Subgrade Support Characteristics as Indicated by Measurements of Deflection and Strain

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● IN 1949 the State Highway Department of Indiana and the Bureau of Public Roads constructed an experimental road for the purpose of investigating means of preventing pavement pumping on highways that carry a high traffic volume with a normal distribution of heavy axle loads (*1*).

This project is located in northwest Indiana on a section of US 41. The test road is entirely within the Valparaiso morainic area, and is underlain by mixtures of silt and clays deposited during the Wisconsin glacial stage.

Each mile of the project has nine subbase sections as follows:

1. Soil-cement mixture 3 in. thick.
2. Soil-cement mixture 5 in. thick.
3. Open-graded crushed stone 3 in. thick.
4. Open-graded crushed stone 5 in. thick.
5. Open-graded crushed stone 8 in. thick.
6. Dense-graded crushed stone 3 in. thick.
7. Dense-graded crushed stone 5 in. thick.
8. Dense-graded crushed stone 8 in. thick.
9. Pavement on natural soil.

Many types of observations have been made to determine the differences in performance of the concrete pavements on the various subbase treatments. Among these are pavement roughness, differential levels, visual inspections, and moisture cell readings (*2*).

Included in the original plan for the test road was a series of deflection studies to be conducted by the Joint Highway Research Project with the cooperation of the Division of Engineering Sciences of Purdue University.

This pavement deflection project, which is the subject of the present report, was initiated in October 1953 with the twofold purpose (a) of developing a multi-channel instrument capable of measuring dynamic pavement deflections and (b) of investigating the behavior of pavement-earth systems under various load conditions.

PRELIMINARY STUDIES

Pavement deflection measurements were made by means of differential transformers. The transformers were attached to the pavement and the transformer cores were connected to reference rods driven into the bottom of cylindrical holes in the earth. The relative vertical motion of the pavement with respect to the reference rods was measured.

In the early stages of this investigation much effort was directed to the development of necessary equipment for obtaining the desired pavement deflection measurements. A major task was the design and construction of a 14-channel recording device.[2] Other problems which were considered dealt with such items as proper design of a transducer[3] holder

[1] Now of the Raytheon Manufacturing Co., Wayland, Mass.

[2] The development of this instrument is covered in a paper "A Fourteen Channel Displacement Measuring Device Utilizing Magnetic and Paper Tape Recording," by G. R. Partridge, J. W. Dunkin, R. L. Anderson, and R. C. Geldmacher, submitted to the American Institute of Electrical Engineers for publication.

[3] As used in this report the term "transducer" refers to the device used to indicate vertical movements of the concrete pavement. It is also referred to as a "deflection gage."

and calibrating device, effect of variation in the speed of the test vehicle, effect of the lateral placement of the test vehicle, effect of changes in environment, and the proper depth for the placement of reference rods. These items are discussed under separate headings.

A holder was developed from which the transducer could be removed when not in use. The holder was composed of a 2-in. diameter by 7-in. long brass (Figure 1), sleeve which was mortared to and moved with the pavement. The differential transformer was placed in a hollow micrometer-threaded cylinder of nonmagnetic steel, which screwed into the brass sleeve. One revolution of this transformer holder provided a displacement of 0.025 in.

Reference rods were made of ½-in.

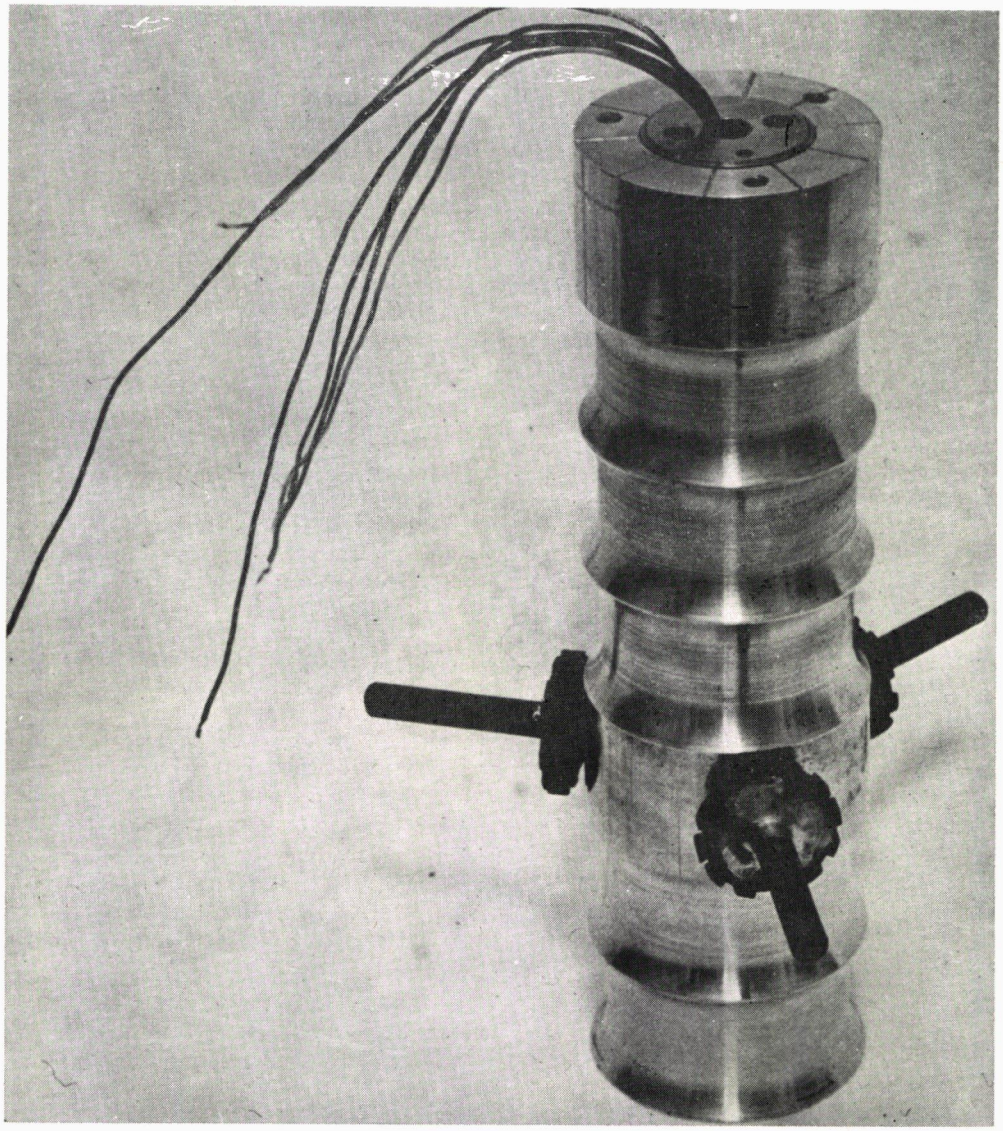

Figure 1. Brass sleeve and core holder.

diameter steel having one end sharpened and the other end turned to a hemispherical surface upon which the transformer core holder could rest.

The transformer core holder was made of a thin brass rod connected to a socket fitted onto the top of the reference rod.

