Value of Automobile Transit Time in Highway Planning

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Economic analysis of proposals for highway projects necessitates assigning a dollar value to the transit time of private automobiles. The current approach is to use a prescribed dollar figure, such as $1.35 per automobile-hour suggested by the AASHO.

This paper rests on the premise that there is no single dollar value for transit time of private automobiles which can be used all over the country. For most usual applications it is inappropriate to begin a problem with a prescribed dollar figure, therefore the need is established for a flexible technique which can be used in different situations for assigning an appropriate value.

Finally, the paper shows how the local concept of satisfactory highway conditions can be used for assigning an appropriate dollar value to transit time. The technique's superiority over the current approach lies in the fact that it provides for some validity tests for judgment used in the selection of satisfactory highway conditions.

• THIS PAPER results from an attempt to determine if analytic techniques can be devised for aiding planners in making better decisions about highways best suited to the needs of their communities. Since the topic of decision-making in highway planning is very broad, the scope of this paper will be limited to a specific problem which arises in the engineering economy studies of highway projects. This problem is the assignment of an appropriate monetary value to the transit time of private automobiles.

The AASHO (*1*) has recommended the use of benefit ratios (ratio of direct benefits received by users to the costs incurred by highway departments) for making economic analyses of proposed highway projects. Several other criteria, such as the total transportation cost (*2*) or the rate of return (*3*), are also used for making economic analyses of highway projects. The advantages and disadvantages of these different criteria are not discussed here; however, it is important to note that whichever criterion is used, a critical problem is the choice of a dollar value for the transit time of private automobiles.

Usually, the problem of assignment of an appropriate dollar value to the transit time of private automobiles is avoided by the use of a prescribed dollar figure. Various figures have been recommended for this purpose, such as $1.35 per automobile-hour suggested by the AASHO (*1*). Obviously, $1.35 per automobile-hour cannot be an appropriate figure for use in engineering economy studies all over the country.

Highway engineers have long recognized that highway conditions and the degree of congestion which may be considered satisfactory vary from community to community. If this is true, the use of a prescribed dollar figure for transit time does not enable a highway planner to take the local needs for traffic conditions explicitly into consideration in the engineering economy analyses of highway projects. Therefore, there is a need for a flexible technique which can be used individually in each community for assigning an appropriate dollar value to transit time. One such technique will be discussed in this paper.

Frequent references will be made to the roles of two entities, the planner and the analyst. The planner's function is to exercise judgment in the choice of highway conditions considered satisfactory for his community. Assisting the planner with the mathematical analysis is the analyst. This distinction is drawn on the basis of functions; however, it is possible that in any specific case, a single entity will perform the dual roles of planner and analyst.

The essential concept of the proposed technique is the following:

> Before beginning the engineering economy analysis for evaluating a proposed highway project, the planner considers the existing highways in his jurisdiction and selects one (to be called the "reference highway") which he considers satisfactory for the needs of his community, taking into consideration the actual traffic volumes and transportation cost. The analyst then asks: What dollar value must be assigned to transit time so that the design of the reference highway will be an optimal design? The value thus derived is then used for evaluating proposals for the highway.

However, before this question can be meaningful, the analyst has to ensure that the selected design for the reference highway will, in fact, be optimal for the reference highway for some finite, positive dollar value of transit time. In practice, it is only by accident that this criterion will be satisfied. In such a case, the analyst's problem is to find some designs which can be optimal, to let the planner designate one of these as satisfactory, and to use this design for assigning a dollar value to transit time. All the mathematical analysis is directed towards these questions.

It is necessary for the purpose of developing the technique to abstract from the real situation and define an idealized highway. In the following, some terms will be defined and notation introduced. The basic assumptions which define an idealized highway will then be stated. Finally, the technique will be illustrated geometrically and a hypothetical example solved. In this treatment, mathematical rigor will be sacrificed for the purpose of ease of exposition.

