Measured flow = 10.40 cm^3/sec at atmospheric pressure
Differential manometer reading = 128.5 mm H_2O
Outflow manometer reading = 10.0 mm Hg

$$\mu = 186.7 \text{ mp or } \frac{186.7 \times 10^{6} \text{ gm} - \text{sec}}{980.7 \quad \text{cm}^2}$$

P_2 = Outflow manometer reading plus atmospheric pressure = $(10.0 \times 1.36) + 1018.0 = 1031.6$ gm/cm^2

$(P_1 - P_2)$ = Differential manometer reading = 128.5 mm H_2O = 12.85 gm/cm^2

$$\text{Flow at Mean Pressure} = \frac{10.4 \times 1018.0}{1038.1} = 10.20 \text{ cm}^3/\text{sec}$$

$$K = \frac{186.7 \times 10^{-6}}{980.7} \times \frac{10.20 \times 4.10}{79.34 \times 12.85} = 7.8 \times 10^{-9} \text{ cm}^2$$

II. Calculation of pore diameter

Carmen (*10*) derives the following relationship for average pore size.

$$d = 4m = 4\left[\frac{\epsilon}{(1 - \epsilon)S_0}\right]$$

m = hydraulic radius, the ratio of area of cross-section to perimeter of cross-section
ϵ = porosity
S_0 = specific surface—cm^2/cm^3
and
d = average pore diameter—cm

Specific surface values were approximated from a table of surface area equivalents given by Hveem (*12*).

For a typical case, a specimen of grading A has a porosity of 0.25 and a specific surface of 126.3 cm^2/cm^3.

$$d = 4\left[\frac{\epsilon}{(1 - \epsilon)S_0}\right] = \frac{4 \times 0.25}{0.75 \times 126.3} = 0.010$$

or $d = 100\mu$

Significance of Variation as Related to Asphalt-Aggregate Mixes

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● IT is convenient to think of all matters related to the quality of a prepared asphalt-aggregate mix in terms of specification, production and inspection. Ordinarily if we comply with the requirements of these three terms, it may be stated that satisfactory control prevails. However, when specification or inspection requirements are not being met, this is evidence that the quality or the quality characteristics of the product has been subjected to a certain amount of variation, which may be due to 1. the result of change, or 2. assignable causes. Variation is an inherent fundamental characteristic of any manufactured product including asphalt-aggregate mixes. Therefore, it should be recognized, understood and applied by engineers concerned with either design or acceptance specifications.

Samples are taken for the primary purpose of predicting the quality of the remaining aggregation from which the sample was drawn. Let us assume that this aggregation of material varies in its physical properties, and this we can ordinarily prove by taking samples and testing until we obtain several test results that are different. Therefore, we must know something about the variability of the overall aggregation of material before we can make a valid prediction of its true test characteristic. Let us take an example of 100 Marshall briquets made up and tested for stability during the day's run on a hot mix plant. If the range of values obtained were divided into equal class intervals and a count made of the number of test results in each class interval, we would then be able to classify the results according to the frequency of the individual test. A plot of the number of individual tests versus the class interval average would result in a frequency distribution curve such as shown in Figure 1. In practice the curve seldom comes out in such a symetrical shape as that just shown. However, for practical considerations this symetrical bell curve is sufficiently representative for our purposes. Even though we attempt to manufacture briquets that are supposedly alike, each measureable quality characteristic is really a sample from a frequency distribution. The same will apply for each test characteristic such as flow, bitumen content, or the content of any gradation sizes of the stone or aggregate used. It is characteristic of data classified and distributed according to their test characteristic that the largest group of them will fall close to the mathematical average of the entire lot as illustrated by the average line ($\bar{x}$) in this figure. Thus the frequency distribution (sometimes called the central limit theorem) pictorially describes the pattern of variation.

