DEPARTMENT OF DESIGN

Selection and Design of Semi-Flexible and Conventional Type Pavements

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Flexible type pavements carry their loads by granular interaction and their design is predicated on the condition of each layer being of sufficient thickness to not overstress the underlying layers. A marked similarity is noted in several popular methods of design using this stress-strength principle with different types of strength tests.

Rigid type pavements carry their loads by slab strength and the strength of the subgrade is seldom the controlling factor. Design is based on elastic theory of continuously supported slabs, and is predicated on maintaining tensile stresses in the slab within the allowable flexural strength of the slab material.

Pavements constructed of soil-cement, lime-clay, asphalt stabilized bases, and many other materials are in a category intermediate between flexible and rigid, in that they possess considerable slab strength, and at the same time deflect sufficiently to transmit sizeable stresses to the subgrade. The design, in the case of such semi-flexible type pavements, may be governed by the strength of either the subgrade or that of the pavement material. A design procedure is outlined whereby both elements are considered.

Physical properties of various types of pavement materials ranging from true rigid type to true flexible type are correlated with water-cement ratio for ease in using the various design procedures.

Cost and performance data are presented showing economic advantages for semi-flexible type pavements.

PAVEMENT SELECTION

● IN designing pavements for any traffic facility, the Engineer has at his disposal a variety of materials from which to choose. Some require greater initial cost and less maintenance, while the reverse is true in other cases. However, to be economically sound, the decision must be based on the estimated ultimate cost, including the construction cost plus the cost of maintenance for the anticipated life of the pavement.

The construction cost can be estimated with reasonable accuracy from a knowledge of available materials and their cost. In most cases, the choice of material is governed by availability of local materials. Cost of maintenance is more difficult to determine and frequently involves such intangibles as cost to the facility users due to interruption of service, traffic hazards, etc. Until recently, it has been common practice to classify pavements as either rigid or flexible and to design each by entirely different methods. Actually, there are a large group of materials that are not in either the flexible or the rigid category. Mis-classification of such materials has no doubt in many cases led to improper designs, resulting in an erroneous design or erroneous choice of pavement type. In the interests of promoting a better understanding of the nature of the various materials available, the author has established a third category of pavement types called "semi-flexible" and has assembled data correlating the various

physical properties of all three types with their respective contents of stabilizing agent (for example, Portland Cement).

As a corollary service, design procedures have been developed for semi-flexible type pavements. This method considers both subgrade strength and slab strength and insures against overstressing either. It is hoped that the new tools will assist road designers in their difficult task of proper selection of pavement types.

FLEXIBLE PAVEMENT

This type of pavement consists of one or more layers of granular material to which is applied an asphaltic wearing surface, and is distinguished from a rigid type pavement in that the loads are carried principally by individual interaction between the particles of the granular base course. Each layer transmits a load of diminishing intensity to the underlying layer or to the subgrade, at the same time undergoing a certain amount of deflection or deformation in proportion to the loads transmitted; the resultant yielding of the pavement surface under the action of a wheel load is responsible for the term "flexible" in this type of pavement. The thickness of each layer required for stability under any given wheel load is determined from the requirement that the stress or unit load transmitted to the underlying layer or to the subgrade shall not overstress the latter. Thus it is apparent that each layer depends not only on its own strength but also on that of its next-door neighbor, and, therefore, the designer must know something of the characteristics of the subgrade soils and of each of the materials or mixes contemplated for use.

Testing and Design of Flexible Pavements

Until recently, the designer judged the qualities of each base material on the basis of its grading, past performance, plasticity index, percent soil binder, and other physical appearances; such tests and qualities were sufficient in a qualitative sense, but gave no hint as to the required thickness. Now, however, there are available methods of testing and design that enable direct determination of the strength of subgrade soils and of any desired type of select or base course material, and a subsequent determination of the required thickness of each layer of the pavement.

The United States Army Corps of Engineers and others use the California Bearing Ratio Test for determining the strength of the subgrade and of the various layers of pavement (*1*, *2* and *3*). The Texas Highway Department has developed triaxial compression testing equipment suitable for both subgrade soils and base materials, and a classification chart based on triaxial strength (*4*, *5*). More recently there have been published thickness design curves based on the Texas Highway Department Triaxial Classification (*6*). The Asphalt Institute expresses subgrade, subbase and base material strengths in terms of subgrade value "to which all evaluation systems have been related" (*7*). All of these agencies have provided thickness design curves based on their respective strength scales and calculations of stresses at various depths for various wheel loads. Since the details of each method have been previously described in detail in the various literature references cited, it would seem superfluous to repeat these here. However, it is believed pertinent to show here the design curves for the three methods cited. The marked similarity of these is quite evident from an inspection of Figures 1, 2, and 3.

Comparison of Flexible Design Procedures

An example of the use of these three sets of design curves is shown by means of the broken lines on Figures 1 through 3; the example is for a flexible pavement designed for a wheel load of 15,000 pounds and to be con-

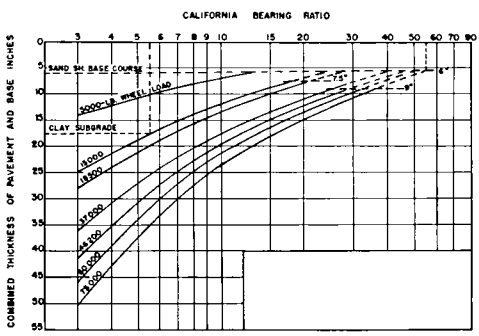

Figure 1. CBR thickness design curves. (From Exhibit 3, Engineering Manual, Chapter XX, Part II.)

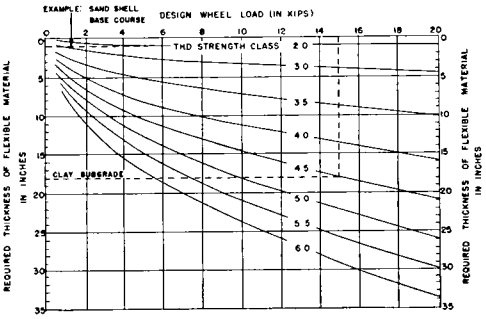

Figure 2. Texas Highway Department flexible pavement design chart.

structed on a heavy clay subgrade having a Triaxial Strength Classification of 4.5 using sand-shell flexible base material having a Triaxial Strength Classification of 2.0 and a hot-mix asphaltic concrete surfacing. The corresponding CBR values are 5.5 and 55, respectively, and the Asphalt Institute "Support Values" are 35 and 85 in the same order. It is to be noted that the thickness of cover required for the clay subgrade by all three methods is very nearly 18 inches in all cases, but that the required thickness of surfacing varies from 1 inch, using the Texas Highway Department procedure, to 6 inches using the CBR procedure and 8 inches using the Asphalt Institute procedure.

Effect of Testing Method

The important conclusion to be drawn from the examples cited is as follows: even though all three methods are obviously based on identical theory, very great differences in design and cost can be occasioned through differences in testing methods used. The Texas Highway Department feels that the triaxial compression test duplicates actual field conditions more closely than any other known method of test and hence has accepted this method of test as standard.

Most flexible type pavements are provided with an asphaltic surface layer. This may be intended to carry a substantial portion of the wheel loads applied, in which case the thickness is determined by application of flexible design procedure using strength values consistent with the thickness of surfacing anticipated. In other cases, a high strength is built into the base course and the only function of the surfacing is to supply an abrasion-resistant surface; in such cases, the stability is an inverse function of the thickness and the thinnest surfacing that can be permanently applied is the most economical and efficient.

It appears to the writer that the most fruitful field for investigation in flexible pavements lies in the development of bituminous or other binder materials that have little or no reduction in strength with increase of temperature. In this connection it is pointed out that most satisfactory asphaltic mixtures have strengths less than a THD Strength Class 1 Base material at 140 F., but at room temperature the strength is many times greater.

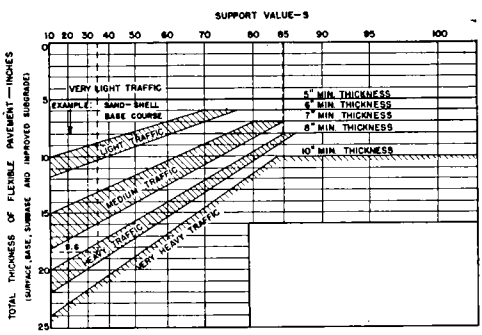

Figure 3. Asphalt Institute thickness design curves. (From Figure 2, page 13, "Thickness Design, Flexible Pavements for Streets and Highways," The Asphalt Institute Manual Series No. 1, January, 1955.)

RIGID PAVEMENT

This type of pavement consists of a relatively non-yielding monolithic layer of Portland Cement concrete which acts as a single unit, rather than by interaction between individual particles. Loads are carried by beam action, the slab acting as a continuous beam on a uniform support. Wheel loads cause the slab to deflect but generally to a much lesser degree than a flexible pavement. Perhaps the outstanding fundamental difference in behavior is that the concrete pavement can crack permanently with a resultant loss in strength unless the crack is properly controlled, whereas the granular nature of the flexible base course normally precludes the possibility of a permanent crack and even after cracking the structural qualities are not impaired to the same extent.

Design of Rigid Pavements

Design of a rigid type pavement involves the determination of tensile stresses produced in top or bottom of the slab by design wheel loads placed at various positions on the slab, and also of tensile stresses produced in the slab by temperature and moisture effects.

Load Stresses

In the Westergaard analysis (*8*), the slab is assumed to be continuously supported by a subgrade material whose supporting value is constant and expressed in terms of its "k-value," or ratio of unit stress applied at any point on the subgrade to the deflection of the

k-value: $$k = \frac{C}{1.5a} \quad (1)$$

Radius of relative stiffness: $$1 = \sqrt[4]{\frac{Eh^3}{12(1-\mu^2)k}} \quad (2)$$

Equivalent radius of resisting section: $$b = \sqrt{1.6a^2 + h^2} - 0.675h \quad (3)$$

Stresses for load at:

Interior: $$S_i = \frac{Q_iW}{h^2} = \frac{0.3162W}{h^2}[4 \log 1/b + 0.633] \quad (4)$$

Protected edge: (Full subgrade support) $$S_e = \frac{Q_eW}{h^2} = \frac{0.572W}{h^2}[4 \log 1/b + 0.359] \quad (5)$$

Unprotected edge: (Deficient subgrade support) $$S_e = \frac{Q_eW}{h^2} = \frac{0.572W}{h^2}[4 \log 1/b + \log b] \quad (5a)$$

Protected corner: (Full subgrade support) $$S_c = \frac{Q_cW}{h^2} = \frac{3W}{h^2}[1 - (1.414\, a/1)^{0.6}] \quad (6)$$

Unprotected corner: (Deficient subgrade support) $$S_c = \frac{Q_cW}{h^2} = \frac{3W}{h^2}[1 - (a/1)^{0.6}] \quad (7)$$

Warping stresses·

Infinite slab: $$\sigma_0 = \frac{Eet}{2(1-\mu)} \quad (8)$$

Center of finite slab: σ_c — See "Public Roads," Vol. 8, No. 3, May 1927

Edge of finite slab: $$\sigma_e = \sigma_c(1 - \mu) \quad (9)$$

Terms and units:

C—Elastic modulus of pavement subgrade, psi/in/in.
a—Radius of loaded area of subgrade in formula (1), inches. Radius of tire contact area in formulas (3) thru (7), inches.
E—Elastic modulus of concrete, psi/in/in.
h—Thickness of slab, inches.
μ—Poisson's ratio of concrete.
W—Design wheel load, lbs.
e—Coefficient of thermal expansion of concrete, in/in/°F.
t—Temperature differential in slab, °F.

Figure 4. Formulas for rigid pavement design.

subgrade at that point. Details of this procedure as used by the author are given in the Appendix, and are summarized on Figure 4.

Warping Stresses in Rigid Pavements

In addition to tensile stresses due to load, rigid slabs are subject to tensile stresses produced by constrained warping due to differences in moisture content and temperature at top and bottom of the slab. The higher moisture content prevailing in the bottom of any pavement slab compared to that at the surface of the slab induces a warping effect in the opposite direction to temperature warping effects that might add to load stresses. Study of available literature on moisture differentials and their effect (*cf. 15*) has led the author to use a "moisture-compensated temperature differential," t, of 1.5 degrees Fahrenheit per inch thickness of slab. Warping stresses are then evaluated using the formulas and references cited on Figure 4.

Total Allowable Stress in Rigid Pavement

Regardless of the method used for determining tensile stresses due to load, the total stress for any load case is the sum of the stress due to load and stress due to warping, since in any load case the warping stress may occur at the same time as the maximum load stress. The total allowable stress should be held to half the ultimate flexural strength of the slab for loads that will be repeated 200,000 times or more. For loads expected to occur less than 200,000 times, a smaller safety factor may be used (*11*).

Sub-Bases for Concrete Pavement

In addition to stress-strength requirements, rigid type pavements require a " non-pumping" subgrade. Various investigations have been made to determine the requirements of subgrade materials to prevent pumping (*16, 17*). The results of these investigations can be summarized by the following criterion:

A layer of granular material of adequate thickness having no more than 45 percent passing the 200 mesh sieve placed under a rigid pavement slab will adequately control slab pumping.

Considerable economy can often be effected in construction operations by use of granular base courses affording also a certain degree of subgrade reinforcement and waterproofing. Cement-stabilized mixtures have proven particularly valuable in this manner (*9*).

