expected under certain conditions, it will be possible to estimate accurately what may be accomplished by improvements. The cost benefits accompanying these improvements may also be estimated.

In presenting these results, it is realized that other methods of measuring the quality of traffic flow may be derived. In fact several have been considered and rejected. One of these was:

$$Q = \frac{\text{overall speed}}{\text{running speed}}$$

another was:

$$Q = \frac{\text{Actual travel time}}{\text{Legal travel time}}$$

a third consisted of three parts:

$$Q = \frac{\text{crest speed}}{\text{running speed}} + \frac{\text{moving time}}{\text{elapsed time}} + \frac{1}{3\sqrt{f}}$$

wherein $f$ = frequency of speed changes per mile. There is also the ratio of average speed to the standard deviation. This too is a dimensionless number and it is a measure of the variance in speed and therefore a measure of turbulence of flow.

In any case it is hoped that this presentation will serve to stimulate thought and to hasten the time when quality and efficiency indices may be used to achieve better design for improvements and better control of traffic movements.

# Study of Traffic Flow by Simulation

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The paper embraces certain philosophies and approaches in the utilization of modern high-speed automatic computers to solve traffic problems. Individuals and research groups in both the United States and England have applied their efforts in this direction. Under the policy of cross-fertilization of fields of specialization at the University of California, a Communication Systems Research team undertook certain traffic studies. It is concluded that computers, used as simulators, offer considerable promise in the solution of such traffic problems as investigating the effects of traffic control devices in advance of installation and predicting the effect of proposed changes on the capacity of a facility.

The concept of vehicle flow rate or, alternately distribution of gaps, finds general utility in approaches of both analysis and simulation. Ideally, the behavior of a physical model (simulator) resembles that of the real situation under study by virtue of the postulates laid down by the investigator. Such a model encompasses both the structure (fixed facility) and the dynamics of the movement of intersecting streams of vehicles in terms of flow paths, queuing, waiting, proceeding ahead and turning subject to delays caused by cross traffic and pedestrians. Computers can be programmed to simulate such structure and dynamics.

Activity to date at the University of California has been as follows: Formulation of a trial problem for solution on a general-purpose discrete-variable computer, Standards Western Automatic Computer (SWAC), completion of the logical design of a special-purpose discrete-variable computer, and considerable progress on the construction of accessory equipment to permit the use of a general-purpose continuous-variable computer.

● WHEN an engineer attacks a new problem he attempts to express the behavior of the system in terms of mathematical relationships. When there has been no body of theory built up, however, he resorts to empirical studies. Some such studies may be performed on the real system. In many cases, however, the operation of the real system is inferred from some physical model,[1] and in some instances this may be a scale model of the real system. Since the development of large-scale electronic computers during and following World War II there has been considerable interest in the simulation of various physical systems by means of computers.

In simulation one seeks physical systems whose behavior has something in common with that of the situation under study. There are many routes to such analogous situations, two of the major ones being mechanization of mathematical operation (e.g., the integration operation of a describing equation) and representation of structure and dynamic components. Applying this to the traffic situation, the flow of vehicles on a given net of streets is simulated, in one manner or another, subject to certain rules of conduct and controls. Then, if the flow of a random sample of traffic entering the net is introduced, the effect of control devices, etc., may be observed. Confidence in results is assured by employing sufficiently large samples. These samples may be deduced and prepared from partly empirical and partly theoretical considerations, or from purely empirical data. This sort of experimental study is an example of a general approach to the solution of physical or mathematical problems which has become popular since the advent of the general-purpose high-speed computer.

Under the encouragement of Dean L. M. K. Boelter, the Institute of Transportation and Traffic Engineering in February, 1953, provided support for an investigation into the simulation and theory of vehicular traffic flow. In line with the policy of cross fertilization among the various engineering disciplines, the personnel assigned to this task were people whose background was in communication theory.[2] This paper summarizes the work of this group in the field of simulation.[3]

[1] A logical description of a physical system originating in the mind of the investigator and adequately accounting for what he considers to be significant behavior, we here term a **model.**

BLOCK DIAGRAM

The formulation of a model (either mathematical or physical) is greatly aided by the construction of a functional diagram incorporating certain postulates. In the formulation of a model of traffic, vehicular and pedestrian, one takes the viewpoint of an observer standing beside the road. To such an observer the arrival rate, the interferences encountered, and the direction taken by cars on leaving the intersection appear to be random processes. Once the functional diagram has been drawn it serves as a guide in the writing of mathematical expressions or in the design of suitable units for a simulator.