NOTATION AND TERMINOLOGY

A "policy" means a set of design parameter values which specifies the essential characteristics of the highway. Let $x_1, x_2, \ldots, x_n$ be the generic symbols which represent the essential physical characteristics of a highway (with n finite) and let X be the generic set (vector) which represents the highway policy. By definition,

$$X = (x_1, x_2, \ldots, x_n). \qquad (1)$$

To illustrate, if a highway has a pavement width of 50 ft, maximum grade of 4 percent, maximum horizontal curvature of 3 deg for 50 percent of its length, etc., then

x_1 = pavement width = 50 ft
x_2 = maximum grade = 4 percent
x_3 = maximum curvature = 3 deg
x_4 = horizontal alinement = 50 percent
.
.
.

and the highway policy, X, can be specified by a vector: (50 ft, 4 percent, 3 deg, 50 percent, . . .).

"Traffic pattern" for a highway means the traffic demand for the use of the highway classified according to the type of vehicle, direction, and length of travel during each time unit of a period. Thus, if one hour is taken as the time unit and one year as the period, traffic pattern for a highway will specify the demand for the use of the highway during each of the 365 × 24 hours. Alternatively, a traffic pattern can be described by means of frequency curves.

Associated with each policy, X, and traffic pattern is the transportation cost which arises (or, as in the case of a proposed highway, will arise) from the construction, maintenance, operation, administration, modernization, and use of the highway. Let $C(X)$ denote the transportation cost (per annum). For the purpose of this technique, $C(X)$ is classified under the following two headings:

1. Cost attributable to the transit time of all private automobiles using the highway. Let T be the transit time of private

automobiles (expressed in hours per year) and let S = dollar value attributed to one hour of transit time of private automobiles. Then, as soon as a dollar value is assigned to S, the cost attributed to the transit time of private automobiles becomes S times T. Therefore,

$$S\,T = C_1 \text{ (by definition)}. \qquad (2)$$

The problem is to assign an appropriate value to S.

2. Transportation cost other than C_1. It is assumed that this cost is measured in dollars (per year). This cost is denoted by C_2. Then,

$$C = C_1 + C_2 = S\,T + C_2. \qquad (3)$$

T and C_2 are functions of highway policy. The problem of determining an optimal policy involves finding a policy X^* which has associated with it the minimum transportation cost, *i.e.*,

$$\underset{X}{\text{Min}}\; C(X) = C(X^*). \qquad (4)$$

Assuming that $T(X)$ and $C_2(X)$ are known, X^* cannot be determined until a dollar value is assigned to S. In general, X^* will change every time the dollar value assigned to S is changed. It follows that when an inappropriate dollar value is assigned to S, solving for an optimal policy amounts to answering a wrong question. This explains the objection to the current approach of using a prescribed dollar value for S.

"Rationality" in decision-making means consistency of choice. Thus, for any three policies, X_1, X_2, and X_3 for the reference highway, if the planner either prefers X_1 to X_2 or is indifferent between the two and either prefers X_2 to X_3 or is indifferent between the two, then when he chooses between X_1 and X_3 he must either prefer X_1 to X_3 or be indifferent between the two (*4*). It is assumed, implicitly, that:

1. Any two policies X_1 and X_2 are comparable.
2. The planner's choice for any pair of policies X_1 and X_2 is not influenced by any other policy.

BASIC ASSUMPTIONS

The following assumptions are in the nature of stipulative definitions and define an idealized highway and environment which are the subject of this paper.

Continuity

This assumption means that each of the policy variables, the x_i's, is continuous within the region of interest. When the design parameters take on discrete values, the assumption of continuity is not satisfied. However, when the number of discrete values for a parameter is large, the parameter can be assumed to be continuous.

Technology

Technology is used in a specialized sense. It means that for any traffic pattern the functional relations $T(X)$ and $C_2(X)$ are known completely.

Differentiability

This assumption is that, within the region of interest, the functions $T(X)$ and $C_2(X)$ are single-valued and at least twice continuously differentiable with respect to each x_i. Most functions met with in practice satisfy this assumption.

Invariance of Traffic Pattern

The traffic pattern is independent of the policy. This assumption is similar to that used by the AASHO (*1*). It is also similar to that of Nicholson who has discussed the difficulties which arise when this assumption does not hold in reality (*5*).