Installation of Transducer

Holes were made in the pavement with a core drill. After the pavement was cored, a soil auger was used to make holes of varying depth in the subgrade just large enough for a 1½-in. galvanized pipe casing. The pipe length was chosen so that the top of the pipe was about 2-in. below the bottom of the pavement.

The reference rod was then inserted in the casing and driven into the subgrade at the bottom of the hole. The rod length was such that from 1 to 2 ft extended into the subgrade below the bottom of the hole and the top of the rod came to the bottom of the pavement. The sleeve mortaring operation completed the installation. Figure 2 shows the installed sleeve and an exploded view of the core holder, transducer holder, transducer, electrical leads, and sleeve cover.

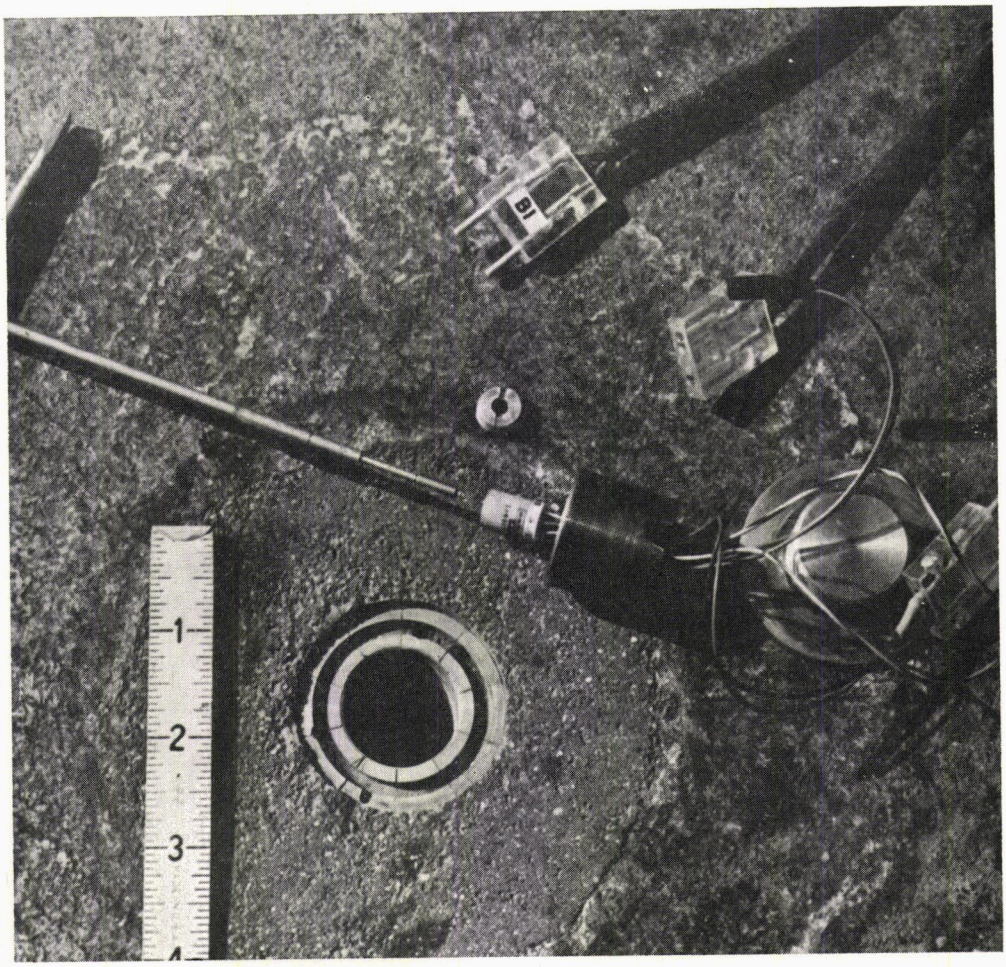

Figure 2. Exploded view of transformer installation.

Calibrating Device

A device was developed which provided direct calibration in the field. This device (Figure 3) consisted of four principal parts: a housing which fitted to the brass sleeve imbedded in the pavement, two rotating sections which afforded a means of fixing the calibrator at a null point from which differential transformer motion relative to a core could be measured in increments of 0.0025 in., and a connecting link that was used between the transducer holder and the bottom of the internal rotating part of the calibrator.

Figure 3. Calibrator.

Initial Installations

In September 1954, 16 transducer holders were installed in a section of the US 41 test road near Cook, Ind. This site was in a fill section having a 5-in. dense-graded stone base beneath the portland cement concrete pavement. Acting on the best information available at the time, 4-ft reference rods were used.

A series of preliminary measurements was taken at this site. Upon analysis of the data, the question arose as to the proper depth at which reference rods should be placed. Because the answers to this question were of primary importance to the design of the over-all experiment, a theoretical and experimental study of earth motion was begun.

Exploratory observations were made by spanning the pavement with a 50-foot television antenna mast supported at each end and having a differential transformer attached at its center. As a result of measurements made with this arrangement, it was discovered that the reference rod moved on the order of one-half as much as the pavement. This meant that with 4-ft reference rods absolute deflections of the pavement were not being obtained.

A series of tests designed to measure earth motion at increasing depth was made in both a cut and a fill section of US 41. All tests made in the fill section were repeated in the cut section. To evaluate any change in the load bearing characteristics of the pavement due to environmental conditions, a control and a test reference rod were provided for each location. During the measurements at each location, nothing was changed in

the control rod installation, but changes were made in the test rod installation so that it measured deflection at the time of each test. Changes indicated by the control rod were then evaluated and applied as corrections to the test installation. Four series of measurements were made beginning at daybreak, two in the cut section and two in the fill section. It was only possible to run one test a day, but the tests made in the fill on two different days gave almost identical results. This was also true of the cut section.

This series of depth of influence measurements was performed by running the test truck over the gage and recording the deflection and reference rod length. The procedure was repeated for increasing reference rod lengths and a curve of deflection versus rod length was plotted (Figure 4). The depth of influence appeared to be greater in the fill section.

A series of preliminary measurements also was made to determine how the load bearing characteristics of the pavement changed with environmental changes. For this purpose, a transducer was installed in the center of the driving lane and a test truck passed over the transducer at creep speed at 15-minute intervals. During the period of the test (from 10 AM to 2 PM), it was observed that the pavement deflection with this same load increased steadily until it was finally approximately 2½ times as great as the original deflection. It was also observed that during this period the center of the pavement was continually moving upward.

Pilot Section Tests

As a result of the depth of influence studies, a pilot section was established just south of the original site in a slab having a 5-in. dense-graded stone base. This pilot section was equipped with 10 transducer holders installed in line across the pavement. The reference rods used in this section were 12 ft long, with the upper 10 ft free from contact with the soil.

