced the difference by about half.

As can be seen from this example, manual procedures are entirely impracticable for other than extremely simple problems, such as the one illustrated. However, it can also be seen that the procedures are repetitious and each is, in itself, relatively simple. Because of this, an extensive problem of any conceivable complexity can be readily set up for rapid analysis by business-machine methods.

COMPARISON OF RESULTS OBTAINED WITH PROPOSED METHOD AND WITH AVERAGE-GROWTH-FACTOR METHOD

If it is determined that the trips into and out of a zone will change in a definite way, the proposed method will provide a solution compatible with the anticipated change. For the four-zone example described above, the results by the method of successive approximations and by the method of averaging growth factors would be as follows:

Estimated Interzonal Trips

	A-B	A-C	A-D	B-C	B-D	C-D
Computed at end of fourth Approximation by Successive Approximation Method	54.8	13.0	13.2	33.8	23.3	2.0
Computed by method of Averaging Growth Factors	25.0	21.0	27.0	31.5	28.0	7.5

and the totals for the zones would be:

	A	B	C	D
Desired Totals	80	114	48	38
Totals at the end of fourth Approximation by Successive Approximation Method	81.0	111.9	48.8	38.5
Totals which would be obtained by the method of averaging growth factors	73	84.5	60	62.5

For the above example the totals obtained by the method of averaging growth factors is at considerable variance with the respective totals desired. The future volume of Zone D, for example, was increased about 90 percent by the averaging method, although the total volume of Zone D is expected to remain unchanged.

COMMENTS

The method of successive approximations described herein will undoubtedly raise many questions in the minds of readers. A few of these questions are anticipated below.

Can a negative number be obtained for an interzonal traffic movement?

No. The growth factors are always positive, being less than one when the activity for a zone is anticipated to be less in the future than at present. Consequently, although the volume for any interzonal movement may diminish, the proposed method cannot produce a negative number.

How does interzonal accessibility affect the solution?

The suppression of trips because of inadequate accessibility is not accounted for in the procedure but must be taken care of in the facility-generation factor applied after a definite route is laid out to connect the two zones.

Does a change in the volume moving along one desired line of travel affect the volumes along other lines of travel?

The method distributes the future trips in accordance with assumed conditions. For different assumed conditions, the interzonal volumes must also be different, inasmuch as the total number of trips is predetermined. The apparent influence is one of effect and not cause.

ACKNOWLEDGMENTS

This approach to the distribution of traffic is believed to be unique, although methods for solving similar complex problems by successive

approximations are well-known and extensively used in structural engineering, hydraulics, and other fields. The use of the successive-approximation procedure in the analysis of traffic problems was proposed to the writer by Hardy Cross in 1941. Its application then was considered in connection with the selection of alternative routes by motorists; but a satisfactory procedure was not found, because of the great number of unknowns involved in such problems. In the present instance, for interzonal desire-line volumes, fewer unknowns are involved and, for given assumptions, the method has been found to produce a rational distribution of trips.

The methods described in this article were conceived and applied in connection with the planning for a county-wide highway system for the Cleveland area, undertaken by Knappen-Tippetts-Abbett-McCarthy for the County of Cuyahoga, Ohio. The author is indebted to many persons for inspirations which resulted from discussions with them. Particular appreciation is expressed to Glen E. Brokke, of the staff of Knappen-Tippetts-Abbett-McCarthy.

DISCUSSION

Alan M. Voorhees, *Traffic Planning Engineer, Automotive Safety Foundation*—The underlying assumption upon which Fratar is basing his technique to assign future interzonal vehicular trips should be considered, I think, in light of recent studies made by the Bureau of Public Roads and other agencies. His technique, as well as other procedures now being used to forecast future traffic volumes (excepting San Juan), stems from the assumption that trips between zones will change in accordance with variations in a "growth factor" predicted for each zone regardless of modifications that may occur when the percentage of various types of trip purposes change between zones.

However, such an assumption seems to be contrary to recent studies. Studies made by Gordon Sharpe and others, indicate that the generating characteristics of different types of land use vary with the distances from the generating zone. That is to say, the various categories of trip purposes have different patterns of frequency, as illustrated in Figure A. For example: As shown on the chart, within 1 or 2 miles of a shopping center the number

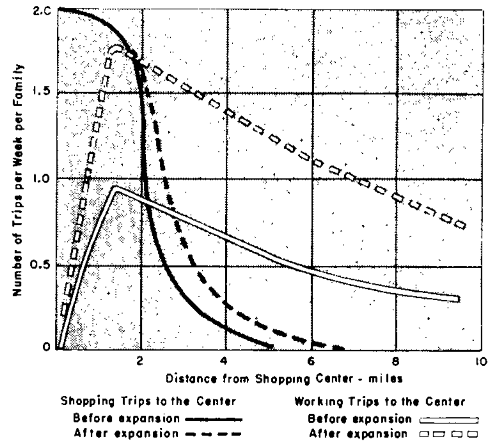

Figure A.

of shopping trips to the center is approximately two trips per week per family, but beyond that area the shopping trips per family drop to less than one trip per month. On the other hand, the pattern of work trips is quite different, the maximum occurring near the center and then decreasing at a more-even rate.

