Stress and Displacement Characteristics of a Two-Layer Rigid Base Soil System: Influence Diagrams and Practical Applications

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The complete problem of stresses and displacements in layer no. 1 of a two-layer rigid base soil system is presented. This problem represents a closer approximation to many actual conditions encountered in foundation practice. The characteristics of the distribution of stresses throughout layer no. 1 are illustrated and discussed with particular reference to the favorable and unfavorable aspects and their implications in stress investigations. The realm of validity of this problem and its limitations in foundation practice are considered. A complete series of influence diagrams for stresses and displacements are presented. Practical applications in investigations of stresses imposed by the foundation loads of structures are made to illustrate the practical use and importance of this problem in soil mechanics and foundation engineering. A major problem in stress investigations is to raise the conceptions and the standard of excellence in practices regarding adequacy and reliability. This is a question essentially of attitudes and conceptions, knowledge and training, and experience and good judgment, rather than merely of acquiring facility in the use of stress influence diagrams.

● ADEQUATE and reliable investigations of the magnitude and distribution of stresses and displacements imposed in the supporting soils by the foundation loads of structures are fundamental and essential aspects of foundation studies. The basic problem is to make estimates of stresses, which are in closest possible agreement with the probable actual stress condition imposed. Such estimates can provide a clear understanding and correct conceptions regarding stress and displacement responses of soils for the proper evaluation of all physical and economic conditions that control and are inherent in a situation.

In the design and construction of foundations the engineer is dealing with real soils in natural deposits, which, more often than not, are layered in character. A strong surface layer has a considerable reinforcing and load spreading capacity on a weaker underlying layer, which is a very favorable aspect in foundation work. On the other hand, a weak compressible surface layer underlaid by a strong base layer, which is the subject of this present paper, causes a concentration and increase in stress beneath the loaded foundation throughout the depth of the upper compressible layer, which is a relatively unfavorable aspect. The strong base layer, however, definitely limits appreciable settlements to the upper compressible layer—a very favorable aspect.

Prior to about 1920, the Boussinesq Problem (*1*, *2*, *3*) was the only working hypothesis available for two- and three-dimensional stress investigations. Since 1930 theoretical advances of great importance and practical value have been made in the determination of stresses in layered soil systems with a rigid base layer. Marguerre in 1931 (*4*), Biot in 1935 (*5*), and Picketts in 1938 (*6*), contributed solutions by different approaches for the simpler special case of the two-layer rigid base problem for stresses at the surface of the base layer only. These problems gave the first clear indications of the importance of layering on the magnitude and distribution of stresses imposed by the foundations of structures. These solutions, however, were not in a form that would permit a general solution of the two-layer rigid base problem. In 1943 and 1945 the writer solved the three-dimensional general two- and three-layered system problems (*7*, *8*) in a form permitting evaluations of all stresses and displacements throughout the depths of all layers, and for all relative strength conditions of the layers.

In 1954 a complete series of influence diagrams of stresses and displacements in a two-layer rigid base soil system (*9*) were numerically evaluated and were presented by the writer in a report of a study sponsored by the Corps of Engineers, U. S. Army, Office of the Chief of Engineers. The study was carried out in the Department of Civil Engineering and in the Watson Scientific Computing Laboratory of Columbia University, during the years July 1, 1949 to June 31, 1954, using I.B.M. computing machines and desk calculators. The numerical evaluation of influence coefficients for stresses and displacements and the preparation of influence diagrams and tables involved some 1500 man-days of work during this period. The results of this study in the form of influence diagrams are presented in this paper.

This two-layer rigid base problem represents a closer agreement with actual subsurface soil conditions commonly encountered in foundation practice. It discloses the character and importance of layered system action and responses, and extends the realm of validity and usefulness of foundation stress investigations to such thick layered soil deposits. This problem, by means of a series of influence diagrams, provides methods of practical importance and value for investigating and estimating the stress and settlement conditions imposed by foundations of spread footings and mat foundations located near the surface of the ground.

BASIC ELASTICITY CONDITIONS

Although problems in the theory of elasticity are based upon conditions of ideal materials and upon ideal boundary and other conditions, they have been found to be of practical use in the range of imperfectly elastic and somewhat non-homogeneous materials, such as soils. Each stress problem has its special inherent conditions. These conditions determine its individual character and impose definite limitations on its valid use, which have been discussed in detail elsewhere (*10*).

The term "conditions" is to be preferred to the term "assumptions" and should be consistently used. Conditions refer to a carefully set up framework or reference system, which in the nature of things must be idealized to some degree in order to obtain a solution to the problem at all. Hence in each application of the problem, this framework of conditions must be compared and validated with respect to actual natural conditions that prevail in a situation with regard to the necessary degree of agreement for valid use. Too often making a number of assumptions which are then promptly forgotten may lead to unrealistic results. But the investigator making practical applications should always be fully cognizant of the limitations, realm of validity, and applicability of each stress problem in its applications to practical problems in foundation engineering, and should adequately take them into account (*10*).

The general conditions (*1*) inherent in and common to all elasticity problems are summarized below:

A. Boundary conditions—(1) The load (point or distributed) is applied at the plain surface of a semi-infinite soil deposit; (2) the surface of the soil deposit is free of normal and shearing stresses beyond the limits of the loaded area; and (3) all stresses and displacements become equal to zero at infinite depth.

B. Soil material, structure, and property conditions—(1) The soils of the deposit or layer are considered to be homogeneous with depth and in horizontal extent with regard to character of the soil material, degree of compactness, and natural coherence of the structure of granular soils, natural consistency and natural structure of clay soils, and strength properties of the soils; (2) the soils are considered to possess the response characteristics of elastic and isotropic materials with linear stress-strain relations in accordance with the infinitesimal strain theory of elasticity and with complete strain recovery upon the removal of stress.

C. Additional general conditions—(1) The imposed stresses are moderate in relation to the failure strength of the soils; (2) the principles of superposition of stresses and strains are considered to be valid; and (3) uniformly distributed loadings obtained by applications of the principles of superposition are used as being more realistic of foundation loading conditions in practice.

The special conditions inherent in the problem:

D. Two-layer rigid base soil system conditions—(1) Complete continuity of vertical and shearing stresses and of vertical and horizontal displacements are considered to exist across the interface between the upper compressible layer and the immediately underlying, rigid base layer; (2) the shearing stresses are considered to be fully active at this interface, as called for by theory; and (3) the ratio of the elastic moduli of the lower rigid base layer 2 and the upper layer 1, E_2/E_1, is considered to be equal to infinity, but for practical purposes should exceed a value of 100 to assure the necessary validity.

BASIC STRESS AND DISPLACEMENT EQUATIONS

The two-layer rigid base soil system with a concentrated load, P, applied at the surface of the ground is illustrated in Figure 1. The solution of this elasticity problem with its special boundary and interface conditions requires two basic physical parameters, r/h and z/h, which are the ratios of radial and vertical distances to the thickness of the upper compressible layer 1 of Equation 1(a), and requires two basic strength parameters of Equations 1(b), which involve ratios of the elastic constants of the two layers, namely—the modulus, E, and Poisson's ratio, μ. For conditions of a rigid incompressible base layer, as the limiting case of this present paper, the general strength parameters reduce to the form of Equations 1(c) involving only the elastic constant, μ of layer 1. The solution of the two-layer rigid base system problem for the three-dimensional case of a point load, P, in cylindrical coordinates yields the stress and displacement Equations 3 through 8. Each equation has a characteristic numerator bracket, N, and the common characteristic denominator bracket, D, of Equation 2. Equations 2 through 8 reveal the fundamental and systematic nature and dependence of the physical and elastic relations existing between the basic physical and elastic parameters inherent in this layered system, which govern its stress and displacement responses under imposed foundation loads. For comparative

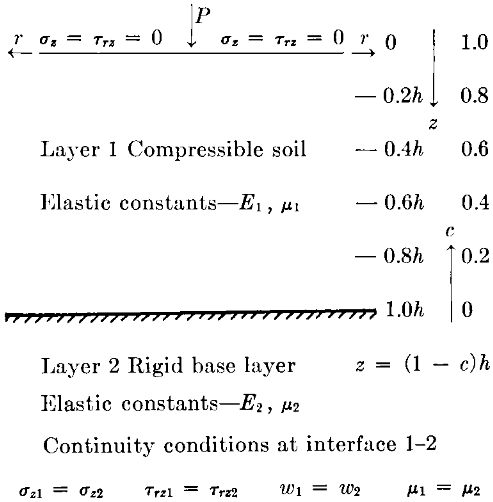

Figure 1. Two-layer rigid base soil system. Burmister problem.

purposes the terms in the numerator brackets of Equations 3 through 8, which would yield the solutions for the Boussinesq stresses and displacements for a homogeneous soil deposit, are enclosed in brackets. Since Equations 1(b) become equal to zero for $E_1 = E_2$ in a homogeneous deposit, all terms involving the strength ratios become equal to zero and the denominator bracket reduces to unity for the Boussinesq problem.

Two-layer Rigid Base Soil System Stress and Displacement Equations: Burmister Problem

Basic parameters of two-layer system. Figure 1.

Ratio, r/h of radial distances to thickness of layer 1

Ratio, z/h of depths to thickness of layer 1.

$$z = (1 - c)h \tag{1a}$$

Two-layer strength ratios.

$$\left.\begin{aligned} n &= \left[\frac{E_2}{E_1}\frac{1+\mu_1}{1+\mu_2}\right] \\ M &= \left[\frac{1-n}{1+n(3-4\mu_1)}\right] \\ L &= \left[\frac{(3-4\mu_2)-(3-4\mu_1)n}{(3-4\mu_2)+n}\right] \end{aligned}\right\} \tag{1b}$$

For conditions of a rigid incompressible base layer:

$$\left.\begin{aligned} E_2 &\rightarrow \infty \qquad n \rightarrow \infty \\ M &= \frac{-1}{(3 - 4\mu_1)} \\ L &= -(3 - 4\mu_1) \end{aligned}\right\} \tag{1c}$$

Two-layer denominator.

Common to all stress and displacement equations

$$D = [1 - (L + M + 4M\alpha^2)e^{-2\alpha} + MLe^{-4\alpha}] \tag{2a}$$

where $\alpha = mh$ is a parameter of the functional relations involving Bessel functions and integrations.

Denominator for rigid base system becomes:

$$D = \left[1 + \left(3 - 4\mu_1 + \frac{1}{3 - 4\mu_1}\right)e^{-2\alpha} + \frac{4\alpha^2}{3 - 4\mu_1}e^{-2\alpha} + e^{-4\alpha}\right] \tag{2b}$$

Equivalent reciprocal denominator for $\mu_1 = 0.4$

$$\frac{1}{D} = [1 - 1.62\alpha^3 e^{-2.2\alpha} - 2.35\alpha e^{-2.4\alpha} - 0.7569e^{-4\alpha} - 0.23\alpha e^{-4\alpha}] \tag{2c}$$

Stress and Displacement Equations for a point load, P.

Vertical Stress, σ_z

$$= \frac{-P}{2\pi h^2}\int_0^\infty \left[\frac{\text{Numerator 1}}{\text{Denominator}}\right]\alpha J_0\left(\frac{r}{h}\alpha\right)d\alpha$$

[Numerator 1] General:

$$\left[\begin{aligned} &+ [e^{-(1-c)\alpha} + (1 - c)\alpha e^{-(1-c)\alpha}] \\ &- 0.5(L + M)e^{-(1+c)\alpha} \\ &- M(1 + c)\alpha e^{-(1+c)\alpha} - 2Mc\alpha^2 e^{-(1+c)\alpha} \\ &- 0.5(L + M)e^{-(3-c)\alpha} \\ &+ M(1 + c)\alpha e^{-(3-c)\alpha} - 2Mc\alpha^2 e^{-(3-c)\alpha} \\ &+ MLe^{-(3+c)\alpha} - ML(1 - c)\alpha e^{-(3+c)\alpha} \end{aligned}\right] \tag{3a}$$

[Numerator 1] Rigid base system: (3b)

$$\left[\begin{aligned} &+ [e^{-(1-c)\alpha} + (1 - c)\alpha e^{-(1-c)\alpha}] \\ &+ 0.5\left(3 - 4\mu_1 + \frac{1}{3 - 4\mu_1}\right)e^{-(1+c)\alpha} \\ &\qquad + \frac{1 + c}{3 - 4\mu_1}\alpha e^{-(1+c)\alpha} + \frac{2c}{3 - 4\mu_1}\alpha^2 e^{-(1+c)\alpha} \\ &+ 0.5\left(3 - 4\mu_1 + \frac{1}{3 - 4\mu_1}\right)e^{-(3-c)\alpha} \\ &\qquad - \frac{1 + c}{3 - 4\mu_1}\alpha e^{-(3-c)\alpha} + \frac{2c}{3 - 4\mu_1}\alpha^2 e^{-(3-c)\alpha} \\ &+ e^{-(3+c)\alpha} - (1 - c)\alpha e^{-(3+c)\alpha} \end{aligned}\right]$$

Vertical Settlement,

$$w = \frac{-P}{2\pi h}\frac{1 + \mu_1}{E_1}\int_0^\infty \left[\frac{\text{Numerator 2}}{\text{Denominator}}\right] \cdot J_0\left(\frac{r}{h}\alpha\right)d\alpha \tag{4}$$

[Numerator 2]

$$\left[\begin{aligned} &+ [(2 - 2\mu_1)e^{-(1-c)\alpha} + (1 - c)\alpha e^{-(1-c)\alpha}] \\ &- (2 - 2\mu_1)e^{-(1+c)\alpha} \\ &\qquad - \left(\frac{3 - 4\mu_1 - c}{3 - 4\mu_1}\right)\alpha e^{-(1+c)\alpha} \\ &\qquad\qquad - \left(\frac{2c}{3 - 4\mu_1}\right)\alpha^2 e^{-(1+c)\alpha} \\ &+ (2 - 2\mu_1)e^{-(3-c)\alpha} \\ &\qquad - \left(\frac{3 - 4\mu_1 - c}{3 - 4\mu_1}\right)\alpha e^{-(3-c)\alpha} \\ &\qquad\qquad + \left(\frac{2c}{3 - 4\mu_1}\right)\alpha^2 e^{-(3-c)\alpha} \\ &- (2 - 2\mu_1)e^{-(3+c)\alpha} + (1 - c)\alpha e^{-(3+c)\alpha} \end{aligned}\right]$$

$$\text{Radial Stress, } \sigma_r = \frac{-P}{2\pi h^2}\int_0^\infty + \left[\frac{\text{Numerator 3}}{\text{Denominator}}\right]\left[\alpha J_0\left(\frac{r}{h}\alpha\right) - \frac{h}{r}J_1\left(\frac{r}{h}\alpha\right)\right]d\alpha + \left[\frac{\text{Numerator 4}}{\text{Denominator}}\right]\alpha J_0\left(\frac{r}{h}\alpha\right)d\alpha \tag{5}$$

[Numerator 3]

$$\left[\begin{array}{l} +\ [(1-2\mu_1)e^{-(1-c)\alpha} - (1-c)\alpha e^{-(1-c)\alpha}] \\ -\ (1-2\mu_1)e^{-(1+c)\alpha} \\ \qquad + \left(\dfrac{3-4\mu_1-c}{3-4\mu_1}\right)\alpha e^{-(1+c)\alpha} \\ \qquad\qquad - \left(\dfrac{2c}{3-4\mu_1}\right)\alpha^2 e^{-(1+c)\alpha} \\ -\ (1-2\mu_1)e^{-(3-c)\alpha} \\ \qquad - \left(\dfrac{3-4\mu_1-c}{3-4\mu_1}\right)\alpha e^{-(3-c)\alpha} \\ \qquad\qquad - \left(\dfrac{2c}{3-4\mu_1}\right)\alpha^2 e^{-(3-c)\alpha} \\ +\ (1-2\mu_1)e^{-(3+c)\alpha} + (1-c)\alpha e^{-(3+c)\alpha} \end{array}\right.$$

[Numerator 4]

$$\left[\begin{array}{l} +\ [2\mu_1 e^{-(1-c)\alpha}] \\ + \left(\dfrac{2\mu_1}{3-4\mu_1}\right) e^{-(1+c)\alpha} + \left(\dfrac{4\mu_1}{3-4\mu_1}\right)\alpha e^{-(1+c)\alpha} \\ + \left(\dfrac{2\mu_1}{3-4\mu_1}\right) e^{-(3-c)\alpha} - \left(\dfrac{4\mu_1}{3-4\mu_1}\right)\alpha e^{-(3-c)\alpha} \\ +\ 2\mu_1 e^{-(3+c)\alpha} \end{array}\right.$$

Tangential Stress, $\sigma_\theta = \dfrac{-P}{2\pi h^2}\displaystyle\int_0^\infty$

$$+ \left[\frac{\text{Numerator 3}}{\text{Denominator}}\right]\frac{h}{r} J_1\left(\frac{r}{h}\alpha\right) d\alpha \qquad (6)$$

$$+ \left[\frac{\text{Numerator 4}}{\text{Denominator}}\right]\alpha J_0\left(\frac{r}{h}\alpha\right) d\alpha$$

Horizontal Displacement,

$$u = \frac{P}{2\pi h}\,\frac{1+\mu_1}{E_1}\int_0^\infty \left[\frac{\text{Numerator 3}}{\text{Denominator}}\right] \qquad (7)$$

$$\cdot J_1\left(\frac{r}{h}\alpha\right) d\alpha$$

Shearing Stress, $\tau_{rz} = \dfrac{-P}{2\pi h^2}\displaystyle\int_0^\infty$

$$\cdot \left[\frac{\text{Numerator 6}}{\text{Denominator}}\right]\alpha J_1\left(\frac{r}{h}\alpha\right) d\alpha \qquad (8)$$

[Numerator 6]

$$\left[\begin{array}{l} +\ [(1-c)\alpha e^{-(1-c)\alpha}] \\ -\ 0.5\left(3-4\mu_1-\dfrac{1}{3-4\mu_1}\right)e^{-(1+c)\alpha} \\ \qquad + \left(\dfrac{1-c}{3-4\mu_1}\right)\alpha e^{-(1+c)\alpha} \\ \qquad\qquad - \left(\dfrac{2c}{3-4\mu_1}\right)\alpha^2 e^{-(1+c)\alpha} \\ +\ 0.5\left(3-4\mu_1-\dfrac{1}{3-4\mu_1}\right)e^{-(3-c)\alpha} \\ \qquad + \left(\dfrac{1-c}{3-4\mu_1}\right)\alpha e^{-(3-c)\alpha} \\ \qquad\qquad + \left(\dfrac{2c}{3-4\mu_1}\right)\alpha^2 e^{-(3-c)\alpha} \\ +\ (1-c)\alpha e^{-(3+c)\alpha} \end{array}\right.$$

Components of Stresses in the x, y, z directions. Figure 2, integrated over one quadrant of a uniformly loaded circular area.