A series of measurements was then made on the pilot section. Tests were made for five lateral vehicle positions and

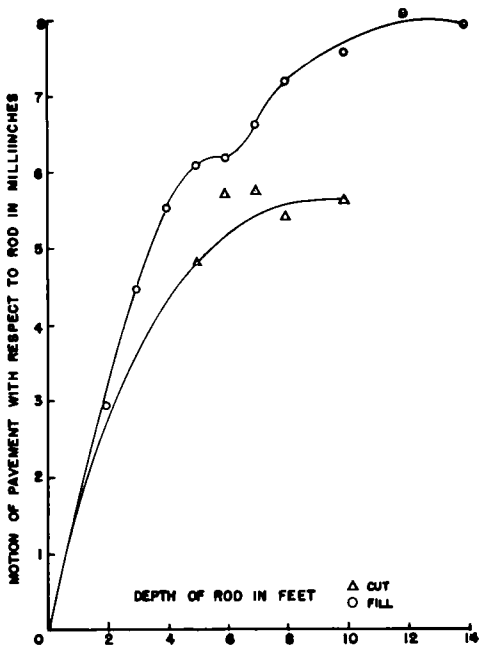

Figure 4. Depth of load influence (US 41).

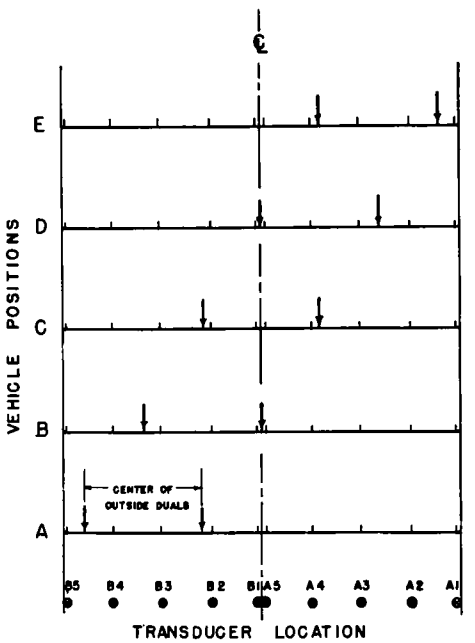

Figure 5. Transducer locations and lateral vehicle positions.

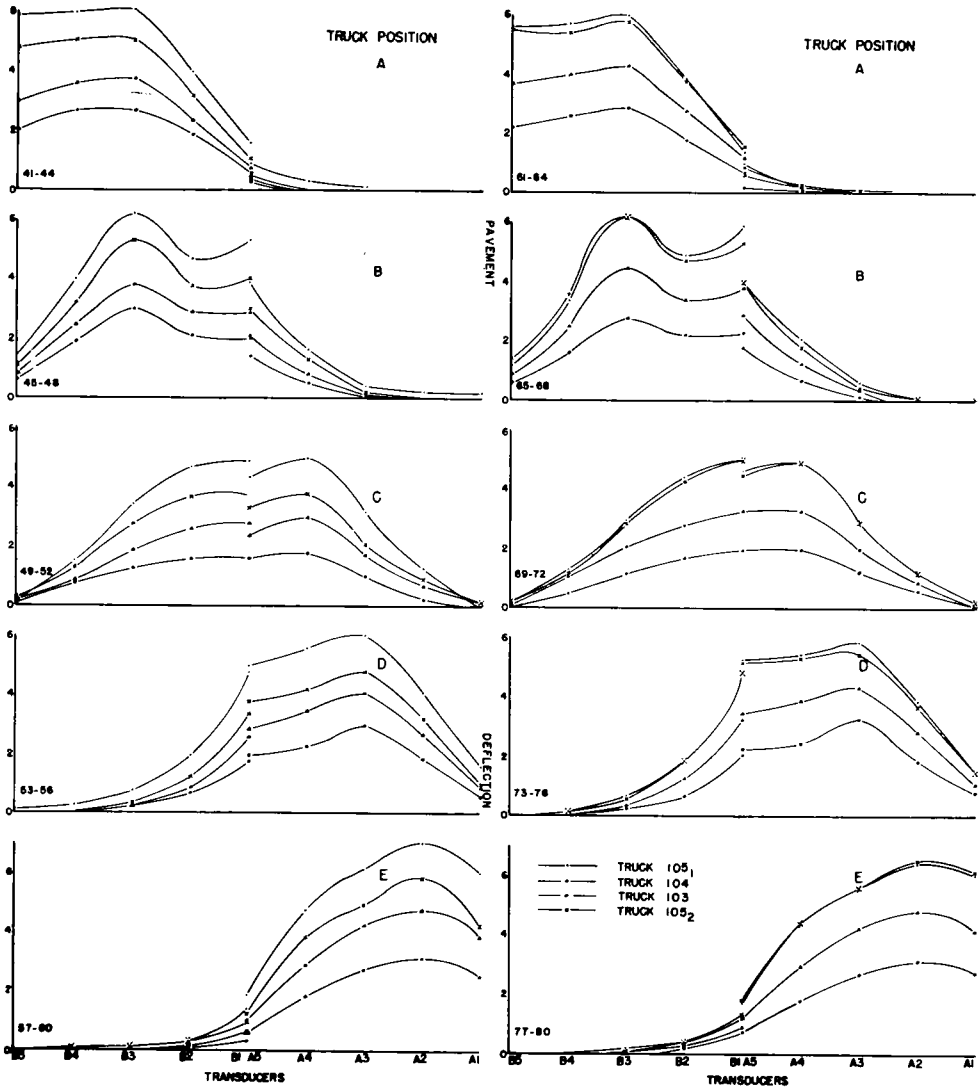

Figure 6. Typical pavement deflection profiles (deflections in chart units, each chart unit being 0.0015625 in.)

five vehicle speeds: creep, 5, 10, 20, and 30 mph. In addition, in order to determine the effect of variability in vehicle speed and lateral position, one vehicle made 30 consecutive runs in the same position and at the same speed.

This series of measurements on the pilot section consisted of more than 2,250 individual records. The positions of the transducers and the lateral vehicle positions are given in Figure 5.

Deflection Profiles

The transverse deflection profiles of the pilot section for each lateral position at creep speed are shown in Figure 6. Truck 105_1 and 105_2 are the same vehicle; however, the curve labeled truck 105_1 was obtained from a seating run made preliminary to the runs of trucks 104, 103, and 105_2. The weights of the vehicles are given in Table 1.

TABLE 1
WEIGHTS OF TEST VEHICLES USED ON PILOT SECTION

Component	Weight, lb		
	Truck 103	Truck 104	Truck 105
Front axle	8,610	8,610	8,610
Right front wheel	4,370	4,490	4,490
Left front wheel	4,230	4,100	4,320
Combined	8,600	8,590	8,810
Rear axle	13,620	8,610	18,910
Right rear wheel	7,070	4,440	9,720
Left rear wheel	6,750	4,300	9,160
Combined	13,820	8,740	18,880

Linearity of Pavement Earth System

As previously noted, Table 1 gives the load at each wheel of each vehicle during the pilot section tests. Loads were arranged so that each vehicle, as nearly as possible, had the same front wheel load. As a result, the deformation of the pavement at a point beneath the rear wheels of all vehicles had equal components contributed by the front wheels (all vehicles had the same wheel base). As it later developed, this arrangement did not affect the results in any appreciable way, as analysis of records showed the contribution of the front wheels to be quite small and any small differences in front wheel loading produced a negligible effect at the rear wheels.