There are a number of ways of mathematically describing this variation characteristic. A listing of the maximum and minimum test value or a listing of the range as the difference between the maximum and minimum test value is frequently used. Perhaps the most useful measure of variation is standard deviation which is a measure of the variation of individual observations about their average. The symbol for standard deviation is the Greek letter σ (sigma) and is computed as the root-mean-square deviation about the average. Helpful in getting an understanding of these terms and their significance are other terms such as dispersion or dispersion characteristics. These are synonymous with variation and variation characteristics such as are mathematically reported as range or standard deviation.

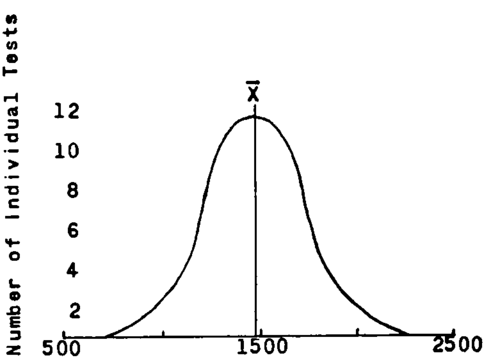

Figure 1. Normal frequency distribution on Marshall stability.

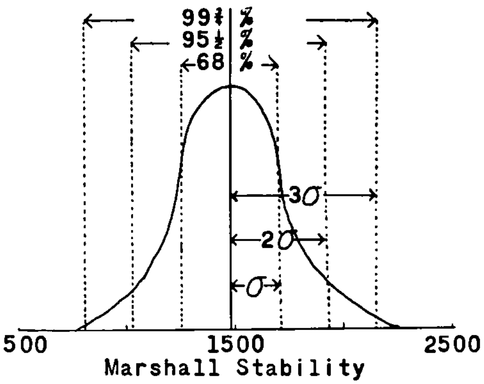

Figure 2. Standard deviation and the frequency distribution.

Mathematical statisticians have proven that a frequency distribution can be divided into six zones equal in width, each equal to σ or one standard deviation. For practical purposes it may be considered that all individual test results will fall within $\pm 3\sigma$, 95½% within $\pm 2\sigma$ and 68% within $\pm 1\sigma$. It thus becomes evident that as the process becomes more variable, the range and the standard deviation of the product become numerically larger. Consequently the calculated range and/or standard deviation thus provide a practical measure of variation of the process or product. Because range may be poorly estimated as the result of one highly deviating result, standard deviation is a more reliable measure of dispersion. Figure 2 illustrates the points just made.

We have tried to point out that an exact repetitive operation is unrealistic since no two objects are exactly alike, assuming of course that our test method is capable of distinguishing reasonable physical differences. Likewise the same concept is applicable when considering asphalt-aggregate mixes, which we know are not going to show exact duplication from sample to sample or batch to batch. Just to emphasize this point a little more strongly, let us say that the output of any plant is not going to be uniform but is essentially variable in nature. The observed average of test results that we obtain on the product is likewise subject to uncertainties that arise from the natural

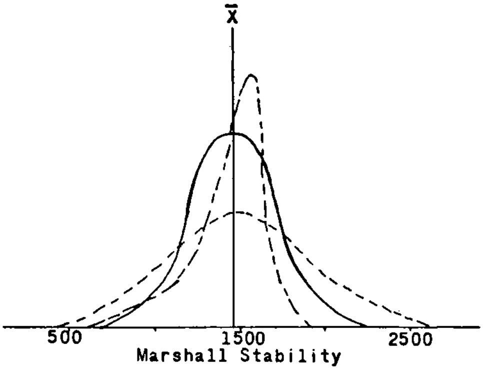

Figure 3. Different frequency distributions may have the same average ($\bar{x}$).

variability. Consequently our observed average may or may not be the true average. It is also possible to obtain the same average for different types of distribution as illustrated by Figure 3. As pointed out earlier we seldom obtain the perfectly symetrical bell shaped curve which is called a normal distribution. However, for practical purposes we can assume that a normal distribution exists.