Rational Decision-Making

The planner has an intimate knowledge of the existing highways in his jurisdiction and that he can rank, rationally (in the sense rationality is defined here) the policies of these highways in their order of desirability to his community. In some respects, this assumption is similar to that of Dixon (*6*).

THE TECHNIQUE

The proposed technique enables an analyst to aid an intelligent planner in designing a proposed highway. To develop this technique, the question is asked: How does the human mind work when faced with the design problem and not knowing anything about a prescriptive formula? It is reasonable to presume that the following steps, or a variant of them, are followed:

1. Examination of some or all highways of the same nature as the proposed highway in the same or identical communities.
2. Selection of some highways which appear to be operating satisfactorily.
   (a) Pinning down the features which give to the selected highways the desirable attributes.
   (b) Incorporation of these features in the proposed design.

All this is done on the basis of subjective judgment. This raises the question: Can some aid be given to the planner to enable him to improve his decisions and quantify his judgment? To answer this, considering an existing highway (the reference highway) selected by the planner as representing his notion of a satisfactory highway for his community, it is supposed that x_1 and x_2 are the only two design parameters which have a significant effect on $T(X)$ and $C_2(X)$ for this reference highway, where x_1 is the degree of control of access and x_2 is the pavement width in feet÷100. In actual practice, more than two parameters will have a significant effect on $T(X)$ and $C_2(X)$ and such cases can be handled mathematically (*2*). Since this paper explains the technique geometrically, the case of a two-parameter policy alone will be treated.

Assuming x_1 to have values between 0 and 1, when x_1 is 0, there is no control of access; that is, vehicles enter or leave anywhere along the highway. When x_1 is 1, there is full control of access. In between these two values, there can be various degrees of partial control (7). (How the degree of control of access will be measured in any actual case is not of concern in this hypothetical illustration.) Also, a foot is assumed to be the minimum possible pavement width (this restriction on x_2 can arise from the physical characteristics of the vehicles which use the highway). The bounds on the parameters are expressed by:

$$0 \leqq x_1 \leqq 1 \tag{5a}$$

$$x_2 \overset{=}{>} a/100. \tag{5b}$$

Mapping

Figure 1 shows the policy space. The thick lines indicate the bounds on this region within which any policy, X, can be represented by a point (x_1, x_2). Associated with each X is a unique value of each T and C_2 which can be computed. Thus, each policy can be plotted in two ways:

1. As a point (x_1, x_2) in the policy space.

2. As a point (T, C_2) in the input space (see Fig. 2). Any policy can be mapped from policy space (Fig. 1) into the input space (Fig. 2). Mapping all points from the policy space into the input space gives points as shown in Figure 3.

Admissibility

Among all the points in the input space, some points have a special signifi-

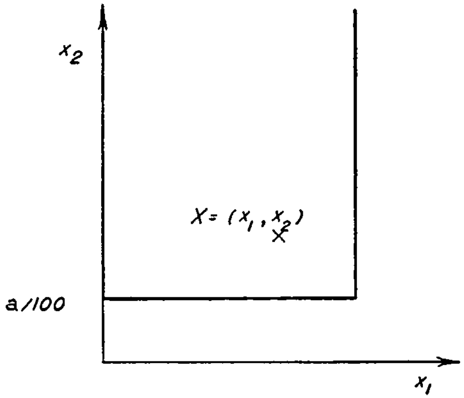

Figure 1. Policy space.