All vehicles made four passes at creep speed at each of the five lateral positions. Figure 7 shows a representative plot of the maximum deflections at each transducer location for the three test vehicles on the pilot section at lateral position B.

On the basis of the information gained it was concluded that over the range of loadings used and under the conditions existing in the pilot section tests, the pavement-earth system behaves quite linearly.

Effect of Changes in Vehicle Position

One of the factors contributing to the variance of measurements in the field was control over vehicle lateral position. As a consequence, part of the series of measurements made on the pilot section

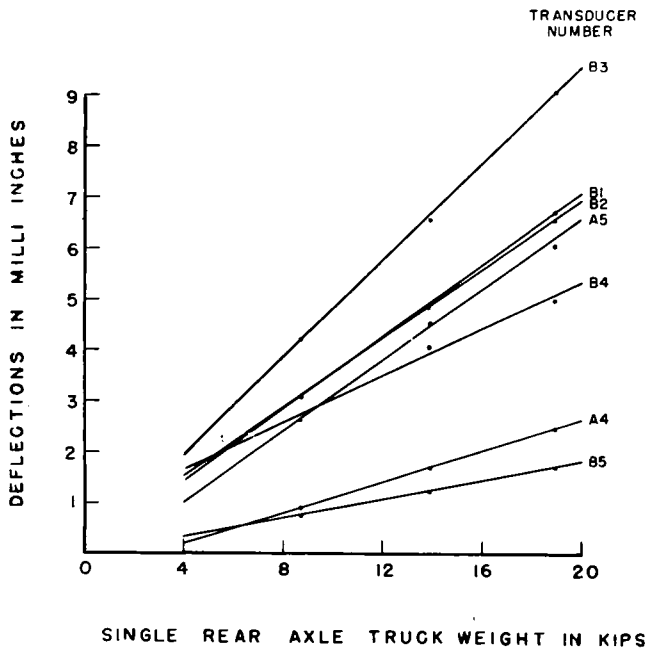

Figure 7. Representative pavement deflections for vehicle position B.

was planned to provide an estimate of the effect on pavement deflection of a small change in the lateral placement of a vehicle.

These determinations were made by measurement of the deflections at each of the ten transducers as vehicle 105 passed four times at creep speed over each of the five chosen load paths.

The maximum deflections at each transducer position were plotted in a manner indicating the rate of change of deflection with respect to lateral displacement. Typical curves are shown in Figure 8.

Some care should be used in interpreting these measurements, because (a) each measured deflection resulted from simultaneous loads at two lateral placements (the two sets of adjacent wheels), and (b) the increment of lateral placement was 3.55 ft. The first condition would probably tend to reduce the rate of change of deflection with respect to lateral displacement at points located between the wheels (between points of application of loads). The second condition leaves the curves undefined between points of load application.

As might be expected, the greatest rate of change of deflection with respect to lateral position occurred at the edges of the pavement.

It was found that to control the variance in deflection at the edge of the pavement (when a vehicle is near the edge) to within ± 1 percent, the position of the vehicle must be controlled to stay within ± 0.4 in. of the prescribed path. This degree of control applies to a vehicle traveling at creep speed along the edge of the pavement. At the other positions in the pavement when a vehicle is traveling at creep speed the degree of control is not as critical.

Control over lateral vehicle placement becomes more difficult as the speed is increased; therefore, one would expect this factor to produce a larger component of variance in measurements made at higher speeds and the experimental data show this to be true.

Because the pavement-earth system behaves as a linear elastic system it is possible to devise an experiment that will

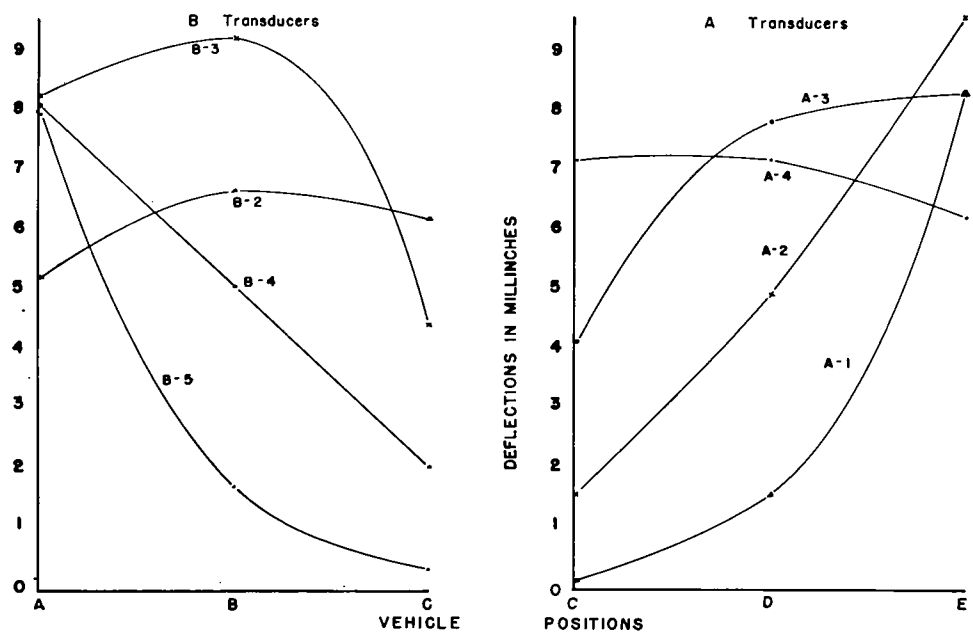

Figure 8. Maximum deflection vs vehicle position at creep speed.

take advantage of this property in such a way that an equivalent uniform transverse load may be obtained by using a single wheel load at closely spaced increments of lateral displacement. All that is necessary to accomplish this is to provide vehicle paths such that by algebraically superposing deflections a resultant deflection is obtained which is equivalent to that caused by a uniform transverse load. Such a procedure would enable one to obtain a more accurate representation of the effect of changes in lateral placement. This principle was used to obtain an equivalent uniformly distributed transverse load for the earth motion study, discussed in a later section.

Effect of Changes in Vehicle Speed

A part of the series of tests on the pilot section was made to obtain an estimate of the component of variance contributable to changes in speed and to provide information useful in designing a full-scale experiment for more carefully exploring the relationship between vehicle speed and pavement deflection.

The runs consisted, in addition to seating runs, of three passes by the test truck at speeds of creep, 5, 10, 20, and 30 mph at placement A on the pilot section.

Representative results from the pilot section are plotted in Figure 9, from which it may be seen that the effect of a change in speed on changes in deflection is quite small compared to the changes in deflection resulting from a change in lateral placement (Figure 8).

An attempt was made to minimize the effect of environmental changes on the measurements made at the pilot section by making the runs at the five different speeds within a 15-minute period.

Effect of Changes in Environment

A part of the series of tests on the pilot section was designed to provide information relative to the effect of environmental changes on pavement deflection.