SEMI-FLEXIBLE PAVEMENTS

It is almost self-evident that there are many materials now being used for the primary load-carrying element in road construction that are neither flexible nor rigid type pavements. In this category are such materials as soil-cement, clay-lime, lime treated gravels, soil-asphalt mixtures, bituminous concrete and cement-stabilized sand-shell mixtures, in which the controlling factor in design may be either the slab strength of the base material or the supporting value of the subgrade soil. Just as the modern automobile is intermediate between the airplane and the horse and buggy, so also is the semi-flexible pavement intermediate between the rigid and the flexible type pavement. No self respecting engineer would design a road for modern vehicular traffic on standards of the horse and buggy era, nor would he attempt to design a similar highway for wheel loads encountered in modern airport pavement design. The need for an intermediate classification and method of design seemed to the author to be necessary.

Limits of Semi-Flexible Category

It seemed prudent to first define the limits of semi-flexible pavements. Such pavements should first of all have a sufficiently low elastic modulus and coefficient of thermal expansion as to reduce temperature expansion, contraction, and warping to the extent that jointing is not necessary. These can be achieved by lowering the strength below that of Portland Cement concrete and by a judicious choice of aggregates. On the other hand, from the standpoint of thickness and economy it is desirable to provide as high a strength as possible. The California Department of Highways (*18*) reports that soil-cement materials showing 7-day compressive strengths of up to 650 psi showed no objectionable cracking. Investigations made by the author on cement-stabilized sand-shell mixtures indicate that such mixes having 7-day compressive strengths of 650 psi ultimately attain compressive strengths of approximately 1200 psi, and that no objectionable cracking was observed in such mixtures having ultimate compressive

strengths less than 1200 psi. It was also noted that similar mixtures showing ultimate compressive strengths of less than 200 psi also resulted in objectionable crack formation (*9*). Hence it appears reasonable to assume that Portland Cement sand-shell mixtures showing ultimate compressive strengths of between 200 psi and 1200 psi fall in the semi-flexible range and will not exhibit cracking tendencies requiring jointing. A similar range of limits, no doubt, exists for asphalt and lime stabilized mixtures and sand-cement mixtures and the the limits may or may not coincide with those stated above for cement-stabilized sand-shell mixtures. From the viewpoint of design, semi-flexible pavements may be defined as those constructed of bases having appreciable slab strength, but not sufficient slab strength to assure that the subgrade soil will not be overstressed.

Sem -Flexible Pavement Design

Design of a semi-flexible pavement involves two separate and independent phases:

a. Determination of the thickness of base required to prevent development of tensile stresses in the slab in excess of the allowable flexural strength.
b. Determination of the thickness of base required to prevent overstressing the subgrade soil or other underlying layer of the pavement.

The critical thickness is the maximum thickness determined by either criteria. The first phase is accomplished by applying rigid pavement design procedure as outlined in the preceding section on Rigid Pavement. The second phase is accomplished by determining first the thickness of base and surfacing required to prevent overstress in the underlying layer, assuming no slab strength in the base, and then applying a thickness reduction in proportion to the slab strength. The details of the Texas Highway Department flexible design procedure are given in the Appendix.

Base Thickness Reduction Due to Slab Strength

In seeking a convenient means of evaluating and expressing various degrees of slab strength, initial efforts were confused by thought habits resulting from: the previous classification of pavement materials as either flexible or rigid; by the apparently unrelated physical properties used in design; and by

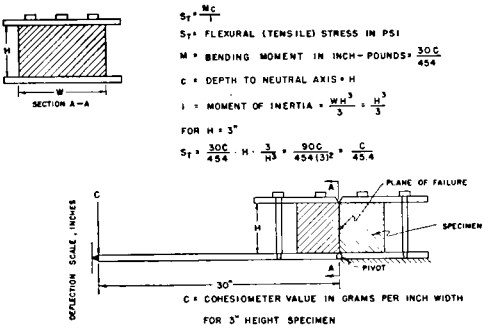

Figure 5. Hveem cohesiometer test.

completely different design procedures for these two categories of pavement. The California Division of Highways appears to be the first agency to consider slab strength and the Hveem Cohesiometer test was devised to measure it (*19*). Figure 5 shows a sketch of the cohesiometer test apparatus. The test specimen is 4-inch diameter and the test is made by clamping rigidly one-half of the specimen and causing the sample to split by applying load to a lever arm clamped to the other half of the specimen. The test result is expressed as grams of weight per inch width of 3-inch height specimen required to deflect the specimen a stated amount when that weight is applied at the end of a 30-inch lever arm. Thickness reductions are made by means of the nomographic chart, Figure 6, which is taken directly from reference (*18*).

California Cohesiometer Method

The broken lines of Figure 6 show an example of the use of the California method using cohesiometer values. The example is the same as that used in the second section of this paper, except that it is now considered desirable to investigate the use of cement-stabilized sand-shell base material containing 1.5 sacks of cement per ton of mix in lieu of the sand-shell mix for base course. It is assumed that this cement content and the method of placement contemplated will result in a base course having an ultimate cohesiometer value of 3000 grams per inch width of 3-inch height test specimen. It will be noted that this design method indicates a 50 percent reduction in thickness of cover required for the heavy clay subgrade—18 inches for a purely flexible base material and 9 inches for

a base material having a cohesiometer value of 3000. The economic implications of this design result are such as to demand the careful scrutiny of any engineer charged with the responsibility of expending public funds for highway construction.

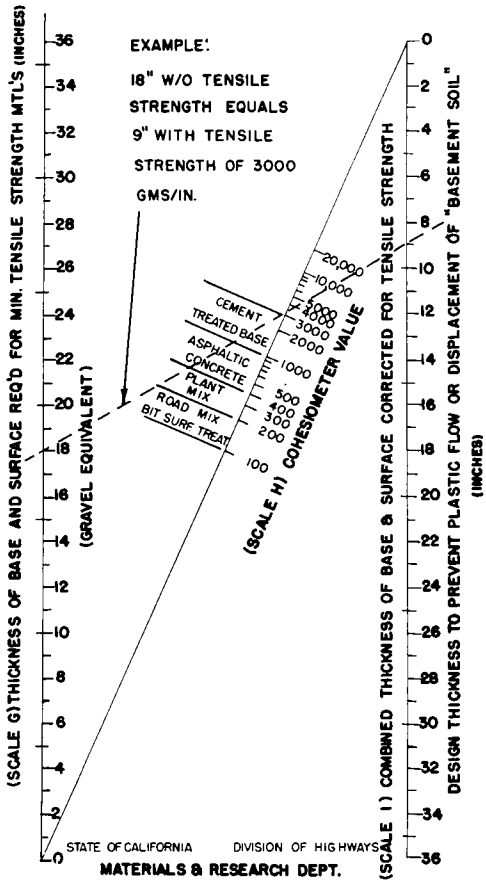

Figure 6. California thickness design chart (partial).

Modified California Method

The Texas Highway Department has for a number of years used flexural tests of 6-inch square beams for control of concrete pavement strengths, and has expressed the results in terms of unit flexural stress in the extreme fiber at failure. In attempting to correlate the cohesiometer values shown on Figure 6 for various materials with the known approximate physical properties of these materials, it occurred to the writer that the cohesiometer test was essentially a flexural strength test and that its results could be expressed in terms of unit flexural strength, provided the dimensions of the apparatus and test specimen were properly evaluated in the basic formula for flexural stress shown on Figure 5.

Attempts to correlate cohesiometer values with values of flexural strength in this academic fashion were not very satisfactory and it was concluded that any reliable correlation should be based on a comparison of observed cohesiometer values of various materials with flexural strength values of similar materials. Considerable data on cohesiometer values were available from publications of the California Division of Highways (*20*); these were compared with known values of flexural strengths of various materials being used in Texas and appearing to be approximately equivalent to the California products reported (*9*). The comparisons are shown in Table 1. In this manner the author has modified the California thickness design chart into a Tentative Semi-Flexible Thickness Design Chart employing flexural strength as the measure of slab

TABLE 1
COMPARISON OF COHESIOMETER VALUES AND FLEXURAL STRENGTHS

California pavement material	Reported cohesiometer value, C*	Calculated flexural strength $S_T = \frac{90C}{(\text{thickness})^2}$ p.s.i.	Comparable Texas Highway Department material	Usual flexural strength, p.s.i.
PCC pavement, 5 sack	15,000	362	5 Sack gravel concrete	700
PCC pavement, 4 sack	7,500	169	5 Sack shell concrete	400–600
Cement treated base, class "A"	3,000	73	Rolled cement-stabilized sand-shell	200+
Cement treated base, class "B"	1,500	36	"Poured" cement-stabilized sand-shell†	80+

* From Table "Typical Cohesiometer Values," page 5, "Explanation and Instructions for use of Pavement Thickness Design Chart," California Division of Highways, Materials and Research Department, July 30, 1955.
† Placed with sufficient water to allow placement similar to concrete.

strength in lieu of cohesiometer value. Figure 7 shows the modification proposed as a tentative measure. This chart is not submitted as a proposed design procedure, but merely as a tentative procedure that may prove reliable and that may have wider application by virtue of employing an expression of slab strength that is independent of specimen size.

Basic Flexural Strength Method

It is believed that semi-flexible pavements should be designed on the basis of their slab strengths as expressed by their flexural strength in pounds per square inch independent of specimen size or method of test. One approach to the problem is to calculate by Westergaard methods the load that can be carried by slabs of various thicknesses and various flexural strengths without any assistance from the subgrade soils; mathematically, this can be handled by assuming k-value equal to unity—a value that is for all practical purposes equivalent to no subgrade support.

Such an investigation is now in progress and will be presented at some time in the future in the form of a family of curves giving thickness reductions for various base thicknesses and flexural strengths. It is hoped that interested individuals normally employing the flexural strength concept will apply this method and the Modified California Method and, whenever possible, compare design results with those obtained using cohesiometer values by the California Method.

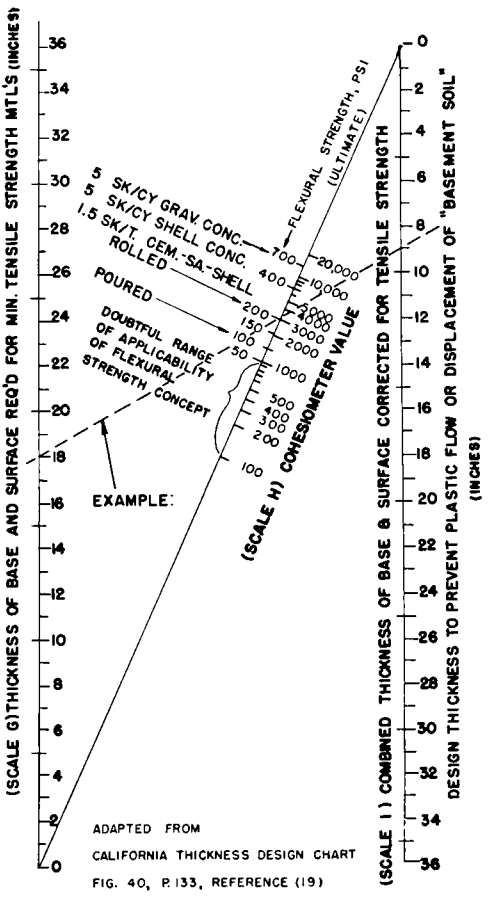

Figure 7. Tentative semi-flexible thickness design chart.

PROPERTIES OF RIGID, SEMI-FLEXIBLE AND FLEXIBLE PAVEMENT MATERIALS

Since semi-flexible pavement materials require treatment by both rigid and flexible design procedure, it seemed advisable to compare all physical properties of these with corresponding materials in the rigid and flexible range, and to determine the relation, if any, of these properties in one range to those in another. Such a comparison would at least enable analysis of any given material by either rigid, flexible or semi-flexible procedure.

Cement-Stabilized Mixtures

While data on all types of stabilization were not available, the writer has been able to compile fairly complete data on the complete range of Portland Cement mixtures including the strictly rigid category of gravel concrete, an intermediate rigid pavement (shell concrete), two types of cement-stabilized sand-shell base materials, and finally sand-shell base materials with zero cement content. Table 2 presents this data in tabular form. It will be noted that a wide variety of cement-stabilized materials are represented in the table and that strength and rigidity and other associated properties decrease with water-cement ratio, according to Abram's Law. The relations appeared so clear cut that it appeared feasible to plot smooth curves of properties against water-cement ratio. Figures 8 through 11 show curves of density, compressive strength, elastic modulus, flexural strength, thermal expansion coefficient, rate of strength increase, and ratio of compressive to flexural strength, versus water-cement ratio. The

TABLE 2
CHARACTERISTICS OF PAVEMENT MATERIALS

Summary of average test results and other data on gravel concrete, shell concrete, cement-stabilized sand,shell, and sand-shell flexible base.

Material	Gravel Concrete	Shell Concrete	Cement-stabilized Sand-shell			Sand-shell flex. Base
Control number	500-3	508-1-4	500-3-26	271-14-5	177-11-2 177-11-3	500-3
Highway	US 75	State 73	US 75	Loop 137	US 59	US 75
Cement, sks/cy	5.0	5.0	2.8	2.8	2.8	0
Total water, gal/sk	6.0	10.5	13.7	19.6	21.7	∞
Method of placement	Formed	Formed	Rolled	Vibrated wet	Vibrated wet	Rolled
Total (wet) density, lb/cf	150	141	135	126	122	137
Dry density, lb/cf		130	125	115	111	122
Moisture content, %		8.5	7.6	10.3	11.6	12.3
Air voids, %		9	14	16	17	6
Absorption, %		9.5	9.0	11.0	12.3	
Ultimate compressive strength, psi	5000	1800	1290	618	433	120
Ultimate elastic modulus, psi/in/in	2,250,000	800,000*	168,000	73,000	45,000	20,000
Ultimate flexural strength, psi	900	500	423	285	207	
Ratio of compressive to flexural strength	6.2	3.6	3.05	2.26	2.10	
Ratio of 28-day to 7-day strength	1.38			1.69	1.89	
Coefficient of thermal expansion, in/in/°F, × 10^6	8.0	6.5†			3.5	

* 1,400,000 horizontally.
† 4.0 horizontally.