Vertical Stress. σ_z. Equation 3.

$$\sigma_z = \frac{-p}{2\pi h^2}\int_0^\infty d\alpha \int_0^r \frac{N1}{D}\,\alpha J_0\left(\frac{r}{h}\alpha\right) r\,dr$$

$$\int_0^\theta d\theta = -p\int_0^\infty \frac{N1}{D}\,\frac{r}{h}\,J_1\left(\frac{r}{h}\alpha\right)\frac{\theta}{2\pi}\,d\alpha \qquad (9)$$

$$= -p\sum\left[I_1\frac{\theta}{2\pi}\right]_0^\theta$$

Horizontal Stresses, σ_x and σ_y. Equations 5 and 6.

$$\sigma_x = \frac{-p}{2\pi h^2}\int_0^\infty d\alpha\int_0^r$$

$$\left[\frac{N3}{D}\,\alpha J_0\left(\frac{r}{h}\alpha\right) - \frac{N3}{D}\,\frac{h}{r}\,J_1\left(\frac{r}{h}\alpha\right)\right.$$

$$\left. + \frac{N4}{D}\,\alpha J_0\left(\frac{r}{h}\alpha\right)\right] r\,dr\int_0^\theta \cos^2\theta\,d\theta\ +$$

$$\left[\frac{N3}{D}\,\frac{h}{r}\cdot{}_1\left(\frac{r}{h}\alpha\right) + \frac{N4}{D}\,\alpha J_0\left(\frac{r}{h}\alpha\right)\right] r\,dr$$

$$\int_0^\theta \sin^2\theta\,d\theta$$

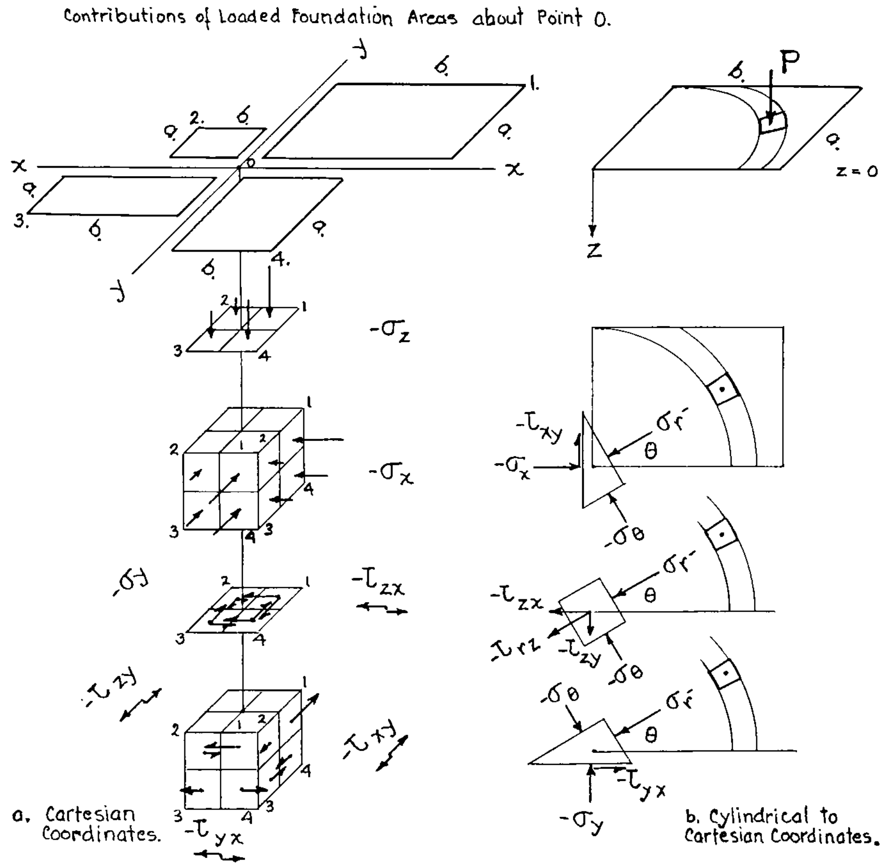

Figure 2. Components of stresses in the x, y, and z coordinate directions with directional and sign notations. Normal stress (-) is compressive stress.

$$= -p \int_0^\infty \left[\frac{\theta}{2\pi}\right]\left[\frac{N3}{D}\frac{r}{h}J_1\left(\frac{r}{h}\alpha\right) + 2\frac{N4}{D}\frac{r}{h}J_1\left(\frac{r}{h}\alpha\right)\right]d\alpha +$$

$$-p \int_0^\infty \left[\frac{\sin\theta\cos\theta}{4\pi}\right] \left[\frac{N3}{D}\frac{r}{h}J_1\left(\frac{r}{h}\alpha\right) + 2\frac{N3}{D}\frac{1}{\alpha}J_0\left(\frac{r}{h}\alpha\right)\right]d\alpha\Bigg]_0^r$$

$$= -p \sum \left[I_3\frac{\theta}{2\pi} + I_4 \sin 2\theta\right]_0^\theta \qquad \text{(10a)}$$

$$\sigma_y = -p \sum \left[I_3\frac{\theta}{2\pi} - I_4 \sin 2\theta\right]_0^\theta \qquad \text{(10b)}$$

Shearing stresses. $\tau_{xy} = \tau_{yx}$. Equations 5 and 6.

$$\tau_{xy} = \tau_{yx} = \frac{-p}{2\pi h^2}\int_0^\infty d\alpha \int_0^r \cdot\left[\frac{N3}{D}\alpha J_0\left(\frac{r}{h}\alpha\right) - 2\frac{N3}{D}\frac{h}{r}J_1\left(\frac{r}{h}\alpha\right)\right] r\,dr \cdot \int_0^\theta \sin\theta\cos\theta\,d\theta \qquad (11)$$

$$= -p\int_0^\infty \left[\frac{\sin^2\theta}{4\pi}\right] \cdot\left[\frac{N3}{D}\frac{r}{h}J_1\left(\frac{r}{h}\alpha\right)\right.$$

$$+ 2 \frac{N3}{D} \frac{1}{\alpha} J_0 \left(\frac{r}{h} \alpha \right) \Bigg] d\alpha \Bigg]_0^r$$

$$= -p \sum \left[I_5 \frac{1 - \cos 2\theta}{2} \right]_0^\theta$$

Shearing stresses, τ_{zx} and τ_{zy}. Equation 8.

$$\tau_{zx} = \tau_{xz} = \frac{-p}{2\pi h^2} \int_0^r r\, dr \int_0^\infty \cdot \frac{N6}{D} \alpha J_1 \left(\frac{r}{h} \alpha \right) d\alpha \cdot \int_0^\theta \cos \theta \, d\theta \quad (12a)$$

$$= -p \int_0^r r\, dr \int_0^\infty \left[\frac{\sin \theta}{2\pi} \right] \frac{1}{h^2} \frac{N6}{D} \cdot J_1 \left(\frac{r}{h} \alpha \right) d\alpha = -p \sum [I_6 \sin \theta]_0^\theta$$

$$\tau_{zy} = \tau_{yz} = \frac{-p}{2\pi h^2} \int_0^r r\, dr \int_0^\infty \cdot \frac{N6}{D} \alpha J_1 \left(\frac{r}{h} \alpha \right) d\alpha \int_0^\theta \sin \theta \, d\theta \quad (12b)$$

$$= -p \int_0^r r\, dr \int_0^\infty \left[\frac{1 - \cos \theta}{2\pi} \right] \cdot \frac{N6}{D} \cdot \frac{1}{h^2} J_1 \left(\frac{r}{h} \alpha \right) d\alpha$$

$$= -p \sum [I_6 (1 - \cos \theta)]_0^\theta$$

Settlement, w. Equation 4.

$$w = \frac{-p}{2\pi h} \int_0^\infty d\alpha \int_0^r \frac{N2}{D} J_0 \left(\frac{r}{h} \alpha \right) r\, dr \cdot \int_0^\theta d\theta \times \frac{1 + \mu_1}{E_1}$$

$$= -p \frac{1 + \mu_1}{E_1} \int_0^\infty \frac{N2}{D} \frac{r}{\alpha} J_1 \left(\frac{r}{h} \alpha \right) \frac{\theta}{2\pi} d\alpha \quad (13)$$

$$= + p \frac{ph}{E_1} \sum I_2 \frac{\theta}{2\pi} \Bigg]_0^\theta$$

INFLUENCE DIAGRAMS FOR STRESSES AND DISPLACEMENTS

The numerical evaluations of these stress and displacement Equations 9 through 12 for loads uniformly distributed over circular areas applied at the surface of the ground were carried out by special computational methods using I.B.M. computing machines and desk calculators in the Watson Scientific Computing Laboratory. The numerical solutions of Equations 9 through 12 could not be obtained by direct integration methods but were obtained in three steps: first, by developing a five-term equivalent equation for the reciprocal of the denominator, $1/D$, of Equation 2 accurate to within 0.5 percent over the entire working range of the basic computational parameter, α from 0 to 10; second, by multiplying the terms of the numerator brackets of Equations 3 through 8, respectively, by the terms of the equivalent reciprocal denominator equation; and third, by direct integrations of the resulting equations, respectively, term by term, involving the products of exponentials, appropriate Bessel functions, and the basic physical and elastic parameters. The stresses and displacements for the three-dimensional case in cylindrical coordinates with a surface loading uniformly distributed over a circular area were then computed at depth of z equal to $0.2h$, $0.4h$, $0.6h$, $0.8h$, and $1.0h$ in layer 1 for a suitable range of the basic parameter, r/h.

Influence diagrams for stresses and displacements in rectangular coordinates (x, y, x) beneath the corner of a uniformly loaded rectangular area were developed and prepared by taking proper vector components, as illustrated by the stress components and sign conventions in Figure 2, and by using special circular diagram methods (*11*). A sufficient number of values of the basic parameters of rectangular areas, namely—the ratio of the width of the area to the thickness of layer 1, b/h, and area side ratio, b/a, were computed at close intervals over the range from 0.01 to 10 to permit accurate plotting of each influence curve for a constant value of z/h. The overall accuracy of the influence diagrams is well within the physical limits of interpolations in the influence curves, that is, better than 1 to 2 percent, except possibly in the region of b/h greater than about 1.0, where the stress phenomena becomes controlled by tensile stresses.

Where the applications of the principles of superposition are simple, as in the case for vertical stresses and settlements, influence diagrams were prepared for vertical stresses and settlements beneath the corner of uniformly loaded rectangular areas. Each influence diagram gives a family of influence curves for a constant value of the depth ratio, z/h, with each curve drawn for a constant value of the side ratio, b/a, for the basic argument, b/h, to define the influence values, I. These influence curves form a systematic and characteristic pattern, which discloses the nature of the stress and displacement phenomena in a two-layer rigid base system and the governing and sensitive influences of the basic parameters. A key figure and the stress of settlement equation is given in each case.

Where the applications of the principles of superposition are not simple, as in the case for horizontal and shearing stresses, circular influence diagrams were prepared which readily permit making graphic integrations with respect to the argument, θ, for any shape of loaded area, located anywhere within the limits of the diagram, and for any distribution of foundation loading. This situation arises from the nature of the integrations with respect to θ in cylindrical coordinates in Equations 10 through 12, where vector components have to be used to obtain stresses in the (x, y, z) directions in accordance with the notations in Figure 2. Three ranges of influence values for the ratio, r/h, are given for each depth ratio, z/h, covering the range of r/h from 0 to 0.6, 0 to 3, and 0 to 15, so that the stress conditions for any size foundation can be investigated in detail. Influence diagrams of stresses and displacement were prepared as follows:

Influence Diagrams, Groups of Charts and Tables.[1]

A. (1) Vertical stress, σ_z beneath the corner of a uniformly loaded rectangular area for Poisson's ratio of 0.4 and at depths of z equal to $0.2h$, $0.4h$, $0.6h$, $0.8h$ and $1.0h$. Charts 1 to 5, I_1

(2) Settlement, w, at the corner of a uniformly loaded rectangular area for Poisson's ratio of 0.2 and 0.4 at the surface of the ground, $z = 0$. Charts 6 to 7, I_2

B. Stresses beneath the corner of a circular influence diagram for any shape of loaded area, located anywhere within the limits of the diagram, and for any distribution of foundation pressure. Stresses for Poisson's ratio of 0.4 and at depth of z equal to $0.2h$, $0.4h$, $0.6h$, $0.8h$, and $1.0h$.

(1) Horizontal Stresses: σ_x and σ_y
  Influence Coefficient—I_3
    Circular Influence Diagram, Chart 8
    Tables of Influence Coefficients, Tables B-I_3
  Influence Coefficients—I_4
    Circular Influence Diagram, Chart 9
    Tables of Influence Coefficients, Tables B-I_4

(2) Shearing Stresses, τ_{xy} and τ_{yx}
  Influence Coefficients—I_5
    Circular Influence Diagram, Chart 10
    Tables of Influence Coefficients, Tables B-I_5

(3) Shearing Stresses, τ_{zx} and τ_{zy}
  Influence Coefficients—I_6
    Circular Influence Diagram, τ_{zx}, Chart 11
    Circular Influence Diagram, τ_{zy}, Chart 12
    Tables of Influence Coefficients, Tables B-I_6

CHARACTERISTIC STRESS AND DISPLACEMENT PATTERNS

A clear understanding of the fundamental nature of the two-layer rigid base system phenomena and correct conceptions regarding the action and stress-deflection responses are essential in making proper applications to practical problems in foundation engineering. This may be accomplished through a study and interpretation of characteristic stress and displacement patterns. It is, however, essential to have a working hypothesis, such as the present problem, which can establish the nature and basic form of the physical and elastic laws governing the phenomena. Important advances in science and engineering have always come from theoretical developments, however imperfect the idealized form may be for the full expression of the real physical phenomena. These theoretical de-

[1] This material appears on pages 791–814.

velopments present new ideas, bring a phenomena into the realm of greater certainty, and serve as a guide to stimulate experimentation and investigations. It should be realized that no theory or statement of a physical law in any field of science is complete in its present form, because it can not fully and adequately explain, take into account, or include present apparent "exception to the rule," which are outside of the realm of validity of the limiting boundary, interface, and elasticity condition for which the problem was developed.

The two-layer rigid base problem provides a working hypothesis regarding the basic functional form of the physical laws that govern the stress-settlement action and responses of this layered system, which are mathematically and dimensionally correct and complete, as a first requirement. This working hypothesis, through Equations 3 through 8, reveals not only the nature of the dependence of stresses and displacements upon the basic physical parameters and elastic strength coefficients inherent in the layered system, but also the fundamental and systematic nature of the functional relations existing between these basic quantities, which govern the stress and displacement responses of the system under imposed loadings. The layered system is very sensitive to small changes in the basic physical and elastic parameters, as evident in the influence diagrams.

The theory of elasticity is based upon and has, as governing conditions, the response characteristics of ideal materials possessing homogeneous, isotropic, and elastic properties, and linear stress-strain relations in accordance with infinitesimal strain concepts. In contrast to either unquestioning acceptance or to skeptical rejection without foundation in fact, the major problems now are these: first of all, of thoroughly testing a working hypothesis against observed phenomena, both in the field and in carefully controlled experiments; second, of comprehending and establishing the nature and importance of the limitations and the reasonable realm of validity; and third and most important of all, of evaluating and reliably establishing the regions in and the degree to which real soils and real conditions may be expected to agree with and/or to depart from the idealized conditions of the working hypothesis, particularly with regard to the stress-strain responses of layered systems. A working hypothesis can tell the investigator better how and what to observe significantly through the fundamental parametric relationships. Then experimental investigations in the laboratory or in the field, now working to advantage, can be made to yield experimental coefficients, which will bring the idealized working hypothesis more into line with actual observed phenomena. Thus maximum usefulness and reliability of a working hypothesis can be realized, and it can be made to become a "powerful and competent scientific tool" in analyzing and interpreting layered system phenomena for foundation design purposes.

Characteristic stress distribution and displacement patterns are presented in order to have a clear understanding of the fundamental nature of two-layer rigid base system phenomena and correct conceptions regarding the mechanics of its action and stress-deflection responses. The significant characteristics and the favorable and unfavorable aspects are discussed with regard to their influences on the settlement responses of the supporting soils of layer 1 under foundations loads.

Vertical Stress Distribution, σ_z

Certain significant characteristics, and favorable and relatively unfavorable aspects of the well known Boussinesq vertical stress distribution in a homogeneous deposit in Figure 3 are to be noted first, because the Boussinesq stress distribution patterns are to be used as a basis for comparison. Stress conditions are here considered favorable or relatively unfavorable as they tend to decrease or to increase, respectively, the settlement contributions in the depth region being considered. The variation in the magnitude of any vertical stress distribution curve in Figure 3 is a function of the basic ratio, z/b, where b in this case is the least half-width of a rectangular area, and of the side ratio, a/b, of rectangular areas. Considering first square foundation with $a/b = 1$, the high stresses within a depth region, z/b of 0 to about 1.0 represents a relatively unfavorable stress condition in its influences on the settlement contributions. The total settlement

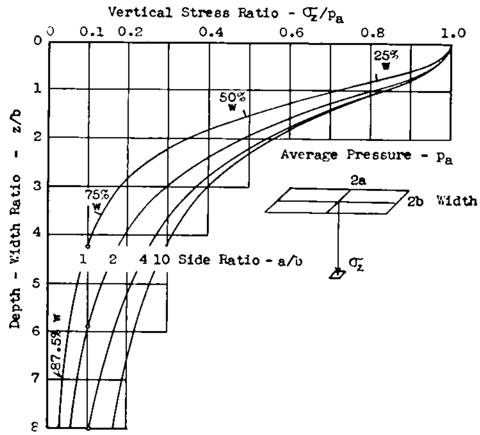

Figure 3. Homogeneous soil deposit. Boussinesq problem. Vertical stresses imposed in the supporting soils by a surface load uniformly distributed over rectangular areas. Fadum vertical stress influence coefficients. Approximate settlement contribution, *w* in percent down to value of *z/b* indicated.

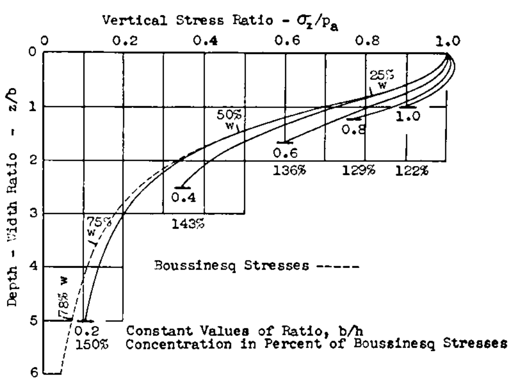

Figure 4. Two-layer rigid base soil system. Burmister problem. Vertical stresses imposed in layer 1 by a surface load uniformly distributed over square areas for constant values of the ratio, *b/h*. Burmister vertical stress influence coefficients.

contribution in percent for Poisson's ratio of 0.4 is noted in Figure 3 down to the depth ratio indicated. The marked decrease in stresses with depth below z/b of about 2.0 is a favorable aspect. About 75 percent of the settlement is contributed within a depth region z/b of 3.6. It is to be noted, however, that the stress decrease with depth becomes smaller and the stress conditions less favorable with increase in size of foundation because of the geometric relationships. For a given value of z/b, but with increase in size of foundation, the same stress is found at a corresponding greater physical depth, $z = (z/b)b$. The shape of the foundation area indicated by the side ratio, a/b, but having the same width, b, also has important influences on the stress conditions in Figure 3, making the stress distribution less favorable below z/b of 2.0, as the length of the rectangular foundation increases. These facts show that a valid comparison of stress distributions can only be made on the basis of the same size and shape of foundations. Practical and useful Boussinesq vertical stress influence coefficients beneath the corner of uniformly loaded rectangular areas for stress investigations have been presented by Fadum (*12, 13*).

The magnitude and distribution of vertical stresses throughout layer 1 of a two-layer rigid base soil system for the three-dimensional case with Poisson's ratio of 0.4 are illustrated in Figures 4, 5, and 6, using the Boussinesq vertical stress distribution as the basis for comparison. The stress conditions are determined by the physical parameters, z/b and b/h, by the layer strength ratio, E_2/E_1 equal to infinity, or for practical purposes greater than a value of 100, and by rectangle side ratio, a/b. These figures were obtained from the vertical stress influence coefficients, I of Charts 1, 2, 3, 4, and 5 beneath the corner of uniformly loaded rectangular areas. Preceding these Group A Charts are tabulations of the influence values obtained from Charts 1 to 5 for the construction of Figures 4, 5, and 6, as illustrative examples of the influence diagram method of stress analysis.

The vertical stress distributions beneath the center of uniformly loaded square areas for the two-layer rigid base system for different values of the ratio, b/h (constant for each curve), show in comparison with the Boussinesq stress pattern for the same size of square foundation area that a concentration of stress exists beneath the loaded area practically throughout the depth of layer 1. The concentration of stress, as a percent of the Boussinesq vertical stress at the same depth, is greatest at the surface of the base layer, decreasing somewhat with decrease in layer thickness, h as noted in Figure 4. For values of b/h greater than 0.6, or thinner layers, the concentration of stress occurs throughout the depth of layer 1, and even exceeds slightly the foundation loading, p_a, as shown in Figure 4. This represents a relatively unfavorable stress condition with increased settlement contributions in the region of stress concentration. But a rigid base layer, however, does restrict settlement contributions to the more compressible layer 1, which is a very favorable condition. The settlement contribution for the Boussinesq stress pattern below the level of the rigid base layer for the ratio, b/h equal to 0.2 is noted for comparison, and indicates how favorable the two-layer rigid base system is in reducing settlements as an overall favorable aspect. Conditions become more favorable for thinner layers. But the settlements contributed by layer 1 itself well be greater than those contributed by the Boussinesq stress pattern within the depth range of layer 1.

With increasing size of square foundation area for a constant thickness of layer 1 in Figure 5, the vertical stresses in the two-layer rigid base system become more nearly uniform throughout the depth of layer 1, but always less favorable than the Boussinesq stress pattern for the same size of square area and for the same depth region. The concentration of stress at the base layer as a percent of the Boussinesq stress, is noted for each value of the ratio, b/h. It should be noted that in Figures 4 and 5, the vertical stresses beneath the center of the loaded areas exceed the applied foundation pressure, p_a (average) for values of b/h greater than about 0.6. This is a part of the stress concentration phenomena contributed by the two-layer rigid base system where the stress phenomena become controlled by tensile stresses in the region of b/h greater than the peak of the influence curves in Charts 1 to 5, as evidenced by the decrease in stress intensity following the peak influence. These stress phenomena arise from the fact that the total stresses integrated over any horizontal plane must equal the applied foundation loading.

The influences of increasing length of rectangular foundation area on the distribution of vertical stresses are shown in Figure 6 for a constant ratio, b/h equal to 0.4 in this case, that is, a constant least width, b, and a constant layer thickness, h. The influences of increasing length of foundation on the unfavorable character of the vertical stress distribution are greatest for small values of b/h with a certain limiting curve being defined for infinite length, depending on the ratio, b/h, as the two-dimensional case of the two-layer rigid base problem.

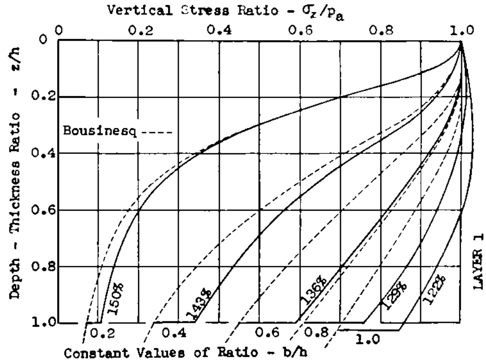

Figure 5. Two-layer rigid base soil system. Burmister problem. Vertical stresses imposed in layer 1 by a surface load uniformly distributed over square areas for constant values of the ratio, *b/h* with *h* constant and *b* increasing.

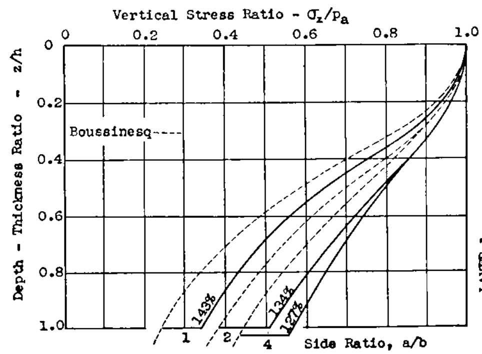

Figure 6. Two-layer rigid base soil system. Burmister problem. Vertical stresses imposed in layer 1 by a surface load uniformly distributed over rectangular areas for a ratio, *b/h* equal to 0.4.