It is well known that a concrete pavement will warp when the temperature

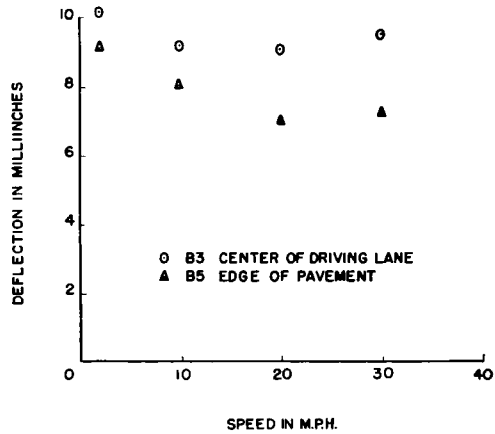

Figure 9. Speed vs deflection (US 41).

of its surface changes. Consequently, if a load is repeatedly passed over a given path of the pavement one might expect the deflection at a particular point to change if the environment of the pavement changed. To explore this problem, measurements over a 3-hour period at midday were made at each transducer position of the pilot section for all lateral positions of the three test loads. Points representing maximum deflection at vehicle position A have been taken as representative and are shown on Figure 10.

Attention should be called to the fact that each point represents one observation and therefore is not necessarily representative of the mean. However, the standard error for large samples of measurements of this sort was found to

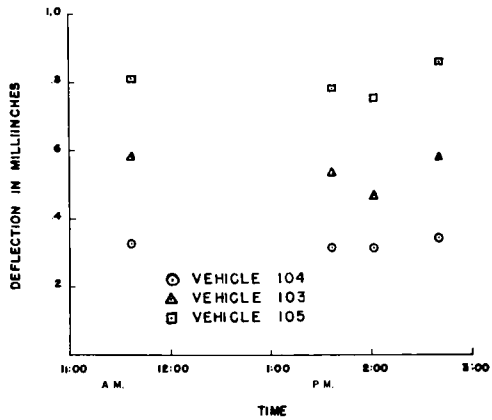

Figure 10. Deflection vs time, position A.

be of the order of 1 percent of the mean.

The effect of observed changes in environment can be summarized as follows:

1. The support characteristics of the pavement-earth system changed appreciably as a result of changes in environment.
2. There is some evidence that the system remained fairly linear over the load range used, even though a change occurred in the amount of deflection caused by a particular load.
3. Not enough is yet known about the relationship between initial conditions of the pavement system (condition of warping) and deflection due to moving loads to be able to predict accurately magnitudes of deflection.

If it can be shown that the pavement-earth system behaves linearly at any arbitrarily chosen time, an accurate statistical description of the boundary condition may perhaps lead to a somewhat general solution of the problem of performance prediction and comparison. Such a statistical description of the boundary conditions would undoubtedly require a well planned series of measurements made over a period of several months.

It should be observed that, in addition to the cyclic changes in the system, long-term changes are also taking place — the pavement cracks, pumping action takes place, and the modulus of elasticity may change. All these conditions contribute a non-stationary component, which complicates the use of statistical methods. However, it may be that over periods of a year or less the non-stationary component will not be noticeable.

ADDITIONAL DEPTH OF INFLUENCE STUDIES

The proper depth for placement of the reference rods was considered to be of such basic nature that further time and effort were devoted to this subject before the evaluation of the subbase type was initiated.

At the same time the experimental work was being carried out in the field, a theoretical study was begun to evaluate earth motion beneath a loaded pavement. Some basic assumptions of the study were to consider the earth as a semi-infinite, homogeneous, isotropic, elastic medium whose upper surface is displaced in the form of an infinitely long trench having a profile with a specified form. A comparison between theoretical results and experimental results is presented in Figure 11. The theoretical curves in Figure 11 are labeled Problem 1 and Problem 2. The first relation (Problem 1) is based on the assumption of zero shear stress between the earth and the pavement at their interface. The second relation (Problem 2) is based on a condition of no slip at the interface. Possible reasons for the discrepancies between theory and experiment are in the assumption of a homogeneous, isotropic and elastic medium and also in the selection of boundary conditions. These assumptions are more idealized than the conditions that actually exist.

As an addition to the knowledge gained in tests at Cook, Ind., another depth of influence experiment was made at the Purdue University Airport during December 1955. In this experiment simultaneous measurements of the relative motion between the pavement and five specified depths were made. A row of five differential transformers with 1 ft between centers was installed in the middle of a concrete slab measuring 45 ft 10 in. by 13 ft 9 in. Reference rods for transducers were anchored at depths of 3, 6, 10, 15, and 19 ft below the top of the pavement. Eight test lanes were laid out so that the lateral position of the test truck could be controlled.

Two types of tests were performed. One set of 15 identical runs was made to determine the variability of the complete sensing and measuring system. In the other test, one run was made in each of the test lanes at the same speed. Because of the way the test lanes were chosen, the wheel positions, considering all eight runs, were almost uniformly spaced; hence, they represented a close approximation to a uniform transverse load. The

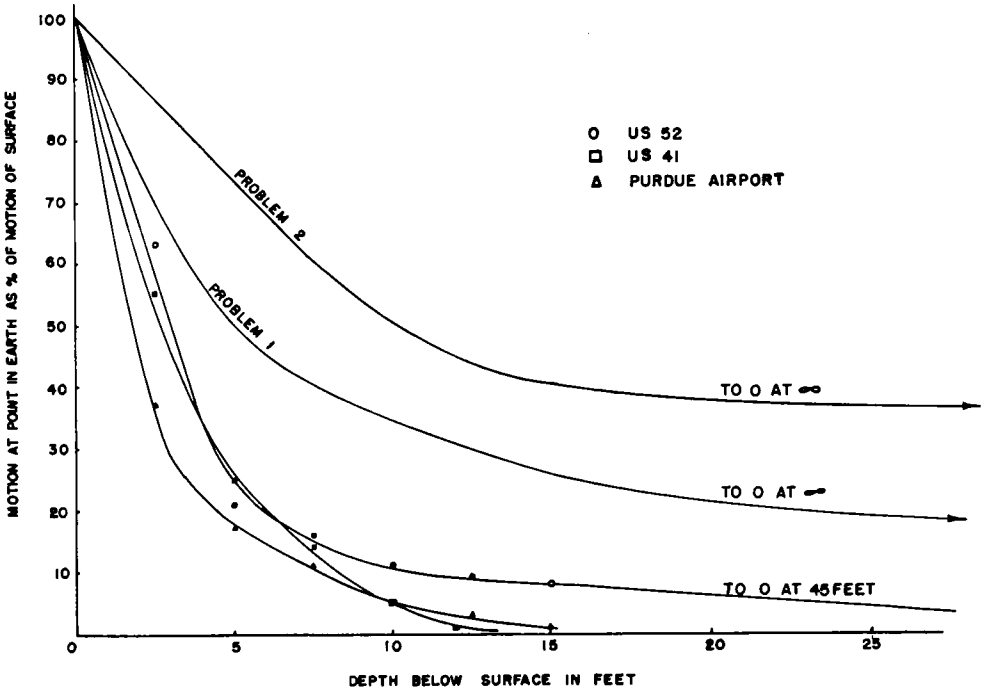

Figure 11. Depth of influence curves (see text for explanation).

purpose of this was to give a boundary condition on the top of the earth under the pavement that would be similar to the one assumed in the theoretical development.