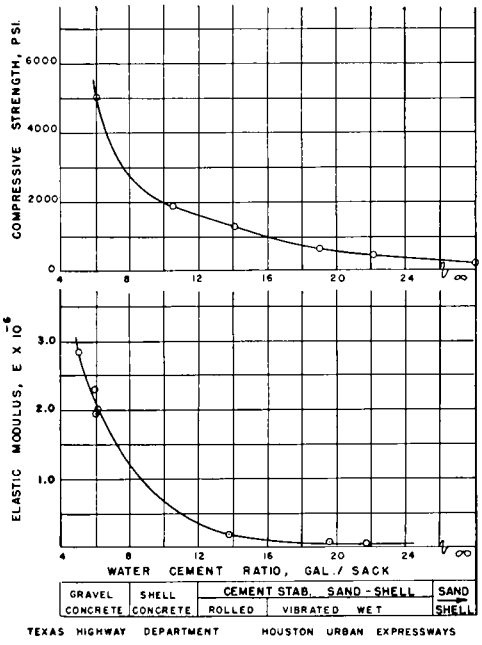

Figure 8. Characteristics of pavement materials.

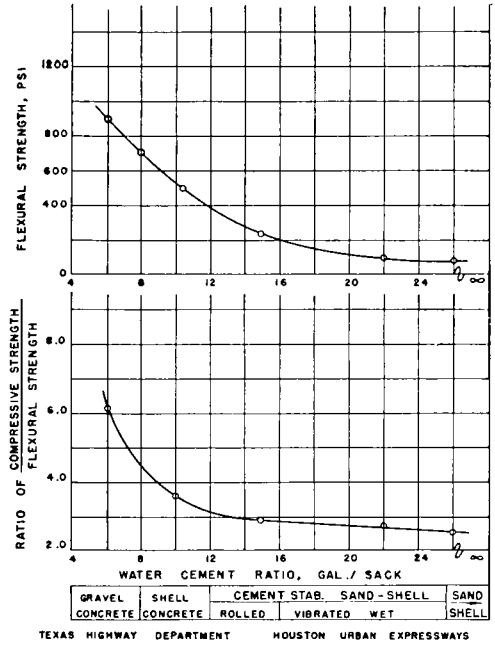

Figure 9. Characteristics of pavement materials.

comparison has more recently been made for cohesiometer and cohesion values, as shown on Figure 12.

Interrelation of Properties

These figures all show a rational transition of properties from truly rigid to truly flexible and show that the previously appearing unrelated properties in the various categories are actually not unrelated but on the other hand show a very close relation to the amount of stabilizing agent. It is to be hoped that other engineers interested in this problem will make similar correlations for materials in their ex-

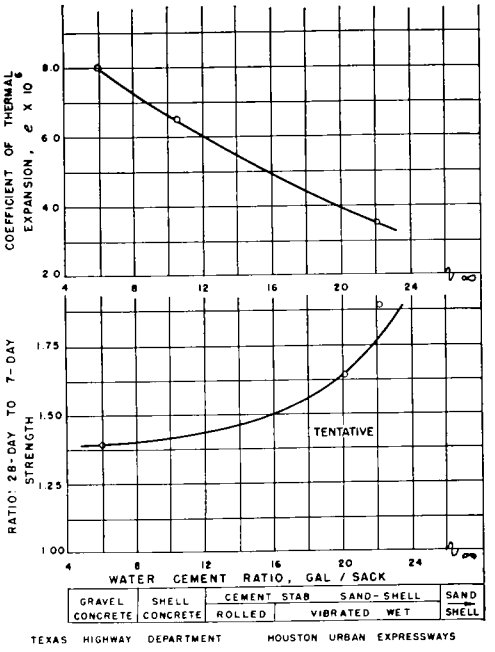

Figure 10. Characteristics of pavement materials.

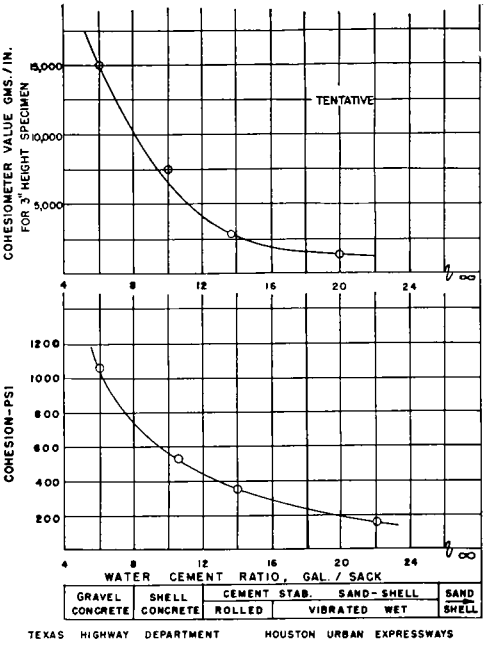

Figure 12. Characteristics of pavement materials.

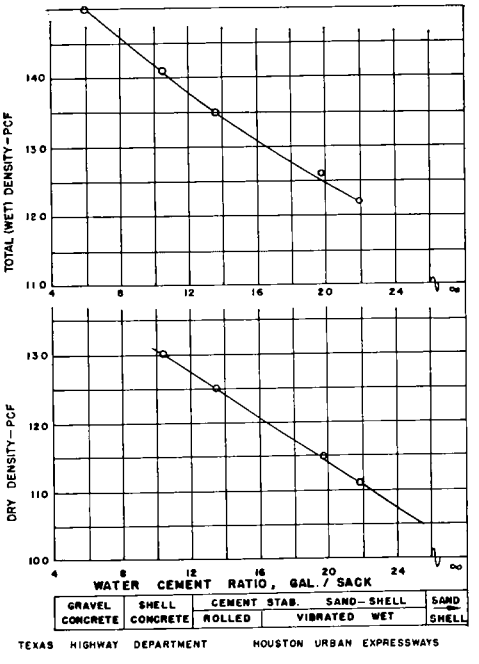

Figure 11. Characteristics of pavement materials.

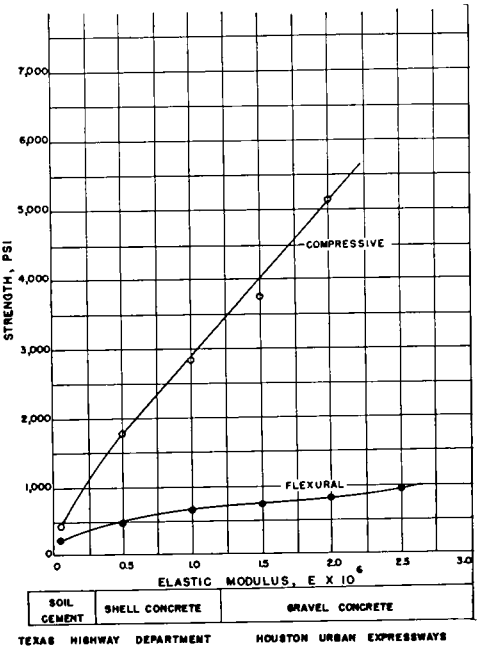

Figure 13. Characteristics of pavement materials.

perience involving other stabilizing agents such as asphalt and lime. In addition, the use of these curves and the semi-flexible concept have done much to eliminate the previous difficulty with preconceived thought habits.

Figure 13 shows the relation between elastic modulus and strength; it will be noted that the elastic modulus is essentially in direct proportion to either compressive or flexural strength. These curves are very valuable in that they make it possible to calculate tensile stresses by Westergaard method using elastic modulus and to compare such stresses with the corresponding flexural strength of any cement-stabilized mixture. Such a process is now being employed with an assumed k-value of unity in the development of the basic flexural strength method for thickness reductions in semi-flexible design, as outlined under "Basic Flexural Strength Method" in the preceding section.

Interrelations Shown by Mohr's Diagrams

As a final gesture toward showing the basic interrelations of properties, the reader is reminded that all strength properties of any material can be completely described in terms of either cohesion and friction, or in terms of compressive and flexural (or tensile) strength, by means of Mohr's diagrams. Figure 14 shows an idealized Mohr's diagram, and Figure 15 Mohr's diagrams for several types of materials represented in Table 2 and Figures 8 through 12. The cohesion values shown on Figure 15 for the various materials were used in plotting the curve on the lower half of Figure 12 relating cohesion and water-cement ratio.

SEMI-FLEXIBLE EXPERIENCE IN TEXAS

Several types of semi-flexible pavements have been used in Texas with good success and in almost all instances have resulted in considerable economy, either directly or through reductions in contractor's labor and time required to complete projects. Also, in almost all instances the products themselves were produced for purposes of achieving some expediency or overcoming some construction or supply problem and in many cases were not designed as semi-flexible pavements. The advantages that have been realized could have been increased had we recognized these as such at the time of design and construction and had designed them accordingly.

Various examples of semi-flexible pavements built in Texas are cited in the succeeding paragraphs along with data on the quality of

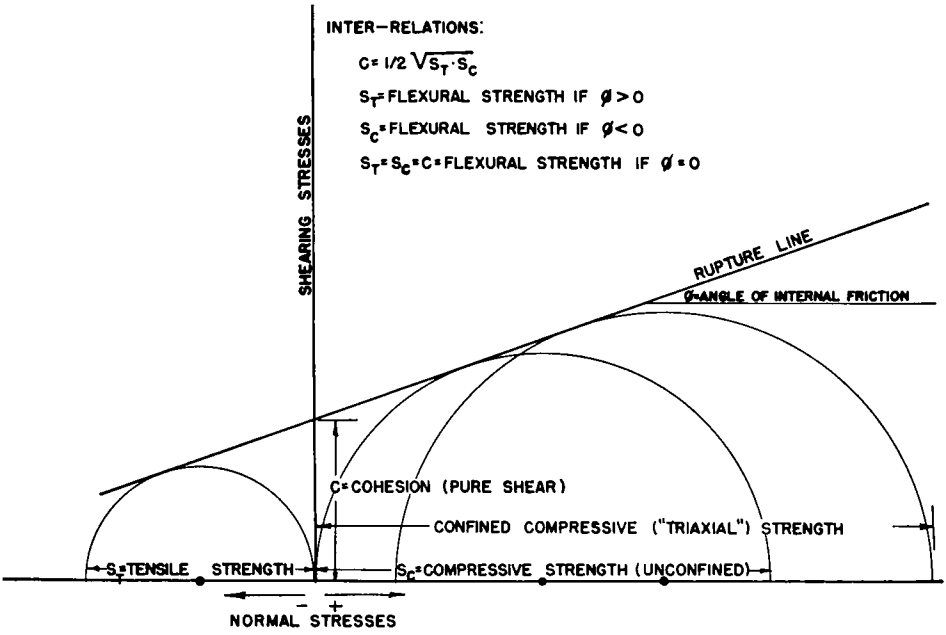

Figure 14. Idealized Mohr's diagram.

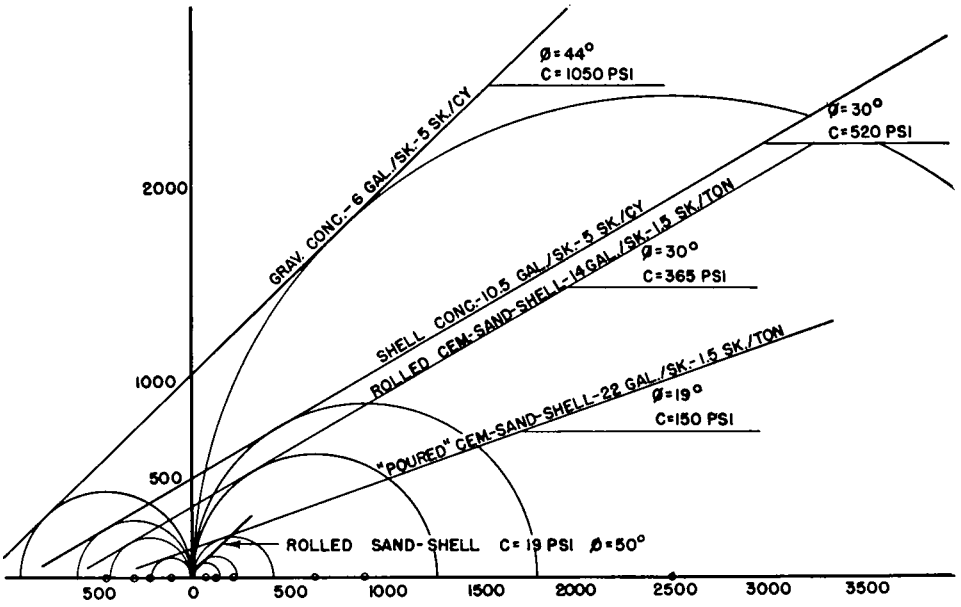

Figure 15. Mohr's diagram for various pavement materials.