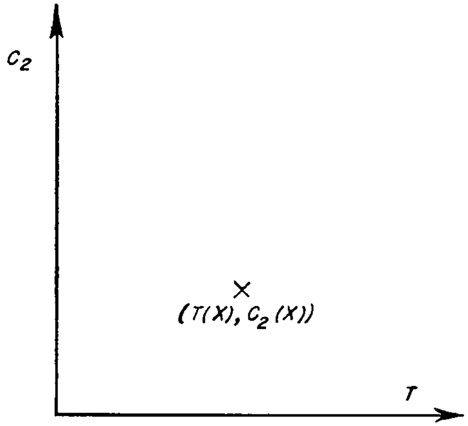

Figure 2. Input space.

cance. Each such point, X', satisfies the following two conditions:

1. Among all points which have the same value of T as $T(X')$, X' has associated with it the minimum value of $C_2(X)$; *i.e.*,

$$\operatorname*{Min}_{X} C_2(X) = C_2(X') \qquad (6a)$$

subject to

$$T(X) = T(X') \qquad (6b)$$

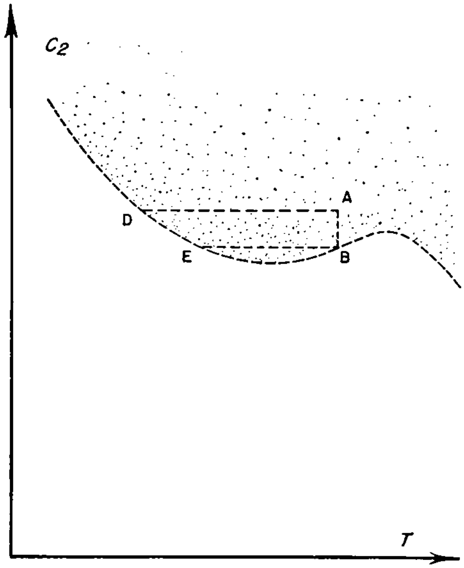

Figure 3. Input space.

2. Among all points which have the same value of $C_2(X)$ as $C_2(X')$, X' has associated with it the minimum value of $T(X)$; *i.e.*,

$$\operatorname*{Min}_{X} T(X) = T(X') \qquad (7a)$$

subject to

$$C_2(X) = C_2(X') \qquad (7b)$$

Each policy which satisfies the above two conditions is called an "admissible policy" and the locus of all admissible policies is called the "transportation isoquant." Each admissible policy can be characterized by the marginal rate of substitution $\left(-\dfrac{dC_2(X)}{dT(X)}\right)$ associated with it, and this marginal rate is S (by definition); *i.e.*

$$S = -\frac{dC_2(X)}{dT(X)} \qquad (8)$$

If T and C_2 are considered as two inputs for the highway, the following property about each admissible policy follows: No other policy can be found which has associated with it less of any one input without a concomitant increase in the other input.

Thus a specified traffic pattern can be satisfied by the consumption of various different combinations of inputs T and C_2. What is the implication of one admissible policy being judged (subjectively) better than another for a specified traffic pattern? Since the functional relations $T(X)$ and $C_2(X)$ are considered fixed, the only allowable latitude is in the choice of S. Hence, if the planner prefers policy Y to policy Z, it follows that according to his judgment, the dollar value of S corresponding to policy Y is more appropriate than the dollar value of S corresponding to policy Z. Finally, if among all admissible policies, the planner chooses policy W as representing his notion of satisfactory highway conditions for his community, the use of S corresponding to policy W for the proposed highway enables him to incorporate into the future design all the desirable features of the reference highway.

It would be desirable to give to the

planner complete information about all admissible policies, but this may require more effort than can be spent. Hence, to economize effort, it is convenient to specify a subregion in the input space which is to be explored for admissible policies. This can be done by letting the planner select a policy, for the reference highway, which he considers satisfactory for his community. Then, it is necessary to verify if this policy is admissible. If it is, the value of S associated with this policy is computed by using

$$S = -\frac{dC_2(X)}{dT(X)} = -\frac{\dfrac{\partial C_2}{\partial x_i}}{\dfrac{\partial T(X)}{\partial x_i}}, \ (i = 1, 2). \tag{9}$$

As is to be expected in most cases, the policies designated by planners on the basis of judgment will turn out to be inadmissible. In fact, it is only by accident that a policy selected on the basis of judgment alone will turn out to be admissible.