Figure 1 is a functional or block diagram for a typical intersection. This diagram is based on the following postulates:

1. The intersection is signalized
2. There is one lane in each direction
3. Pedestrians are present
4. Right turns are permitted only on green
5. Left turns are permitted.

The diagram is drawn for only one quarter of the intersection—that representing traffic entering the intersection from the eastbound approach. Traffic entering from other approaches would be represented by similar diagrams.

In Figure 1 solid lines represent the flow of traffic; broken lines represent the flow of interference information. Upon referring to the figure it may be seen that arriving vehicles pass into an initial waiting zone. Vehicles are delayed here if the signal is red or (since one-lane operation is considered) if the car ahead encounters delay in performing a turn. Having passed the initial delay, the car enters a direction selector which determines whether the car is turning right, turning left, or going straight ahead. Cars going straight ahead proceed without further delay. Cars turning right may be delayed by pedestrians. Cars turning left may be delayed by oncoming traffic or pedestrians. The broken lines from the right and left-turn wait blocks to the initial wait block represent the feedback of informa-

[2] The group was composed of H. Davis, J. Heilfron, E. C. Ho, A. Rosenbloom and D. L. Trautman.

[3] The complete report has been published by the Institute of Transportation and Traffic Engineering as Research Report No. 20.

tion to prevent a car from entering the intersection when one is delayed within the intersection.

## DISCRETE-VARIABLE SIMULATOR

Consider now the construction of a special-purpose discrete-variable electronic computer for mechanization of the model of Figure 1. In the design of such a computer each vehicle may be represented by a voltage pulse. Distance and time may be quantized. The unit of distance is taken as the distance which can contain at most one vehicle. The unit of time is taken as the time required to travel unit distance at some prescribed speed. Timing is accomplished by means of timing pulses supplied from an appropriate timing-pulse generator. Design is carried out in terms of counters and circuits known as gates which perform the logical operations "or" and "and." The symbols and operating details of these circuits are given in Appendix 1.

Figure 2 is a diagram of the wait circuit, which serves as the basic building block. Here, pulses representing cars arrive at the input and enter the counter C. If there are any pulses stored in the counter, a control voltage will appear on the counter input to the AND-NOT gate AN. Timing pulses are applied to the "Timing Pulses" input from the pulse generator. These pulses will pass through the gate and leave the intersection via the output, provided the output of the counter is maintained. Note that as each pulse is gated through AN, it is also fed back and subtracted from those stored in the counter, signifying that one car has passed through the intersection. The intersection may be blocked