TABLE 3
COMPARATIVE BID PRICES ON SUB-BASE MATERIALS

Highway	Completion Date	Sub-base	Total Price Per S.Y.
Gulf Freeway	October, 1948	9″ Sand-shell with Seal Coat	1.35
Gulf Freeway	July, 1951	6″ Sand-shell with Seal Coat	1.01
La Porte Freeway	March, 1952	6″ Sand-shell with Seal Coat	1.12
Gulf Freeway	August, 1952	6″ Cement-sand shell	1.42
Loop 137	January, 1954	6″ Cement-sand-shell	1.52
Eastex Freeway	January, 1954	5″ Cement-sand-shell	1.51
Eastex Freeway	February, 1955	6″ Cement-sand-shell	1.70
U S 75 North	July, 1955	6″ Cement-sand-shell	1.55
Gulf Freeway	August, 1955	6″ Cement-sand-shell	1.40
State Highway No. 35	November, 1955	5″ Cement-sand-shell	1.40
Eastex Freeway	December, 1955	6″ Cement-sand-shell	1.30
State Highway No. 73	1956	5″ Cement-sand-shell	1.35
U S 90A (69th Street Bridge)	1956	4″ Cement-sand-shell	1.25

performance achieved. Attempt will also be made to show the savings actually effected, and the possible additional savings that could have been realized had the design principles outlined herein been known and used.

Cement-Stabilized Sand-Shell Sub-Base

The first several miles of expressway construction in Houston consisted of concrete pavement placed on a sub-base of 6 to 9 inches of a locally available sand-oyster shell mix for control of pumping; a single asphalt surface treatment was placed on the sand-shell sub-base for a working table and waterproofing mat during construction. In order to reduce time of compaction and curing of this low plasticity type of granular material and to eliminate the asphalt surface treatment thereon, it was decided to try the addition of one and one-half sacks of cement per ton of this mix.

Cost Data—Cement-Sand-Shell Sub-Base. In spite of initially higher bid prices, the savings in labor costs and construction time have resulted in a steady lowering of bid prices and the Houston Urban Expressway office is now securing 5-inch and 6-inch thickness cement-stabilized sub-bases at about the same cost as the 9-inch sand-shell sub-base with asphalt surface treatment originally used, and about $0.25 per square yard more than 6-inch sand-shell base with asphalt surface treatment. Table 3 shows chronologically the respective bid prices before and after the change.

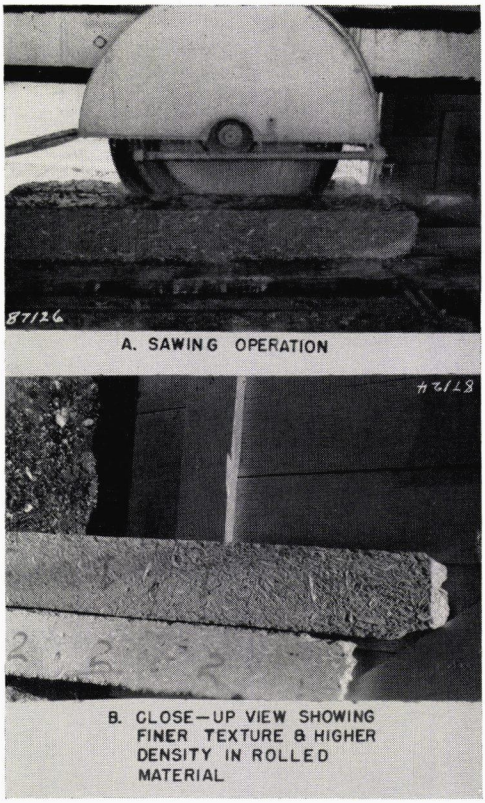

Figure 16. Photographs of cement-stabilized sand-shell base. (*a*)Slab being sawed into beams for flexural strength test. (*b*)Close-up view showing finer texture and smoother sawed surface in rolled material indicating higher strength.

At the time of acceptance of this material as a better product, it was determined that the additional strength gained resulted in increasing k-value from 110 to 500 psi per inch, which justified a 1-inch reduction in concrete slab thickness according to Westergaard analysis; at an average bid price of $17.00 per cubic yard for concrete pavement, this amounts to a saving of $0.47 per square yard. We have placed approximately 386,000 square yards of pavement using this type of sub-base which would have resulted in a saving of $182,000 had we made the thickness reduction effective immediately. On a considerable portion of this pavement we initially preferred to use the same thickness of concrete and use the additional strength as an added margin of safety. Using Westergaard analysis, the latter amounted to increasing design wheel load from 14,000 to 18,000 pounds or from 20,000 to 24,000 pounds.

Physical Appearance of Cement-sand-shell Sub-base. Figure 16 shows two photographs of some of this material being sawed into beams for flexural strength tests; these were sawed from slabs of the material removed from completed sub-base by coring a large number of holes on a square pattern to isolate a 3-foot square section of the base and then lifting this slab from the subgrade. It is apparent that the material possesses considerable strength to allow such handling; also apparent from the photographs is the fine texture of sawed surfaces in the material placed by rolling at optimum moisture indicating high mortar strength, compared to the relatively rough texture of material placed by spreader box machine at high water content without rolling.

Performance Data—Cement-Sand-Shell Sub-Base. Annual condition surveys of all concrete expressway pavements in Houston show a marked reduction in intensity of cracking following substitution of cement-stabilized sand-shell sub-base for sand-shell sub-base. Table 4 summarizes this experience.

It is apparent that joint spacing and thickness of sub-base also have a very definite effect on crack formation; nevertheless, it is quite evident that we have found the right combination of joint spacing and sub-base to control cracking to a high degree. Comparing the North Loop experience with that of the section of the Gulf Freeway having almost comparable joint spacing, the 6-inch cement-stabilized base is at least as effective as the 9-inch sand-shell sub-base.

Cement-Stabilized Sand-Shell Base.

On a recent expressway contract, a semi-flexible type pavement was provided for service roads using 14 inches of cement-stabilized sand-shell base with 1¼-in. hot-mix asphaltic concrete surfacing; freeway lanes on the same project consisted of 10-inch concrete pavement with 5-inch cement-stabilized sub-base.

Cost Data—Cement-Sand-Shell Base. Bid prices on this contract showed a saving of approximately one-third on all of the service roads on this contract, as shown by the tabu-

lation below:

10-Inch concrete pavement plus 5-inch cement-stabilized sub-base	$5.88/sq. yd.
14-Inch cement-stabilized base plus 1¼-in. hot-mix asphaltic concrete surfacing	$4.00/sq. yd.
Saving	$1.88 or 32%

For the 9150 square yards of service road pavement on this contract alone (a relatively small contract), the saving amounts to $17,000. Had the semi-flexible design principles been applied, the base course thickness could have been reduced to 12 inches, and the saving on this contract would have been $21,600 or 40 percent of the cost of the equivalent concrete pavement. Attention is invited to the apparent inconsistency in the above comparison, in which by semi-flexible design procedure less total thickness of pavement is indicated for the asphalt pavement for service roads than for the concrete pavement for freeway lanes designed for the same wheel load. This is in fact not an inconsistency, but the result of taking full advantage of both subgrade strength and that of the base and results primarily from three circumstances:

1. Semi-flexible pavements do not crack to the extent of requiring joints and hence require no non-pumping select material.

2. In semi-flexible pavements, both subgrade and base act together to carry their respective portions of the load, whereas in concrete pavements the slab is designed to carry practically all of the load with the subgrade strength (k-value) having a comparatively small effect on slab thickness requirements.

3. Curb and gutter design was such as to prevent the occurrence of an edge load, hence interior stresses controlled in the Westergaard analysis of the base course.

Performance Data—Cement-Sand-Shell Base. Coincident with the adoption of the use of cement-stabilized sand-shell for sub-base on expressway construction in Houston, the same material was supplied as base course for asphaltic pavement forming emergency parking lanes adjacent to concrete expressway lanes. On the last section of the Gulf Freeway constructed using the sand-shell sub-base, emergency repairs were necessary within one year from date of opening to traffic due to a dangerous drop in grade that developed in the asphalt pavement at the edge of the concrete; the asphaltic surfacing and sand-shell base had apparently consolidated or had been compacted by traffic sufficiently to result in the difference in elevation at the edge of the concrete. On the other hand, no repairs have been necessary in any of the subsequent construction using the cement-stabilized sand-

TABLE 4
INTENSITY OF CRACKING IN HOUSTON EXPRESSWAYS

Highway	Concrete Pavement		Sub-base	Joint Spacing	Intensity of Traffic V.P.D.	Yrs. of Traffic	Lin. Ft. Cracks Per 100 S. Yds.
	Total S. yds.	Thickness					
Gulf Freeway—Dowling St. to Telephone Road	136,000	8″	9″ Sa. Sh.	20′	50,000	1	0.5
					64,000	2	0.75
					70,000	3	0.95
					73,000	4	1.1
					77,000	5	1.4
					80,000	6.2	2.1
Gulf Freeway—Telephone Rd. to Park Place	119,000	9″	6″ Sa. Sh.	30′6″	19,000	1	1.2
					34,000	2	2.6
					38,000	3	4.1
					44,000	4	5.5
					46,000	5	6.5
La Porte Freeway	163,000	9″	6″ Sa. Sh.	30′6″	15,000	1	0.7
					16,000	2	1.4
					18,000	2.5	1.7
Eastex Freeway—Quitman St. to Kelly Street		9″–11″	5″ Cem. Sa. Sh.	30′6″		1	0.5
						1.25	0.6
North Loop		9″	6″ Cem. Sa. Sh.	15′		1	0
						1.25	0

"Sa. Sh." = Sand-shell mix; "Cem. Sa. Sh." = Cement stabilized sand-shell mix, 1½ sacks per ton.

shell base material, nor has any noticeable drop in grade developed.

Sand-Cement Base

On a recent project connecting a major expressway in Houston with existing streets a considerable quantity of silty sand selectively stock-piled from a roadway underpass cut was used in constructing asphalt pavements by stabilizing it with two sacks of cement per ton of sand. The project was originally designed for 13 inches of sand-cement to be placed in three layers with 3-inch asphaltic surfacing, which would have cost $5.54 per square yard at the bid prices obtained. However, the supply of silty sand from the cut was not sufficient for the full 13 inches and the presence of clay balls also made it desirable to provide a cleaner base material in the upper 5 inches of base; cement-stabilized sand-shell base material with one and one-half sacks per ton was accordingly used in the upper 5 inches of base. The resulting cost of the completed pavement, including extra payment to the contractor for additional expense of providing the better base material in the upper 5 inches, was $5.83 per square yard compared to $6.00 per square yard for equivalent concrete pavement provided in the adjoining project by the same contractor. For the 13,630 square yards involved, this represents an apparent saving of $2320. Due to small quantities involved and unbalanced bids, the contract prices for the soil cement base were unusually high; a similar thickness semi-flexible pavement using cement-sand-shell base course (see "Cost Data—Cement-sand-shell Sub-base" of the previous section) was provided by another contractor for $4.00 per square yard, in spite of smaller contract quantities involved. It was subsequently determined that the strength of the sand-cement and sand-shell-cement compacted materials was such as to increase design wheel load from 20,000 pounds to 30,000 pounds using the semi-flexible design procedure, or that total thickness could have been reduced 4 inches for the 20,000-lb. design wheel load actually used in the original design. The latter would have resulted in a saving of $2.53 per square yard or a total of $33,700 on this contract, using actual contract prices.

Other Soil-Cement Roads in Texas

In 1952 the Portland Cement Association compiled a report (*22*) of all soil-cement road construction in Texas, in which they reported 3¼ million square yards or 272 miles under traffic for 19 years or less. Performance has been generally good and maintenance costs reasonable. It was pointed out that 22 of 25 Texas highway districts use soil-cement. A typical statement in the numerous job data records given reads as follows: "Latest traffic count is 2080 to 2510 vehicles per day. There have been no soil-cement base failures during its 10 years of service." Another extract from this report reads as follows: "Probably the first road in Texas that could be labeled 'soil cement' was constructed in 1933, in Haskell County, on what is now F.M. 617. It extends from State Highway 283, west, two city blocks on Main Street of Rochester. It was a plastic mix soil-cement and is in near perfect condition after 19 years of service."

Lime-Gravel Bases

With an ever increasing threat of diminishing supplies of high grade flexible base materials for road construction, Texas highway engineers have been sorely taxed to find suitable materials for their projects or to improvise stabilization measures for locally available but otherwise unsuitable materials. A notable example of the solutions effected is described in a paper by Mr. J. P. Cooper, Senior Resident Engineer, Tarrant County, Fort Worth, Texas (*21*); in this case an unsatisfactory existing road constructed of local gravel was converted into a road having a base course material much better than Texas Highway Department Strength Class I base material at considerably less cost than that of reconstruction, employing imported crushed limestone base material, the cheapest alternate suitable material available. During its first 1½ years of service this farm-to-market road carrying about 2600 vehicles per day and designed for 10,000-pound wheel load has required about $100 maintenance, compared to an annual maintenance cost of approximately $7000 per year prior to reconstruction and compared also to an annual maintenance cost of approximately $1000 per year on an adjacent road of

similar traffic and design wheel load constructed using crushed limestone base material.

Texas has built approximately 200 miles of such lime-stabilized gravel and caliche base courses. A comprehensive report of this work has been made by Chester A. McDowell (*23*) and later by T. S. Huff (*25*). In addition to the tremendous savings in construction and maintenance costs involved in this 200 miles of high quality roads, the serious reader will be astounded to reflect on the many thousands of cubic yards of first class base materials conserved thereby for use elsewhere in the overall program.

