Direct Measurement of Shear Stresses in Soil Mass

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● THE Waterways Experiment Station, Corps of Engineers, has for some time been conducting a program of research in stress distribution in soil masses. Under this program test sections are loaded and stresses and deflections measured using pressure cells and deflection gages buried in the sections. Some of the results of tests on a homogeneous clayey silt test section were reported in *Research Report 12-F*, published in January 1951, and in a "Supplement to Stress Distribution in a Homogeneous Soil" published in the 1949 PROCEEDINGS.

The complete definition of stresses at a point in a mass requires the determination of shear stresses. In the past these have been determined by the use of pairs of pressure cells placed perpendicular to one another. Half the difference between readings from the two cells is taken as the shear stress acting on a plane at 45 deg. to the cells. This means of determination of shear stresses has given reasonably good results as data presented in the above-referenced reports will testify.

Early in 1948 R. R. Philippe, then director of the Ohio River Division Laboratories, Corps of Engineers, suggested an instrument for the direct measurement of shear stresses. Because instruments which measure normal stress are called pressure cells, the suggested instrument was called a shear cell. D. W. Taylor, professor of soil mechanics at Massachusetts Institute of Technology, suggested the general form for such a cell, and Arthur Ruge, of the Ruge-de-Forest Company, Cambridge, Massachusetts, completed detailed designs and fabricated a number of shear cells for the Ohio River Division Laboratories. Figure 1 is a drawing of the shear cell.

The shear cell consists essentially of two parallel plates connected by a dynamometer section which records the effort (shear stress) tending to move the plates with respect to one another. Spring-hingelike frames are used between the plates to provide lateral stability, and springlike pins maintain the spacing between plates.

Four of these cells were furnished to the Waterways Experiment Station for inclusion in a homogeneous dry sand test section which was constructed in 1950 and tested during 1950 and 1951. Figure 2 shows a plan of this test section. The complete results of this testing have not yet been published, but some of the data pertaining to the analysis of shear cell operation are presented here.

For this presentation it is necessary to identify a few symbols. Reference to Figure 3 will help with this identification. Normal stresses are designated as σ and shear stresses as τ with their subscripts indicating directions; σ_z is vertical stress and σ_x horizontal; σ_u and σ_v are diagonal stresses at 45 deg. to the horizontal and perpendicular to one another;

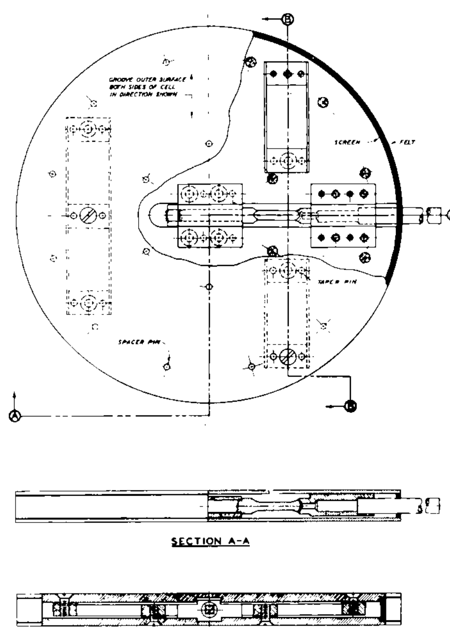

Figure 1. Earth shear cell.