The vertical stress influence coefficients defining the magnitude and distribution of vertical stresses in a two-layer rigid base system beneath the corner of uniformly loaded rectangular areas are given in the Group A Charts 1 to 5 in terms of the basic layered system parameters. Preceding these charts are tabulations in Tables A-1, A-2, and A-3 (see p. 792) of the influence coefficients obtained from Charts 1 to 5 at respective depths in layer 1, which were used for the construction of Figures 4, 5, and 6 respectively, as illustrative examples of the influence chart method of analysis.

The principles of superposition of influence coefficients and of stresses are used for a foundation of any shape, distribution of foundation pressure, and location of points beneath which vertical stress distributions are required. Proper account can and should be taken of additive and subtractive areas, as done in the illustrative example of Table A-4. The work should be carefully planned and organized in a systematic and logical manner, in order to expedite the work and to permit adequate checking, since a relatively large number of steps and mathematical operations may be involved. Free use should be made of the small diagrams of plus and minus areas, with dimensions and pressures, as shown in Table A-4, for each step in the computations of vertical stresses at each elevation in layer 1.

Surface Settlements, W

The significant characteristics and the very favorable aspects of the two-layer rigid base soil system with regard to its action and surface settlement responses are disclosed in Figures 7 and 8 by comparison with the Boussinesq settlement relations. Since settlements are also considerably influenced by Poisson's ratio, two figures are given, Figure 7 being for Poisson's ratio of 0.2 and Figure 8 for 0.4. A decrease in Poisson's ratio causes an increase in surface settlements. In order to have a basis for comparison with the Boussinesq settlement coefficients, the two-layer

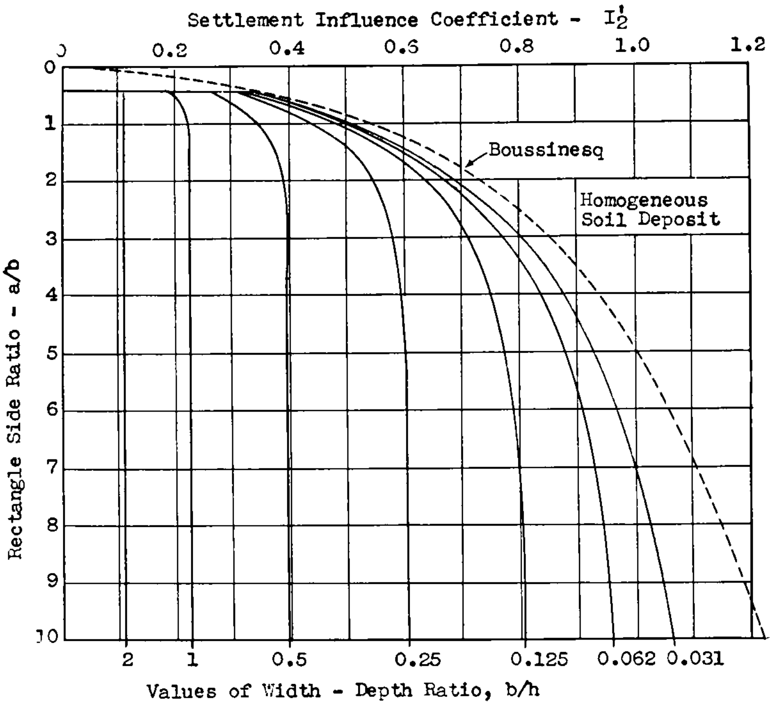

Figure 7. Two-layer rigid base soil system. Burmister Problem. Surface settlements imposed on layer 1 at the corner of a surface load uniformly distributed over rectangular areas for constant values of the ratio, *b/h*, and for Poisson's ratio of 0.2

Two-layer rigid base system	**Boussinesq Problem**
$I_2' = (I_2\, h/b)$	$I_2' = 2(1 - \mu^2)C$

Surface settlement, $w = \dfrac{p_a\, b\, \Sigma I_2'}{E_1}$ **Eq. 13a.**

rigid base settlement equation, $w = (p_a h I_2)/E_1$ is modified to the form of Equation 13*a*, as follows: $w = p_a b(I_2 h/_b)/E_1$.

The Boussinesq settlement coefficient, I_{2B} is considerably influenced by the rectangle side ratio, a/b, the greatest increase occurring at small values of the side ratio, but increasing continuously within the limits of the figure. On the other hand the settlement coefficient in a two-layer rigid base system is governed by the width of footing-thickness of layer ratio, b/h, decreasing markedly as the thickness of layer 1 decreases. This phenomena follows the vertical stress pattern of Figure 4, where it is evident that only layer 1 contributes to surface settlements in the two-layer rigid base system, whereas the entire homogeneous deposit contributes settlements in the percentages noted for the Boussinesq stress distribution. The rigid base does, however, restrict settlement contributions to layer 1, but the stress concentration inherent in the rigid base system increases the settlements above those contributed by the Boussinesq stress distribution down to the same depth.

These overall extremely favorable settlement aspects of the two-layer rigid base system in Figures 7 and 8, particularly for the thinner thickness of layer 1, are composite of influences of the stress conditions in Figures 4, 5, and 6. It is to be noted that the settlement coefficient for the two-layer rigid base system becomes constant on each b/h curve for side ratios greater than a certain value in both Figures 7 and 8, and becomes independent of the size of the foundation. At this stage the settlement phenomena reduces to normal consolidation, for which the coefficient becomes equal to $(1 - 2\mu)(1 + \mu)/(1 - \mu)$, or 0.90 for Poisson's ratio of 0.2 and 0.47 for Poisson's ratio of 0.4. To substantiate this fact, in Table A-5 the influence coefficients used in the construction of Figures 7 and 8 are given as an illustrative example. Values of I_2 of 0.226 and 0.119 at large values of b/h with h small or b large are one-fourth of the above consolidation coefficients, where the diagrams are constructed for settlements at the corner of a rectangular area.

The surface settlement coefficients of Figures 7 and 8 at the corner of loaded rectangular areas may be used as influence values to determine the distribution of surface settlement for any point in a foundation area by principles of superposition. A comparison

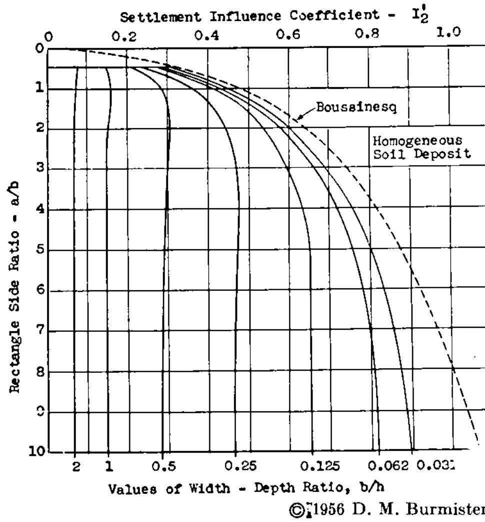

Figure 8. Two-layer rigid base soil system. Burmister problem. Surface settlements imposed on layer 1 at the corner of a surface load uniformly distributed over rectangular areas for constant values of the ratio, b/h, and for Poisson's ratio of 0.4.

Two-layer rigid base system — $I_2' = (I_2\, h/b)$

Boussinesq Problem — $I_2' = 2(1 - \mu^2)C$

Surface settlement, $w = \dfrac{p_a\, b\, \Sigma I_2'}{E_1}$ **Eq. 13a.**

of settlement estimates with actual observed settlements may be made to have very important and practical uses in establishing the realm of validity of the two-layer rigid base soil system and in establishing more accurate values of the elastic constants of soils, namely —the modulus, E, and Poisson's ratio, μ, for cases where a rigid base layer definitely exists and where the thickness of layer 1 is relatively shallow.

Horizontal and Shearing Stresses

Influence coefficients for horizontal and shearing stresses are given in the Group B influence tables and charts. The influence coefficients for vertical, horizontal, and shearing stresses permit the investigation and study of the complete stress conditions imposed in a two-layer rigid base soil system by foundation loadings. Where the applications of the principles of superposition are not simple, as is the case for horizontal and shearing stresses, circular influence diagram methods were prepared, which permit making graphic integrations with respect to the argument—θ. This situation arises from the nature of the integrations with respect to θ where two influence coefficients are involved in the stress equations with different θ-functions. Illustrative examples of the circular influence diagram method of stress investigation are given for the horizontal and shearing stresses in Table B-1, preceding the Group B influence tables and charts (see p. 802).

In the past, only the vertical stress conditions have been considered to have significance in stress investigation. In some cases shearing stress may become critical. It is becoming recognized, however, that the horizontal stresses may play a very important role in the actual stress conditions and particularly in the settlement phenomena. The reason for making stress investigations is to provide the necessary data for making settlement estimates. Furthermore, it is common practice to use the consolidation test far beyond its realm of validity of horizontal strains and displacements equal to zero. It may be that one important cause of disagreement between settlement estimates and the actual observed settlements is the result of ignoring the influences of the horizontal stresses upon settlement phenomena. Applications of consolidation within its strict realm of validity may be limited to certain stress regions only. Everywhere else the stress conditions are dominated by a triaxial state of stress considerably different from consolidation. It is evident in Figures 7 and 8 for certain ranges of the basic parameter, h/b and a/b for the two-layer rigid base system that consolidation may be valid, but the stress conditions should be checked to establish this realm of validity in a given case.

The stress conditions which actually exist in different stress regions in a soil mass supporting a foundation load, particularly the horizontal stresses and the accompanying natural lateral restraint conditions contributed by the surrounding soil mass through the Poisson's ratio effects, may have equally important, and possibly greater influences in governing strains and displacements than does the E-value. This important and significant fact may be indicated approximately by the following form of Hooke's law:

$$\epsilon_z E = \sigma_z[1 - \mu(\sigma_x + \sigma_y)/\sigma_z]$$

As the horizontal stresses, σ_x and σ_y, increase with respect to the vertical stress, σ_z, the strains become smaller without any increase in the value of E.

In addition, and unlike the case in the common structural materials, the stressing and straining of soils never starts from the unstressed and unstrained state, but always starts from a natural state of prestress equal at least to the present weight of overburden. It is well recognized that prestressed concrete is greatly improved in its deflection responses by the imposed state of prestress. Similarly in natural soil deposits, the normal state of prestress under the weight of overburden greatly improves the settlement responses of the soils. A state of overconsolidation and hence a higher state of prestress in a soil deposit improves the settlement responses still more, as indicated in the following form of Hooke's law using total stresses:

$$\epsilon_z E = (\sigma_z + p_v)[1 - \mu(\sigma_x + \sigma_y + 2p_h)/(\sigma_z + p_v)]$$

The influences of these actual stress conditions upon settlement responses should be adequately and reliably investigated so that full advantage of their favorable aspects can be taken. If conditions are relatively unfavorable, certainly they should be adequately taken into account in stress and settlement investigations.

CRITICAL DEPTH CONCEPTIONS

In order to provide a basis for judgments, it is necessary to define certain tentative critical depth limits *(10)* for evaluating the validity and applicability of the two-layer rigid base problem in comparison with the Boussinesq problem. It has been found by experience in making stress and settlement investigation that only within certain rather critical depth limits are the imposed foundation stresses sufficiently significant to cause large enough strains, which would contribute appreciable surface settlements of structures. The critical depth is a variable and an approximate relation, which should be revised in the light of increase in knowledge and experience. It is dependent in each situation on the interrelation, relative dominance, and favorable and unfavorable aspects of the following conditions: (1) the size and shape of the foundation area and the average foundation pressure; (2) the basic stress distribution pattern; (3) the magnitude and distribution of the overburden stresses and excavation stresses with depth, depending on the unit weights of the soils above and below ground water; and (4) the character, relative compressibility, and stratification of the soils of the deposit and depth to rock.

The critical depth is not only useful for forming judgments regarding the validity of the Boussinesq and two-layer rigid base problems of stress investigations in a situation, but also for estimating the approximate minimum required depth of borings for spread foundations to establish whether the Boussinesq, the two-layer rigid base, or some other stress problem is valid and applicable in a situation. As a first approximation before subsurface soil conditions have been disclosed by the first boring, estimates of the minimum depth may be based on a tentative range of values in the above four conditions. As actual subsurface soil conditions are disclosed during the progress of drilling borings over a site, adjustments can be made accordingly in the estimated critical depth. If at this depth the bottom of a boring is still within a compressible clay layer, one boring at least should penetrate the layer to fully establish the character of the soils with depth. Thus adequate depths of explorations can be more consistently assured, and also savings can be realized, if conditions are definitely found to be more favorable than first anticipated.

In accordance with these conceptions the critical depth is defined as a first approximation and basis for judgment, as follows: (1) for individual spread footings, where influences of prestress under the weight of overburden are of relatively minor importance, this depth is taken equal to that where the imposed stresses decrease to values less than one-tenth of the applied footing pressure, but not greater than 0.2 tons per square foot; and (2) for the entire foundation of a structure, where influence values of prestress become of dominating importance, the critical depth is taken as that at which the imposed vertical stresses decrease to a value equal to the following tentative percentages of the overburden stress, namely—20 percent for relatively incompressible granular soils and 10 per cent for relatively more compressible clay soils; and (3) the excavation stress due to the unloading of a soil deposit by the weight of excavation may be deducted from the

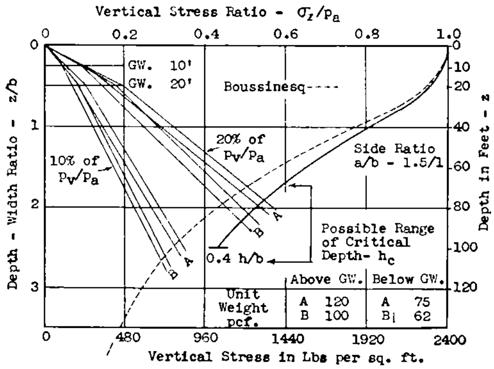

Figure 9. Critical depth concepts. Evaluation of tentative critical depths for stresses imposed in a two-layer rigid base soil system by foundation loads for preliminary soil investigations and for validation of the problem, using the Boussinesq problem as a basis for comparison. Burmister concepts. Critical depth is that at which the foundation stresses become less than 20 percent of overburden stresses for granular soils and 10 percent of overburden stresses for clay soils.

vertical stress distribution in making the estimates for critical depth, but not to exceed one-half of the vertical stress, due to the approximate character of this relation.

These critical depth relations are illustrated for the Boussinesq and two-layer rigid base problems in Figure 9 in order to disclose their significance and implications. The influences of variations of unit weights, ground water elevation, and average foundation pressure on the critical depth are illustrated, and indicate what should be considered conservative in the evaluation. If a rigid base layer occurs within the critical depth for the Boussinesq stress distribution for the same size and shape of foundation and foundation pressure, then the two-layer rigid base problem is valid and applicable, provided that the situation satisfies the other conditions. If on the other hand a base layer does occur just below the critical depth, but was not disclosed due to the fact that borings were stopped at this critical depth, then it is evident that the stress concentration effects might be significant, as indicated in Figure 9. One boring at least in a foundation area should be driven to greater depths, for example 25 feet, more definitely to disclose the presence or absence of such a base layer.

RATING BASIS FOR JUDGMENTS

The value of thorough and competent investigations is to learn and to understand more completely the real nature of a situation, to evaluate the conditions that control, and to estimate reliably and adequately the "probable." The validity of the theoretical conditions should not be taken for granted, nor should skill and facility in the use of influence diagram methods of stress analysis be considered synonymous with adequacy, competence, and good judgment in stress investigations.

No stress investigation should be considered complete and adequate without an appraisal and evaluation in each situation regarding: (1) the kind and degree to which and the regions in which the actual conditions agree with and depart from the theoretical conditions of a stress distribution problem; (2) the probable kind and degree of departures of the probable actual stresses and settlements from the estimated values caused thereby; and (3) the significant characteristics, and the favorable and unfavorable aspects of the stress distribution in each situation.

The degree of agreement of the probable actual stresses in a particular situation with the stress distribution pattern used can be evaluated most effectively on a rating basis (*10*). The rating concept in contrast to broad generalizations is a powerful tool in soil mechanics where exact numerical evaluations of many soil phenomena can not be made in the present state of knowledge. An appraisal and reasonable evaluation of all known facts and conditions inherent in a situation leads to a considered judgment of the rating, which should eliminate to an increasing degree elements of uncertainty and guessing as knowledge and experience grow. The Boussinesq stress distribution pattern for a homogeneous soil deposit is used as the comparative basis for the rating. The rating may be made on a percentage basis between the limits of the two-layer rigid base stress distribution, as the 100 percent upper limit or substantially complete agreement, and of the Boussinesq stress distribution for the same size and shape of foundation, as the zero percent lower limit or substantially no agreement. When the general two-layer system has been evaluated numerically for strength ratios of E_2/E_1 of 5, 10, 20, 50, and 100, a strength ratio basis for a rating would be more satisfactory and effective.

The degree of agreement of the theoretical conditions of a stress distribution pattern, with respect to the actual surface and subsurface soil conditions, can be judged on the basis of proper interpretations and evaluations of records of borings made under competent supervision. First of all, the soils of layer 1 down to the base layer should be reasonably homogeneous in character—(a) no thin layering of soils as in the Westergaard problem (*14*); and (b) no thick layering of soils of different compressibilities as for the general two- and three-layer system (*10*). Adequate and reliable boring records with regard to the stratification of a deposit and accurate and complete identifications (*15*, *16*) of good quality soil samples taken at intervals of 5 feet in depth by the common 2-inch—1⅜-inch split barrel sampler, can provide the basic information required for judging whether a reasonably homogeneous soil condition exists throughout the depth of layer 1, and whether the two-layer rigid base system is valid and applicable in this respect. Second, the strength of the base layer relative to that of the overlying layer 1, expressed by the layer strength ratio, E_2/E_1 should meet certain minimum requrements, namely—a tentative value greater than about 100, in order that the two-layer rigid base system be valid and applicable. Proper interpretations and evaluations (*17*) of reliable and adequate field determinations and records of the driving resistances of the sampler in blows per 6 inches and of the casing in blows per foot, laboratory triaxial and consolidation tests, and field loading tests can provide the basic information required for judging the validity and applicability of the two-layer rigid base soil system in this respect to the situation.

The degree of agreement may be judged and expressed on a rating basis considering the percentage ratings—0 to 25 percent as not usually sufficiently significant; 25 to 50 percent as moderately significant and to be taken into account; 50 to 75 percent as having increasingly good and important significance; and 75 to 100 percent as approaching complete agreement with the two-layer rigid base system. The approximate tentative spread between the moduli, E_1 and E_2 of the two layers for 75 to 100 percent rating agreement should not be less than the following: (1) the spread of compactness or relative density (*17*) of granular soils should not be less than a range of Loose + to Medium Compact + for layer 1 to a corresponding range, respectively, of Compact to Very Compact for layer 2; or (2) the spread of relative consistency (*17*) expressed on a shearing strength basis for clay soils should not be less than a range of Firm + to Medium Hard + for layer 1 to a corresponding range, respectively, of Hard to Very Hard for layer 2. The change in character and compactness or consistency from layer 1 to layer 2 should be relatively sharp. If the ratio of the moduli is less than 100, as indicated by a smaller spread in compactness or consistency, or if there is a more gradual increase in driving resistance of the sampler with depth, an evaluation of the degree of agreement may be made on a percentage rating basis between the limits of the Boussinesq and the two-layer rigid base system vertical stress distributions. In this case, the base layer 2 would contribute appreciable surface settlements, but less than for the Boussinesq stress condition because of lower compressibility, which may be taken into account approximately on a rating basis.

Where exactness is unattainable, due to the nature of soil phenomena, a range of stress conditions corresponding to the limits, respectively, of the percentage rating—25 to 50, 50 to 75, or 75 to 100 percent should be evaluated in stress investigations in order to properly bracket by two answers the possibilities inherent in the actual situation. A single or average evaluation should not be considered sufficiently reliable, adequate, or realistic. A modified vertical stress distribution may then be used accordingly with a reasonable justification. This may be accomplished by computing the stress distributions for both the Boussinesq and the two-layer rigid base problems at a sufficient number of key points of a foundation loading to establish the imposed pattern of stress conditions, and by interpolating between these two limiting theoretical stress distributions in accordance with the percentage rating adopted. The modified stress distributions thus obtained would serve as reasonably reliable, though approximate bases, for the probable actual stress conditions in the investigations.

The advances in stress distribution problems have now reached a point where the conceptions and practices regarding adequacy in stress investigations should correspondingly be revised upward in order to make the advances fully effective. The favorable aspects of a probable stress distribution or settlement pattern should not be disregarded, if a rating made on a proper basis exceeds about 25 percent. Just being conservative can not now be justified if it penalizes the favorable aspects in a probable actual stress distribution or settlement pattern. But the taking into account of the favorable aspects should not go beyond a justifiable rating. Certainly any unfavorable aspects must be adequately taken into account, but not to an excessive degree beyond a justifiable rating.

A complete statement of this appraisal and evaluation with reasons therefor, as an essential and integral part of every stress and settlement investigation, should be carefully, systematically and conscientiously prepared for each situation and should be filed with the investigation. This would serve to clarify one's thinking, and to assure oneself of a proper and adequate basis for judgments and decisions, which can be reviewed, checked and revised against actual observed responses and performances of foundations of structures. Furthermore such appraisals would serve to build up a reliable and authoritative body of knowledge and experience for future use.