Lime-Clay Bases

Conservation is a word of which one does not hear so much in recent years; the writer has not forgotten so quickly the lean years of World War II (lean in such items as sugar, bacon, and asphalt) in which conservation of almost everything was stressed. It has been reported that a maximum of 12 years' supply of concrete aggregates is available within economic haul distances for Houston; contrary to contentions of oyster shell producers, it is not believed that there is an unlimited supply of dead oyster reefs on the Texas Coast. The day may soon come when we must build our roads of clay.

It is not surprising, then, that lime-clay stabilization in Texas has been confined principally to the coastal regions having no gravel or caliche deposts.

District 12 with headquarters in Houston has constructed approximately 12 miles of farm-to-market roads using lime-clay stabilization in combination with sand-shell base material. Cost of such roads is about $1000.00 per mile less than that of similar roads constructed entirely of sand-shell base material where local sands are available for road-mixing with the shell. In other areas not blessed with local field sands the cost differential is much greater. After being in use up to three years, the maintenance cost for the lime-clay roads has been zero. District 20 with headquarters in Beaumont has either constructed or has under contract slightly over 17 miles of lime-clay roads. In this particular section of the state very unstable organic soils are encountered and many are such as to prevent even construction traffic; lime has been successfully employed to stabilize such soils by applying a lime slurry in successive layers in a technique described in detail by Mark S. Swain (*24*). It is truly remarkable to witness the "drying up" effect of a water slurry of lime applied to a sticky, wet fat clay subgrade soil; this is accomplished through chemical reactions resulting in increases in both optimum moisture content and plastic limit, coincident with a lowering of the liquid limit and a very noticeable decrease in physical plasticity. Cost analyses made by District 20 personnel on several such projects indicate that the lime-clay stabilization can be accomplished at costs ranging from 45 to 90 percent of the cost of equivalent stabilization by means of imported select courses, depending on the distance to the source of borrow.

DISADVANTAGES OF SEMI-FLEXIBLE PAVEMENTS

The preceding section of this paper has been presented to show the economic advantage of semi-flexible pavements and what performance data are available at this time. It must not be assumed from the data presented that semi-flexible pavements are without faults and that such a pavement will be the most economical or most judicious choice in any case. On the contrary, each pavement type has its peculiar advantages and disadvantages and each case must be analyzed on its own merit. Moreover, some of these pavement types have not been used extensively and in any new product it is well to know the limitations, as well as the advantages. The following listed shortcomings come to mind in comparison with either flexible or rigid pavements

Lack of Adequate Wearing Surface

This is of prime importance in design of expressway pavements and other high volume traffic facilities where maintenance operations are costly and difficult. As stated in the last paragraph of the section on "Flexible Pavement", development of permanently stable bituminous wearing courses or of a semi-flexible pavement material having an adequate wearing surface is badly needed.

Layered Construction for Thick Bases

Where thicknesses greater than about 6 inches are to be provided, the construction is

usually made in two or more layers. This leads to horizontal planes of weakness, which affect design assumptions and may influence behavior. Layered construction also leads to possible damage to one layer while placing the upper layer. Use of construction equipment capable of compacting thicker layers may be the remedy in some cases; final rolling with pneumatic equipment has alleviated somewhat the condition of horizontal planes on recent projects. It is of prime importance in this type of construction to construct the base in layers as thick as possible, and to provide vertical headers at the end of a day's run to avoid feather-edging and the resultant multi-layered construction.

Difficulty of Re-Working

In this respect semi-flexible pavements are subject to somewhat the same limitations as rigid pavements, in that in the event of failure due to under-design or increased traffic loads, the broken-up base can be utilized only to the extent of providing a base of strength equivalent to the same thickness of flexible base; flexible bases, however, can usually be scarified and re-worked (with or without admix of surfacing material) to produce a product equal in value to that of the original flexible base. This difficulty can be overcome, of course, by adequate initial design.

Dependence of Design on Proper Construction

While this paper is concerned primarily with selection and design of pavement types, it might be well to mention that in most semi-flexible pavement materials faulty or improper construction can often affect the results to a larger degree than in other types of construction and can actually nullify the advantages considered in the design. Some of the factors have been mentioned in the discussion of layered construction for thick bases. Others that come to mind are:

Re-working of material that has already received its initial set. This frequently happens in fine-grading operations and it is necessary to require wasting any such material removed in fine-grading.

Selection and Processing. Improper selection or processing of material acquired from on-site areas; for example, excessive mud balls or other extraneous material not removed during processing. Selective stockpiling of material from roadway cuts for use in semi-flexible pavements requires close supervision and stern inspection policies. Likewise, the designer should anticipate such difficulties and perhaps use another material if construction difficulties appear too formidable.

Improper Curing. Any lime or cement stabilized material derives the major portion of its strength from the cementing value of the material after proper curing; these are similar to concrete pavement in this respect and should be cured with the same care as would be afforded concrete pavement. Light asphaltic surface membranes will serve a dual purpose of providing a tack coat for subsequent surface courses in addition to the curing effects. Lime-clay mixtures seem to be susceptible to loss of strength due to loss of moisture even after a reasonable curing period; hence, in both design and construction, provision must be made for prevention of moisture loss.

SUMMARY

Summarizing the material presented in this report, it appears that many materials in common use today in road construction fall into the semi-flexible category. Considerable economy can be achieved in use of such materials by proper designs giving adequate credit in the design for slab strength of the base material. Such design also gives full credit to subgrade strength, whereas in rigid design construction the subgrade contributes little to the load-bearing capacity. A design procedure has been outlined for semi-flexible pavements which determines thickness required to prevent flexural overstress in the base and shearing overstress in the subgrade or underlying layer; balanced designs giving essentially the same thickness for both conditions are the most economical. Since such design involves both rigid and flexible procedure, data is presented correlating all physical properties pertinent to either procedure for a large variety of materials from the rigid to the flexible category, and a rational progression of all properties through the various categories is noted. Use of Mohr's diagrams will assist in comparing physical properties of various materials in any of the categories. In addition to economy in construction, semi-flexible pavements lead to low

maintenance costs and performance to date has been very satisfactory.

ACKNOWLEDGMENTS

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REFERENCES

1. Porter, O. J., "Foundations for Flexible Pavements," Proceedings Highway Research Board, Vol. 22, 1942, p. 110.
2. Middlebrooks, T. A., "Design of Flexible Pavements Foundations," Roads & Streets, Vol. 86, 1943.
3. War Department, Office of the Chief Engineer, "Engineering Manual," Chapters XX and XXI, Procedings Highway Research Board, Vol. 22, 1942, p. 154.
4. McDowell, C. A., "Progress Report on Development And Use of Strength Tests for Subgrade Soils And Flexible Base Materials," Proceedings Highway Research Board, Vol. 26, 1946, p. 484.
5. Committee on Flexible Pavement Design, "Thickness of Flexible Pavements," Highway Research Board Bulletin 8-R, 33rd Annual Meeting, January 12–15, 1954.
6. McDowell, C. A., "Triaxial Tests in Analysis of Flexible Pavements," Highway Research Board Research Report 16-B, 33rd Annual Meeting, January 12–15, 1954.
7. The Asphalt Institute, "Thickness Design—Flexible Pavements for Streets and Highways," Manual Series No. 1, January, 1955.
8. Westergaard, H. M., "Computation of Stresses in Concrete Roads," Proceedings Highway Research Board, Fifth Annual Meeting, 1925, pp. 90–112.
9. Harris, F. A., "Selection of Pavements for Expressways," Texas Highways, Vol. II, No. 1, November, 1954, p. 182.
10. Bradbury, R. D., "Reinforced Concrete Pavement," The Wire Reinforcement Institute, Washington, D. C., 1938.
11. Pickett, G., "A Study of Stresses in the Corner Region of Concrete Pavement Slabs Under Large Corner Load," Concrete Pavement Design, Portland Cement Association, 1951, p. 77.
12. Hank, R. J., and Scrivner, F. H., "Some Numerical Solution of Stresses in Two- and Three-Layered Systems," Proceedings Highway Research Board, Vol. 28, 1948, p. 457.
13. Acum, W. E. A., and Fox, L., "Computation of Load Stresses in a Three-Layer Elastic System," Geotechnique, Vol. 2, No. 4, 1951, p. 293.
14. Westergaard, H. M., "Analysis of Stresses in Concrete Roads Caused by Variations of Temperature," Public Roads, Vol. 8, No. 3, May, 1927, p. 54.
15. Coons, H. C., "Investigational Concrete Pavement in Michigan," Highway Research Board, Research Reports No. 3-B, p. 35–57.
16. Portland Cement Association, "Progress Report, Investigation of Pumping of Concrete Pavements—Causes, Means and Methods of Prevention," November, 1946, Chicago, Illinois.
17. "A Survey of Pumping of Concrete Pavements in Illinois," Highway Research Board, Research Reports No. 1-D-1948, Supplement.
18. Withycombe, Earle, "Base Stabiliza-

tion With Portland Cement," Proceedings, Fifth California Street and Highway Conference, Highway Research Abstract, Vol. 24, No. 2, February, 1954.

19. HVEEM, F. N., AND CARMANY, R. M., "The Factors Underlying the Rational Design of Pavements," Proceedings Highway Research Board, Vol. 28, 1948, p. 101.
20. California Division of Highways, Materials and Research Department, "Explanations and Instructions for Use of Pavement Thickness Design Chart," July 30, 1951, Table "Typical Cohesiometer Values of Various Materials."
21. COOPER, J. P., "Lime Stabilization of Base Material," Texas Highways, Vol. II, No. 7, May 1955, p. 86.
22. Portland Cement Association, "Soil Cement on Texas Highways," Austin, Texas, 1952.
23. MCDOWELL, C. A., "Development of Lime Stabilization," Texas Highways, Vol. II, No. 7, May 1955.
24. SWAIN, M. S., "Lime Stabilization of Subgrade," Texas Highways, Vol. II, No. 7, May 1955.
25. HUFF, T. S., "Use of Lime Stabilization on Roads of the Texas Highway System," 41st Annual Convention, AASHO, December 1955.

APPENDIX

A. FLEXIBLE PAVEMENT DESIGN

1. *Basic Assumptions of the Texas Highway Department Procedure*

The basic principles of flexible pavement design have been outlined in Section 2, "Flexible Pavement," of the paper. The fundamental difference in behavior of flexible and rigid pavements has also been pointed out, and examples given showing the use of thickness design charts as published by several different agencies. In answer to numerous requests by interested individuals for detailed derivation of the Texas Highway Department Classification Chart (Figure 21) and Thickness Design Curves (Figure 24), it appeared prudent to include such an explanation in this Appendix.

Figure 17 shows graphically the three kinds of wheel loads that may be imposed on a flexible pavement. Of the three possible loading conditions shown, the loading of Figure 17 (*c*), that of a moving vehicle with brakes applied, is by far the most severe and was used in the basic determination of stresses. Figure 18 shows generally the basis of determination of stresses due to this type of loading. Obviously the stresses A, B, and C shown with the subscripts on Figure 18 are the sum of stresses due to two sources:

a. Stresses due to a static wheel load such as shown in Figure 17 (*a*).
b. Stresses due to the horizontal component of the loading shown in Figure 17 (*c*). This horizontal component is the reaction of the pavement surface to the force produced by the deceleration of the vehicle.

Stresses for each of the individual effects *a* and *b* are determined separately and the total stress at any point determined by addition, according to the principle of superposition. According to this principle, the total stress at any point in an elastic medium subjected to stresses due to two or more sources is equal to the algebraic sum of the stresses from each source.

It is assumed that the flexible pavement materials are uniform, elastic, and isotropic. Few, if any, flexible pavement materials possess these qualities; therefore, the computations of stresses are considered to be merely fairly reliable estimates and have been modified according to the dictates of practical experience from time to time.

2. *Stress Calculations*

Theoretical stress *a* can be obtained by using published solutions of the basic Boussinesq formula for the case of a load uniformly distributed over a circular contact area. While all wheel load contact areas are approximately elliptical in shape, very little error is occasioned by assuming the actual contact area to be circular and to have an area equal to that of the actual contact area. Solutions for this case of loading are numerous in the literature of soil mechanics; examples are the works of Jurgensen (*26*) and Love (*27*). The solutions published give influence values for major principal stress, minor principal stress, and shear stress at various points below such a circular load.

Stresses *b* due to braking effect cannot be solved by means of published solutions. Hence, detailed stress analyses were made for this component of applied load. The pavement reaction can be determined if the speed and weight of the vehicle and the coefficient of friction of the tire-pavement contact area are known

or assumed; the latter was assumed to be sufficiently high to prevent skidding. The horizontal shearing stress at the surface was assumed to be equal to the braking force divided by the tire contact area; for depths below the surface the stress was assumed to decrease in proportion to depth.

Stresses *a* and *b* were then added algebraically and the results plotted by means of Mohr's diagram graphic construction to obtain stress circles such as A_2–A_1, B_2–B_1, etc., Figure 18. Each such circle represents the stress at some point on a horizontal plane at some assumed depth Z; therefore, the common stress envelope drawn tangent to all of the stress envelopes concerned represents all possible combinations of stress due to the assumed wheel load at the assumed depth Z.