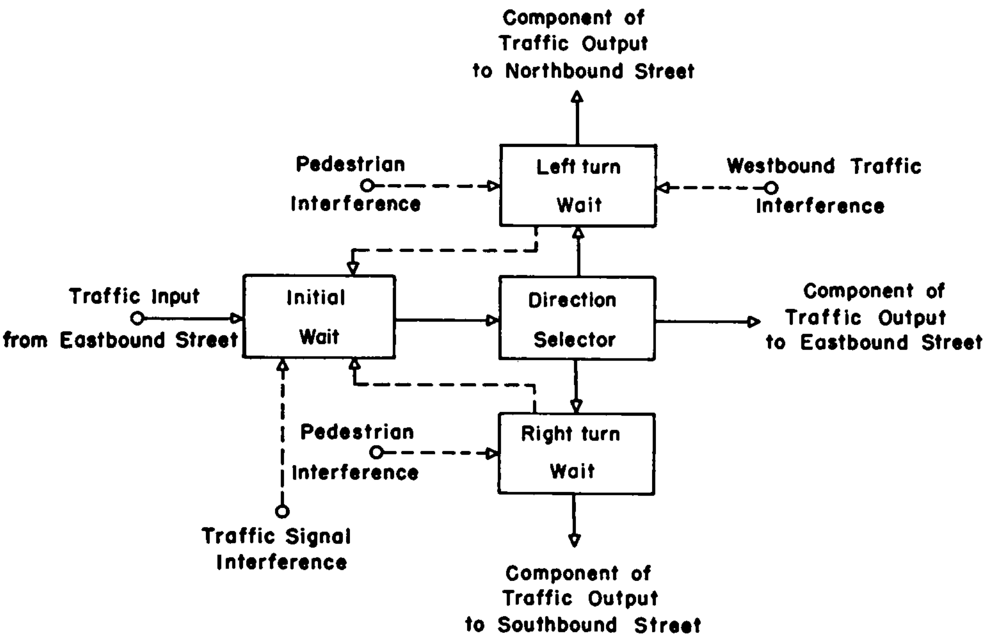



**Figure 1. Block diagram of one approach of a generalized intersection having one lane in each direction.**

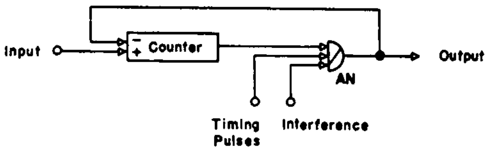



**Figure 2. Wait element of discrete-variable intersection simulator.**

to traffic flow by applying a voltage to the input "Interference". Figure 3 shows how the model of Figure 1 may be assembled. Inputs "Right" and "Left" are connected to random-pulse generators. The generator connected to "Right" is set to the percentage of vehicles making right turns, and the generator connected to "Left" is set to the percentage of left turns. The input marked "Red Signal" receives a voltage when the traffic signal is red. The gates 0-1 and 0-2 prevent interaction between the various inputs connected to them. The streets between intersections may be delay lines which cause pulses leaving one "intersection" to be delayed a predetermined time before arriving at the input of the next "intersection."

The foregoing should give a general idea of how one arrives at logical designs representing almost any type of intersection.

## CONTINUOUS-VARIABLE MODEL

In the model just described the discrete nature of vehicles is preserved. For approximate studies to give general effects in a large network it may be feasible to use a continuous-variable computer. A suitable computer for this purpose may be built around the operational-amplifier type of integrator. This integrator may be used as the storage element of a wait circuit of Figure 1. Figure 4 (See Appendix 2 for notation) shows the wait and the direction selector sections. Since in this model individual vehicles are not recognized but instead more gross effects are considered, it is not necessary to provide separate right-turn and left-turn waits. These effects may be lumped into the initial wait, being taken care of by the response of the Output Rate Generator to the various interferences.

In Figure 4, relay 1 operates during red; relay 2 operates when there are cars waiting at the intersection; and relay 3 operates when there is interference. Thus, if the signal is green, no vehicles are waiting and there is no congestion, there is a direct connection between the input and output, and also there is no voltage on either input to the integrator. If there is interference from opposing traffic or from pedestrians (relay 3 energized), or if there is delay caused by cars waiting to enter the intersection (relay 2 energized), the input is fed to "storage" in the integrator. When the green appears (relay 1 deenergized), traffic is released (subtracted from storage) at