Considering the hot mix itself we can find sources of variation in:

1. Gradation of the aggregate
2. Character of the aggregate
3. Moisture content of the aggregate & mix
4. Manipulation of the mix by the machine
5. Temperatures
6. Grade and percentage of asphalt
7. Operating personnel and their influence on controls
8. Other intangible sources

These sources or causes of variation may then be translated into statistical language and broken down into two types:

1. *Chance causes* (due to random variation in the process)
   a. Are always present
   b. Are neither identifiable nor removable
   c. Are not within our power to regulate
2. *Assignable causes* (due to non-chance variations)
   a. Are potentially identifiable and removable
   b. Are within our power to regulate
   c. Usually merit prompt investigation

These two types of causes of variation are important concepts in our thinking on this subject for the reason that they will be helpful in explaining why and when action should be taken toward the correction of excessive variation.

So far we have attributed all of the variation to the product itself. However, we may also have variation in our test characteristic which may be attributed directly to testing technique or to chance causes inherent in the test method. An error in measurement may be an assignable cause of variation in the test values resulting from the measurement. Any method of measurement of a quality characteristic will have some pattern of variability, which by repeating a measurement many times on a quality characteristic that remains unchanged will show the normal distribution pattern. Therefore, the system that we are measuring includes testing as well as production variations.

Design specifications may be defined as the desired quality goal. Acceptance specifications may be defined as those which describe the quantity and kind of evidence which will be accepted as satisfactory proof that the product will meet the design objectives. Irrespective of whether a building engineer follows this policy or whether he insists on 100% or 90% of the product falling within the specification limits, he should have some idea of the variability of the product in order to pass optimum judgment. One, two or three tests will not insure that the product at that time, prior to, or subsequent therefrom will be a true picture of that particular quality characteristic. However, if the engineer has both an appreciation and an evaluation of the variability of the product, he will pass much sounder judgment. Figure 4 will help to illustrate this, wherein four possible relations between the distribution of individual test results and a typical specification are shown. Within a broad specification such as % passing the #10 sieve, a variable product (one with a comparatively high σ) is possible as long as the average of all tests is centered reasonably well. On the other hand, if the average comes close to the specification limit a fraction of the product must necessarily be outside the limit as shown by that portion under the curve extending beyond the specification limit. Figure 5 illustrates the circumstance in which the specification or the job mix tolerance is a relatively close band.

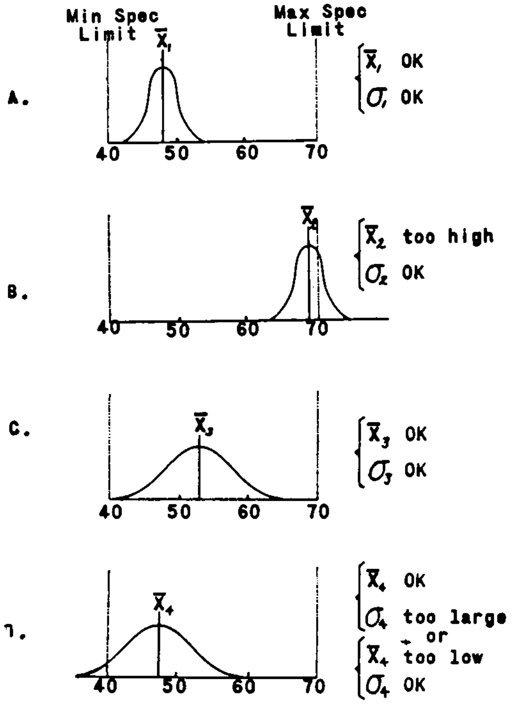

Figure 4. Possible relations between distribution of individual test results and specification limits on percent of aggregate passing #10 sieve.

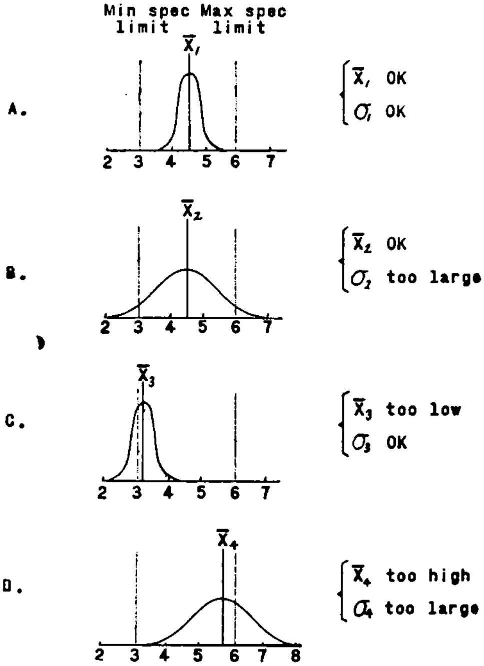

Figure 5. Possible relations between distribution of individual test results and specification limits on percent voids.