3. *Strength Determinations*

The next step in the formulation of a rational design procedure involved the determination of the shearing strength of the flexible pavement materials concerned. Since the influence of confinement or lateral pressure is of paramount importance in testing of most flexible pavement materials, the Texas Highway Department has developed triaxial compression testing equipment compatible with aggregate sizes normally encountered (*4*). Figure 19 represents graphically such tests on three separate specimens of an assumed flexible pavement material; the lower half of this figure shows a circle of stress for each of the three tests representing stress conditions at failure for the three specimens each tested under different lateral confining pressures. As in the case of the stress circles of Figure 18, the common stress envelope (termed "rupture line" in case of strength circles) represents the strength of the flexible pavement material for any combination of stresses.

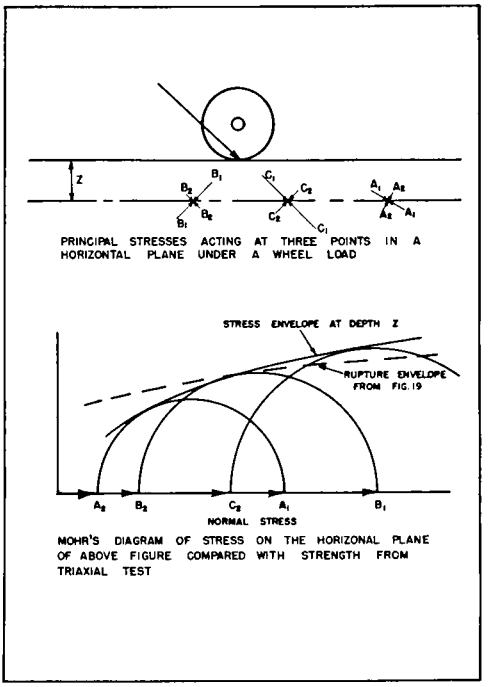

Figure 18. Development of stress circles.

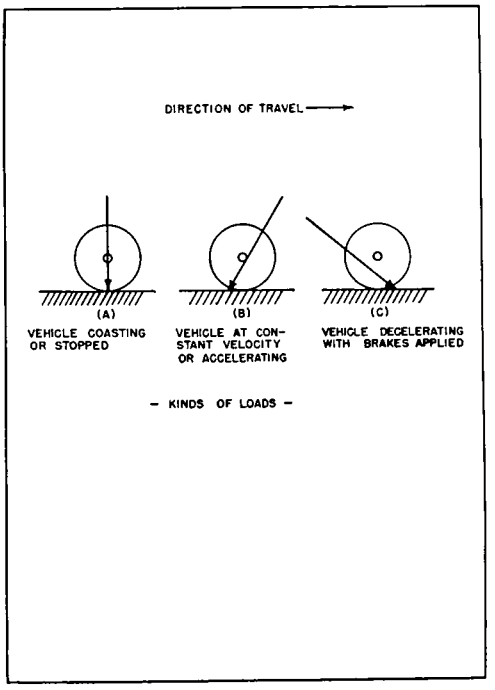

Figure 17. Basis of flexible pavement design procedure.

4. *Strength Classification Chart*

A pavement material will be stable at any depth, Z, under any given wheel load if its rupture line coincides with, or lies below, the stress envelope pertaining to the load and depth concerned. In the case illustrated on Figure 18, the material concerned would not be stable at the depth and under the wheel load concerned, since a portion of the rupture line lies above the common stress envelope pertaining to the load and depth.

In this manner there has been evolved a design procedure by which the stability of any flexible pavement material under any wheel load can be determined for any assumed depth below pavement surface. Figure 20 shows stress envelopes for stresses existing at various depths in a pavement subjected to a wheel load of 8000 pounds. A rupture line of an actual pavement material is shown superimposed on the plot of stress circles for this load; in this

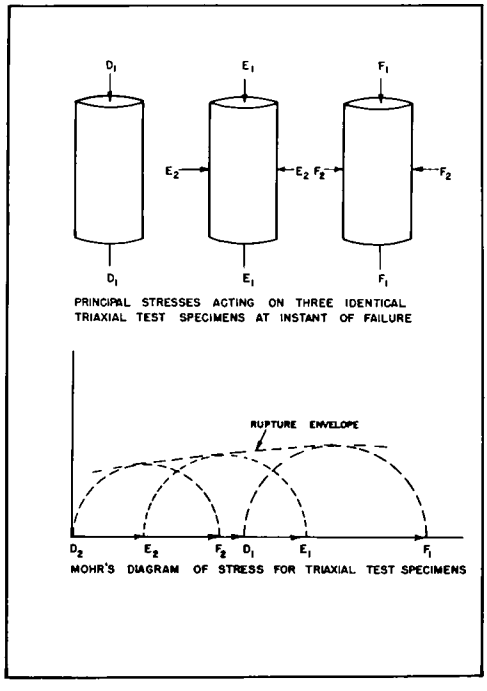

Figure 19. Development of strength circles.

case the pavement material concerned would be stable at depths of 11 inches or more and would be unstable if placed less than 11 inches from the pavement surface.

A material showing a rupture line lower than that shown on Figure 20 would be in a lower strength classification, and would be stable only at greater depths; conversely, any material having a higher rupture line would be a higher quality material and would require less depth for stability. Such a line of reasoning has led to the strength classification chart shown in Figure 21; in this chart the depth figures have been replaced with "Triaxial Strength Classifications" applicable to any wheel load. The corresponding required depths of cover for 8000-pound wheel load are also shown on this figure for comparative purposes and to further clarify the details of the derivation of the Classification Chart; these required depths are taken directly from Figure 20. The strength classification numbers are roughly proportional to the depth of cover required for stability (but not in arithmetic proportion) and allow the use of a single classification value applicable to all loads. Assignment of these numbers was made on the basis of providing a grouping of all soils into as sufficient a number of general classifications as was deemed necessary or desirable for practical usage.

5. *Thickness Design Curves*

During the early stages of its use, the strength classifications were used to assign depth and thickness requirements in a general manner by means of tables of recommended depths such as shown in Table 5; this table is taken from McDowell's report (*28*). It will be noted that this table allows the designer considerable latitude in choice of thickness to be selected.

In order to more clearly define the depths of cover required for materials having various strength classifications, and to enable thickness determinations for loads other than those shown in Table 5, there was subsequently developed a set of thickness design curves relating wheel load, required depth of cover, and strength classification. Figure 24 shows the final design curves; these curves were developed as described in the following paragraphs.

The required thickness of cover for 8000-pound wheel load and materials having strength classifications from 2.0 to 6.0 are indicated in Figure 21. The values of thickness and classification shown furnish one point on each of the curves of Figure 24; these points are shown by closed circles on the vertical line representing 8000-pound wheel load.

Similar points representing required depth of cover for the eight strength classifications

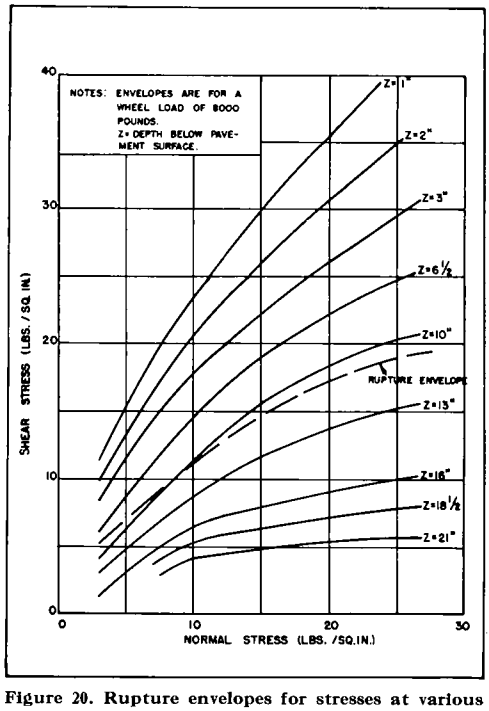

Figure 20. Rupture envelopes for stresses at various depths.

shown in Figures 20 and 24 and other wheel loads can easily be obtained by extrapolating the data of Figure 20 on the premise that the required thicknesses are proportional to the square root of the wheel loads concerned. This retlaion is mathematically rigorous if the material represented by any two depths $Z1$ and $Z2$ are identical, homogeneous, elastic, and isotropic. Mathematical derivation of this relation between load and required thickness is given in Figure 22.

In this manner, points on each of the curves were calculated for the eight classifications and various wheel loads and plotted on charts such as Figure 24. Points representing materials having the same strength classification were joined to obtain the design curves of Figure 24.

6. *Example of Use of Design Curves*

Figure 23 shows the Classification Chart with a complete pavement design example superimposed thereon. Materials A, B, and C are respectively, a flexible base course material, an intermediate "select material," and the compacted subgrade on which the proposed flexible pavement must be constructed. The strength classifications are respectively, 2.2, 3.6, and 4.2.

It is now desired to determine a design for proposed flexible pavement utilizing materials

TABLE 5

TABLE FOR INTERPRETATION OF CLASSIFICATION OF SUBGRADE AND FLEXIBLE BASE MATERIAL

Class of Material	General Description of Material	Coverage of Sub-base, Base and Surfacing Required for Various Classes of Materials		
1	Good flexible base material	Good—light bituminous surfacing acceptable		
2	Fair flexible base material	1 to 4 inches of bituminous surfacing or a 3 to 5-inch stable layer of class 1 material covered with a good light surfacing		
		8000 lb. Wheel load	12,000 lb. Wheel load	16,000 lb. Wheel load
3	Borderline base and sub-base materials	4–10″	5–12″	6–14″
4	Fair to poor subgrade	10–16″	12–20″	14–23″
5	Weak subgrade	16–21″	20–26″	23–30″
6	Very weak subgrade	21″+	26″+	30″+

Where braking stresses occur frequently, such as in dense traffic or at stop signs, additional depths of at least 2 to 4 inches of pavement and/or surfacing are indicated.

Note: This table taken directly from McDowell, "Roads and Streets," reference (*28*).

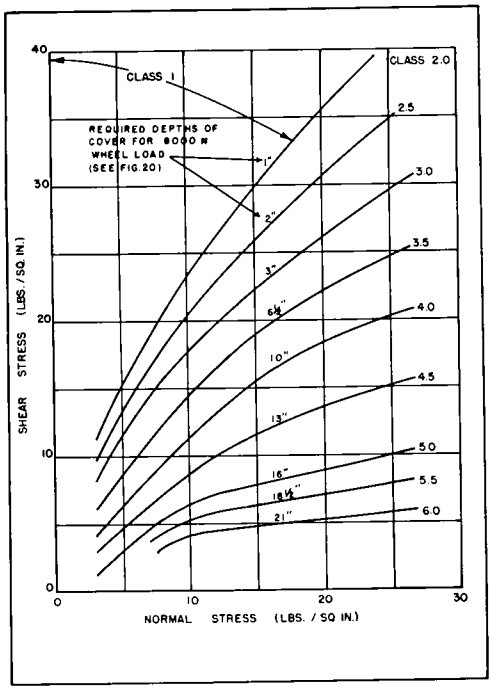

Figure 21. Classification chart.

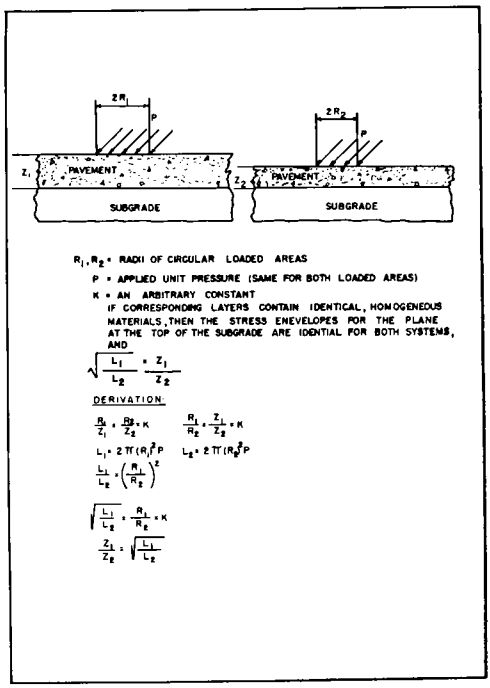

Figure 22. Variation of pavement depth with load.

A, B, and C when subjected to a wheel load of 16,000 pounds. Entering Figure 24 with this wheel load and proceeding vertically to the strength classifications noted above and horizontally to the depth scales gives the following listed required depths of cover:

Material A-2″
Material B-9″
Material C-16″

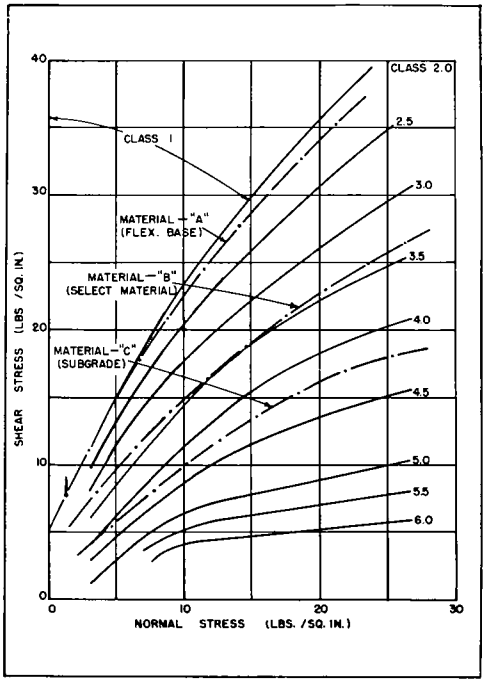

Figure 23. Example showing use of classification chart.