Based on information now available, there is evidence that the expansion of land use producing these various types of trips will magnify these variations in patterns of frequency. As illustrated in Figure A, an enlargement of the shopping facilities would have little effect on basic patterns of frequency, since shopping trips by automobile seem to follow Reilly's law of retail gravitation. However, if office spaces were enlarged at the center the pattern of work trips generated would be quite different. Therefore, it would seem that the number of trips between any two zones will largely depend upon the type of trips generated by the land-use expansion and the distance between the two zones.

Morton S. Raff, *Mathematician, Highway Transport Research Branch, Bureau of Public Roads*—Fratar has developed an ingenious procedure, which it will be interesting to test against actual survey results. This can be done by making a second origin-and-destination survey in some area which has already been surveyed. The zone totals from the second survey can be used in conjunction with all the data from the first one to compute estimated values for the interzonal flows, which can then be compared with the values actually found in the second survey.

Numerical experiments with a few simple examples indicate that the process does not converge quite as rapidly as might be desired. Table A illustrates the rate of convergence in Fratar's own four-zone example:

Approximation	AB	AC	AD	BC	BD	CD
Initial values	10	12	18	14	14	6
1st	38.9	18.9	18.8	35.8	23.7	4.0
2nd	49.7	15.5	15.5	34.1	22.7	2.4
3rd	53.5	13.8	14.0	33.8	22.9	2.0
4th	54.7	12.9	13.3	33.9	23.2	1.9
5th	55.2	12.5	12.9	34.1	23.5	1.8
10th	55.7	11.9	12.4	34.4	23.9	1.8
Limit	55.7	11.9	12.4	34.4	23.9	1.7

At the first approximation, half the flows are in error by more than 50 percent of their respective true values. This is reduced to 25 percent at the second approximation and 13 percent at the third. These errors are not large in comparison with the possible errors in predicting future zone totals, but they are not exactly negligible either.

One pitfall against which the user of the method ought to be on guard concerns pairs of zones between which there is no travel in the initial survey. The use of zero for the initial value of any flow forces the final value of that flow to come out zero also. Hence, if there is any possibility of future travel, the process should be started using some small positive number like 0.1 or 0.01. The difference between zero and 0.01 may lead to a large difference in the final result.

T. J. Fratar, *Closure*—The method of successive approximations is by no means a simple averaging procedure as suggested by Voorhees' initial paragraph. The technique which Vôorhees discusses, is as a matter of fact, the technique which the procedure was devised to avoid.

As stated in my paper, the method of expanding an interzonal movement by multiplying it by the arithmetic or geometric mean of the factors representing the probable traffic growth of the two zones involved, creates obvious errors. It is my understanding that in recognition of these errors, the San Juan survey was adjusted by very careful, by necessarily arbitrary, transfers of interzonal trips. In recognition of the basic errors of the averaging method and to avoid the tedious and arbitrary transfers of interzonal volumes, the successive approximations method was devised.

The method recognizes that the attractiveness of a zone is modified by the size of the zone but the technique does not stem from that relationship nearly as simply as Voorhees states. The method also recognizes the influence of distance in that the basic origin and destination data used reflects that effect. Voorhees' chart illustrates that the effect of distance is reasonably proportional for various sizes of a generating area.

For the example illustrated by my paper, a comparison is given of the results obtained with the successive approximations method and the method of averaging growth factors. It is demonstrated that the totals for each zone which would result from the use of the averaging growth factor method would be at considerable variance with the correct totals. For example, the future volume of Zone D was increased about 90 percent by the averaging method although that volume should remain unchanged. It was also found that the travel between Zones D and A should be diminished because of the greater relative attractiveness of other possible interzonal movements, although by the averaging method the volume would be substantially increased.

Raff's points are well taken for the example illustrated. In actual practice two analyses were made for the Cuyahoga County survey. The first involved zones and groups of zones and we found that two approximations were desirable in order to obtain a tolerance of less than 5 percent for almost all of the zones. In the second analysis each zone was treated separately (there were 256 zones) and we found that except for a few strays the tolerance for each zone was satisfactory at the end of the first approximation. As Raff suggested for interzonal movements where there is no travel shown by the origin and destinations survey but where travel would be anticipated in the future, factors must be introduced based on good judgment.