In this case, the variation characteristic as well as the average level must be closely controlled.

Although the cases pictured in these figures were hypothetically drawn for purposes of illustration, the amount of variation as shown is typical of many hot mix products. It is common practice to pass upon a product whose average test value is just within the specification limit, such as illustrated by Figure 4 B. Also it is not unusual to find a product with a well centered average but yet so variable that both a maximum and a minimum limit are being exceeded such as shown in Figure 5 B. Corrective action to be taken in each of these cases is quite obvious. On the other hand engineering judgment may still pass these products, and the pavement may be completely satisfactory. However, we do occasionally have pavement irregularities or failures which must have a cause. In the large part design specifications are written as the result of one or more successful experiences. Consequently many of these irregularities or failures must be attributed to deviation from some specification requirement.

One means of assuring better compliance with a given specification or job mix formula is by using the Shewhart control chart (*1*). The mechanics of this technique (*2*, *3*) and the application in the control of hot mix plants (*4*) has been published. In the application of the control chart, the out of control points or those in which assignable causes are indicated, are normally eliminated from the final calculations. The result is the approached value of average and the approached value of dispersion (for which we use the estimated value of standard deviation or σ'). Thus the control chart gives us a practical estimate of the centering of the process along with a useful measure of its normal variability.

To better exemplify these points actual plant control data were taken from different plants in the process of preparing three types of asphalt-aggregate hot mixes. The result is an interesting comparison of different plants

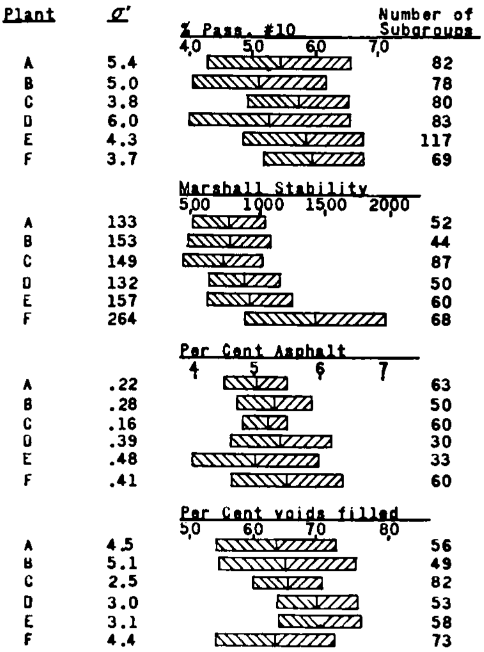

Figure 6. Typical variation characteristics of a sand-gravel hot mix, illustrating the average and control chart limits on four testing variables.

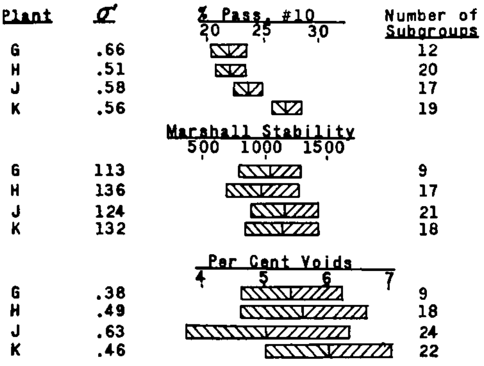

Figure 7. Typical variation characteristics of a binder mix, illustrating the average and control chart limits on three testing variables.