However, the inadmissible policy can serve as a starting point for the analyst to explore the input space and see if there are admissible policies in the neighborhood of the inadmissible policy originally designated by the planner. The analyst can only assist by exploring the input space and present some admissible policies to the planner. The final choice of a policy is that of the planner. In exploring the input space, the analyst can determine, for example, an admissible policy that

1. Has the same value of T as the inadmissible policy;
2. Has the same value of C_2 as the inadmissible policy;
3. Has a specified value of T;
4. Has a specified value of C_2; and
5. Has a specified value of any parameter, x_i.

Admissible policies in some of the above categories might not exist. For example, corresponding to X' (mapped by A in Fig. 3), there does not exist an admissible policy with the same value of T as $T(X')$.

Finally, the analyst presents a number of admissible policies to the planner and when the latter selects rationally, one of these as satisfactory (in the sense that considering the actual traffic pattern and T and C_2 associated with the policy, he feels that this policy would best represent the traffic conditions considered satisfactory for the community) the analyst can determine the value of S by using Eq. 9.

Example

In the terminology of this paper, assuming that the planner has judged tentatively the existing policy of an existing highway as satisfactory and that the policy of this highway consists of two parameters — control of access = 0.6, and pavement width = 50 ft, *i.e.*, $x_1' = 0.6000$ and $x_2' = 50.00/100 = 0.5000$; and also assuming that $a = 10$ ft is the minimum pavement width, *i.e.*, $x_2 = 0.1000$, the analyst asks: If X' is the minimum-cost policy, what dollar value must be assigned to S?

First Step. To determine functions T and C_2 for this highway and to verify if the basic assumptions are satisfied: The functions are assumed to be:

$$T = \frac{1}{x_1} + e^{(1-x_2)} \tag{10}$$

$$C_2 = x_1^2 + 2x_2 \tag{11}$$

where T is in million hours per year and C is in million dollars per year. It is assumed that the condition of continuity is satisfied. It is obvious that assumptions of technology and differentiability are satisfied. For the purpose of this illustration, the assumptions of invariance of traffic pattern and rational decision-making also are satisfied.

As the control of access, x_1, is increased, T decreases and C_2 increases. Also as the pavement width, x_2, is increased, T decreases and C_2 increases — as should be expected in an actual case.

The total transportation in million dollars per year is

$$C = S\,T + C_2 \tag{12}$$

where S is still unknown.

$$T(X') = \frac{1}{0.6000} + e^{(1-0.5000)}$$
$$= 3.3154 \times 10^6 \text{ hr/yr}$$

$$C_2(X') = (0.6000)^2 + 2\ (0.5000)$$
$$= \$1.3600 \times 10^6 \text{ per year.}$$

Second Step. To verify if policy $X' = (0.6000, 0.5000)$ is admissible: Policy and input spaces are drawn (see Figs. 4 and 5 and the Appendix). On mapping X' from policy into input space, it is obvious that:

1. For all X such that $T(X) = T(X')$, there exists an X with $C_2(X) < C_2(X')$ and
2. For all X such that $C_2(X) = C_2(X')$, there exists an X with $T(X) < T(X')$. Hence X' is inadmissible.

Third Step. Some admissible policies in the neighborhood of X' are computed and presented to the planner.

The admissible policy with

$$T(X) = T(X') \text{ is } x_1 = 0.7877, x_2 = 0.2842.$$

The admissible policy with

$$C_2(X) = C_2(X') \text{ is } x_1 = 0.8065, x_2 = 0.3550.$$

The admissible policy with

$$x_2 = x_2' \text{ is } x_1 = 0.8465, x_2 = 0.5000.$$

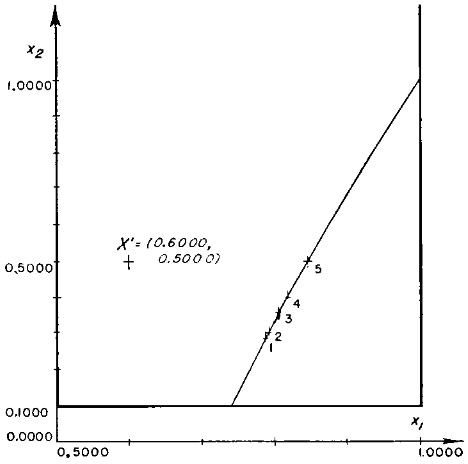

Figure 4. Policy space.