Up to the present, the Boussinesq Problems have been the only method of stress analysis for which sufficient influence coefficients have been available for practical use. As the only available stress analysis tool, it has been used far beyond its reasonable realm of validity and applicablility. Under such circumstances, there was some justification for such rough approximations in order to obtain some conception of and information on the nature of imposed foundation stresses. The serious aspects, however, in many of the present conceptions and practices are the result of the acceptance and general use of rough approximation, broad generalizations and other expediencies as a matter of course with the attitude that this approach is permissible in a material such as soil. These habits of thought and action have tended to give rise to the idea that adequacy of stress analysis, and excellence and competence of judgment are merely matters of skill and facility in the use of stress influence coefficient methods. Little or no attempt was made to estimate the kind and degree of departures from the probable actual stress and settlement conditions caused by the use of these rough approximations. With the knowledge available since about 1935 from the Marguerre, Biot, and Picketts' solutions of stresses at a rigid base layer regarding the stress concentration effects, the minimum requirements for "adequacy," where the Boussinesq approximations were used in such a situation, should have been the recognition of the fact that stresses were underestimated, at least in the vicinity of the base layer and hence settlement were also underestimated by cutting off the settlement contributions below the base layer. Some recognition of these facts could have been made by increasing the settlement estimates somewhat.

These habits of acceptance and general use of rough approximation in conceptions and practices have resulted in attempts to fit each new advance into the general existing framework, whereas actually each new advance always makes certain aspects of the old scheme of things obsolete. This attitude tends to bring the advance down to the lower level of existing conceptions and practices and to perpetuate them beyond their usefulness. On the contrary each new advance should in the nature of things broaden the horizons, and should cause a re-evaluation of all related aspects in a field with a general revision upward of these aspects in order to take full advantage of each new advance and to make the most effective use of it. It should be recognized that advances do not do away with the essential need for judgment in the intelligent, competent and adequate use of an advance. Each advance should remove certain ignorance factors and rough approximations from conceptions and practices and should bring natural phenomena more into the realm of certainty. The most important forward step in soil mechanics and foundation engineering is "to treat real soils under essentially real conditions."

CONCLUSION

1. The value of thorough and competent stress and settlement is to understand more completely the real nature of a situation, to evaluate fully the conditions that control, and to estimate reliably and adequately the probable.

2. The two-layer rigid base problem is in close agreement with actual conditions encountered in foundation practice within its realm of validity and applicability.

3. An essential and integral part of each stress and settlement investigation is to appraise and to establish the validity and applicability of a stress problem and to take adequately into account the favorable and unfavorable aspects.

4. A major aspect of an appraisal of conditions is an adequate evaluation of subsurface conditions and of soil test data. These related aspects in soil mechanics should be consistently improved and advanced in order to provide adequate and reliable information on the strength characteristics of soils, which will permit making full and effective use of advances in stress and settlement investigations.

5. The evaluation of the validity and applicability of the two-layer rigid base problem and of the probable actual stress and settlement conditions can be made most effectively and significantly on a rating basis between the limits of the Boussinesq and the two-layer rigid base system problem.

6. On the basis of reliable ratings, a range of stress and settlement conditions can be evaluated, which can be made to properly bracket the possibilities inherent in a particular situation.

ACKNOWLEDGMENTS

The numerical evaluation of the series of influence diagrams of stresses and displacements in a two-layer rigid base soil system reported in this paper was sponsored by the Corps of Engineers, U. S. Army, Office of the Chief of Engineers during the years 1949 to 1954. The work was carried out in the Department of Civil Engineering, Columbia University, New York City. Acknowledgment is made to the Watson Scientific Computing Laboratory, Columbia University and to Dr. Wallace J. Eckert, Director, for the free use of their I.B.M. electronic computing machines, which made possible the completion of this computational work. Acknowledgment is also made to Mr. Vladimir Obrcian, who started the work and developed the equivalent denominator equation needed for the computations, to Mr. Robert L. Schiffman, who did most of the computation work on the I.B.M. computing machines and prepared most of the influence diagrams. and to Messrs. G. Rossoni, Lloyd A Curtis, and Charles A. Baily, who assisted materially in the computations and the preparation of the report.

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APPENDIX A

TWO-LAYER RIGID BASE SOIL SYSTEM INFLUENCE DIAGRAMS AND TABLES OF STRESSES AND DISPLACEMENTS

Group A—Influence Charts 1 to 7

Vertical Stress, σ_z : Beneath the corner of a uniformly loaded rectangular area for Poisson's ratio of 0.4 and at depths of $z = 0.2h.$, $0.4h$, $0.6h$, $0.8h$, and $1.0h$.

Charts 1 to 5, I_1

$$\sigma_z = -\sum (p_a I_1) \quad \text{Equation 14}$$

Surface Settlement, w: At the corner of a uniformly loaded rectangular area for Poisson's ratios of 0.2 and 0.4 at the surface of the ground, $z = 0$. Charts 6 and 7, I_2

$$w = \sum (hI_2p_a/E) = \sum (bp_a/E)(I_2h/b)$$

Equation 13a

Illustrative Examples: Vertical Stresses:

TABLE A-1

VERTICAL STRESS INFLUENCE COEFFICIENTS FOR CONSTRUCTION OF FIGURE 4 BENEATH THE CENTER OF SQUARE AREAS FOR CONSTANT VALUES OF b/h WITH THE THICKNESSES, h OF LAYER 1 VARYING. $z/b = (z/h)(h/b)$. I_1—TWO LAYER. I_B—BOUSSINESQ

b/h, z/h	0.2		0.4		0.6		0.8		1.0		Boussinesq	
	z/b	$4I_1$	z/b	$4I_1$	z/b	$4I_1$	z/b	$4I_1$	z/b	$4I_1$	z/b	$4I_B$
0.2	1.0	0.706	0.5	0.940	0.33	0.996	0.25	1.012	0.20	1.016	1.0	0.700
0.4	2.0	0.344	1.0	0.744	0.67	0.924	0.50	1.008	0.40	1.032	2.0	0.340
0.6	3.0	0.200	1.5	0.540	1.00	0.816	0.75	0.994	0.60	1.004	3.0	0.172
0.8	4.0	0.140	2.0	0.432	1.33	0.708	1.00	0.872	0.80	0.948	4.0	0.108
1.0	5.0	0.108	2.5	0.344	1.67	0.585	0.125	0.700	1.00	0.860	5.0	0.072

Note: Influence coefficients from Charts 1, 2, 3, 4, and 5 for appropriate values of b/h and a/b.

TABLE A-2

VERTICAL STRESS INFLUENCE COEFFICIENTS FOR CONSTRUCTION OF FIGURE 5 BENEATH THE CENTER OF SQUARE AREAS FOR CONSTANT VALUES OF b/h WITH THE THICKNESS OF LAYER 1 CONSTANT AND WITH b VARYING. $z/h = (z/b)(b/h)$

b/h, z/h	0.2, $4I_1$	0.4, $4I_1$	0.6, $4I_1$	0.8, $4I_1$	1.0, $4I_1$	Boussinesq	
						z/b	$4I_B$
0.2	0.706	0.940	0.996	1.012	1.016	1.0	0.700
0.4	0.344	0.744	0.924	1.008	1.032	2.0	0.340
0.6	0.200	0.540	0.816	0.994	1.004	3.0	0.172
0.8	0.140	0.432	0.708	0.872	0.948	4.0	0.108
1.0	0.108	0.344	0.585	0.760	0.860	5.0	0.072

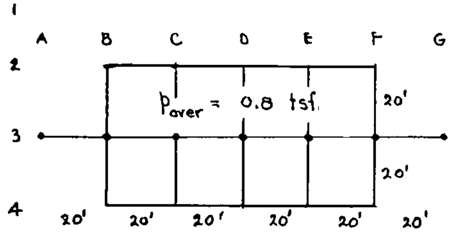
TABLE A-3

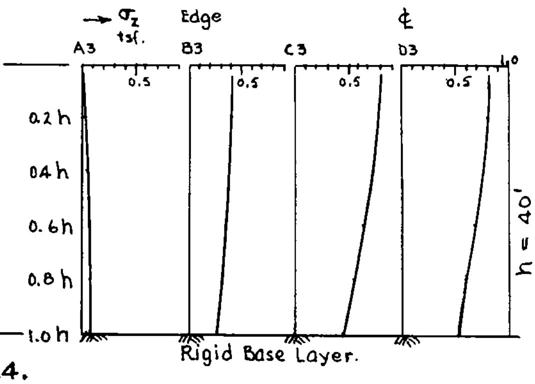
VERTICAL STRESS INFLUENCE COEFFICIENTS FOR CONSTRUCTION OF FIGURE 6 BENEATH THE CENTER OF RECTANGULAR AREAS FOR CONSTANT VALUES OF THE SIDE RATIO, a/b AND FOR A CONSTANT VALUE OF b/h EQUALS TO 0.4. $z/b = (z/b)(h/b)$

a/b, z/h	z/b	1		2		4	
		$4I_1$	$4I_B$	$4I_1$	$4I_B$	$4I_1$	$4I_B$
0.2	0.5	0.940	0.925	0.972	0.956	0.974	0.960
0.4	1.0	0.744	0.694	0.860	0.796	0.860	0.816
0.6	1.5	0.520	0.484	0.720	0.608	0.740	0.668
0.8	2.0	0.432	0.336	0.612	0.480	0.648	0.536
1.0	2.5	0.340	0.240	0.500	0.372	0.560	0.440

TABLE A-4
INVESTIGATION OF DISTRIBUTION OF VERTICAL STRESSES IMPOSED IN A TWO-LAYER RIGID BASE SOIL SYSTEM BY FOUNDATION LOADS

ILLUSTRATIVE EXAMPLE- VERTICAL STRESSES, σ_z BENEATH A FOUNDATION.
FIG. a-FOUNDATION PLAN. FIG. b-MAGNITUDE AND DISTRIBUTION OF STRESSES.





5 NOTES- Stress Direction and Sign Notations- Fig. 2,

n- Number of Loaded Areas of same Dimensions and Pressure, $p_{aver.}$

Stress, $\sigma_z = \sum \pm (n \times p_{aver} \times I_1)$ Eq. 14.

I_1- Influence Values from Charts of σ_z. Areas- + and −, Principle of Superposition.

p_{aver}- $\sum$ Column Loads ÷ Foundation Area for each different Foundation Loading.

COMPUTATION TABLE OF VERTICAL STRESSES, σ_z, h = 40'.

Point	z	Chart	b/h	b/a	$\sum \pm I \times n \times p_a =$	σ_z (tsf.)
A3	0.2h	1	40/40 = 1.0	40/20 = 2	+ 0.249 × 4 × 0.8 =	0.797
	0.4h	2			+ 0.235 =	0.753
	0.6h	3			+ 0.210	0.672
	0.8h	4			+ 0.186	0.594
	1.0h	5			+ 0.158	0.537
B3	0.2h	1	20/40 = 0.5	20/20 = 1	+ 0.244	
			60/40 = 1.5	60/20 = 3	+ 0.248/0.492 × 2 × 0.8 =	0.785
	0.4h	2			+ 0.214	
					+ 0.234/0.448	0.717
	0.6h	3			+ 0.175	
					+ 0.210/0.385	0.615
	0.8h	4			+ 0.145	
					+ 0.189/0.334	0.534
	1.0h	5			+ 0.117	
					+ 0.164/0.281	0.450
C3	0.2h	1	80/40 = 2	80/20 = 4	+ 0.248 × 2 × 0.8 =	0.397
	0.4h	2			+ 0.233	0.372
	0.6h	3			+ 0.210	0.336
	0.8h	4			+ 0.188	0.300
	1.0h	5			+ 0.165	0.246
D3	0.2h	1	100/40 = 2.5	100/20 = 5	+ 0.250	
			20/40 = 0.5	20/20 = 1	− 0.244/0.006 × 2 × 0.8 =	0.0096
	0.4h	2			+ 0.233	
					− 0.214/0.019	0.0304
	0.6h	3			+ 0.208	
					− 0.175/0.033	0.0577
	0.8h	4			+ 0.187	
					− 0.145/0.042	0.0672
	1.0h	5			+ 0.165	
					− 0.117/0.048	0.077

Illustrative Examples: Surface Settlements:

TABLE A-5
SETTLEMENT INFLUENCE COEFFICIENTS FOR CONSTRUCTION OF FIGURE 7 AND 8 USING INFLUENCE CHARTS 6 AND 7, BENEATH THE CORNER OF RECTANGULAR AREAS OF SIDE RATIO, a/b

Two-layer Rigid Base System							Boussinesq $2C(1-\mu^2)$ Poisson's Ratio	
			Poisson's Ratio					
a/b	b/h	h/b	$\mu = 0.2$		$\mu = 0.4$		$\mu = 0.2$	$\mu = 0.4$
			I_2	I_2h/b	I_2	I_2h/b	I	I
0.5	4	0.25	0.226	0.057	0.110	0.030	0.364	0.318
	2	0.5	0.226	0.113	0.151	0.075		
	1	1	0.198	0.198	0.151	0.151		
	0.5	2	0.136	0.272	0.111	0.222		
	0.25	4	0.080	0.320	0.067	0.268		
		8	0.042	0.336	0.037	0.296		
		16	0.023	0.368	0.019	0.306		
		32	0.011	0.352	0.0095	0.205		
1	4	0.25	0.226	0.056	0.121	0.030	0.538	0.470
	2	0.5	0.226	0.112	0.134	0.067		
	1	1	0.225	0.225	0.160	0.160		
		2	0.175	0.350	0.141	0.282		
		4	0.110	0.440	0.092	0.368		
		8	0.060	0.480	0.052	0.416		
		16	0.0316	0.504	0.027	0.432		
		32	0.016	0.513	0.014	0.448		
2	4	0.25	0.226	0.056	0.119	0.030	0.720	0.630
	2	0.5	0.226	0.113	0.129	0.064		
	1	1	0.226	0.226	0.151	0.151		
		2	0.196	0.392	0.151	0.302		
		4	0.136	0.544	0.111	0.444		
		8	0.079	0.632	0.067	0.536		
		16	0.042	0.672	0.036	0.575		
		32	0.021	0.672	0.019	0.610		
3	4	0.25	0.226	0.056	0.119	0.030	0.848	0.744
	2	0.5	0.226	0.113	0.128	0.064		
	1	1	0.226	0.226	0.147	0.147		
		2	0.198	0.396	0.148	0.296		
		4	0.145	0.580	0.119	0.476		
		8	0.089	0.712	0.074	0.592		
		16	0.048	0.768	0.041	0.656		
		32	0.025	0.800	0.021	0.672		
5	4	0.25	0.226	0.056	0.119	0.030	1.000	0.872
	2	0.5	0.226	0.113	0.128	0.064		
	1	1	0.226	0.226	0.146	0.146		
		2	0.199	0.398	0.145	0.290		
		4	0.180	0.600	0.118	0.472		
		8	0.096	0.768	0.081	0.648		
		16	0.055	0.880	0.047	0.752		
		32	0.029	0.930	0.025	0.800		
10		4	0.150	0.600	0.115	0.460	1.225	1.070
		8	0.101	0.808	0.081	0.648		
		16	0.060	0.960	0.051	0.815		
		32	0.033	1.055	0.028	0.898		

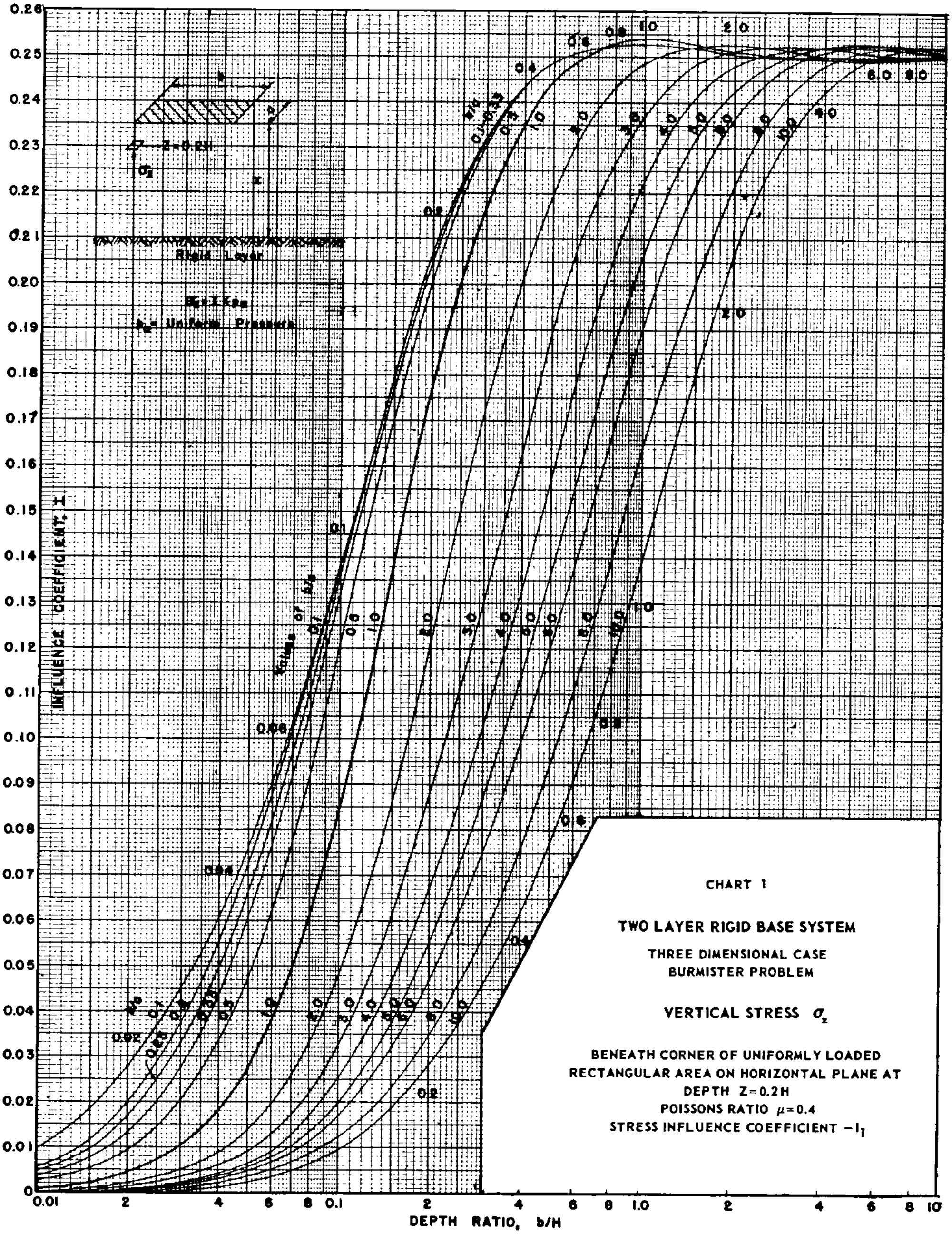


CHART 1
TWO LAYER RIGID BASE SYSTEM
THREE DIMENSIONAL CASE
BURMISTER PROBLEM
VERTICAL STRESS σ_z
BENEATH CORNER OF UNIFORMLY LOADED
RECTANGULAR AREA ON HORIZONTAL PLANE AT
DEPTH Z=0.2H
POISSONS RATIO μ=0.4
STRESS INFLUENCE COEFFICIENT $-I_1$
INFLUENCE COEFFICIENT, I
DEPTH RATIO, b/H
Rigid Layer
Z=0.2H

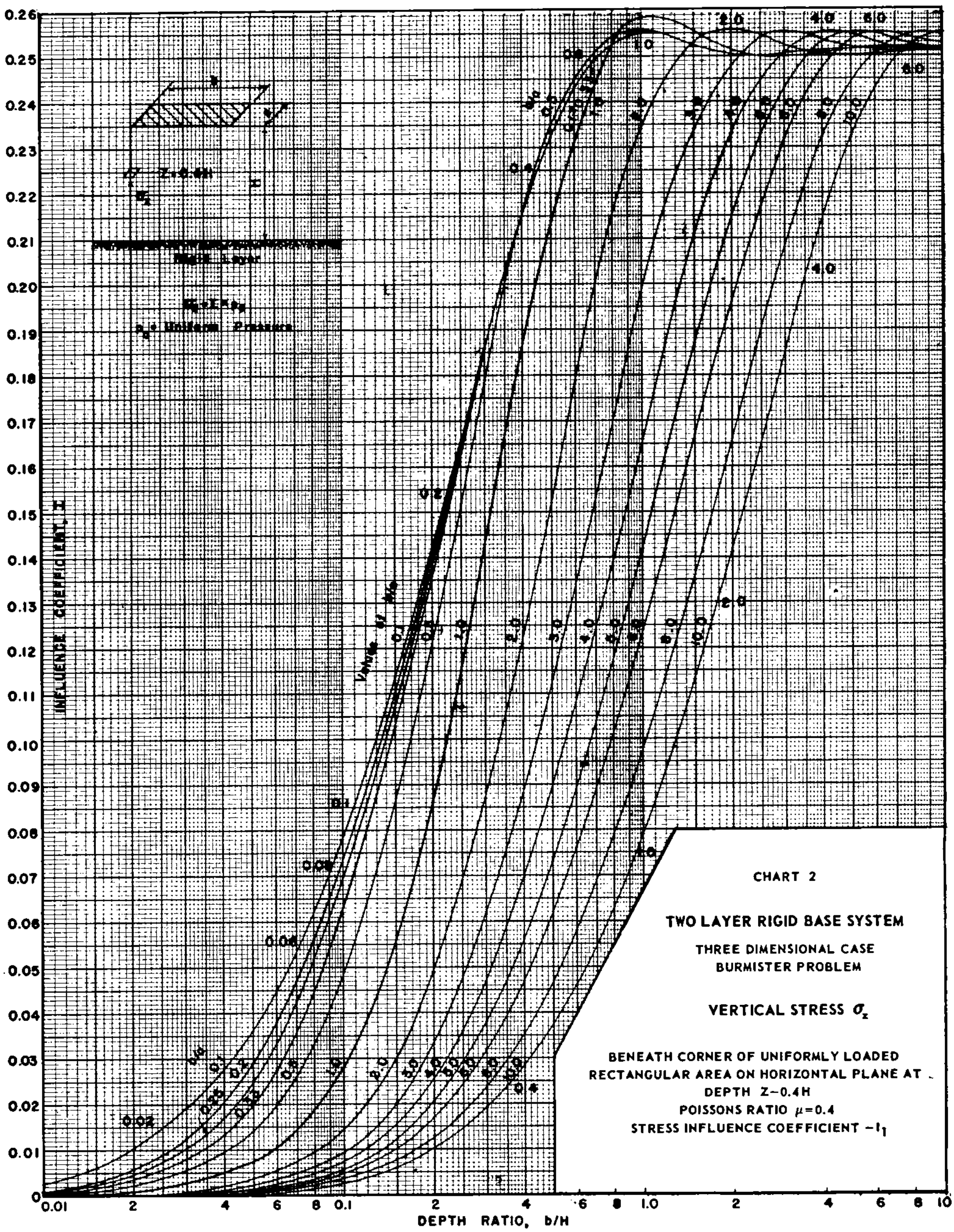


CHART 2
TWO LAYER RIGID BASE SYSTEM
THREE DIMENSIONAL CASE
BURMISTER PROBLEM
VERTICAL STRESS σ_z
BENEATH CORNER OF UNIFORMLY LOADED
RECTANGULAR AREA ON HORIZONTAL PLANE AT
DEPTH Z–0.4H
POISSONS RATIO μ=0.4
STRESS INFLUENCE COEFFICIENT $-I_1$
INFLUENCE COEFFICIENT, I
DEPTH RATIO, b/H
Value of b/a
0.01
0.1
1.0
10