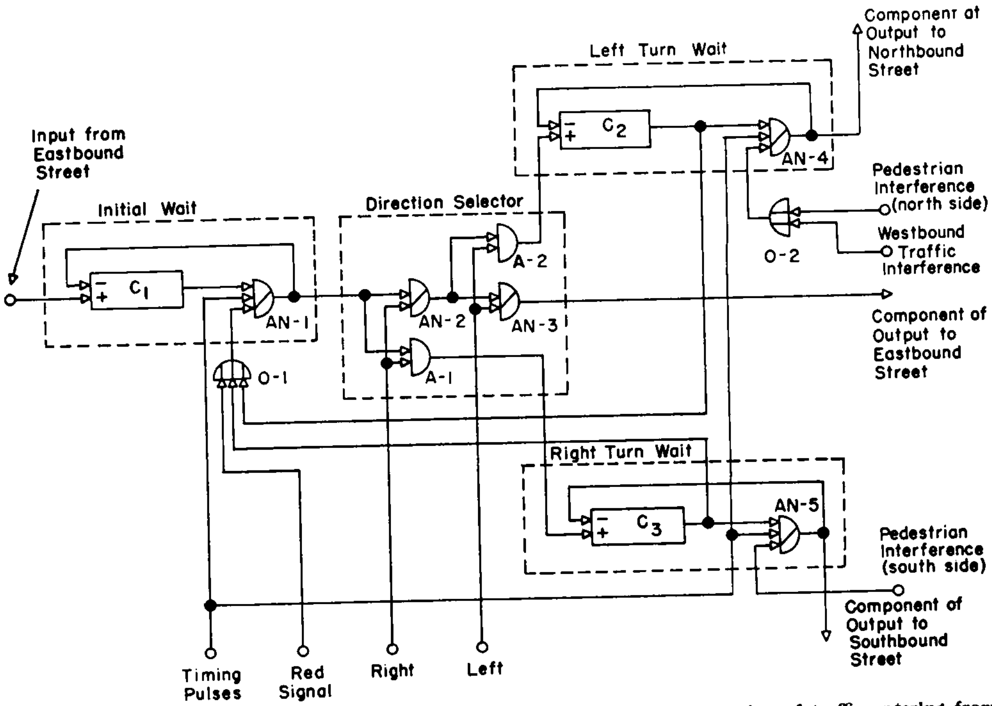



Figure 3. Portion of discrete-variable simulator showing elements for representation of traffic entering from the west.

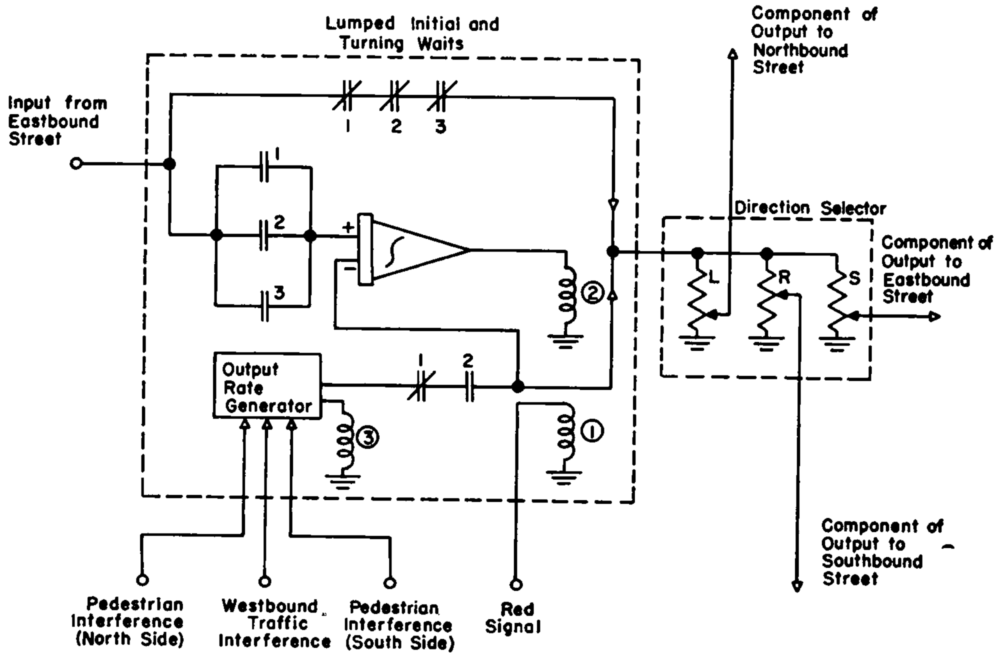



**Figure 4. Portion of continuous-variable simulator showing elements for representation of traffic entering from the west.**

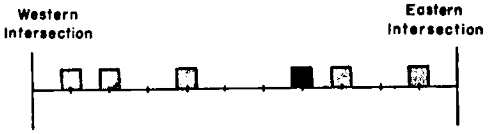
a rate in approximate inverse proportion to the interference, as established by the Output Rate Generator.