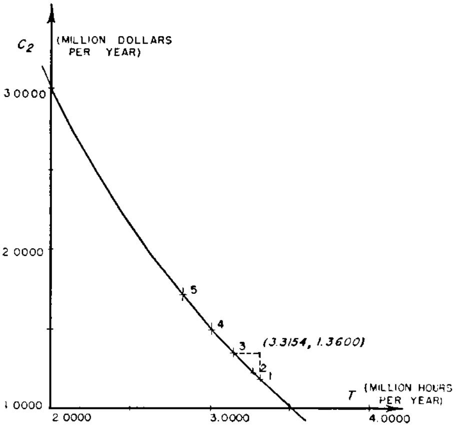

Figure 5. Input space.

In fact it can be shown mathematically that each point located on the curve $x_1^3 = e^{(x_2-1)}$ (Fig. 4) is an admissible policy and that all admissible policies are located on this curve (see Appendix). The curve in Figure 5 is the mapping of all admissible policies from the policy space into the input space.

Table 1 shows some admissible policies together with T and C_2 values associated with them. The number assigned to each policy in the first column of this table corresponds to the way policies have been numbered in Figures 4 and 5.

Fourth Step. The planner ranks the policies that are presented to him in their order of desirability to his community and states that according to him the policy of his choice (policy No. 5, in this case) represents satisfactory highway conditions for his community.

Fifth Step. To compute the value of S, The analyst uses the policy selected in the preceding step for computing S by

TABLE 1

SOME ADMISSIBLE POLICIES

Policy				
	Parameter			
No.	x_1	x_2	T (hr × 10^6)	C_2 (\$ × 10^6)
1	0.7877	0.2842	3.3154	1.1889
2	0.7920	0.3000	3.2764	1.2273
3	0.8065	0.3550	3.1460	1.3600
4	0.8187	0.4000	3.0451	1.4703
5	0.8465	0.5000	2.8300	1.7166

means of Eq. 9. Thus, the value of S corresponding to policy No. 5 selected in the preceding step is:

$$S = -\frac{\dfrac{\partial C_2}{\partial x_2}}{\dfrac{\partial T}{\partial x_2}} = 2/e^{(1-x_2)} = 2/e^{0.5000}$$

$$= (2)(0.6065) = \$1.21$$

The sequence of steps shown in this example is not rigid and may be varied to suit the circumstances of the case.

GENERAL COMMENTS

The technique can be used for policies which contain any finite number of parameters (*2*). When the number of parameters exceeds two, the policy space cannot be represented geometrically and the analysis must be done mathematically. As the number of parameters increases, the complexity of the problem increases. For this reason, the number of parameters should be kept as small as possible.

The planner need not be an individual. Since the choice of satisfactory highway conditions is a social choice, the function of a planner is more likely to be performed by a group than by an individual (*8*).

It seems reasonable to conjecture that the appropriate value for S will vary not only from community to community but also from one type of highway to another (*9*). Motorists are more tolerant of delays on city streets than on freeways. Therefore, the purpose for which the value of S is required should be kept in mind. Thus, if the value of S is required for the purpose of evaluating policies for a freeway, it seems reasonable that the value be assigned on the basis of an existing freeway or at least on the basis of a comparable high-speed facility. On the other hand, if the value is required for making analyses for city streets, it would seem reasonable to derive it on the basis of an existing city street. Facilities can also be distinguished on the basis of urban and rural locations.

The question of costs to be included in C_2 is critical. A full discussion of this topic would need more space than is available here. As already stated, C_2 includes all transportation costs with the exception of the cost attributable to the transit time of private automobiles. Usually, a great deal of confusion exists about the basis on which the right-of-way and construction costs should be included in C_2. In this connection, it should be remembered that the current book value of the reference highway will have no relevance as far as C_2 is concerned.