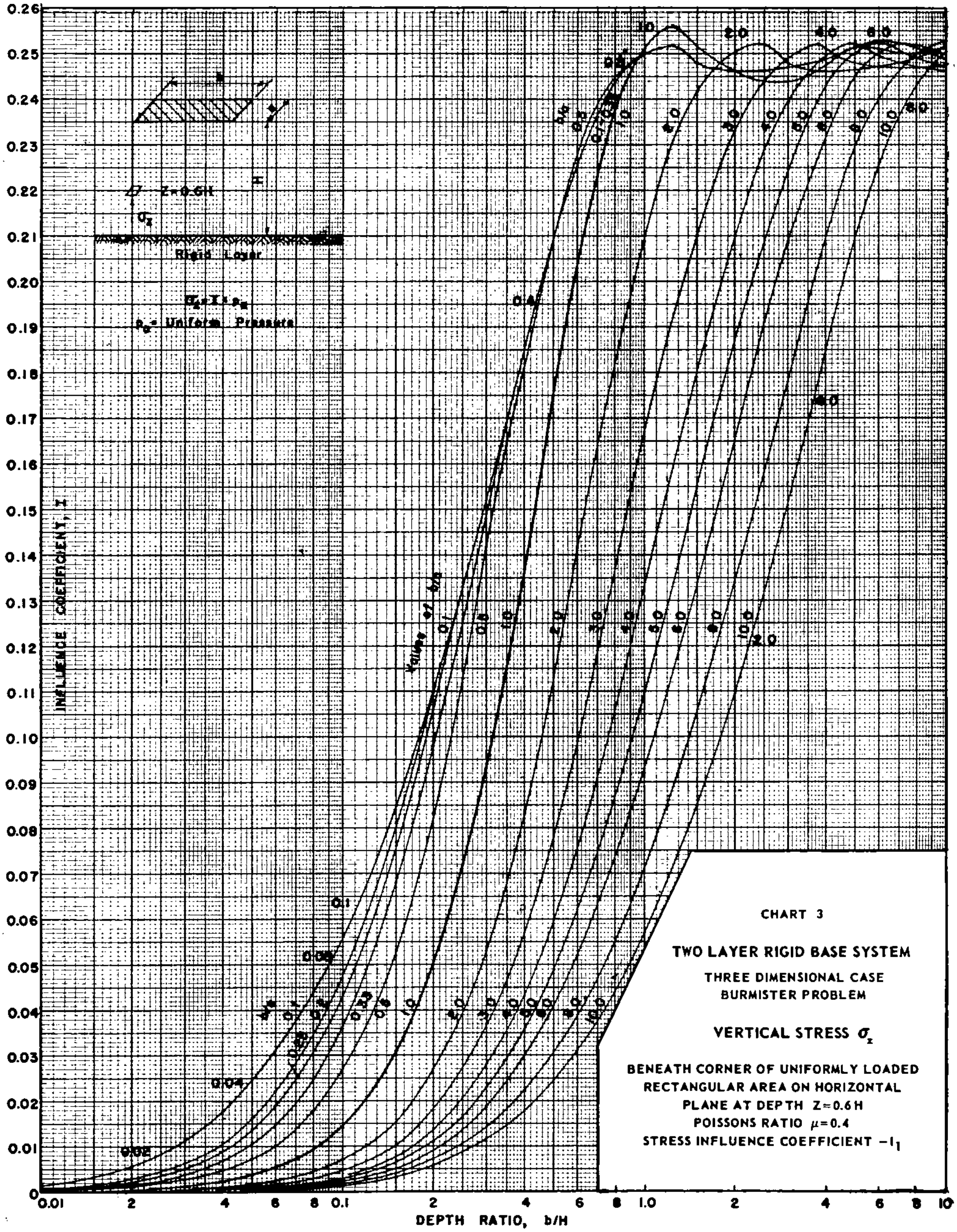


CHART 3
TWO LAYER RIGID BASE SYSTEM
THREE DIMENSIONAL CASE
BURMISTER PROBLEM
VERTICAL STRESS σz
BENEATH CORNER OF UNIFORMLY LOADED
RECTANGULAR AREA ON HORIZONTAL
PLANE AT DEPTH Z=0.6H
POISSONS RATIO μ=0.4
STRESS INFLUENCE COEFFICIENT - I1
INFLUENCE COEFFICIENT, I
DEPTH RATIO, b/H
Values of a/b
Rigid Layer
Z=0.6H
σz = I × p0
p0 = Uniform Pressure

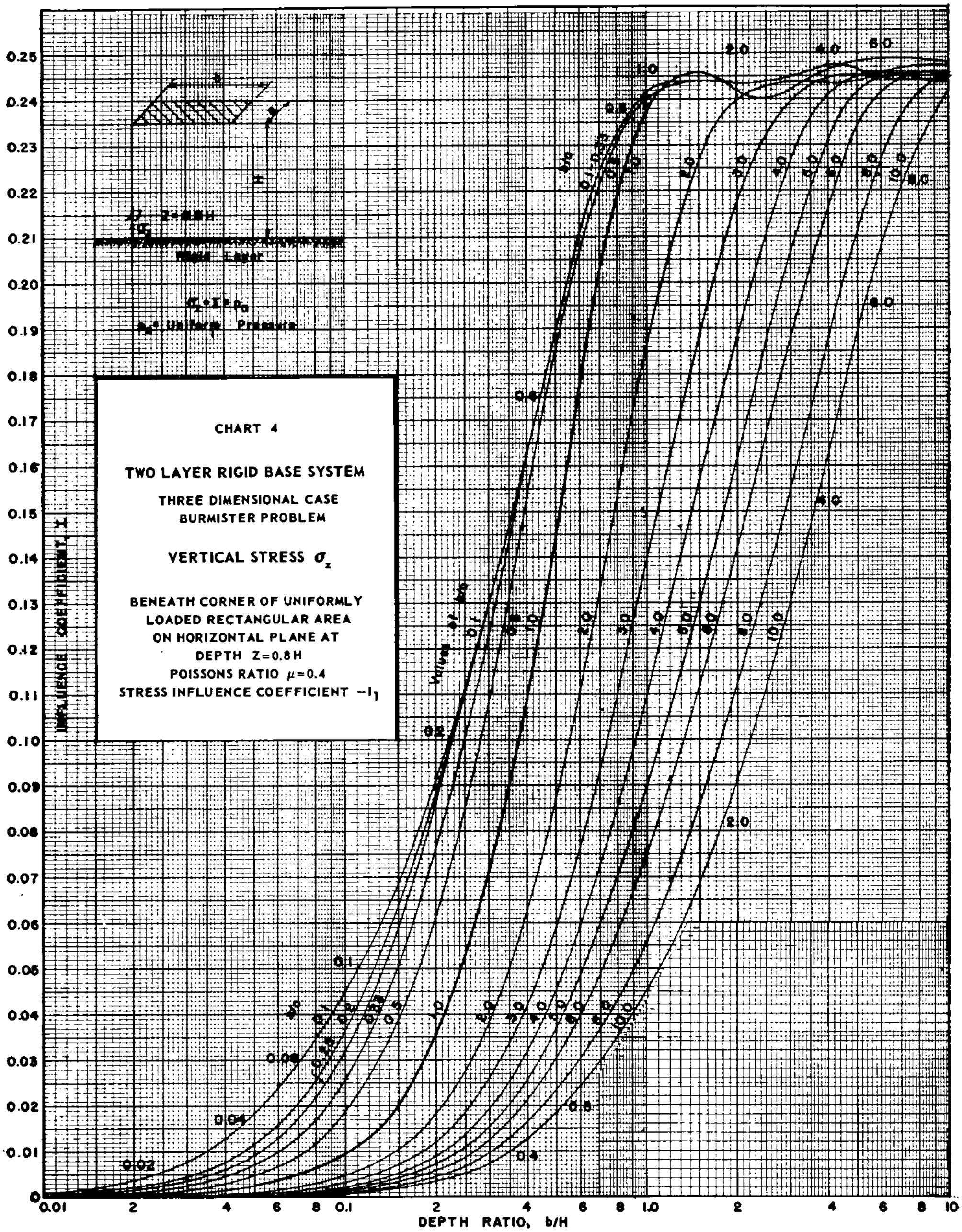


CHART 4
TWO LAYER RIGID BASE SYSTEM
THREE DIMENSIONAL CASE
BURMISTER PROBLEM
VERTICAL STRESS σ_z
BENEATH CORNER OF UNIFORMLY
LOADED RECTANGULAR AREA
ON HORIZONTAL PLANE AT
DEPTH Z=0.8H
POISSONS RATIO μ=0.4
STRESS INFLUENCE COEFFICIENT $-I_1$
INFLUENCE COEFFICIENT, I
DEPTH RATIO, b/H
Rigid Layer
$\sigma_z = I \cdot p_0$
p_0 = Uniform Pressure
0 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.10 0.11 0.12 0.13 0.14 0.15 0.16 0.17 0.18 0.19 0.20 0.21 0.22 0.23 0.24 0.25
0.01 2 4 6 8 0.1 2 4 6 8 1.0 2 4 6 8 10

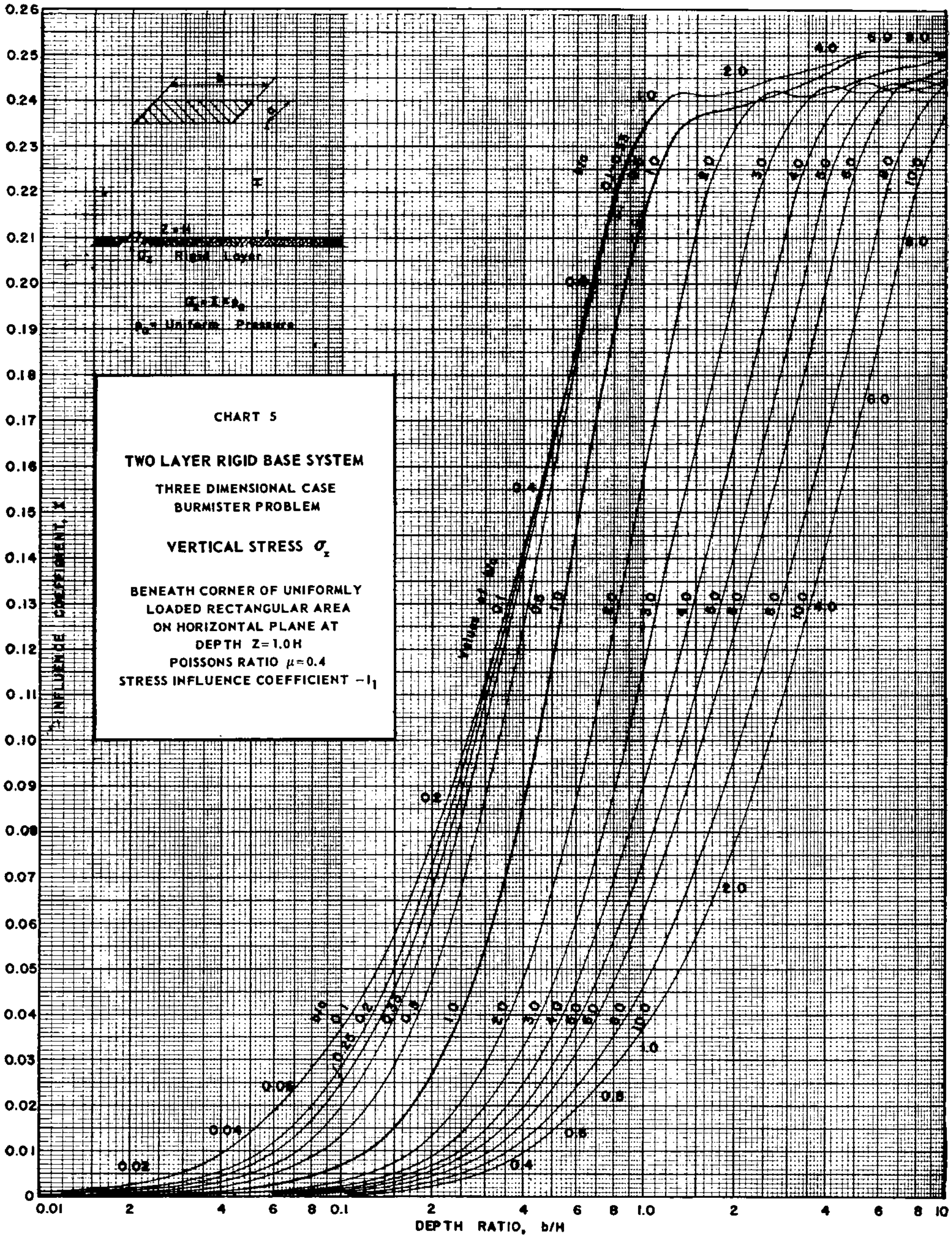


CHART 5
TWO LAYER RIGID BASE SYSTEM
THREE DIMENSIONAL CASE
BURMISTER PROBLEM
VERTICAL STRESS σ_z
BENEATH CORNER OF UNIFORMLY
LOADED RECTANGULAR AREA
ON HORIZONTAL PLANE AT
DEPTH Z=1.0H
POISSONS RATIO μ=0.4
STRESS INFLUENCE COEFFICIENT $-I_1$
Rigid Layer
Uniform Pressure
INFLUENCE COEFFICIENT
DEPTH RATIO, b/H
0.26
0.25
0.24
0.23
0.22
0.21
0.20
0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0.01
2
4
6
8
0.1
2
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1.0
2
4
6
8
10

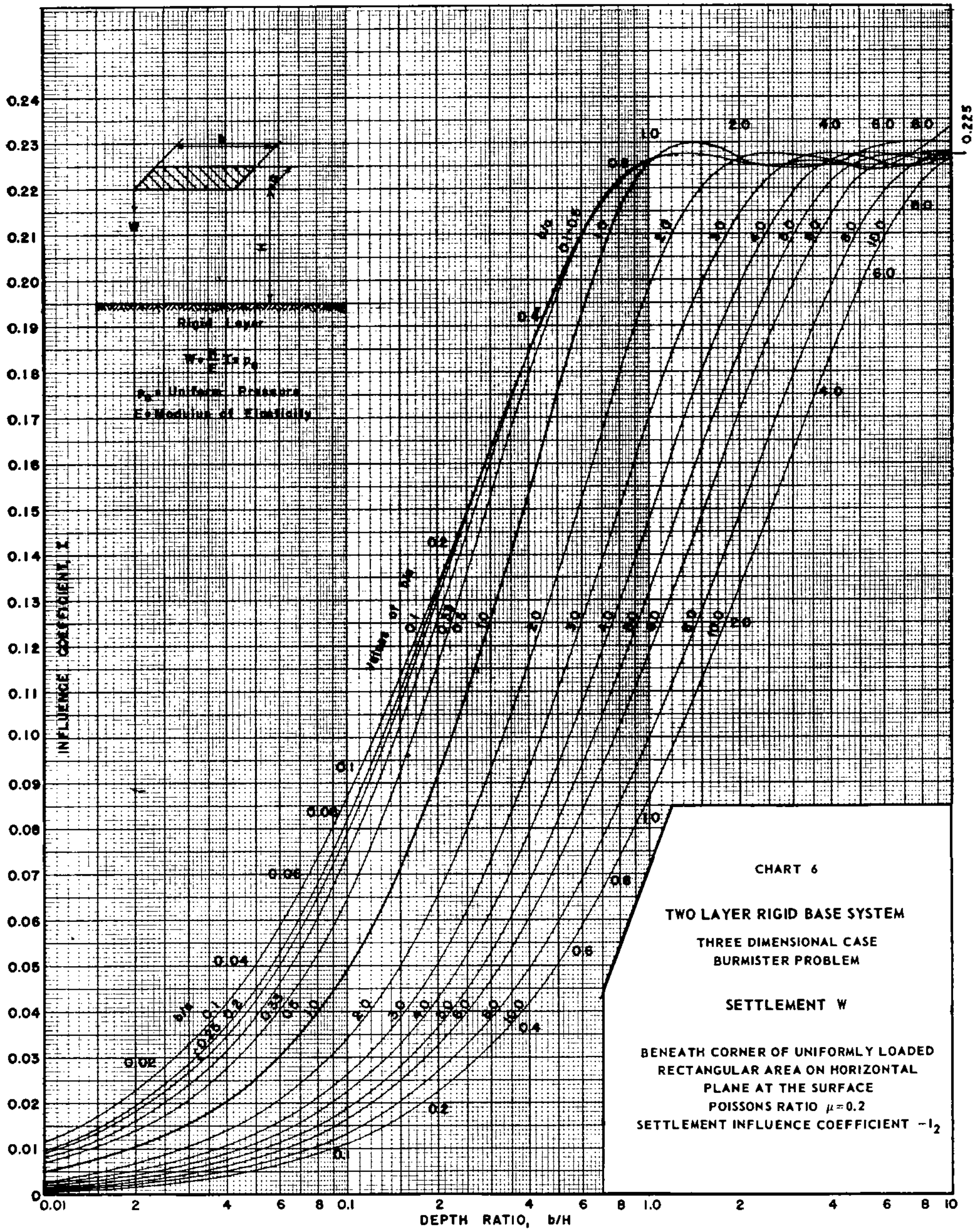


CHART 6
TWO LAYER RIGID BASE SYSTEM
THREE DIMENSIONAL CASE
BURMISTER PROBLEM
SETTLEMENT W
BENEATH CORNER OF UNIFORMLY LOADED RECTANGULAR AREA ON HORIZONTAL PLANE AT THE SURFACE
POISSONS RATIO $\mu = 0.2$
SETTLEMENT INFLUENCE COEFFICIENT $-I_2$
INFLUENCE COEFFICIENT, K
DEPTH RATIO, b/H
Rigid Layer
p_o = Uniform Pressure
E = Modulus of Elasticity
Values of a/b
0.225

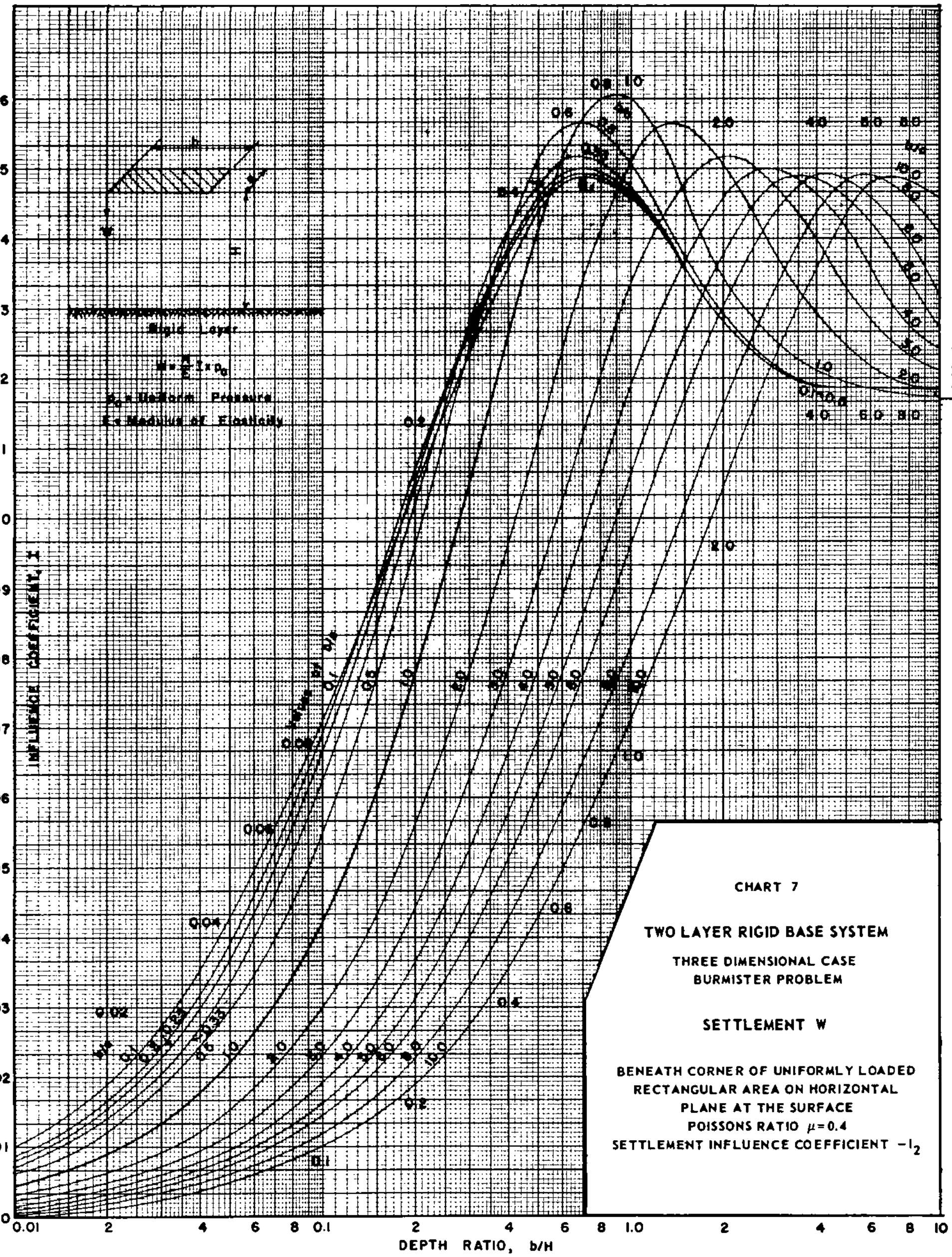


CHART 7
TWO LAYER RIGID BASE SYSTEM
THREE DIMENSIONAL CASE
BURMISTER PROBLEM
SETTLEMENT W
BENEATH CORNER OF UNIFORMLY LOADED
RECTANGULAR AREA ON HORIZONTAL
PLANE AT THE SURFACE
POISSONS RATIO μ=0.4
SETTLEMENT INFLUENCE COEFFICIENT − I2
INFLUENCE COEFFICIENT, I
DEPTH RATIO, b/H
Rigid Layer
0.1167

APPENDIX B

Group B—Influence Charts and Tables

Stresses beneath the corner of a circular influence diagram for any shape of loaded area, located anywhere within the limits of the diagram, and for any distribution of foundation pressure. Stresses for Poisson's ratio of 0.4 and at depths of $z = 0.2h$, $0.4h$, $0.6h$, $0.8h$, $1.0h$.