In an attempt to further improve on the depth of influence tests, a series of measurements was made on a pavement slab on US 52 about ½ mile west of Klondyke, Ind., during June 1956.

The improvements were as follows:

1. A slab was selected in a cut section in the hope of attaining a fairly homogeneous subgrade.
2. By using a power-driven soil auger it was possible to drill to about 43 ft below the pavement.

Six transducers on 1-ft centers were installed on the center line of the driving lane. These transducers were also centered in the direction of the length of the slab. The free lengths of the reference rods were 1 ft 10 in., 5 ft 4 in., 9 ft 5 in., 14 ft 11 in., 27 ft, and 42 ft 7 in.

Depth of influence curves for the Purdue Airport, US 52, US 41, and the theoretical problem are shown in Figure 11. The experimental curves cannot be directly compared, as they involved different pavements and, of course, different subgrades. However, they exhibit the same general shape.

As previously mentioned, it was found from tests made on US 41 that the depth of influence was not the same for a pavement on a fill section as a pavement on a cut section (Figure 4). It was also found that some of the depth of influence curves exhibited a dip or sudden change in slope, as occurs in the curve for the fill section (Figure 4). It was reasoned and verified by examination of soil borings that this occurred at a layer of earth of greater density.

COMPARISON OF NINE SUBGRADE TREATMENTS

As previously noted, each mile of the US 41 test road was divided into nine

sections, with each section having either a different foundation treatment or depth of subbase.

The concrete pavement for the entire project was 24 ft wide, 9 in. thick at each edge, and 8 in. thick at the center. The test sections used for this deflection study were reinforced with 45 lb of welded wire fabric per 100 sq ft and had contraction joints spaced at 40-ft intervals with dowels on 1-ft centers.

Three slabs from each of the nine sections of one mile of the test road were chosen to be tested. In an effort to select slabs as nearly identical as possible it developed that the three slabs were usually not adjacent. Factors which prevented choosing adjacent slabs were that slabs were rejected if they:

1. Showed appreciable differences in location of transverse cracks.
2. Showed evidence of excessive pumping.
3. Adjoined crossovers or driveways.
4. Were near culverts.

The locations of the test slabs, the deflection devices, and strain gages are shown in Figure 12.

The quarter-point deflection gage locations in the center of the driving lane were made 10 ft from the north joint of the slabs because this was approximately

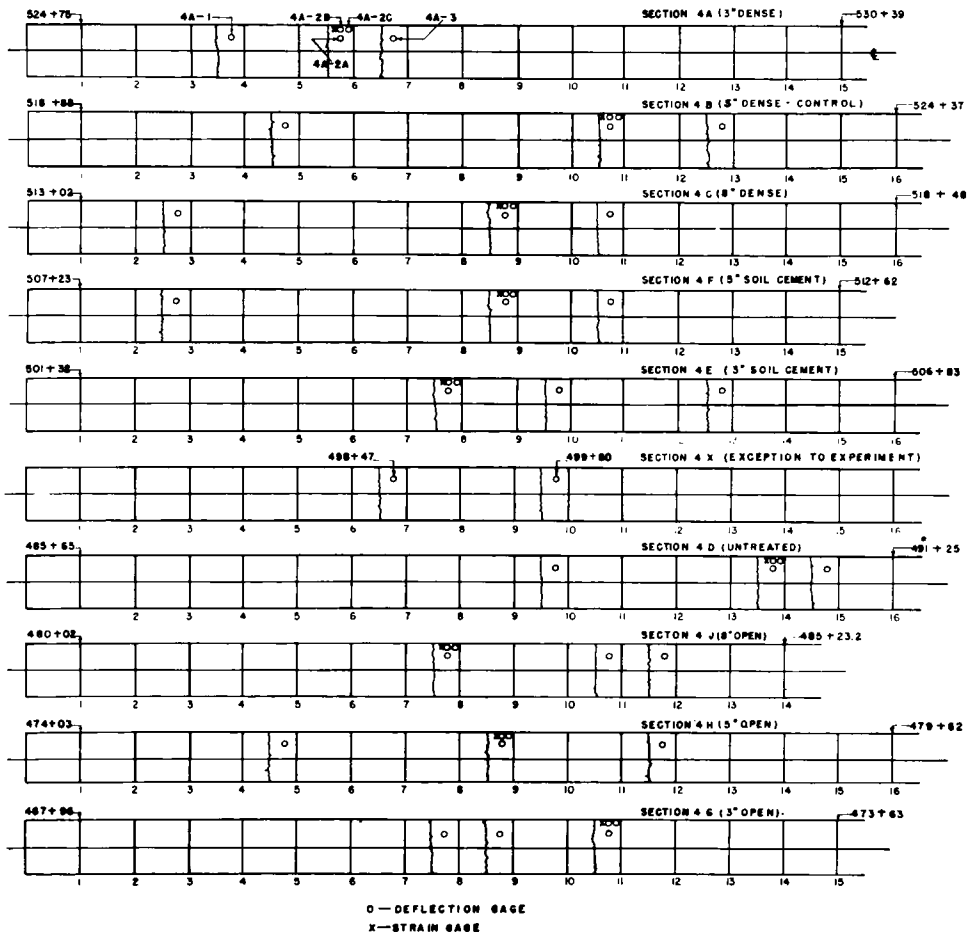

Figure 12. Transducer locations and subbase types (US 41 test section).

TABLE 2

Section Number	Sub-Base	
	Type	Thickness, in.
4A	Dense-graded	3
4C	Dense-graded	8
4D	Untreated	—
4E	Soil-cement	3
4F	Soil-cement	5
4G	Open-graded	3
4H	Open-graded	5
4J	Open-graded	8
4B*	Dense-graded	5
4X†	Fine sand	

* Control section. † Exception.

midway between the joint and the usual transverse crack in the slab. The deflection gage locations along the edge of the slabs were placed so that the center of the gage would be 3½ in. from the edge of the slab. To aid in the investigation, strain gages were applied on the surfaces of the concrete section being studied.

Guide lines for the center of the left front wheel were drawn on the pavement with yellow crayon. The truck drivers, with the help of guides stationed in front of the trucks, used these lines to control lateral placement.

One line was drawn so that the center of the right rear dual assembly was 4 ft from the west edge of the pavement. This placement had been determined by previous study as the normal lateral placement for this particular road.

The other line was drawn so as to get the truck as close to the edge of the pavement as possible without hitting the gages. This distance was 0.5 ft from the west edge of the pavement to the outside edge of tire tread.

For each of the foregoing lateral placements the test trucks made a minimum of 12 passes. This number of passes was chosen on the basis of earlier work, which showed that the mean deflection could be placed within a band width of ± 0.6 percent if 12 passes were sufficiently close together in time.

Because pavement warping caused the load deflection characteristics of the pavement to vary with time, a control section was established. This was quite necessary, as no more than one section could be tested on any given day. Therefore, Section 4B (see Figure 12) with 5 in. dense-graded subbase, was arbitrarily chosen as the control section. At the same time that any given section was being tested, an identical test was performed on the control section. Two nearly identical trucks were used, the only difference being a slight difference in tire size. The trucks were loaded equally with sand.