CONCLUSION

The principle that the concept of satisfactory highway conditions is local in nature has long been recognized by highway engineers. For example, the *Highway Capacity Manual* states:

> Because the conditions that govern the degree of congestion which may be considered as tolerable are so local in character, the Committee chooses to refrain from any specific recommendations for these intermediate terms. Considered more important is the need to inform the reader of the effect of these intermediate traffic volumes. Having been thus informed, the local official will be better able to exercise sound judgment in deciding upon satisfactory or tolerable capacities for use in administering his available funds to the greatest advantage of the public.

. . .

> To determine the practical capacity of a facility it is necessary, first, to determine the operating conditions that the majority of motorists will accept as satisfactory. . . . Thus, in the final analysis, the matter of specifying precise values for practical highway capacities becomes a localized problem.

Thus the technique has not advanced any new concept. However, what the technique has done is to develop mathematical analysis to use this concept. The superiority of the proposed technique over the current approach lies in the fact that whereas there is no test for checking the validity of judgment used in the selection of a dollar figure, the proposed technique provides for a test of validity for judgment used in the selection of satisfactory highway conditions.

The flexibility of the technique exploits the concept that the notion of satisfactory highway conditions is local in nature. The results derived by the use of the technique are strictly hypothetical of the form: If these are the highway conditions which are considered desirable for the community, then this is the appropriate dollar value for transit time of private automobiles.

The use of judgment for the designation of satisfactory highway conditions raises problems such as the following: How far is the human mind capable of choosing rationally among different highway policies? Is the judgment reliable and judicious when the choice situation involves policies which differ a great deal from the existing policies? It is to be noted that the planner's experience usually is restricted to the existing policies of existing highways and his ranking of other policies will necessitate a great deal of extrapolation and might involve the risk of a rash judgment. For this reason, a suggestion could be made that only the neighborhoods (in the input space) of existing policies of highways be explored for admissible policies until some experience is gained with the application of the technique to the problems of highways. Mock trials in experimental setups, using highway policymakers, might also be useful in giving some insight into the decision problems involved (*11*).

The immediate usefulness of the results of the study presented here might be questioned because of the inadequate knowledge of technology, that is, the lack of knowledge about functional relations, T and C_2. However, the technique has focussed attention on the type of data which will be needed if the technique is to be put into use.

This is a simplified version of a technique for assigning an appropriate dollar value to the transit time of private automobiles when some basic assumptions are satisfied. Having derived such a value, how it is used for determining an optimum policy is a separate topic in itself which has not been touched upon in this paper (*2*). This question also has many interesting facets, for example, handling cases involving deterministic traffic patterns and uncertain traffic patterns (*2*).

ACKNOWLEDGMENT

The author takes this opportunity to express his appreciation to Professors D. B. Hertz, S. B. Littauer, R. T. Livingston, and W. S. Vickery of Columbia University and Professors L. G. Mitten, R. F. Reeves, and A. D. Ziebur of the Ohio State University for providing guidance during various phases of research reported in this paper.

The author's interest in this problem was aroused during his work on a research project on urban congestion which was sponsored by the Automotive Safety Foundation at the Ohio State University. During the progress of this project, the author profitted from discussions with Professors Robert F. Baker and L. G. Mitten. Chester E. Ball, Joan Steele and Patricia Heffley of the Ohio State University gave valuable assistance.

Professor C. A. Anderson of North Carolina State College has provided encouragement for the completion of this paper.

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APPENDIX

In this appendix, a necessary condition for admissibility will be investigated. The treatment of this topic will be brief.

For an n-dimensional policy:

$X = (x_1, x_2, \ldots, x_n)$. For a policy X' to be admissible, it is necessary that:

$$\text{(a)} \qquad \underset{X}{\text{Min}}\, C_2(X) = C_2(X'), \qquad (13a)$$

subject to

$$T(X) = T(X') \qquad (13b)$$

and

$$\text{(b)} \qquad \underset{X}{\text{Min}}\, T(X) = T(X') \qquad (14a)$$

subject to

$$C_2(X) = C_2(X'). \qquad (14b)$$

First, condition (a): To minimize $C_2(X)$, form the Lagrangean function:

$$F(X,\lambda) = C_2(X) + \lambda\,\{T(X) - T(X')\}, \qquad (15)$$

where λ is a Lagrangean multiplier.