Horizontal Stresses, σ_x and σ_y
- Influence Coefficients—I_3
  - Tables of Influence Coefficients, Tables B-I_3
  - Circular Influence Diagram, Chart 8
- Influence Coefficients—I_4
  - Tables of Influence Coefficients, Tables B-I_4
  - Circular Influence Diagram, Chart 9

Shearing Stresses, τ_{xy} and τ_{yx}
- Influence Coefficients—I_5
  - Tables of Influence Coefficients, Tables B-I_5
  - Circular Influence Diagram, Chart 10

Shearing Stresses, τ_{zx} and τ_{zy}
- Influence Coefficients—I_6
  - Tables of Influence Coefficients, Tables B-I_6
  - Circular Influence Diagram, τ_{zx}, Chart 11
  - Circular Influence Diagram, τ_{zy}, Chart 12

Where the applications of the principles of superposition are not simple, as is the case for horizontal and shearing stresses, circular influence diagram methods permit making graphic integrations with respect to the argument, θ, for any shape of loaded area, located anywhere within the limits of the diagram, and for any distribution of foundation loading. This situation arises from the nature of the integrations with respect to θ in cylindrical coordinates in Equations 9 through 12, where vector components have to be used to obtain stresses in the (x, y, z) directions in accordance with the notations in Figure 2, and where two influence coefficients are involved in the stress equations with different θ-functions.

Notations—

Stress direction and signs—Figure 2.

n = Number of different loaded areas of the same dimensions and average pressure, p_a.

p_a = Sum of column loads divided by the foundation area for each different foundation loading used in the application of the principles of superposition.

I = Influence coefficient for each numbered *ring* given in the appropriate Tables of Influence Coefficients, B-I for the Ring Numbers shown in Charts 8 through 12. Three influence scales are given for each chart to cover a range of conditions of foundation sizes from $r/h = 0$ to 0.6, 0 to 3.0, and 0 to 15.0.

R = Ring intercept reading interpolated with respect to a foundation plan for graphic integrations, which takes into account the θ-functions of Equations 9 through 12 and carrying the same subscript as the influence coefficient, I. The interpolation methods are shown in the illustrative examples.

z = The influence coefficients for each depth of $0.2h$, $0.4h$, $0.6h$, $0.8h$, and $1.0h$ are obtained from the appropriate Table B-I for corresponding ring numbers for computation tables as given in the illustrative examples.

Horizontal Stresses. Equations 10a and 10b.

$$\sigma_x = -\sum \underset{\text{Chart 8}}{np_a(I_3R_3)} + \underset{\text{Chart 9}}{np_a(I_4R_4)} \quad (15)$$

$$\sigma_y = -\sum np_a(I_3R_3) - np_a(I_4R_4) \quad (16)$$

Note: n is additive from Figure 2.

Shearing Stresses. Equations 11 and 12.

$$\tau_{xy} = \tau_{yx} = -\sum np_a(I_5R_5) \quad (17)$$

Chart 10

$$\tau_{zx} = \tau_{xz} = -\sum np_a(I_6R_{6a}) \quad (18)$$

Chart 11 (Sin θ-function)

$$\tau_{zy} = \tau_{yz} = -\sum np_a(I_6R_{6b}) \quad (19)$$

Chart 12 (1-Cos θ)-function

Note: For Shearing Stresses n may be plus or minus in accordance with the direction and sign notations of Figure 2.

Table B-1. Illustrative Examples Determination of horizontal and shearing stresses at a point beneath a loaded foundation in a two-layer rigid base soil system.

Steps in the analysis of imposed foundation stress conditions.

1) Selection of r/h—Scale and foundation scale for use with Charts 8 through 12.

Depth to Rigid Base layer, $h = 40'$.
Maximum $r/h = 100/40 = 2.5$.
Use influence scale—$3\ r/h \equiv 15$ inches.
$15''$ is length of circular chart.
Then 1.0 inches $\equiv 0.2\ r/h$.
Column Bay Scale—$20/40 = 0.5\ r/h$.
Note: Circular influence Charts 8 through 12 can be drawn to any enlarged scale desired.

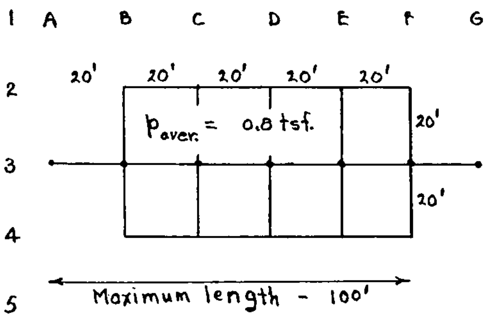


Figure a. Foundation plan.

2) Foundation Plan: Draw foundation plan carefully on tracing paper to this scale—$3\ r/h \equiv 15$ inches or 1.0 inches $\equiv 0.2\ r/h$. This plan is to be used for determinations of all the stresses of Equations 15 through 19. Show all areas of different uniformly distributed pressures if the average pressures vary significantly over the foundation area due to heavier contributory column loads.

3) Select the points or columns where it is desired to investigate the imposed foundation stresses beneath the foundation, or where the stresses may be critical for some reason. Otherwise use a symmetrical plan of points, which greatly shortens the computational work involved in the stress investigations.

4) Location and Definition of Foundation Areas—Figure b. Lay out to the above scales the foundation areas contributory to each point, beneath which stresses are to be determined with appropriate and complete notations, as noted in Figure b, in order to facilitate and to expedite the work.

5) Stress Analyses: Locate each point 0 of the foundation areas of Figure b over the corner of the circular influence chart appropriate for the stress under investigation. Determine the influence values—$np_a(IR)$ for each set of areas about the point having the same dimensions and pressure loading. For example, Point C-3 equal to Point E-3, is illustrated as follows. Area 2 is folded over on to Area 1 to facilitate the work, as shown. Repeat the analysis for each depth, z, and for each stress, using the appropriate influence chart and table of influence coefficients. For the horizontal stresses the area effects are additive in accordance with the Figure 2 notations. But for the shearing stresses, the plus and minus areas must be carefully worked out and noted with respect to the stress directions, as noted in Figures d and e. The work should be carefully and logically organized and systematically carried out, in order to provide adequate facilities for checking the work and to expedite it.

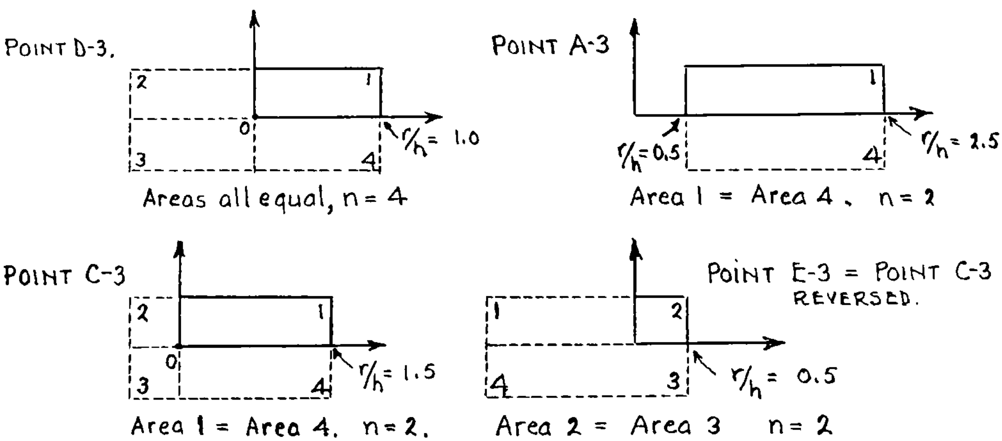


Figure b. Foundation areas and notations.

6) Stress Computations and Estimates: Select the appropriate circular influence chart from the Group B charts for the stress under consideration.

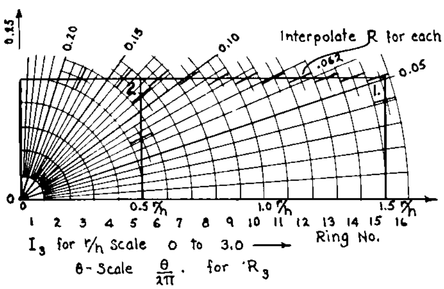
Locate the point 0 of each foundation area of Figure b drawn on tracing paper to the same scale as that of the circular influence chart, as indicated under Step (2) and determine the ring intercept readings, R at the center of each ring (light line arc) for each set of areas having the same dimensions and pressure loading.

This set of R-readings from the circular influence chart for the stress under consideration will serve for all depths, $z = 0.2h$, $0.4h$, $0.6h$, $0.8h$, and $1.0h$. Take the influence values, I from the appropriate Table B-I and tabulate in suitable form for the stress computation, as indicated below, involving computations—$\sum np_a(IR)$.

The work should be organized in table form to compute the products (IR) or $p_a(IR)$ for all depths for the single set of R-readings obtained for the stress under consideration. Repeat for other stresses, as desired.

Figure c. Computations for horizontal stresses, σ_x and σ_y at point C-3.

Chart 8 for ring reading—R_3



Area 2: Ring 6 R = (.178 − .072) = .106
Ring 7 R = (.078 − .052) = .026
Area 1: Ring 16 R = (.051 − .041) = .010

TABLE B-I_3 FOR INFLUENCE VALUE
Computation of $\sum I_3R_3$

Ring	Areas 1–4 I_3	Areas 1–4 R_3	Areas 1–4 I_3R_3	Areas 2–3 R_3	Areas 2–3 I_3R_3
1	0.0024	0.25	0.0006	0.25	0.0006
2	0.0722	0.25	0.0181	0.25	0.0181
3	0.1104	0.25	0.0276	0.25	0.0276
4	0.0881	0.25	0.0221	0.25	0.0221
5	0.0648	0.25	0.0162	0.25	0.0162
6	0.0434	0.176	0.0077	0.106	0.0048
7	0.0278	0.138	0.0039	0.026	0.0008
8	0.0173	0.113	0.0020		
9	0.0105	0.098	0.0011		
10	0.0070	0.085	0.0006		
11	0.0044	0.076	0.0003		
12	0.0039	0.069	0.0003		
13	0.0039	0.062	0.0003		
14	0.0042	0.058	0.0002		
15	0.0048	0.055	0.0002		
16	0.0057	0.010	0.0000		
	$\sum I_3R_3$		0.1012		0.0902

Chart 9 for ring reading—R_4

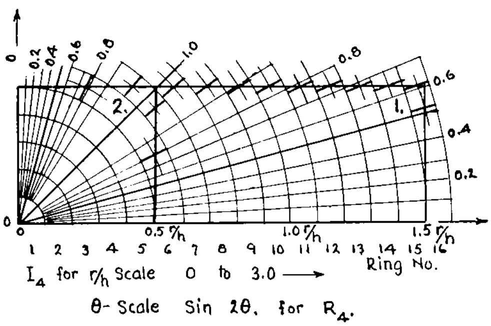


TABLE B-I_4 FOR INFLUENCE VALUES
Computation of $\sum I_4R_4$

Ring	Areas 1–4 I_4	Areas 1–4 R_4	Areas 2–3 I_4R_4	Areas 2–3 $R_4I_4R_4$
1	0.0012	0		0
2	0.0074	0		0
3	0.0098	0		0
4	0.0082	0		0
5	0.0062	0		0
6	0.0045	0.72	0.0032	0
7	0.0035	0.96	0.0033	0
8	0.0027	0.98	0.0026	
9	0.0022	0.93	0.0020	
10	0.0019	0.90	0.0017	
11	0.0017	0.84	0.0014	
12	0.0015	0.78	0.0012	
13	0.0014	0.73	0.0010	
14	0.0013	0.69	0.0009	
15	0.0012	0.65	0.0008	
16	0.0011	0.14	0.0001	
	$\sum I_4R_4$		0.0182	0

Horizontal Stresses: Equations 15 and 16. Stresses at point C-3.

$$\sigma_x = -\sum [2 \times 0.8 \times (0.1012 + 0.0902 + 0.0182 + 0)] = -0.335 \ tsf.$$

$$\sigma_y = -\sum [2 \times 0.8 \times (0.1012 + 0.0902 - 0.0182 - 0)] = -0.277 \ tsf.$$

Figure d. Computations for shearing stresses, τ_{xy} and τ_{yx} at point C-3.

Chart 10 for ring reading—R_5

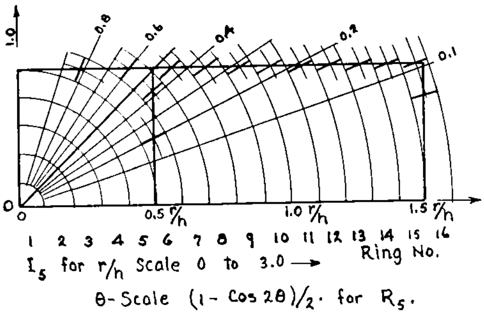


TABLE B-I₅ FOR INFLUENCE VALUES

Computation of $\sum I_5 R_5$

Ring	Areas 1–4			Areas 2–3	
	I_5	R_5	I_5R_5	R_5	I_5R_5
1	0.0024	1.0	0.0024	1.0	0.0024
2	0.0148	1.0	0.0148	1.0	0.0148
3	0.0195	1.0	0.0195	1.0	0.0195
4	0.0165	1.0	0.0165	1.0	0.0165
5	0.0124	1.0	0.0124	1.0	0.0124
6	0.0091	0.82	0.0075	0.62	0.0057
7	0.0069	0.60	0.0041	0.19	0.0013
8	0.0054	0.45	0.0024		
9	0.0043	0.36	0.0016		
10	0.0038	0.28	0.0010		
11	0.0034	0.23	0.0008		
12	0.0030	0.19	0.0006		
13	0.0028	0.17	0.0005		
14	0.0026	0.14	0.0004		
15	0.0024	0.12	0.0003		
16	0.0022	0.11	0.0002		
			0.0850		0.0726

Sign notation for shearing stresses

$$\tau_{xy} = \tau_{yx} = -0.8 \sum [\overleftarrow{+\,0.0856}^{1} - \overrightarrow{0.0856}^{4} + \overrightarrow{0.0726}^{2} - \overleftarrow{0.0726}^{3}] = 0$$

Figure e. Computations for shearing stresses, τ_{zx} and τ_{zy} at point C-3.

Chart 11 for ring reading—R_{6a}

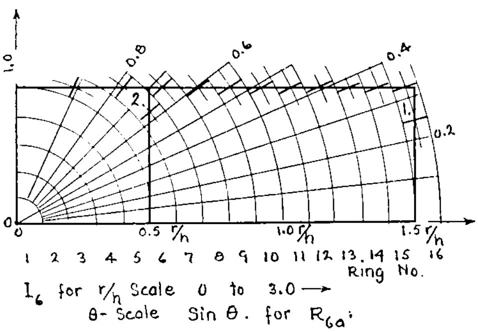


TABLE B-I₆ FOR INFLUENCE VALUES

Computation of $\sum I_6 R_{6a}$

Ring	Areas 1–4			Areas 2–3	
	I_6	R_{6a}	I_6R_{6a}	R_{6a}	I_6R_{6a}
1	+0.0142	1.0	+0.0142	1.0	0.0142
2	0.0418	1.0	0.0418	1.0	0.0418
3	0.0351	1.0	0.0351	1.0	0.0351
4	0.0208	1.0	0.0208	1.0	0.0208
5	0.0119	1.0	0.0119	1.0	0.0119
6	0.0065	0.91	0.0059	0.50	0.0032
7	0.0029	0.78	0.0023	0.16	0.0005
8	+0.0009	0.67	+0.0006		
9	−0.0004	0.59	−0.0002		
10	−0.0012	0.53	−0.0006		
11	0.0015	0.48	0.0007		
12	0.0017	0.44	0.0008		
13	0.0016	0.41	0.0007		
14	0.0015	0.38	0.0006		
15	0.0013	0.35	0.0004		
16	−0.0010	0.32	−0.0001		
	$\sum I_6R_{6a}$		0.1285		0.1275

Chart 12 for ring reading—R_{6b}

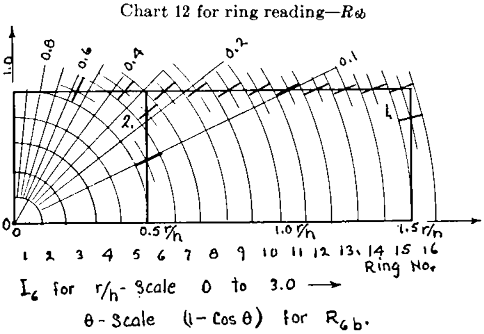


TABLE B-I₆ FOR INFLUENCE VALUES

Computation of $\sum I_6 R_{6b}$

Ring	Areas 1–4			Areas 2–3	
	I_6	R_{6b}	I_6R_{6b}	R_{6b}	I_6R_{6b}
1	+0.0142	1.0	+0.0142	1.0	0.0142
2	0.0418	1.0	0.0418	1.0	0.0418
3	0.0351	1.0	0.0351	1.0	0.0351
4	0.0208	1.0	0.0208	1.0	0.0208
5	0.0119	1.0	0.0119	1.0	0.0119
6	0.0065	0.58	0.0039	0.49	0.0032
7	0.0029	0.35	0.0010	0.14	0.0004
8	+0.0009	0.24	+0.0002		
9	−0.0004	0.19	−0.0001		
10	−0.0012	0.15	−0.0002		
11	0.0015	0.13	0.0002		
12	0.0017	0.10	0.0002		
13	0.0016	0.08	0.0001		
14	0.0015	0.07	0.0001		
15	0.0013	0.06	0.0001		
16	−0.0010	0.02	−0.0000		
	$\sum I_6R_{6b}$		0.1279		0.1274

Sign notation for shearing stresses

$$\tau_{zx} = \tau_{xz} = -0.8 \sum [\overleftarrow{+\,0.1285}^{1} + \overleftarrow{0.1285}^{4} - \overrightarrow{0.1275}^{2} - \overrightarrow{0.1275}^{3}]$$

$$= -0.0016$$

$$\tau_{zy} = \tau_{yz} = -0.8 \sum [\overleftarrow{+\,0.1279}^{1} - \overrightarrow{0.1279}^{4} + \overleftarrow{0.1274}^{2} - \overrightarrow{0.1274}^{3}] = 0$$