The streets between intersections may be simulated by means of magnetic-tape delays. These consist of a magnetic tape having recording and playback heads operating continuously. The time for the tape to travel from the recording head to the playback head constitutes the delay.

## GENERAL-PURPOSE COMPUTER

We may now consider still another approach, namely, that of simulator application through arithmetic representation of traffic. A very natural procedure is to seek a one-to-one correspondence between numbers and instantaneous distributions of vehicles on thoroughfares. For example, we may employ a sequence of ones and zeros representing, respectively, the presence or absence of vehicles at a certain prescribed set of possible locations. The purpose of such a representation is to simulate the flow of vehicles by performing successive arithmetic operations on these numeric representations, each operation yielding the effect of movement in an increment of time. From such basic representation one can proceed to the programming of any general-purpose high-speed digital computer. As a first step it is necessary to assume distance and time to be quantized. As previously indicated, a suitable unit distance may be that distance which can contain at most one car. The unit of time may be the time required to travel unit distance at some specified speed.

For example, now consider a street section $m$ units in length. By virtue of the definition of the space unit it may be stated that this street section can contain at most $m$ cars. Each of the $m$ spaces is either empty or contains one car; thus there will be $2^m$ possible configurations of cars in the street section. These configurations may be conveniently represented by binary numbers. (A brief discussion of binary numbers will be found in Appendix 3). Figure 5a shows an example of one configuration of cars in a street section 10 units long ($m = 10$). Figure 5b shows the corresponding binary number and its decimal equivalent. In the binary representation a "1" denotes the presence of a car while a "0" de-




BINARY REPRESENTATION; X = .1101001101

DECIMAL EQUIVALENT OF X = 0.8173953125

**Figure 5. One of $2^{10}$ possible configurations of traffic on a roadway 10 units long, including binary representation of this traffic and the decimal equivalent of this number.**

notes the absence of a car, and thus there is a one-to-one correspondence between the digit positions in the number and quantized space positions on the street. For any specified configuration on a given street section there will be a unique number $X$ whose binary representation corresponds to the distribution of cars and gaps.

Having represented the distribution of cars and gaps by a number we may devise certain mathematical operations which will cause vehicles to flow along the street in accordance