operating toward the same specification. Figure 6 illustrates four test variables on six different plants preparing a sand gravel hot mix for base courses. First the estimated standard deviation (σ') is listed followed by a graphical picture of the control limit. The vertical line in the center of the shaded block is the value that approaches the true plant average (*X*) for that test characteristic. The

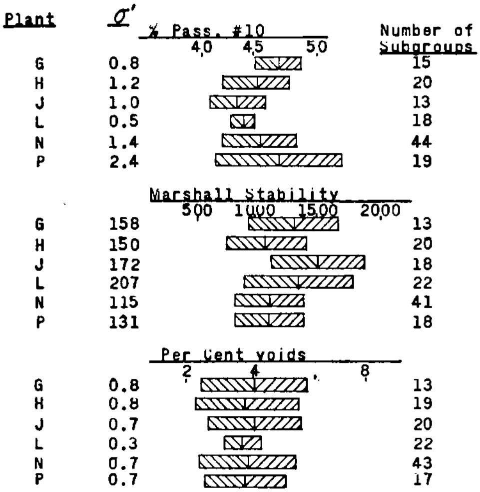

Figure 8. Typical variation characteristics of a surface mix, illustrating the average and control chart limits on three testing variables.

shaded area on each side of this average represents the extent of the control limits for that particular plant with respect to the test variable. The number of subgroups (of two test results each) used in establishing σ' and the control limits is listed at the right. Figure 7 is a continuation of this type of analysis on a binder mix, and Figure 8 is on a surface mix. It should be pointed out that the base course mix was made from pit run gravel whereas the binder and surface mixes were made with selected crushed stone, sand and limestone filler.

The significance of these data is simple, but on the other hand it is very pertinent in our consideration of the product in relation to the specifications. The standard deviation (σ') is a direct estimate of the variability of the product, and as pointed out previously, virtually all of the plant product will fall within ±3σ' of the average. When the plant is operating in control the average of each subgroup will fall within the control limits as pictured by the block diagrams. Taking for example the % passing #10 sieve on sand gravel hot mix (Figure 6) we find σ' to vary from 3.7 to 6.0. Obviously the plant with a low σ' is producing a product of more uniformity as far as this gradation point is concerned. Considering Marshall stability on this mix we find one of the plants with a σ' of 264. Previous experience had indicated that Marshall stabilities should have a σ' in the vicinity of 150 or less, consequently this plant was under suspicion as far as this quality characteristic was concerned. In the case of percent asphalt in the mix we find all plants with average values between 5.0 and 5.5% but with considerable differences in their variation characteristics. In the case of voids filled on the sand gravel mix the plant averages varied over a range of about 7% (63.2% for plant A to 70.3% for plant E). We found a difference of about 24% between the lower control limit of plant A and the upper control limit of plant E, whereas the individual tests for these plants could possibly range over 29% (49.7% as the 3σ minimum of plant A and 79.3% as the 3σ maximum of plant E). These values have been calculated for the purpose of showing that a plant average is only a part of the story. The rest of the picture can only be obtained by an appreciation and an evaluation of these dispersion characteristics of the product. Figures 7 and 8 may be interpreted the same was as the sand gravel mix except, of course, for the magnitude of the variation and the differences in their average level. Thus much information can be gained by comparing the variability characteristics of the different plants, not necessarily for the purpose of condoning one plant or its operator but rather to locate sources of trouble or non-conformance with acceptance specifications. Certainly an engineer, irrespective of whether he represents the producer or the inspection team, will have a much better appreciation of both the process and the product if he understands variability characteristics of his product in relation to the capabilities of the hot mix plant.

SUMMARY

A review has been made of the frequency distribution curve as a means of illustrating the variation characteristics of asphalt-aggregate mixes. Variation may be conveniently measured by use of standard deviation which can be estimated by use of the Shewhart control chart. Thus a tool is available which should be used more frequently by engineers when they are concerned with either design or acceptance specifications. Data on sand gravel, binder and surface mixes have been presented in order to illustrate typical plant variations when dealing with some of the common test characteristics. These examples should also be useful to any engineer faced with judging an asphalt-aggregate product in relation to specifications.

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