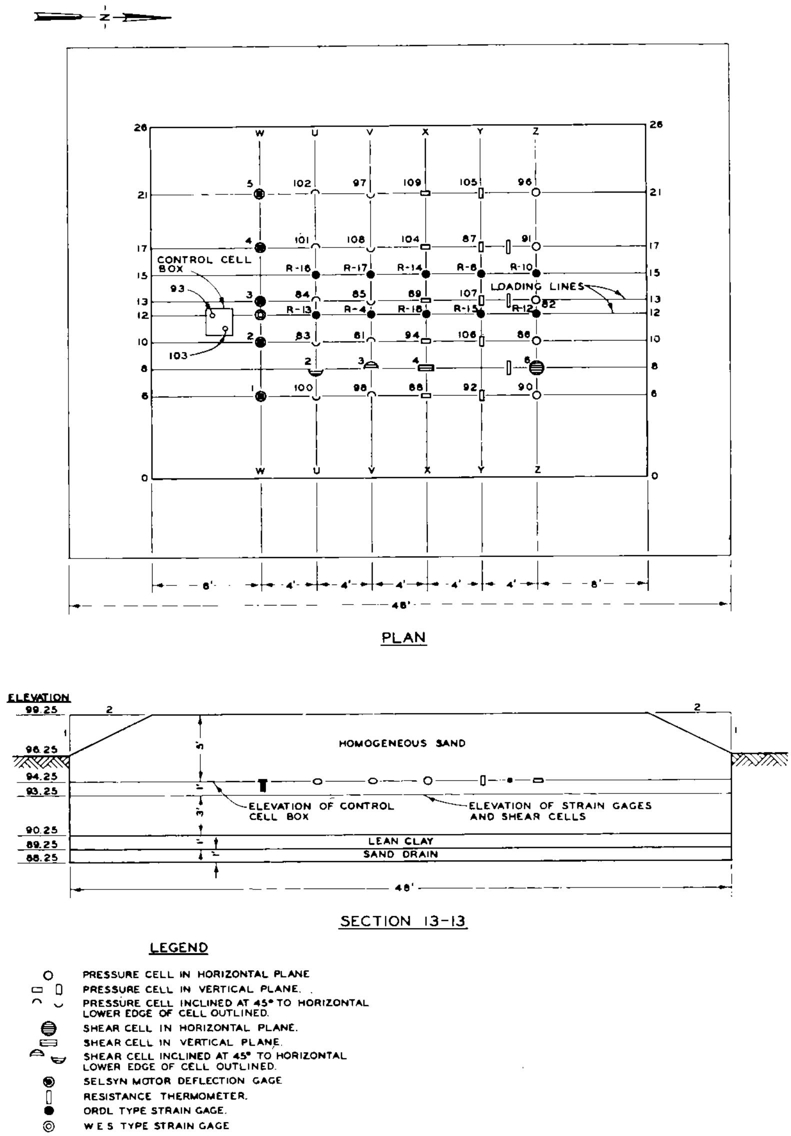

Figure 2. Plan and section, homogeneous sand test section.

τ_{xz} is vertical shear stress on a vertical plane, and τ_{zx} is horizontal shear stress on a horizontal plane; τ_{uv} and τ_{vu} are shear stresses perpendicular to one another on planes which are at 45 deg. to the horizontal.

The four shear cells installed in the homogeneous sand test section were so placed as to measure shear stresses τ_{xz}, τ_{zx}, τ_{uv}, *and* τ_{vu}. From pressure cells at equivalent positions the normal stresses σ_x, σ_z, σ_u, and σ_v were obtained. For stresses at a point $\tau_{zx} = \tau_{xz} = \frac{1}{2}(\sigma_v - \sigma_u)$ and $\tau_{uv} = \tau_{vu} = \frac{1}{2}(\sigma_z - \sigma_x)$, and from these relations it is possible to make the comparisons between measurements shown in Figure 4. Measurements from shear cells in equivalent positions with respect to the load but which are perpendicular to one another are compared in the two lower plots. Perpendicular shear stresses should be equal at any point, and the plots show very good agreement. The two upper plots show comparisons of shear stresses whose means of measurement differ greatly from one another. In the one case shear stresses are directly

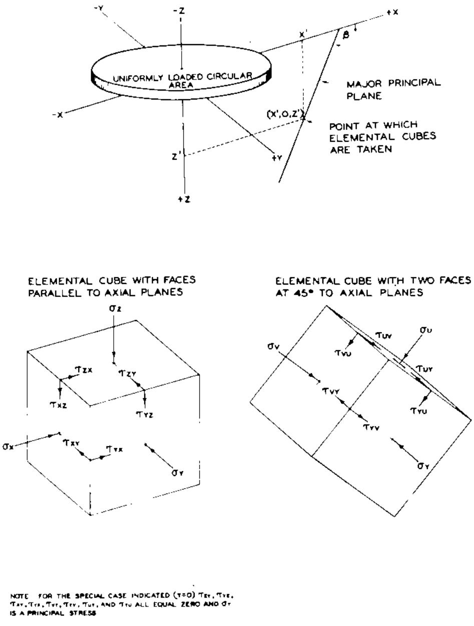

Figure 3. Stress directions.