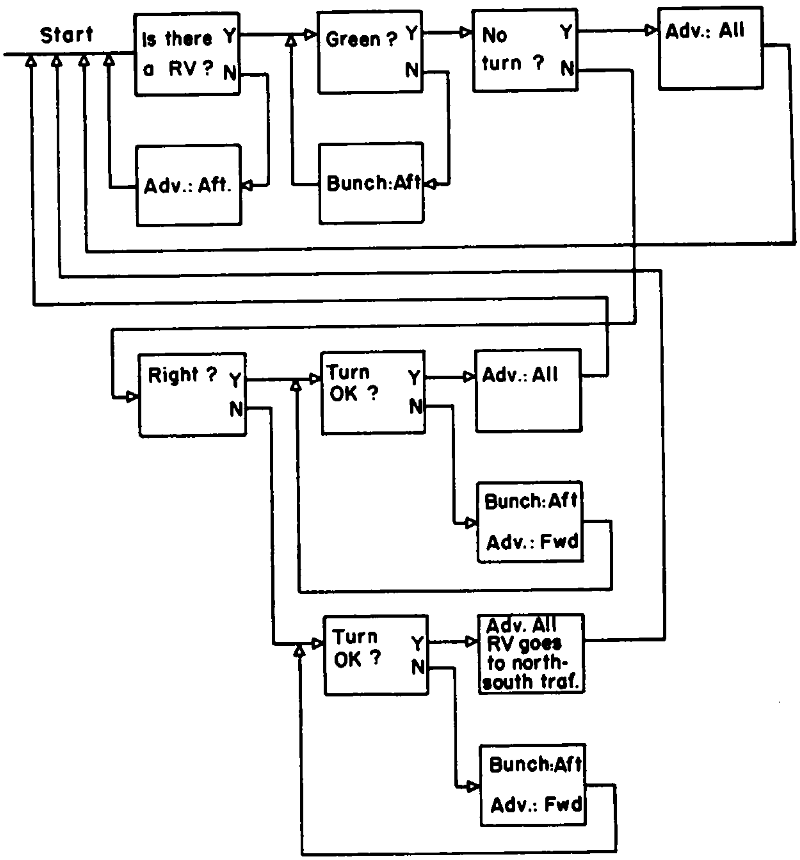



**Figure 6. Diagramatic representation of a program for simulation of one approach of a simple intersection by means of a general-purpose discrete-variable computer. Terms: RV, Ready Vehicle (Eastbound); Green, Signal on east-west phase is green; Turn, RV prepared to turn into north-south traffic; Y, N, Answers to questions: Y = yes, N = no; All, All vehicles (between adjacent signals east of intersection and west of intersection); Aft., All those vehicles which have not yet reached the intersection; Fwd., All those vehicles which have passed the intersection; Adv., Vehicles advance one unit; Bunch, Vehicles west of a stopped vehicle advance; till space between vehicle vanishes, e.g. *Adv. Fwd.* indicates that all vehicles between intersection and adjacent signal east of intersection are to advance one unit.**

with the postulates used in devising the model.[4]

The result of such simulation may be a table of numbers giving the complete time history of traffic within a network, i.e. a record of the various distributions at every instant of time. But a more reasonable approach consists of programming the appropriate sequence of arithmetic operations representing the movement of vehicles through the network and paralleling these computations with the calculation of some suitable figure of merit. Following this procedure, a representative problem has been programmed for the SWAC.

An approach to coding is represented by Figure 6 which can be interpreted as follows: The computer is coded so that the various indicated paths are to be followed as may be appropriate. Beginning at the point marked "start" in the upper left hand corner, the computer is to proceed in one of two ways according to whether or not a vehicle appears at the intersection. If a vehicle does appear, the computer is to proceed in one of two ways according to whether the traffic signal is red or green. Of the many possible paths indicated only two effects on traffic are to be accomplished. Either the stream of traffic advances a unit, or else the traffic "bunches."

It should be noted that every possible path eventually returns to the point marked "start." Thus, this program would be suitable for continuing simulation of one approach to an intersection. For most practical problems, however, it is desired to simulate all approaches to each intersection for a system of several intersections. In such cases instead of returning to "start" and repeating the same routine, attention is shifted to another approach or another intersection by the introduction of another routing at the appropriate stage.

Modern computers perform their normal operations at high speeds (e.g. 64 microseconds for an addition). The complexity of traffic simulation problems is such, however, that a network of a few intersections may require several thousand operations during each unit of simulated time. Thus the time to perform the simulation of traffic on a general-purpose computer programmed in a serial manner may require a longer time than the movement of real traffic through the real system. Even so, the simulator can become a very powerful tool because of its ability to "bring traffic into the laboratory."

## CONCLUSIONS

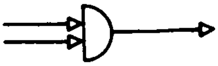
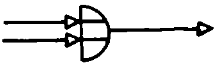
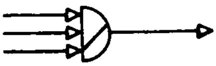
Inasmuch as the greater portion of this paper treats of simulation in some detail it can be concluded that reasonable progress is being made. Furthermore, in the synopsis of this paper brief allusion has been made to simulation work by investigators other than the authors. Thus, it can be further concluded that the solution of traffic problems through simulation is receiving attention from several well qualified investigators. There are, however, two supporting areas in which work is required: Parameter Measurement and Validation. Measurement of significant parameters is essential for realistic interpretation of the results of simulation. Validation consists of setting up a series of problems having a parallel in actuality and comparing (probably statistically) simulator results with actual results. It is expected that work in these phases will proceed in the near future.

[4] Typical operations are discussed in the report listed in footnote 3.