TABLE B-I_3

STRESS INFLUENCE COEFFICIENTS—I_3 FOR HORIZONTAL STRESSES σ_x AND σ_y
TWO-LAYER RIGID BASE SYSTEM. BURMISTER PROBLEM

Depth.....	$z = 0.2h$			$z = 0.4h$			$z = 0.6h$			$z = 0.8h$			$z = 1.0h$		
r/h (max)..	0.6	3.0	15.0	0.6	3.0	15.0	0.6	3.0	15.0	0.6	3.0	15.0	0.6	3.0	15.0
Ring No.															
1	−0.0012	+0.0024	+0.3379	−0.0001	−0.0024	+0.1307	+0.0000	+0.0005	+0.0894	+0.0002	+0.0051	+0.1319	+0.0003	+0.0146	+0.2697
2	−0.0008	0.0722	0.1059	0.0004	+0.0050	0.1253	0.0001	0.0068	0.2429	0.0006	0.0159	0.2572	0.0019	0.0415	0.2789
3	−0.0002	0.1104	0.0218	0.0007	0.0269	0.1094	0.0002	0.0141	0.1523	0.0010	0.0270	0.1419	0.0030	0.0617	0.0749
4	+0.0007	0.0881	0.0259	0.0008	0.0462	0.0577	0.0003	0.0270	0.0657	0.0014	0.0375	0.0506	0.0042	0.0747	0.0078
5	+0.0039	0.0648	0.0421	0.0006	0.0550	0.0397	0.0005	0.0410	0.0335	0.0019	0.0468	0.0231	0.0052	0.0776	0.0043
6	+0.0081	+0.0434	+0.0363	−0.0003	+0.0554	+0.0294	+0.0010	+0.0459	+0.0232	+0.0023	+0.0535	+0.0194	+0.0063	+0.0743	+0.0075
7	0.0116	0.0278	0.0276	+0.0002	0.0512	0.0228	0.0012	0.0513	0.0210	0.0027	0.0542	0.0179	0.0074	0.0665	0.0075
8	0.0151	0.0173	0.0187	0.0009	0.0455	0.0168	0.0015	0.0516	0.0104	0.0032	0.0536	0.0150	0.0084	0.0565	0.0059
9	0.0175	0.0105	0.0133	0.0017	0.0393	0.0118	0.0019	0.0489	0.0084	0.0036	0.0505	0.0120	0.0093	0.0452	0.0042
10	0.0199	0.0070	0.0090	0.0025	0.0339	0.0081	0.0022	0.0450	0.0066	0.0041	0.0454	0.0089	0.0101	0.0358	0.0027
11	+0.0217	+0.0044	+0.0062	+0.0035	+0.0290	+0.0050	+0.0026	+0.0407	+0.0044	+0.0046	+0.0399	+0.0061	+0.0109	+0.0269	+0.0016
12	0.0225	0.0039	0.0041	0.0046	0.0250	0.0036	0.0030	0.0350	0.0028	0.0050	0.0333	0.0041	0.0117	0.0196	0.0005
13	0.0228	0.0039	0.0027	0.0056	0.0214	0.0023	0.0034	0.0297	0.0018	0.0054	0.0272	0.0024	0.0124	0.0136	0.0002
14	0.0223	0.0042	0.0017	0.0065	0.0191	0.0015	0.0038	0.0250	0.0012	0.0058	0.0230	0.0011	0.0131	0.0091	+0.0003
15	0.0211	0.0048	0.0009	0.0073	0.0158	0.0008	0.0042	0.0217	0.0006	0.0062	0.0185	+0.0002	0.0136	0.0059	−0.0001
16	+0.0199	+0.0057	+0.0006	+0.0080	+0.0140	+0.0005	+0.0046	+0.0178	+0.0003	+0.0066	+0.0141	−0.0004	+0.0141	+0.0035	−0.0002
17	0.0188	0.0067	0.0003	0.0086	0.0126	0.0002	0.0051	0.0152	0.0001	0.0071	0.0119	0.0008	0.0146	0.0020	0.0002
18	0.0177	0.0076	0.0002	0.0092	0.0113	0.0001	0.0057	0.0127	+0.0000	0.0075	0.0095	0.0011	0.0150	0.0011	0.0002
19	0.0165	0.0080	+0.0000	0.0097	0.0105	−0.0001	0.0063	0.0109	−0.0001	0.0079	0.0075	0.0013	0.0152	0.0007	0.0003
20	0.0156	0.0083	−0.0000	0.0101	0.0098	−0.0001	0.0070	0.0093	0.0002	0.0083	0.0069	0.0014	0.0154	0.0005	0.0003
21	+0.0147	+0.0086	−0.0001	+0.0106	+0.0092	−0.0002	+0.0078	+0.0080	−0.0002	+0.0087	+0.0054	−0.0014	+0.0155	+0.0005	−0.0003
22	0.0138	0.0087	0.0001	0.0109	0.0086	0.0002	0.0085	0.0073	0.0002	0.0091	0.0049	0.0014	0.0156	0.0007	0.0002
23	0.0130	0.0086	0.0002	0.0111	0.0080	0.0002	0.0089	0.0065	0.0002	0.0094	0.0045	0.0013	0.0156	0.0009	0.0002
24	0.0121	0.0084	0.0002	0.0112	0.0074	0.0002	0.0091	0.0060	0.0002	0.0097	0.0042	0.0013	0.0155	0.0010	0.0002
25	0.0112	0.0082	0.0002	0.0113	0.0070	0.0002	0.0095	0.0054	0.0002	0.0100	0.0041	0.0012	0.0154	0.0012	0.0002
26	+0.0103	+0.0078	−0.0002	+0.0113	+0.0066	−0.0002	+0.0094	+0.0052	−0.0002	+0.0103	+0.0040	−0.0012	+0.0153	+0.0013	−0.0002
27	0.0095	0.0076	0.0002	0.0113	0.0062	0.0002	0.0096	0.0048	0.0001	0.0106	0.0039	0.0012	0.0151	0.0014	0.0001
28	0.0098	0.0072	0.0001	0.0111	0.0059	0.0001	0.0098	0.0046	0.0001	0.0107	0.0039	0.0011	0.0149	0.0015	0.0001
29	0.0080	0.0068	0.0001	0.0110	0.0057	0.0001	0.0099	0.0044	0.0001	0.0108	0.0038	0.0010	0.0146	0.0016	0.0001
30	0.0073	0.0065	0.0001	0.0109	0.0055	0.0001	0.0100	0.0042	0.0001	0.0109	0.0038	0.0009	0.0144	0.0017	0.0001

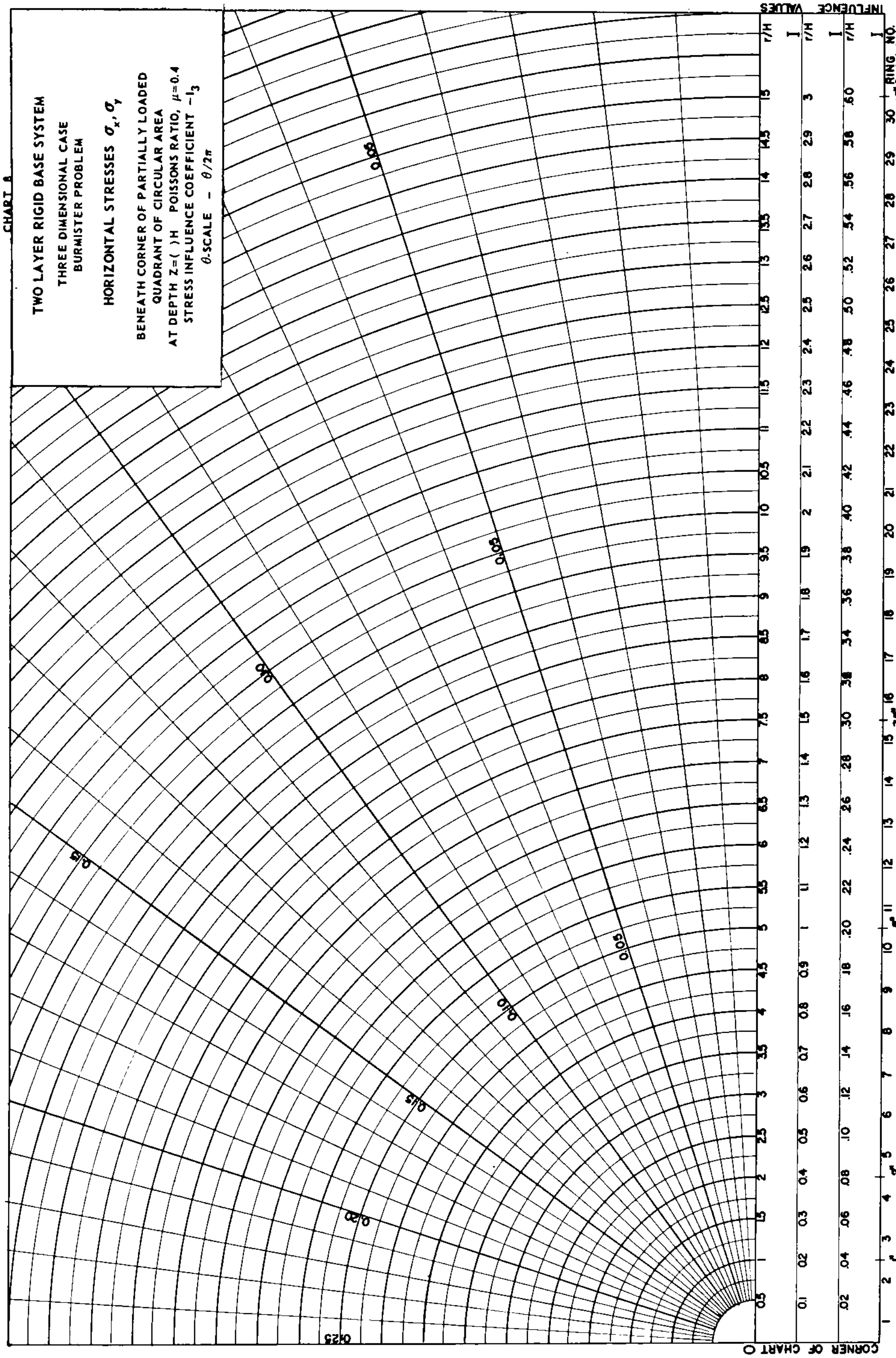


CHART 8
TWO LAYER RIGID BASE SYSTEM
THREE DIMENSIONAL CASE
BURMISTER PROBLEM
HORIZONTAL STRESSES σ_x, σ_y
BENEATH CORNER OF PARTIALLY LOADED
QUADRANT OF CIRCULAR AREA
AT DEPTH Z=( )H POISSONS RATIO, μ=0.4
STRESS INFLUENCE COEFFICIENT $-I_3$
θ-SCALE - $\theta/2\pi$
INFLUENCE VALUES
r/H
I
RING NO.
15" Scale in Inches
CORNER OF CHART O
0.05
0.10
0.15
0.20
0.25

TABLE B-I₄

STRESS INFLUENCE COEFFICIENTS—I_4 FOR HORIZONTAL STRESSES σ_x AND σ_y
TWO-LAYER RIGID BASE SYSTEM. BURMISTER PROBLEM

Depth	$z = 0.2h$			$z = 0.4h$			$z = 0.6h$			$z = 0.8h$			$z = 1.0h$
r/h (max) ..	0.6	3.0	15.0	0.6	3.0	15.0	0.6	3.0	15.0	0.6	3.0	15.0	0.6 3.0 15.0
Ring No.													
1	+0.0000	+0.0012	+0.0328	+0.0000	+0.0001	+0.0138	+0.0000	+0.0001	+0.0062	+0.0000	+0.0000	+0.0025	All these stress influence coefficients are equal to zero since $\sigma_x = \sigma_y$ at $z = 1.0h$
2	0.0001	0.0074	0.0147	0.0000	0.0011	0.0207	0.0000	0.0004	0.0167	0.0000	0.0001	0.0108	
3	0.0001	0.0098	0.0071	0.0000	0.0029	0.0112	0.0000	0.0009	0.0118	0.0000	0.0005	0.0086	
4	0.0003	0.0082	0.0050	0.0000	0.0046	0.0060	0.0000	0.0020	0.0057	0.0000	0.0008	0.0042	
5	0.0007	0.0062	0.0034	0.0001	0.0051	0.0032	0.0000	0.0028	0.0027	0.0000	0.0011	0.0017	
6	+0.0011	+0.0045	+0.0028	+0.0001	+0.0050	+0.0022	+0.0001	+0.0032	+0.0015	+0.0000	+0.0017	+0.0010	
7	0.0013	0.0035	0.0016	0.0001	0.0047	0.0013	0.0001	0.0034	0.0009	0.0000	0.0021	0.0006	
8	0.0015	0.0027	0.0011	0.0002	0.0041	0.0009	0.0001	0.0035	0.0006	0.0000	0.0023	0.0005	
9	0.0017	0.0022	0.0007	0.0003	0.0037	0.0006	0.0001	0.0037	0.0004	0.0001	0.0024	0.0003	
10	0.0018	0.0019	0.0005	0.0004	0.0033	0.0004	0.0001	0.0033	0.0003	0.0001	0.0023	0.0002	
11	+0.0019	+0.0017	+0.0003	+0.0004	+0.0028	+0.0003	+0.0001	+0.0030	+0.0002	+0.0001	+0.0022	+0.0002	
12	0.0020	0.0015	0.0002	0.0005	0.0025	0.0002	0.0002	0.0027	0.0002	0.0001	0.0019	0.0001	
13	0.0020	0.0014	0.0002	0.0006	0.0022	0.0001	0.0002	0.0022	0.0001	0.0001	0.0017	0.0000	
14	0.0020	0.0013	0.0001	0.0007	0.0020	0.000	0.0002	0.0020	0.0001	0.0001	0.0015	+0.0001	
15	0.0019	0.0012	0.0000	0.0007	0.0017	0.0001	0.0003	0.0018	0.0000	0.0001	0.0013	−0.0000	
16	+0.0018	+0.0011	+0.0001	+0.0008	+0.0015	+0.0000	+0.0003	+0.0015	+0.0001	+0.0001	+0.0012	−0.0000	
17	0.0018	0.0011	0.0000	0.0009	0.0014	0.0001	0.0003	0.0013	0.0000	0.0002	0.0009	0.0001	
18	0.0016	0.0010	0.0001	0.0009	0.0012	0.0000	0.0004	0.0011	0.0001	0.0001	0.0008	0.0000	
19	0.0015	0.0009	0.0000	0.0010	0.0010	0.0000	0.0004	0.0010	0.0000	0.0002	0.0008	0.0000	
20	0.0015	0.0009	0.0001	0.0010	0.0009	0.0000	0.0005	0.0009	0.0000	0.0001	0.0006	0.0001	
21	+0.0014	+0.0008	0.0000	+0.0010	+0.0008		+0.0005	+0.0007		+0.0002	+0.0005	−0.0000	
22	0.0013	0.0008	0.0000	0.0010	0.0007		0.0005	0.0006		0.0002	0.0004	0.0000	
23	0.0012	0.0008	0.0000	0.0011	0.0006		0.0006	0.0005		0.0002	0.0003	0.0001	
24	0.0011	0.0007		0.0010	0.0006		0.0006	0.0005		0.0003	0.0003	0.0000	
25	0.0010	0.0006		0.0010	0.0005		0.0006	0.0004		0.0002	0.0003	0.0000	
26	+0.0010	+0.0006		+0.0010	+0.0005		+0.0006	+0.0004		+0.0003	+0.0002		
27	0.0010	0.0006		0.0010	0.0005		0.0006	0.0003		0.0003	0.0002		
28	0.0009	0.0005		0.0010	0.0004		0.0006	0.0003		0.0003	0.0002		
29	0.0008	0.0005		0.0010	0.0004		0.0007	0.0003		0.0004	0.0002		
30	0.0008	0.0004		0.0010	0.0004		0.0007	0.0002		0.0004	0.0001		

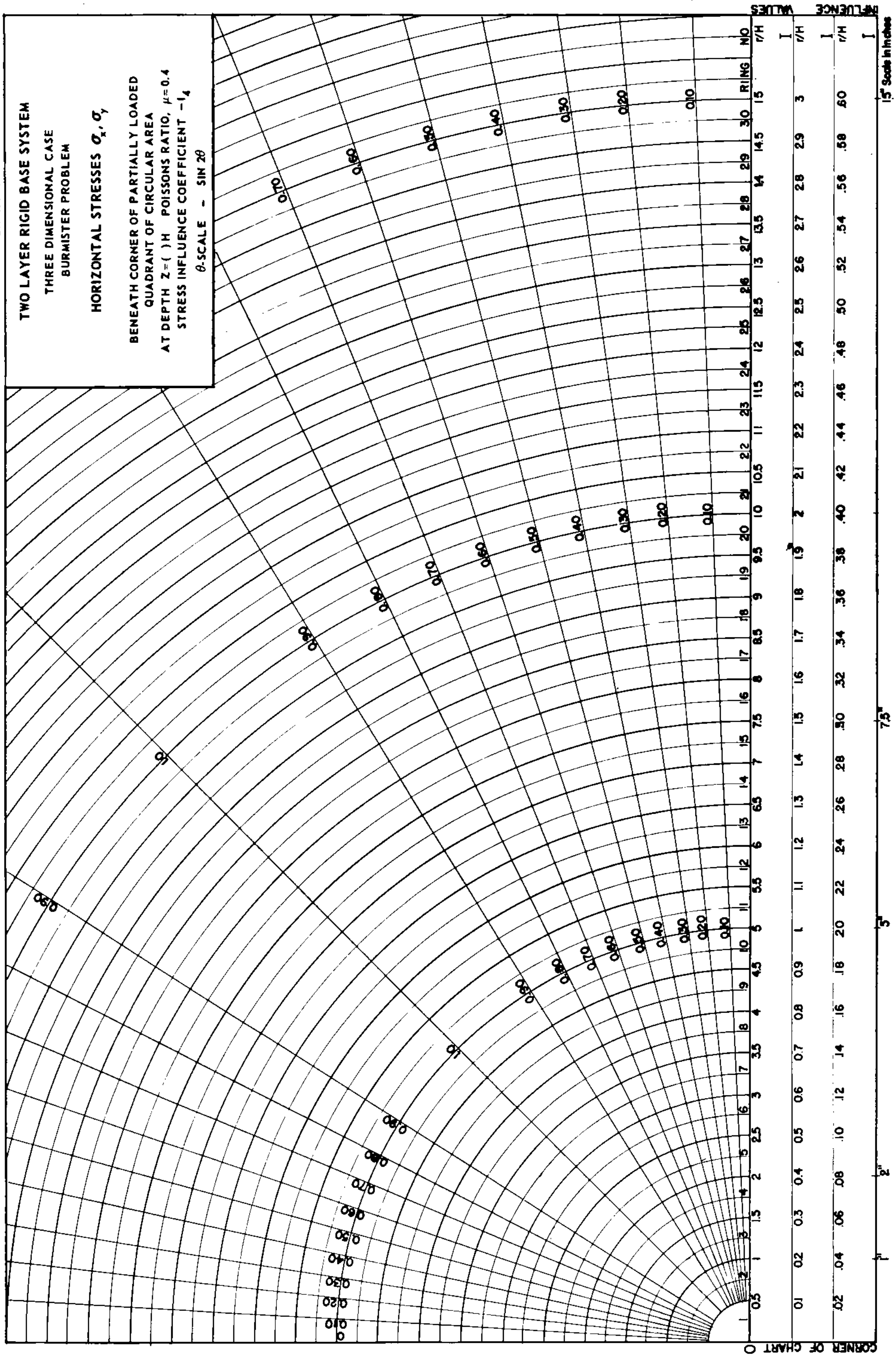


CHART 9
TWO LAYER RIGID BASE SYSTEM
THREE DIMENSIONAL CASE
BURMISTER PROBLEM
HORIZONTAL STRESSES σ_x, σ_y
BENEATH CORNER OF PARTIALLY LOADED
QUADRANT OF CIRCULAR AREA
AT DEPTH Z = ( )H POISSONS RATIO, μ = 0.4
STRESS INFLUENCE COEFFICIENT $-I_4$
θ-SCALE - SIN 2θ
RING NO
INFLUENCE VALUES
CORNER OF CHART O
1.5" Scale in Inches

TABLE B-I_5

STRESS INFLUENCE COEFFICIENTS—I_5 FOR SHEARING STRESSES $\tau_{xy} = \tau_{yx}$
TWO-LAYER RIGID BASE SYSTEM. BURMISTER PROBLEM