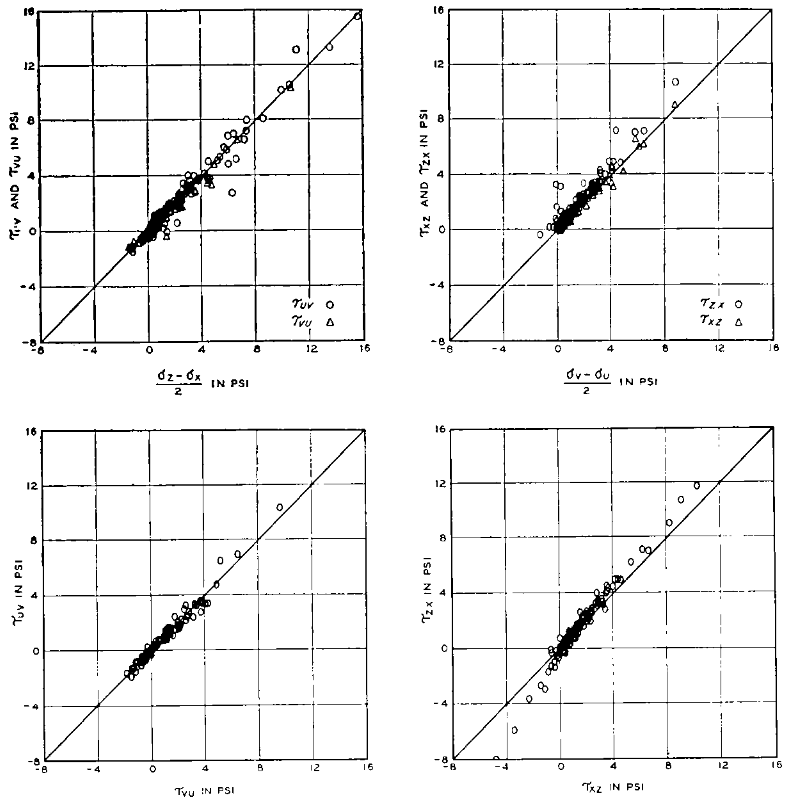

Figure 4. Comparisons of equivalent shearing-stress data.

measured using shear cells; while in the other, shear stresses are derived from pairs of perpendicular normal stresses measured with pressure cells. Here again the agreement between values compared is very good.

It appears that, at least for the conditions present in the homogeneous dry sand test section, the shear cell is capable of measuring shear stresses directly with a reasonable degree of accuracy.

Application of the Elastic Theory to Highway Embankments by Use of Difference Equations

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THE problem solved in this paper by the use of the theory of elasticity consists in the determination of the shearing and normal stresses in a trapezoidal embankment and its foundation consisting of a relatively thin, uniform, natural layer which, in turn, is underlaid by a rigid boundary, such as the top surface of a rock or stiff soil deposit. It is assumed that the materials of the embankment and its foundation are characterized by identical elastic constants. Using the stress function, a series of differential equations is obtained and replaced by finite difference equations from which the stresses may be found. This is done for a numerical example in which Mohr's circle is applied to determine the values of cohesion and internal friction required to prevent an overstressed condition in the given structure.

● THE factor of safety of an embankment slope is usually computed by application of well-known methods involving the assumption of a surface of sliding along which the average shearing stress is computed from statics. Stresses in embankment foundations have been computed from the theory of elasticity by several methods, but those known to the author have failed to satisfy all boundary conditions (*1*, *2*, *3*, *4*).

Zienkiewicz (*5*) has computed the stresses within a concrete dam with proper regard for boundary conditions by the substitution of finite difference equations for the differential equations of elasticity and by using the method of successive approximations known generally as the "Method of Relaxation" (*6*) for solving the difference equations.

In the present case, it is proposed to use an attack similar to that of Zienkiewicz, but with the addition of a rigid horizontal surface at some distance beneath the surface of the ground. Such a condition frequently exists in the coastal region of Texas, where beds of soft clay or muck have been deposited on relatively firm strata existing at depths of 10 to 40 feet beneath the present ground surface.

STATEMENT OF THE PROBLEM

The elastic body of Figure 1 is bounded by the planes $x = 0$, $x = f$, $y = b$, $y = mx$, $y = c$, $y = d$, and $z = \pm\infty$. The embankment and foundation are symmetrical about the plane, $x = 0$. The distance from the embankment to the boundary, $x = f$, is indefinitely large when compared to the vertical dimension, d-c; and the state of stress at the boundary, $x = f$, is assumed to be the same, at least to the degree of accuracy to be achieved herein, as it would have been had the embankment load not been applied. The boundary, $y = d$, is completely rigid and it is assumed that no slipping occurs along that boundary. Poisson's ratio is taken as 0.5 throughout. Gravity, which is assumed to be