The deflections at quarter points, for conditions of loading at the edge and at normal lateral placement, are shown in Figures 13 and 14, for the various subbases and the corresponding simultaneously obtained deflections from the control section. Figures 15 and 16 show deflections at the corners and edges, respectively, for both edge and normal loading. Figure 17 shows the values of strain at pavement edge obtained for each subbase for both loadings. In addition, the ratios of strains in the test sections to strain in the control section against type subbase are represented in Figure 18.

From these data it was observed that:

1. For the open-graded and soil-cement sections the deflections decreased as depth of subbase increased.
2. For the dense-graded subbases the deflections did not appear to vary with depth.
3. The deflections obtained on the untreated and fine sand subbases were of the same order of magnitude as those on the other subbases.

A statistical analysis was made on the data from the tests on the nine sections. This analysis was limited to the center gages located at the quarter points (see Figure 12), because the quarter points were the only points that involved more than one slab from each section.

The ratio of test section deflections to control section deflections for the quarter points, edge load, are shown in Figure 19. A ratio for the 5-in. dense-graded subbase is not shown, as it was the control section.

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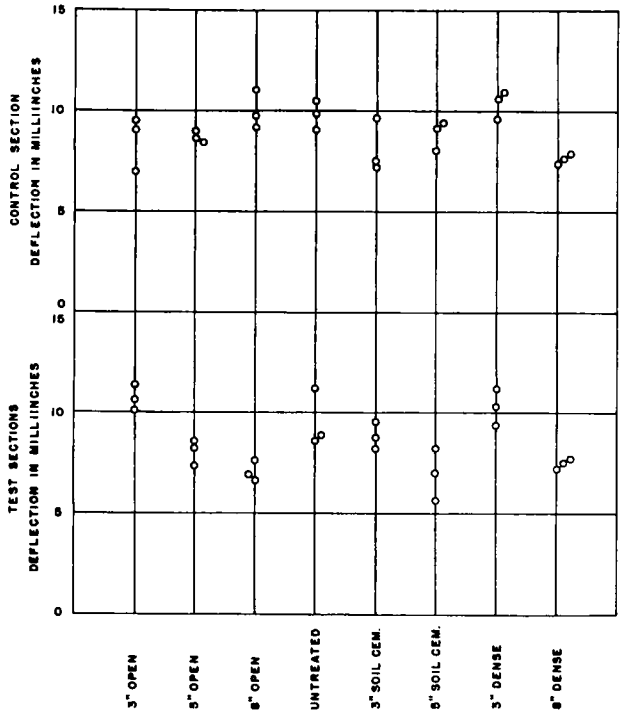

Figure 13. Deflections at quarter points, edge load.

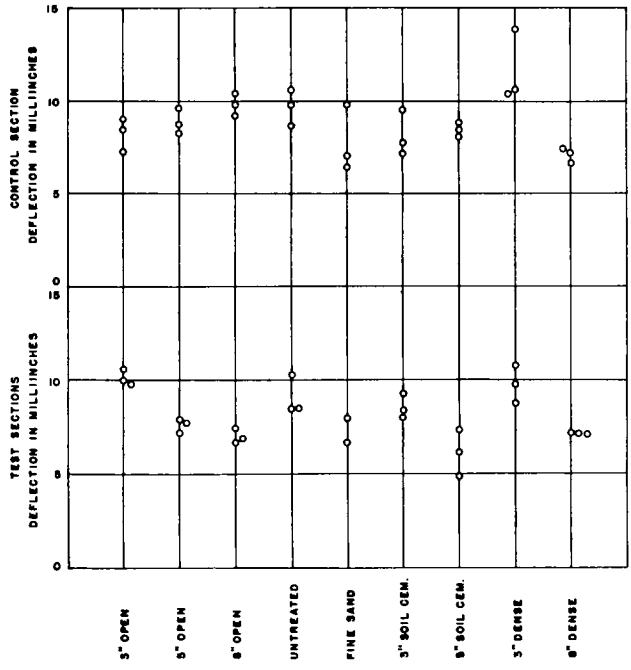

Figure 14. Deflections at quarter points, normal lateral placement.

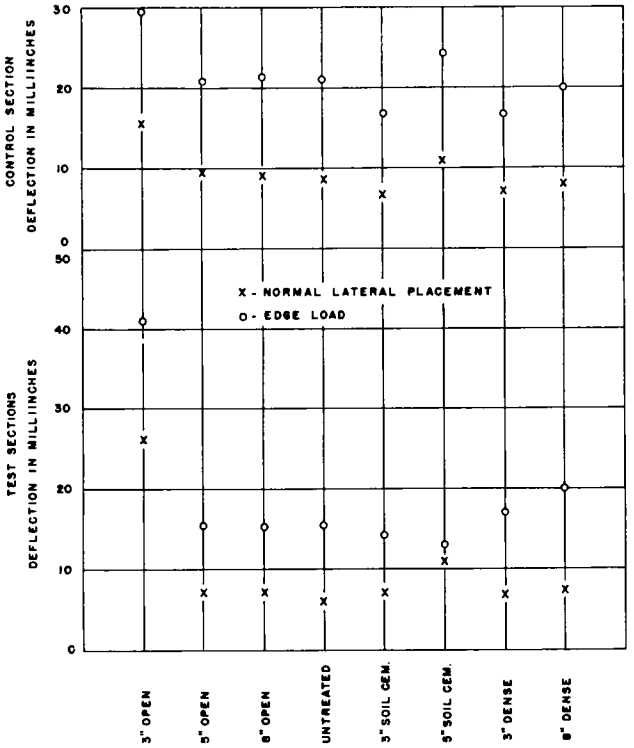

Figure 15. Deflections at corner points.

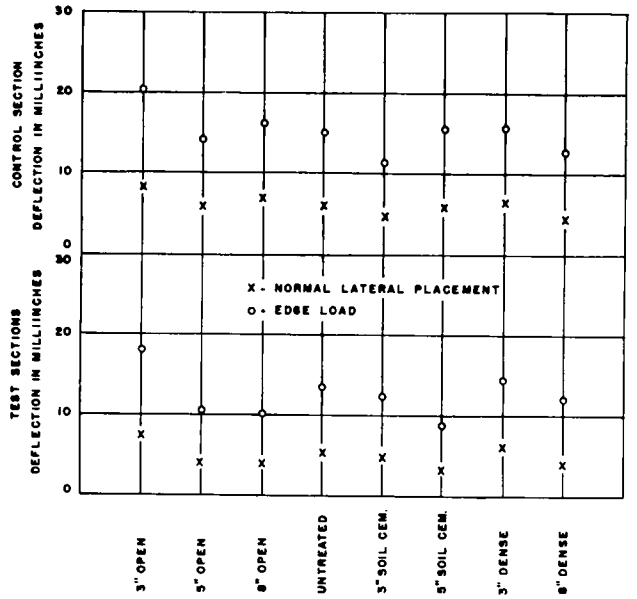

Figure 16. Deflections at edge points.