Since material A will be stable under 2-inch cover, and material B under 9-inch cover, a layer thickness of 7 inches of material A must be required. Similarly, 7 inches of material B must be supplied to place the subgrade material C at a total depth of 16 inches below pavement surface. The final design, as indicated on the left portion of Figure 24 is as follows:

A 2-inch thickness asphaltic concrete surfacing
B 7-inch thickness material A—flexible base
C 7-inch thickness material B—select material

B. RIGID PAVEMENT DESIGN

1. *Basic Consideration*

Since in rigid pavement design the primary consideration is to prevent tensile overstress in the slab, the flexural strength is balanced against anticipated tensile stresses due to a combination of load and warping effects.

Considering first the stresses due to load, Dr. H. M. Westergaard made a study of tensile stresses due to circular loads placed at interior edge, and corner section of a theoretical pavement slab. His results were published in 1926 (*8*); despite numerous revisions in his formulas, the Westergaard method of analysis remains today as the only practical and usable method of design for rigid pavements.

2. *Subgrade Strength K Value*

In the Westergaard analysis the concrete slab is assumed to be a slab continuously sup-

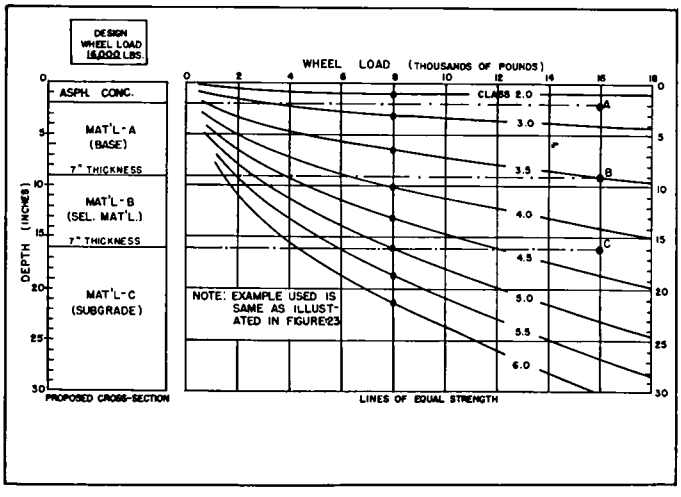

Figure 24. Thickness design curves with example showing a complete design.

ported by a subgrade of known or assumed strength. It was mathematically necessary to define this subgrade strength as the ratio of the unit pressure reaction at any point in the tire contact area to the resultant deflection (or settlement) at that point; it was also mathematically expedient to assume that this ratio of unit contact pressure to deflection at any point in the area of contact was constant. The term "modulus of subgrade reaction," termed "k-value," given to this factor, further substantiates the purpose for which k-value was originally intended.

Various practical investigators have attempted to define k-value in more practical terms such as:

a. Ratio of unit pressures applied to rigid plates of various sizes to the settlement observed under such unit pressures.

b. Ratio of total load applied to such plates to cubic inches displacement of the plate or plates into the subgrade.

Inspection of the units of measurement involved in "a" and "b" above will show that both definitions are physically the same and represent merely different units for expression of results.

c. The load required to deflect a 1-square-inch area of the subgrade 1 inch.

Here again, examination of units will show "c" to be equivalent to "a" and "b."

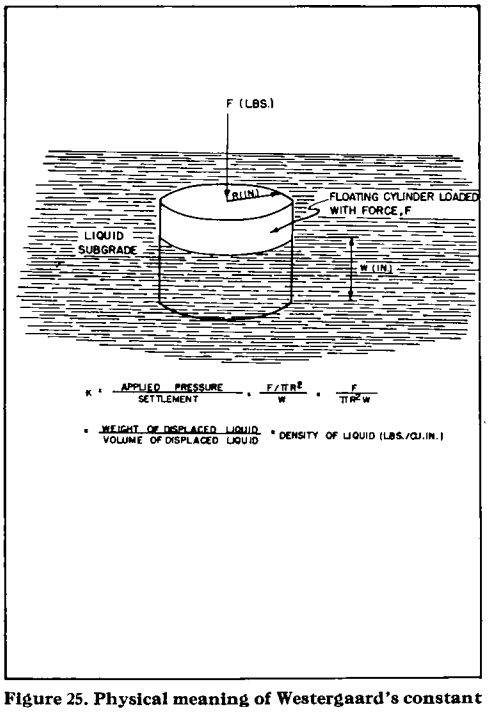

Figure 25. Physical meaning of Westergaard's constant pressure settlement ratio, K.

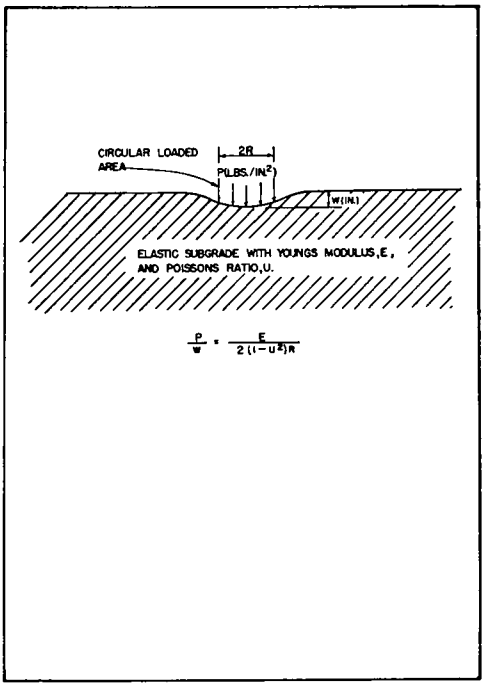

Figure 26. The variable pressure—settlement ratio for a solid subgrade.

It may be well to pause here and reflect for a moment on Dr. Westergaard's original assumption—constant subgrade modulus. Such assumption can only be physically true for the case of a load on a slab resting on, or supported by, a liquid. These basic relations are shown on Figure 25. The distribution of contact pressure is constant, as is also the ratio of contact pressure to deflection or settlement.

For any subgrade material other than a liquid—which has no shearing resistance and no practical properties simulating those of actual pavement subgrades—shearing resistance of the subgrade material will alter the uniform distribution noted for a liquid. The maximum deflection will be incurred at the center of the loaded area and the deflection will decrease toward the edges of the loaded area. The pattern of deflection for any solid or semisolid material will be as shown on Figure 26.

If the dimension $2R$, the diameter of the test plate, is increased in Figure 26, the value of W, the plate deflection or settlement in inches (for the same unit pressure on the plate), is increased proportionally. Thus, it is apparent that k-value is not a constant in any known physical terms and is an extreme variable depending on the size of the test plate, as well as the thickness and properties of the soils affected by such plate loading tests.

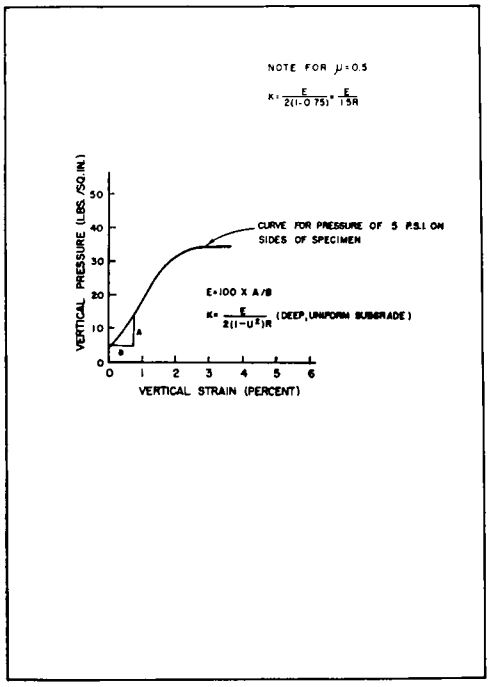

Figure 27. Value of K from triaxial test for a deep, uniform subgrade.

3. *Field Determinations of K-Value*

Such being the use, practical field measurements of k-value have no theoretical meaning since it is impossible to measure accurately at a point of contact the unit pressure and corresponding deflection or settlement. All such methods of measurement can consequently be considered merely as rough estimates of the basic meaning of k-value as defined by Dr. Westergaard. Also, the size of test plate used has been shown by various investigators (*9*) to have a very marked effect on measured k-values. Since measured k-value decreases to a minimum for an infinitely large plate (*9*), and since a concentrated or distributed load on any concrete pavement produces subgrade deflections over a rather large area of subgrade, many investigators have selected as a practical compromise test, plates as large as are possible of test-loading without undue expense; these plate diameters vary from 30 inches to 60 inches.

4. *Laboratory Determinations of K-Value*

Several obvious objections to field measurements of k-value are:

a. Expense of loading and anchoring devices for loading large plates.
b. Expense of constructing test fills or select courses. To be of any value, such tests must be made on the actual subgrade upon which concrete pavement is to be placed.
c. Impracticability of obtaining such data during the design stage, so that design may be based on actual rather than estimated subgrade strengths.

Fortunately, k-value can be determined with a sufficient degree of accuracy by calculation from results of triaxial compression tests of the soils and base materials concerned. Figure 27 shows results of such a triaxial test; the slope of the initial portion of the stress-strain curve obtained in such a test represents the elastic modulus, E, of the material tested. Timoshenko (*29*) has published solutions for deflection of the surface of an elastic solid when loaded with a rigid circular die; this case is mathematically similar to the problem at hand (the deflection of an assumed elastic solid the pavement subgrade) when loaded with a uniform load on a rigid plate. Timoshenko's expression for the settlement, d, is:

$$d = \frac{p \times 2(1 - \mu^2)a}{E}$$

where p = unit pressure in psi.
a = radius of plate or die.
μ = Poisson's ratio of the elastic material.
E = elastic modulus of the elastic material.

Rearranging terms:

$$\frac{p}{d} = k = \frac{E}{2(1 - \mu^2)a}$$

If we assume $\mu = 0.5$, then:

$$k = \frac{E}{2(1 - 0.25)a} = \frac{E}{1.5a} \qquad (1)$$

The relation derived is identical with equation (1), Figure 4 of the paper, except that the more familiar terms, E and R are used in lieu of the terms C and a appearing in Figure 4. The designer may use any value of plate radius which he considers most applicable.

5. *K-Value for Layered Subgrades*

For subgrades of uniform strength to considerable depth, a single determination may be made of k-value. Frequently, however, a relatively soft subgrade will be underlaid at shallow depth with a considerably firmer layer; the reverse is also encountered on occasion. In such cases some method must be used to arrive at the effective k-value for the composite subgrade. Figure 29 presents the results of investigations by Burmister (*30*) on the effect of sublayers of either higher or lower strength than surface layers. The settlement factors represent the ratio of actual settlement to settlement to be expected if the subgrade consisted entirely of the softer material. Since

k-value is in inverse ratio to settlement, the effective k-value is equal to that of the softer layer divided by the "settlement factors" given by Figure 29.

6. *Relative Influence of K-Value*

It may appear to the uninitiated that a hopeless situation exists in rigid pavement design in that the basic measure of subgrade strength, k, cannot be measured, and that all known methods of estimating same are subject to considerable interpretation of results. By way of reassurance, it is pointed out here that rather large variations in k-value occasion only minor variations in required slab thickness. Kelley (*31*), for example cites an increase in stress of 15 to 20 percent for a 300 percent increase in k-value. In the section on "Semi-Flexible Experience in Texas", it was noted that an increase of k-value from 110 to 500 psi justified a reduction in slab thickness of approximately 1 inch. Hence, a very approximate estimate of k-value will suffice for practical purposes.

7. *Calculation of Load Stresses*

Dr. Westergaard (*8*) envisioned three possible load conditions:

a. Interior section—four quadrants of slab effective in carrying loads.
b. Edge section—two quadrants of slab effective in carrying load.

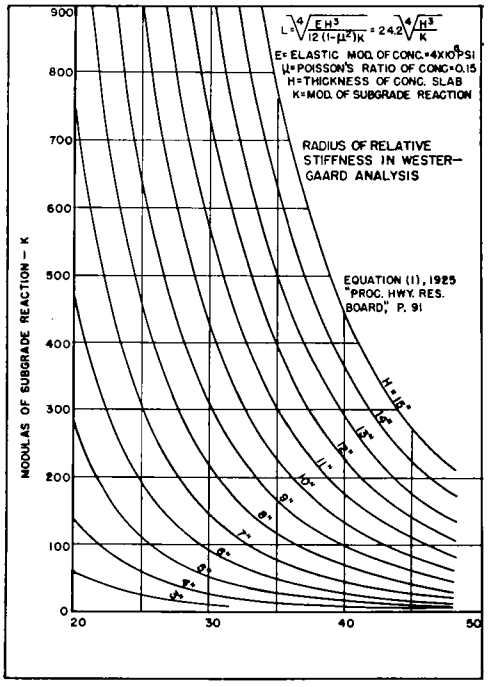

Figure 28. Radius of relative stiffness—L.

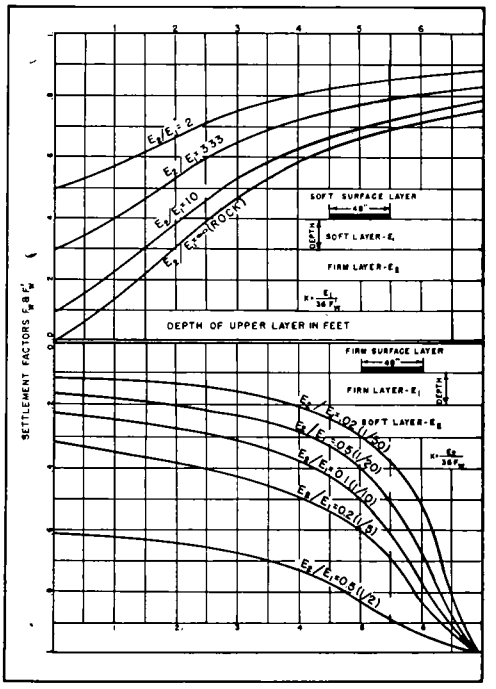

Figure 29. Factors for estimating combined K-values.

c. Corner section—one quadrant of slab effective in carrying load.