Depth.....	$z = 0.2h$			$z = 0.4h$			$z = 0.6h$			$z = 0.8h$			$z = 1.0h$
r/h (max)..	0.6	3.0	15.0	0.6	3.0	15.0	0.6	3.0	15.0	0.6	3.0	15.0	0.6 3.0 15.0
Ring No.													
1	+0.0000	+0.0024	+0.0655	+0.0000	+0.0002	+0.0275	+0.0000	+0.0001	+0.0124	+0.0000	+0.0000	+0.0050	All these stress influence coefficients are equal to zero since $\sigma_x = \sigma_y$ at $z = 1.0h$
2	0.0001	0.0148	0.0295	0.0000	0.0022	0.0415	0.0000	0.0009	0.0333	0.0000	0.0002	0.0215	
3	0.0002	0.0195	0.0142	0.0000	0.0058	0.0224	0.0000	0.0019	0.0236	0.0000	0.0010	0.0172	
4	0.0007	0.0165	0.0100	0.0001	0.0092	0.0120	0.0000	0.0039	0.0114	0.0000	0.0015	0.0083	
5	0.0014	0.0123	0.0068	0.0001	0.0102	0.0063	0.0001	0.0056	0.0053	0.0000	0.0023	0.0035	
6	+0.0021	+0.0091	+0.0055	+0.0002	+0.0100	+0.0044	+0.0001	+0.0064	+0.0030	+0.0000	+0.0034	+0.0019	
7	0.0027	0.0069	0.0033	0.0003	0.0093	0.0025	0.0001	0.0069	0.0018	0.0000	0.0042	0.0012	
8	0.0030	0.0054	0.0021	0.0004	0.0083	0.0018	0.0002	0.0071	0.0012	0.0001	0.0045	0.0010	
9	0.0033	0.0043	0.0014	0.0006	0.0074	0.0012	0.0002	0.0073	0.0009	0.0001	0.0047	0.0005	
10	0.0036	0.0038	0.0010	0.0007	0.0065	0.0008	0.0002	0.0066	0.0006	0.0002	0.0047	0.0004	
11	+0.0038	+0.0034	+0.0006	+0.0009	+0.0057	+0.0006	+0.0003	+0.0060	+0.0004	+0.0002	+0.0044	+0.0003	
12	0.0040	0.0030	0.0005	0.0010	0.0050	0.0004	0.0003	0.0053	0.0003	0.0002	0.0038	0.0002	
13	0.0040	0.0028	0.0003	0.0011	0.0044	0.0002	0.0004	0.0044	0.0002	0.0002	0.0033	0.0001	
14	0.0039	0.0026	0.0002	0.0013	0.0039	0.0002	0.0004	0.0041	0.0001	0.0002	0.0030	+0.0000	
15	0.0038	0.0024	0.0001	0.0015	0.0034	0.0001	0.0005	0.0035	0.0001	0.0002	0.0027	−0.0000	
16	+0.0037	+0.0022	+0.0001	+0.0017	+0.0030	+0.0001	+0.0006	+0.0031	+0.0001	+0.0003	+0.0023	−0.0001	
17	0.0035	0.0021	0.0001	0.0018	0.0027	0.0001	0.0007	0.0025	0.0001	0.0003	0.0018	0.0001	
18	0.0033	0.0020	0.0001	0.0018	0.0024	0.0000	0.0008	0.0022	0.0000	0.0003	0.0016	0.0000	
19	0.0031	0.0019	0.0001	0.0019	0.0021	0.0000	0.0009	0.0019	0.0000	0.0003	0.0014	0.0001	
20	0.0029	0.0018	0.0001	0.0020	0.0018	0.0000	0.0009	0.0017	0.0000	0.0003	0.0012	0.0000	
21	+0.0027	+0.0017	+0.0000	+0.0020	+0.0016		+0.0010	+0.0014		+0.0004	+0.0010	−0.0001	
22	0.0025	0.0016	0.0000	0.0021	0.0014		0.0011	0.0012		0.0004	0.0008	0.0000	
23	0.0024	0.0015	0.0000	0.0021	0.0012		0.0011	0.0010		0.0005	0.0006	0.0001	
24	0.0023	0.0014	0.0000	0.0020	0.0011		0.0012	0.0009		0.0005	0.0006	0.0000	
25	0.0021	0.0013	0.0000	0.0020	0.0010		0.0012	0.0008		0.0005	0.0005	0.0000	
26	+0.0020	+0.0012		+0.0020	+0.0010		+0.0012	+0.0007		+0.0006	+0.0005		
27	0.0019	0.0011		0.0020	0.0009		0.0012	0.0007		0.0006	0.0004		
28	0.0018	0.0010		0.0020	0.0009		0.0013	0.0006		0.0007	0.0004		
29	0.0017	0.0009		0.0020	0.0008		0.0013	0.0005		0.0007	0.0003		
30	0.0016	0.0009		0.0020	0.0008		0.0014	0.0005		0.0008	0.0003		

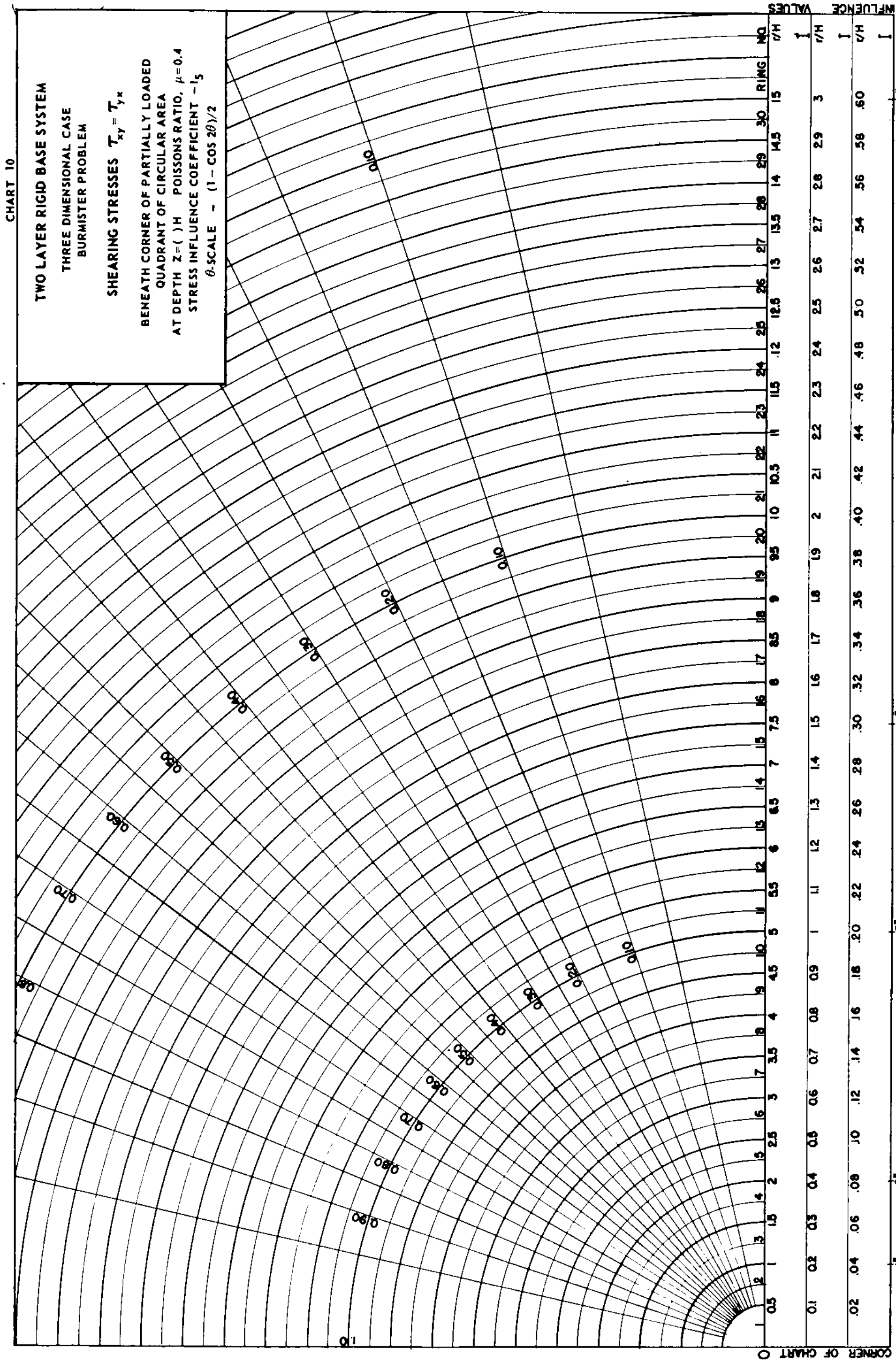


CHART 10
TWO LAYER RIGID BASE SYSTEM
THREE DIMENSIONAL CASE
BURMISTER PROBLEM
SHEARING STRESSES $\tau_{xy} = \tau_{yx}$
BENEATH CORNER OF PARTIALLY LOADED
QUADRANT OF CIRCULAR AREA
AT DEPTH Z=( )H POISSONS RATIO, $\mu = 0.4$
STRESS INFLUENCE COEFFICIENT – I_5
θ-SCALE – $(1 - \cos 2\theta)/2$
INFLUENCE VALUES
RING NO.
r/H
CORNER OF CHART O
15" Scale in Inches

TABLE B-I_6

STRESS INFLUENCE COEFFICIENTS—I_6 FOR SHEARING STRESSES, $\tau_{zx} = \tau_{xz}$ AND $\tau_{zy} = \tau_{yz}$
TWO-LAYER RIGID BASE SYSTEM. BURMISTER PROBLEM

Depth......	$z = 0.2$			$z = 0.4$			$z = 0.6$			$z = 0.8h$			$z = 1.0h$		
r/h (max)...	0.6	3.0	15.0	0.6	3.0	15.0	0.6	3.0	15.0	0.6	3.0	15.0	0.6	3.0	15.0
Ring No.															
1	+0.0001	+0.0142	+0.1238	+0.0000	+0.0022	+0.0703	+0.0000	+0.0006	+0.0355	+0.0000	+0.0002	+0.0192	+0.0000	+0.0002	+0.0207
2	0.0011	0.0418	+0.0087	0.0001	0.0116	+0.0310	0.0000	0.0039	0.0380	0.0000	0.0016	0.0400	0.0000	0.0016	0.0551
3	0.0025	0.0351	−0.0076	0.0004	0.0193	−0.0038	0.0001	0.0081	0.0105	0.0000	0.0038	0.0239	0.0000	0.0038	0.0364
4	0.0043	0.0208	−0.0029	0.0007	0.0202	−0.0006	0.0002	0.0110	0.0054	0.0001	0.0060	0.0123	0.0001	0.0064	0.0165
5	0.0061	0.0119	+0.0008	0.0010	0.0170	+0.0024	0.0003	0.0118	0.0048	0.0001	0.0076	0.0073	0.0001	0.0087	0.0082
6	+0.0075	+0.0065	+0.0016	+0.0015	+0.0126	+0.0029	+0.0004	+0.0109	+0.0041	+0.0002	+0.0085	+0.0050	+0.0002	+0.0105	+0.0053
7	0.0083	0.0029	0.0013	0.0019	0.0086	0.0023	0.0006	0.0093	0.0031	0.0002	0.0086	0.0036	0.0002	0.0113	0.0039
8	0.0087	+0.0009	0.0009	0.0023	0.0054	0.0017	0.0008	0.0075	0.0023	0.0003	0.0083	0.0027	0.0003	0.0116	0.0029
9	0.0088	−0.0004	0.0006	0.0027	0.0030	0.0011	0.0009	0.0058	0.0016	0.0004	0.0077	0.0020	0.0004	0.0112	0.0021
10	0.0085	0.0012	0.0004	0.0031	0.0014	0.0008	0.0011	0.0043	0.0011	0.0005	0.0070	0.0013	0.0005	0.0105	0.0015
11	+0.0081	−0.0015	+0.0003	+0.0034	+0.0002	+0.0005	+0.0013	+0.0032	+0.0008	+0.0006	+0.0062	+0.0009	+0.0006	+0.0094	+0.0010
12	0.0076	0.0017	0.0002	0.0037	−0.0009	0.0004	0.0015	0.0025	0.0005	0.0007	0.0047	0.0007	0.0007	0.0084	0.0007
13	0.0071	0.0016	0.0001	0.0039	0.0012	0.0003	0.0016	0.0019	0.0004	0.0008	0.0041	0.0004	0.0008	0.0072	0.0005
14	0.0064	0.0015	0.0001	0.0040	0.0010	0.0002	0.0018	0.0015	0.0002	0.0009	0.0035	0.0003	0.0009	0.0062	0.0003
15	0.0058	0.0013	0.0001	0.0041	0.0008	0.0001	0.0019	0.0013	0.0002	0.0010	0.0031	0.0002	0.0010	0.0052	0.0002
16	0.0052	−0.0010	+0.0000	+0.0041	−0.0005	0.0001	+0.0020	+0.0012	+0.0001	+0.0010	+0.0027	+0.0001	+0.0011	+0.0044	+0.0001
17	0.0046	0.0008	0.0001	0.0041	0.0003	0.0000	0.0021	0.0011	0.0001	0.0011	0.0024	0.0001	0.0012	0.0038	0.0001
18	0.0041	0.0006	0.0000	0.0041	0.0001	0.0001	0.0022	0.0011	0.0000	0.0012	0.0021	0.0000	0.0013	0.0032	0.0000
19	0.0037	0.0003	0.0000	0.0040	0.0000	0.0000	0.0023	0.0010	0.0000	0.0013	0.0019	0.0001	0.0014	0.0027	0.0001
20	0.0033	0.0002	0.0001	0.0039	+0.0002	0.0000	0.0023	0.0010	0.0001	0.0013	0.0017	0.0002	0.0015	0.0023	0.0000
21	0.0030	−0.0000	+0.0000	+0.0037	+0.0003		+0.0024	+0.0010	+0.0000	+0.0014	+0.0016	+0.0000	+0.0016	+0.0020	+0.0000
22	0.0026	+0.0001	0.0000	0.0036	0.0004		0.0024	0.0010	0.0000	0.0015	0.0014		0.0017	0.0018	0.0000
23	0.0023	0.0002		0.0034	0.0005		0.0024	0.0010	0.0000	0.0015	0.0013		0.0017	0.0016	
24	0.0021	0.0002		0.0032	0.0006		0.0024	0.0010		0.0016	0.0012		0.0018	0.0014	
25	0.0019	0.0003		0.0031	0.0006		0.0023	0.0009		0.0016	0.0013		0.0019	0.0013	
26	0.0016	+0.0003		0.0029	+0.0006		+0.0023	+0.0009		+0.0017	+0.0011		+0.0020	+0.0012	
27	0.0015	0.0003		0.0027	0.0006		0.0022	0.0009		0.0017	0.0010		0.0020	0.0011	
28	0.0013	0.0003		0.0025	0.0006		0.0022	0.0008		0.0017	0.0009		0.0021	0.0010	
29	0.0011	0.0003		0.0023	0.0006		0.0021	0.0008		0.0017	0.0009		0.0021	0.0010	
30	0.0010	0.0003		0.0022	0.0006		0.0021	0.0008		0.0017	0.0008		0.0022	0.0009	

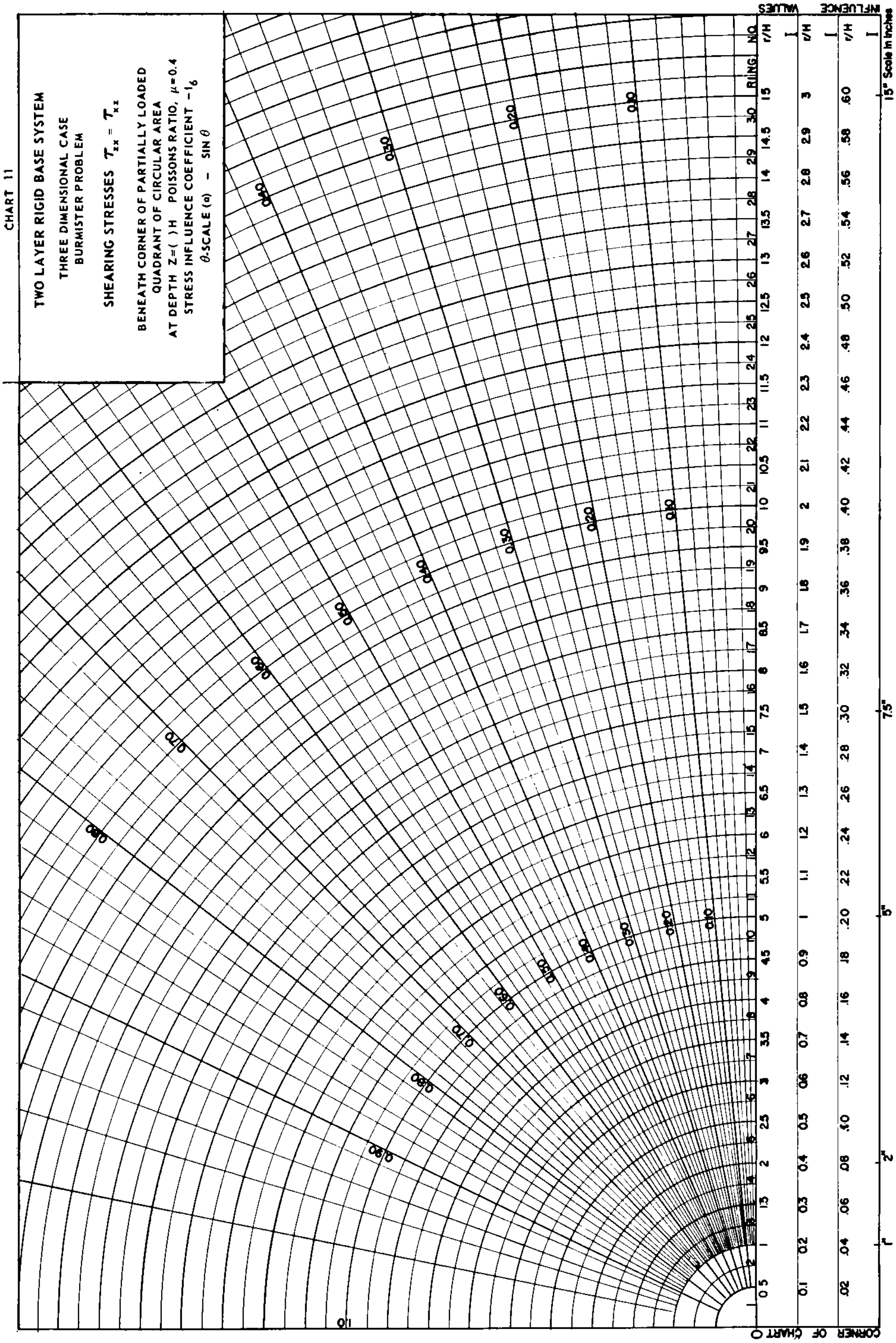


CHART 11
TWO LAYER RIGID BASE SYSTEM
THREE DIMENSIONAL CASE
BURMISTER PROBLEM
SHEARING STRESSES $\tau_{zx} = \tau_{xz}$
BENEATH CORNER OF PARTIALLY LOADED
QUADRANT OF CIRCULAR AREA
AT DEPTH Z=( ) H POISSONS RATIO, μ=0.4
STRESS INFLUENCE COEFFICIENT $-I_6$
θ-SCALE (α) – SIN θ
RING NO.
r/H
VALUES
INFLUENCE
CORNER OF CHART
15" Scale in Inches
7.5"
5"
2"
1"

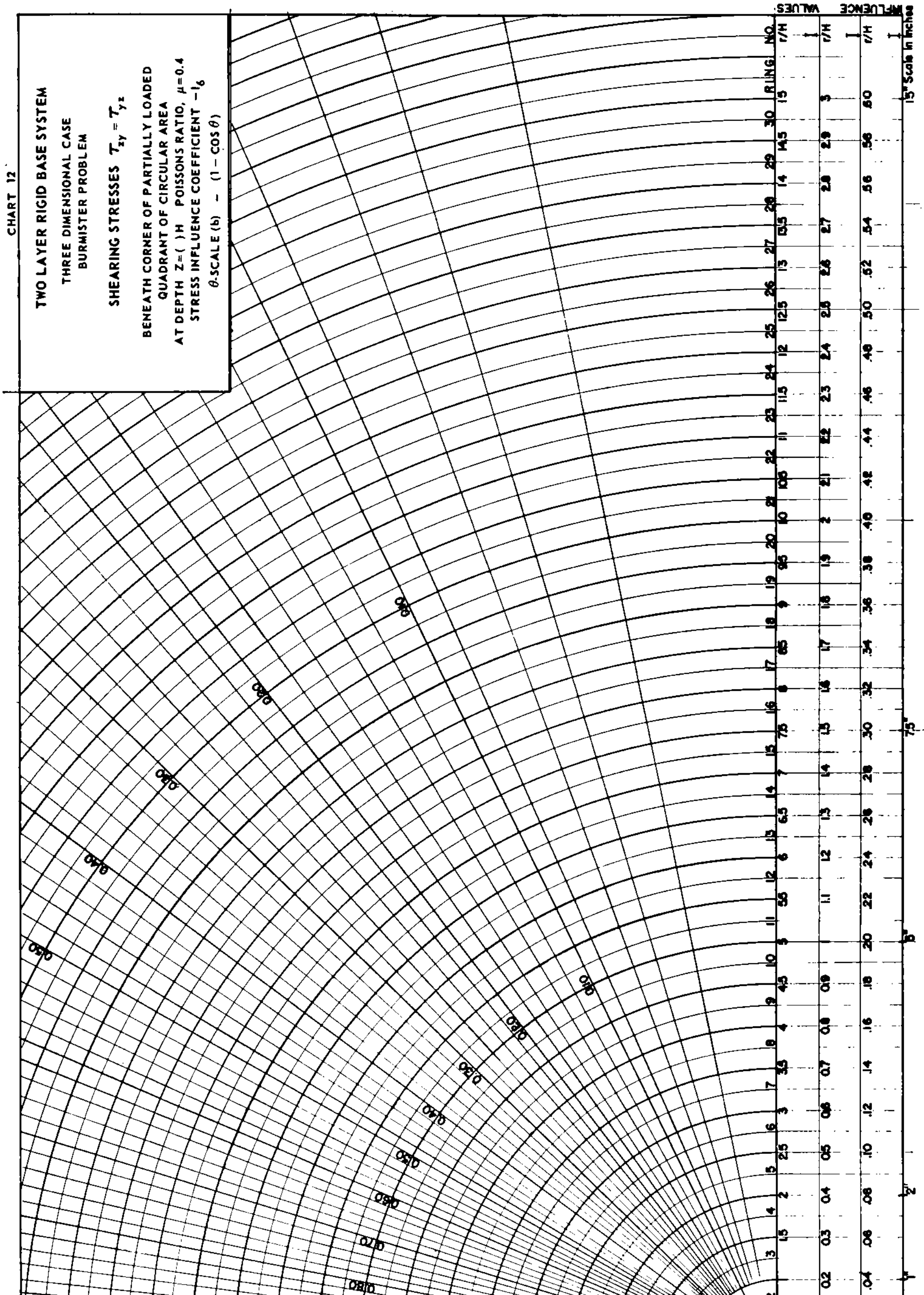


CHART 12
TWO LAYER RIGID BASE SYSTEM
THREE DIMENSIONAL CASE
BURMISTER PROBLEM
SHEARING STRESSES $\tau_{zy} = \tau_{yz}$
BENEATH CORNER OF PARTIALLY LOADED
QUADRANT OF CIRCULAR AREA
AT DEPTH Z=( )H POISSONS RATIO, μ=0.4
STRESS INFLUENCE COEFFICIENT $-I_6$
θ-SCALE (6) – (1 – COS θ)
RING NO.
r/H VALUES
INFLUENCE
CORNER OF CHART O
15" Scale in Inches