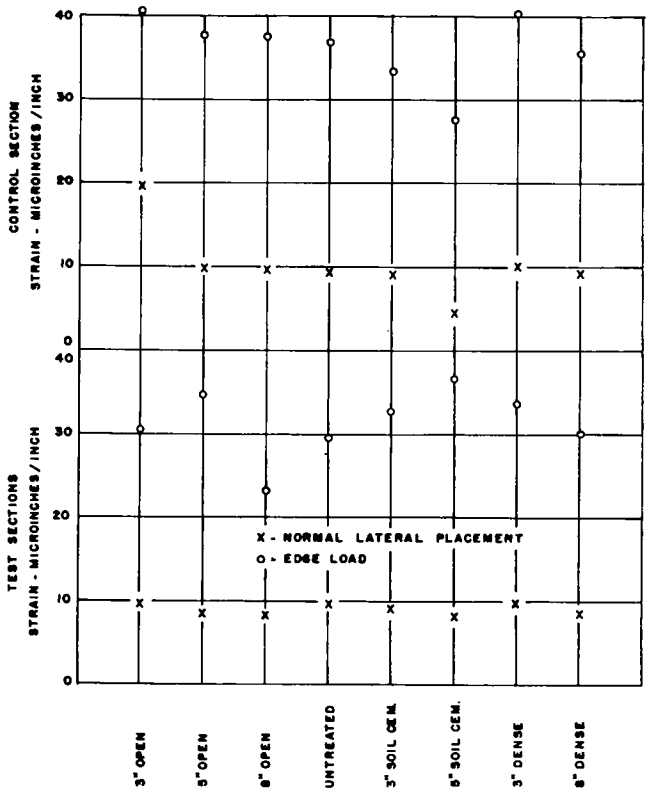

Figure 17. Strain at pavement edge for normal and edge loadings.

quarter points, comparisons of the ratios of test section to control section deflections (as given in Figure 13) were made for like treatments and like thicknesses. For example, 8-in. open was compared with 3-in. open and 8-in. dense, but not with 3-in. dense.

As a result of this analysis, significant differences in deflection ratios were found between the following:

8-in. open and 3-in. open
8-in. open and 5-in. open
8-in. open and 8-in. dense
8-in. open and untreated
5-in. open and 3-in. open

All other comparisons showed differences in deflection ratios that were not significant.

It was then concluded for the nine sections that at the quarter points for edge loading the deflection of the open-graded subbase decreased significantly as subbase thickness increased and the 8-in. open-graded gave significantly less deflection than the 8-in. dense and the untreated subbases.

A statistical analysis was not possible for edge and corner deflections and edge strain; only one slab from each section was tested and therefore the slab-to-slab variance was unknown.

It can be observed (Figures 15, 16, and 17), however, that edge and corner deflections and edge strains generally exhibit similar trends. The magnitude of the edge and corner deflections also tended to be greater than the deflection at the quarter points.

CONCLUSIONS

The following general conclusions are

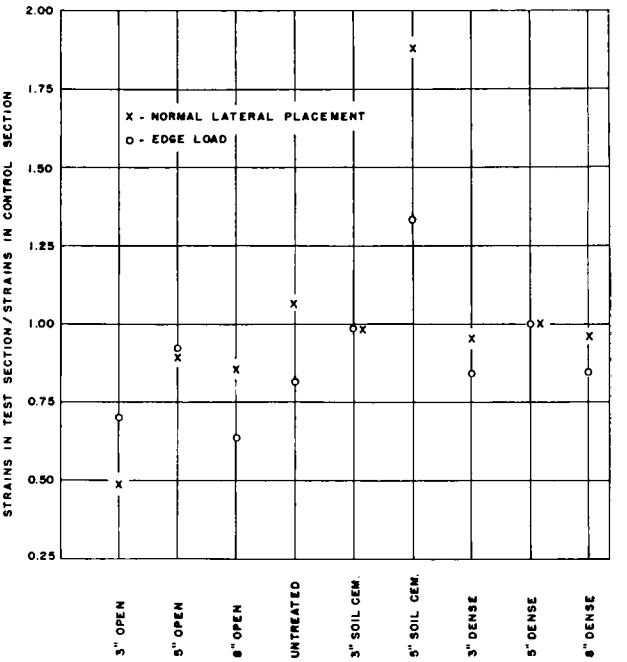

Figure 18. Ratio of test section to corresponding control section strains.

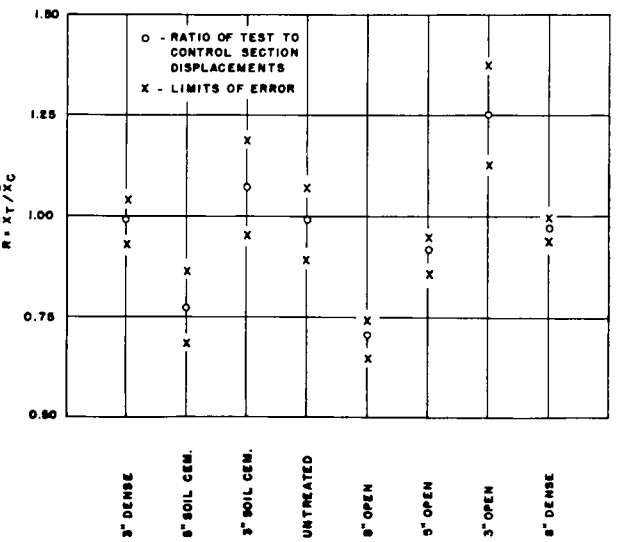

Figure 19. Variance in ratios of tests to control section displacements for quarter points, edge load.

based on the pavement deflection studies:

1. The motion of concrete pavements deflected by moving loads may be measured with a high degree of precision by means of differential transformers.

2. It is possible to provide direct calibration to the measurements of concrete pavement deflections.

3. Over the single rear axle load range of 9,000 to 20,000 lb the load deflection relations of the experimental sections of pavement on US 41 are quite linear.

4. Lack of control of lateral vehicle position can be a major contributing factor to variance of pavement deflection measurements. With the vehicle near the edge of the pavement, changes in deflection of about 1 percent, measured at the edge, were noted with a change of 0.4 in. in lateral position of the vehicle.

5. Pavement deflections decreased with increase in speed of the vehicle in the range of creep to 20 mph. This effect was also more pronounced at the pavement edge than in the center of the driving lane. For one of the conditions of load and lateral placement, as speed was increased from creep to 20 mph, a 10.8 percent decrease in deflection resulted at the center of the driving lane and a 23.1 percent decrease resulted at the edge of the pavement.

6. Earth motion beneath a load provided by a truck with 20,000 pounds on dual rear wheels is influenced by the soil profile. The maximum depth of influence was found to be 15 ft at the test sites on US 41 and 45 ft at one point on US 52.

7. The load support characteristics of the pavement change appreciably as a result of pavement warping caused by changes in environment.

8. On the basis of research to date, the relationship between initial conditions of the pavement system (condition of warping), and deflection due to moving loads, is not clear and precludes the accurate prediction of deflections from the knowledge of loads.

9. The deflections measured on the nine sections of experimental pavement on US 41 at the quarter points in the center of the driving lane for edge loading decreased significantly (in a statistical sense) as subbase thickness increased for the open-graded subbase. In addition, the 8-in. open-graded gave significantly less deflection than the 8-in. dense-graded and the untreated subbases.

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