Reference is now made to Figure 4 of the paper. Formula (1) has been derived in the preceding section in discussion of k-value. Formula (2) is Dr. Westergaard's expression for the "radius of relative stiffness"; it expresses the elastic properties of the pavement in terms of the relative rigidity of the slab compared to that of the subgrade. The thicker the slab and the higher its elastic modulus, the greater will be the value of l; likewise higher subgrade strength (represented by k-value) will result in a lowering of l. Figure 28 presents a family of curves giving the radius of relative stiffness for elastic modulus of concrete of 4,000,000 psi/in./in. and various slab thicknesses and k-values. These curves may be used to solve formula (2) if E is approximately 4,000,000; similar curves may be calculated and plotted for other values of E.

Formula (3) is Dr. Westergaard's expression for "equivalent radius of resisting section." It represents an estimate of the size of a theoretical section assumed to be effective in carrying the stresses imposed by the assumed wheel load. It is a function of the radius of the wheel load contact area and thickness of the slab. Figure 30 presents a family of curves by which formula (3) may be solved for any combination of slab thickness and load normally encountered in highway design.

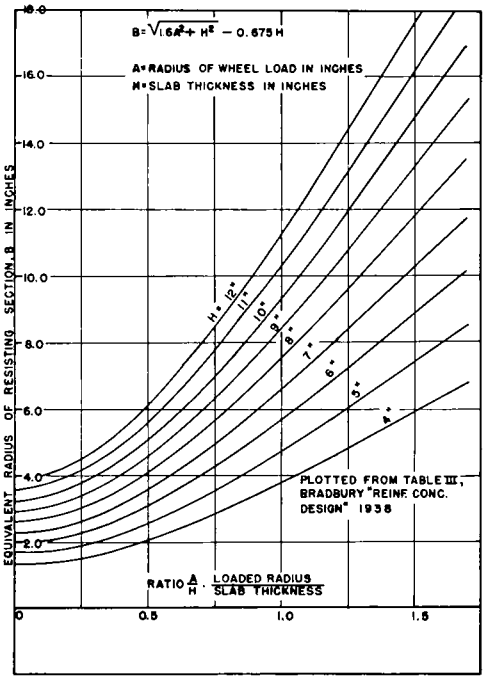

Figure 30. Equivalent radii of resisting section for use in Westergaard analysis.

The factors l and b, Formulas (2) and (3), must be evaluated in all calculations of stresses using the Westergaard analysis, since all of the following formulas for stresses are in terms of l and b.

8. *Interior Load Stresses*

The Westergaard Formula (4) for interior stress has not been questioned since full contact with the subgrade is assured in the central portion of the slab, and also because this load condition generally produces less stress than the other load conditions. However, in some cases of semiflexible design, this load case governs and will be the basis of design.

9. *Edge Load Stresses*

Formula 5 represents Dr. Westergaard's formula for an edge load assuming $\mu = 0.15$ and full subgrade support. When the edges of such a slab are warped upward, subgrade contact is lost in the outer regions and the stresses for such a condition have been empirically determined to be according to Formula 5a (*31*).

10. *Corner Load Stresses*

Formula (6) represents Dr. Westergaard's formula for a corner load with full subgrade support and $\mu = 0.15$. If the slab is warped upward, subgrade contact is lost in the corner region, and this effect has been considered by Bradbury (*10*) to be equivalent to assuming the k-value to be one-fourth of its true value. Such modfication results in Formula (7) for an unprotected corner.

Pickett (*11*) has made a thorough study of the Westergaard and other formulas for stresses in the corner region of a concrete pavement slab. His results appear to point toward a modified corner stress formula which has been adopted by the Portland Cement Association, and which has been the basis of design thickness curves published by the Association (*11*). Formulas 4, 5, 6, and 7 may be solved by means of the curves of Figures 31 and 32.

11. *Warping Stresses*

In considering the effect of warping stresses, two alternate procedures are available:

a. Determining stresses due to load in the warped condition; or
b. Independent determinations of stresses due to load in unwarped position and those due to restrained warping effects.

Due to the relative rigidity of concrete pavement slabs, it is believed that few pavement slabs of modern dimensions will actually warp to any appreciable degree. Warping tendencies will generally be resisted by the stiffness of the slab, with the resultant development of warping stresses.

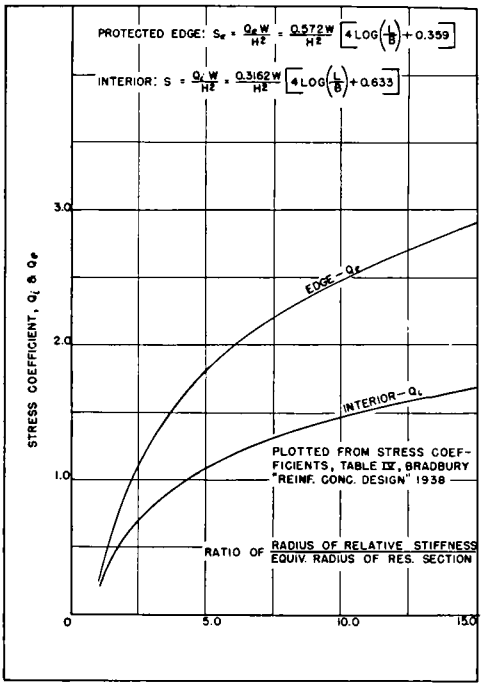

Figure 31. Edge & interior slab stresses by Westergaard formula.

It is primarily the designer's choice as to whether load stresses in the warped upward condition, or combined normal-position load stresses and warping stresses are to be the basis of design. The writer prefers the latter as the more reasonably expected manner of actual behavior.

Dr. Westergaard made a study of temperature effects in concrete pavement slabs (*14*). His studies show that constrained warping tendencies will produce tension in the bottom of a pavement slab when the top of the slab becomes warmer than the bottom of the slab. For the interior section of a slab of infinite dimensions horizontally the warping stress is given by Formula (8), Figure 4:

$$\sigma o = \frac{Eet}{2(1 - \mu)} \qquad (8)$$

For slabs of finite practical dimensions the warping stress at the center is generally less, and may be calculated by multiplying the stress σo by factors given by Figure 33. This curve was plotted from data supplied by Dr. Westergaard (*14*). For slabs having greater length than width, the stress in either coordinate direction will be parallel to the direction of measurement of length or width.

At the edge of the slab, greater freedom of movement (less restraint) is provided and the warping stress is accordingly less; the warping stress at an edge may be taken as:

$$\sigma o = \sigma c\ (1-\mu) \qquad (9)$$

At a corner region, still greater freedom of movement is provided and the warping stress is so small that it may be ignored.

12. *Design Temperature Differential*

Teller and Sutherland (*30*) have investigated warping effects due to moisture difference between top and bottom of slab. They concluded that the wetter condition of concrete slabs during warm weather tends to cause upward warping, and that restraint of this warping produces a compression in the bottom of the slab—which stress is such as to relieve ten-

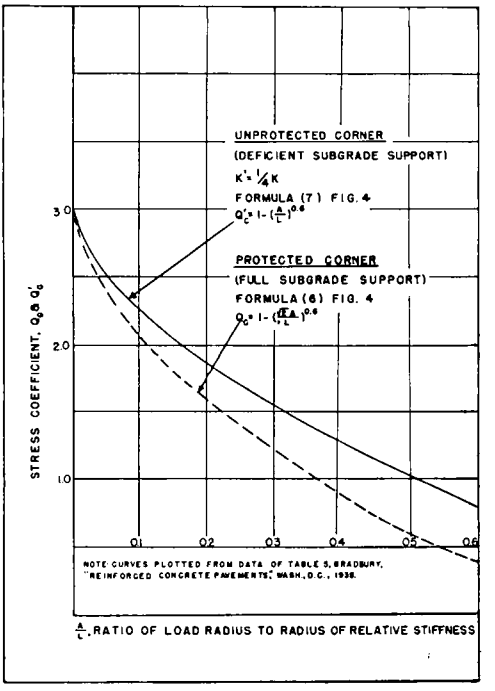

Figure 32. Corner slab stresses by Westergaard and Bradbury.

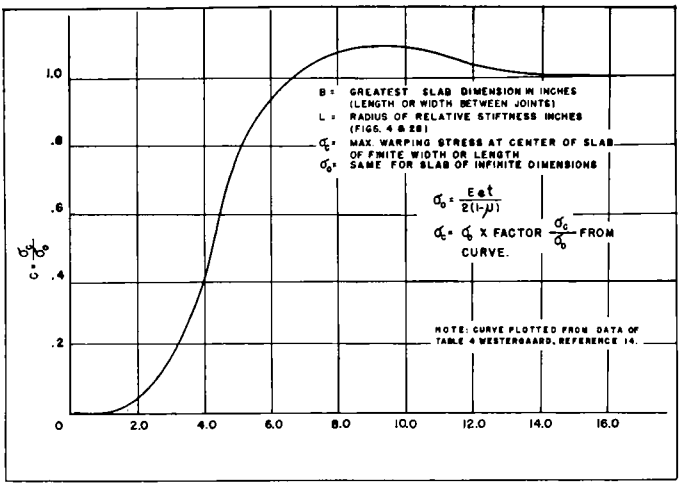

Figure 33. Chart for determining warping stress at center of concrete pavement.

sile stresses produced at the bottom of the slab by temperature warping effects during summer daytime temperature effects. At such times the resultant warping stress will add to tensile stresses due to interior or edge loads; at other times the temperature warping effects will not not be additive to load stress. Hence, the highest total stress will occur at times when moisture warping stress will relieve temperature warping stress.

The work of Teller and Sutherland (*30*) proved the truth of this condition but did not yield sufficient quantitative data to permit accurate valuation of the relief so afforded. Coons (*15*) also made investigations on moisture and temperature warping and furnished additional data on the problem. Careful study of his data and that of reference (*30*) has led the author to conclude that at the critical time, the maximum temperature differential is approximately 3°F. per inch thickness of slab, and that moisture warping effects may be such as to relieve half of the temperature warping effects.

Based on the observations of the preceding paragraphs, the author adopted as a practical design procedure a "moisture-compensated temperature differential" of 1.5°F. per inch thickness of slab.

13. *Total Stress*

The total stress for corner or edge loads will be either:

a. Load stress for the load condition concerned with the slab assumed to be free to warp and to be in a warped upward condition, with deficient subgrade support; or
b. Stress due to load with slab assumed to be in unwarped position, plus warping stress induced by constraining slab from warping. This restraint is enforced by the thickness, weight, and flexural strength of the slab.

For the interior load, complete restraint must be assumed. For this reason and also because modern pavement designs invariably afford a high degree of restraint for all load cases, the author prefers alternative "b" above.

14. *Critical Load Case and Critical Thickness*

Regardless of the method of determining total stress, the latter is ultimately compared with the anticipated ultimate flexural, or tensile strength, of the pavement slab.

The critical load case for any design wheel load is that resulting in the highest total anticipated stress; the critical thickness is that thickness producing a total stress equal to one-half the anticipated ultimate flexural strength for loads expected to be repeated 200,000 times or more. For loads expected to be repeated less than 200,000 times, safety factors less than 2.0 may be used (*11*).

ACKNOWLEDGMENT

It will be apparent to the reader that most of the material presented in this appendix is not the result of work of the author. In respect to flexible pavement design procedure particularly the work of others has been cited; many of the figures were taken directly from Texas Highway Department "Plan Preparation—Book II," Road Design Division. For further details of stress computations the reader is referred to "Wheel-Load-Stress Computations Related to Flexible Pavement Design," C. A. McDowell, Bulletin 114, Highway Research Board.

With respect to rigid pavement design the material presented in this appendix represents the conclusions of the writer in regard to the pertinent issues after review of all available literature on the subject; in addition, there are presented curves allowing convenient solutions of many of the formulas and equations necessary for a rigid type pavement design. No originality is claimed for the basic formulas cited, and acknowledgments of source of information have been given in each case. It is hoped that such curves will eliminate much of the labor of computations involved in the Westergaard type of analysis.

REFERENCES

26. Jurgensen, L. M., "The Application of Theories of Elasticity and Plasticity to Foundation Problems," Journal of Boston Society of Civil Engineers, July 1934, p. 148.
27. Love, A. E. H., "The Stresses Produced in a Semi-infinite Solid by Pressure on Part of the Boundary," Philosophical Transactions of the Royal Society, Series A, Vol. 228, 1929.
28. McDowell, C. A., "Triaxial Testing—Its Adaptation and Application to Highway Materials," Part II, Roads and Streets, March 1950.
29. Timoshenko, S., *Theory of Elasticity*, equation 209. New York, McGraw-Hill, 1934, p. 339.
30. Burmister, D. M., "The Theory of Stresses and Displacements in Layered Systems and Application to the Design of Airport Runways," Proceedings, Highway Research Board, Vol. 23, 1943, p. 126.
31. Kelley, E. F., "Application of the Results of Research to the Structural Design of Concrete Pavements," Public Roads, Vol. 20, No. 5, July 1939, p. 83.