# APPENDIX 1

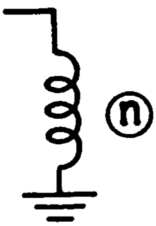
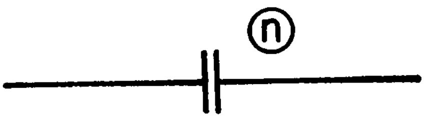
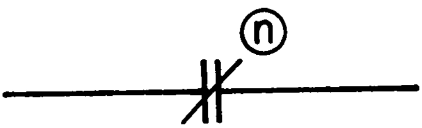
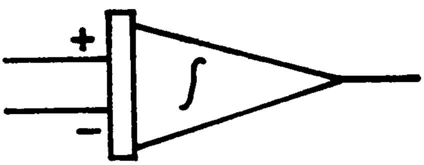
## Symbols for Discrete-Variable Simulator

The majority of the design set forth in this paper has been carried out in terms of the logical operations "And," "Or" and "Not" (complement) and the corresponding devices called "gates" which perform these operations.

| Symbol | Name | Description |
|---|---|---|
|  | And Gate | Voltage at output if and only if voltages appear at all inputs. |
|  | Or Gate | Voltage at output if and only if a voltage appears at *at least one* input. |
|  | And-Not Gate | Voltage at output if and only if voltages at *all* inputs *except* the one marked by a slash (/); no voltage at output if voltage appears at input marked by a slash. |
|  | Counter (Storage) | Voltage at output if and only if the number of pulses into the (+) input exceeds the number into the (−) input. |

# APPENDIX 2

## Symbols for Continuous-Variable Simulator

| | |
|---|---|
|   | Operating coil of relay $n$—normally deenergized |
|   | Normally open contact of relay $n$ |
|   | Normally closed contact of relay $n$ |
|  | Integrator (Storage) |

## APPENDIX 3

### NOTES ON BINARY ARITHMETIC

Whereas decimal numbers (base ten) are made up of ten digits (0 through 9) binary numbers (base two) are made up of only two digits (0 and 1). In decimal numbers the positions of the digits have place significance. The number 549, for instance, represents

$$5 \times 10^2 + 4 \times 10^1 + 9 \times 10^0$$

Likewise in binary numbers the digits have place significance. In this case, however, the multiplier is a power of two rather than a power of ten. The binary number 11010 represents

$$1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 12^1 + 0 \times 2^0$$

In case of decimal numbers, digits to the right of the decimal point are multiplied by appropriate negative powers of ten. Similarly, in binary numbers digits to the right of the binary point are multiplied by appropriate negative powers of two.

When the digits of a decimal number are shifted one place to the left the number is multiplied by ten (the base of the decimal system). When the digits of a binary number are shifted one place to the left the number is multiplied by two (the base of the binary system).

Addition in binary notation becomes very simple. The complete list of combinations is as follows:

$$\begin{aligned} 0 + 0 &= 00 \\ 0 + 1 &= 01 \\ 1 + 1 &= 10 \end{aligned}$$

An overflow in one position (as in $1 + 1 = 10$) is carried to the next position to the left and added in the appropriate manner.

Equally simple rules may be written for multiplication, division, square root, etc. Many automatic computers are designed to operate in the binary system because of the resulting simplicity of equipment design.

# Allocation of Traffic to the Hampton Roads Bridge-and-Tunnel System

WALTER A. BARRY, JR.,
*De Leuw, Cather & Company*, and
MARSHALL RICH,
*Wilbur Smith and Associates*

● IN December 1953 the Virginia Highway Department gave us the interesting and important assignment of estimating traffic and revenues for the proposed Hampton Roads Bridge and Tunnel System and the James River Bridge System, assuming removal of the Newport News-Pine Beach and the Old Point-Willoughby Ferries. The studies formed the basis for the refinancing of the existing bridges and the financing of the proposed new facility through the issuance of $95,000,000 of revenue bonds.[1]

Figure 1 shows the location of the present and proposed toll facilities in relation to major highways in southeast Virginia.

When the proposed Bridge and Tunnel System is completed, traffic from the peninsula will cross Hampton Roads on either the Bridge and Tunnel System or the James River Bridge System. Motorists presently have a choice of three facilities.

It is quite obvious that with such a drastic change in alternate routes, a detailed analysis would be necessary to determine how this change would affect the motorists' selection of future routes of travel.

It was decided to make a complete study of all major factors influencing a motorist's selection of his present routes of travel. Origin-destination surveys were conducted by the Virginia Department of Highways in conjunction with the consultants at five locations, namely the James River Bridge, the Newport

[1] The financing of the Rappahannock River Bridge and the refinancing of the York River Bridge was also included in this bond issue.