Differentiating F partially with respect to x_i's and λ, and setting the partial derivatives equal to zero, gives:

$$\text{(c)} \qquad \frac{\partial C_2(X)}{\partial x_i} + \lambda \frac{\partial T(X)}{\partial x_i} = 0, \qquad i = 1, \ldots, n. \qquad (16)$$

$$\text{(d)} \qquad T(X) - T(X') = 0.$$

From (c):

$$\text{(e)} \qquad \lambda = -\frac{\dfrac{\partial C_2(X)}{\partial x_i}}{\dfrac{\partial T(X)}{\partial x_i}}, \qquad i = 1, \ldots, \text{n}. \qquad (17)$$

Similarly, minimizing $T(X)$, in condition (b), by the use of another Lagrangean function $F(X,\lambda')$ will yield a necessary condition:

$$\text{(f)} \qquad \lambda' = -\frac{\dfrac{\partial T(X)}{\partial x_i}}{\dfrac{\partial C_2(X)}{\partial x_i}}, \qquad i = 1, \ldots, n. \qquad (18)$$

If (e) and (f) are evaluated at the same point X', then

$$\text{(g)} \qquad \lambda = 1/\lambda'. \qquad (19)$$

Further, it can be shown that for X' to satisfy conditions (a) and (b), λ must be positive and must equal to S.

For the case considered in this paper, $n = 2$. Hence, Eq. 17 becomes:

$$-\frac{\dfrac{\partial C_2}{\partial x_1}}{\dfrac{\partial T}{\partial x_1}} = -\frac{\dfrac{\partial C_2}{\partial x_2}}{\dfrac{\partial T}{\partial x_2}} \qquad (20)$$

that is,

$$\frac{-2x_1}{-1/x_1^2} = -\frac{2}{-e^{(1-x_2)}} \qquad (21)$$

or

$$x_1^3 = e^{(x_2-1)} \qquad (22)$$

The curve in Figure 4 is obtained from Eq. 22.

The curve in Figure 5 is obtained by computing T and C_2 values for a number of policies located on the curve in Figure 4.

The admissible policy with $T(X) = T(X')$ is obtained by solving Eq. 22 and

$$\frac{1}{x_1} + e^{(1-x_2)} = 3.3154.$$

The admissible policy with $C_2(X) = C_2(X')$ is obtained by solving Eq. 22 and

$$x_1^2 + 2x_2 = 1.3600.$$

The admissible policy with $x_2 = x_2'$ is obtained by substituting $x_2 = 0.5000$ in Eq. 22.

For second order condition for a minimum of a function, the reader is referred to any text on calculus or Ref. (*2*).

DISCUSSION

G. P. St. Clair, *Bureau of Public Roads.* — Although assured of the validity of the mathematical treatment in this paper, the writer is still assailed with doubts of its logical significance. How can these equations yield a valid solution for automobile transit time, when no treatment of the subjective factors that give value to transit time is included in the steps leading to their formulation?

Further discussion and reflection quieted these doubts and led to what it is hoped is an acceptable interpretation of the author's theme. The concept is that an evaluation of time is inherent in the adoption of highway design standards, which are the "parameters of the road." If a traffic volume of 5,000 vehicles per day is judged to require a 4-lane divided highway, that decision implies a much higher valuation of the time of those using the facility than if a 10-ft 2-lane road were chosen; and a very much lower valuation than if the decision were for a 6-lane highway with full control of access. To put it more technically, if design is based on the traffic volume in the 30th highest hour, or the 50th, that decision implies a much higher appraisal of the cost of traffic delays than if the 200th or the 1,000th hour were chosen. A decision to use the *highest* hour would indicate that an extraordinarily high premium was being placed on the elimination of all traffic delays.

In this design of a mathematical system to aid the decision-maker (highway designer or planner) in choosing the parameters of the road (highway design features) it is observable that the equations as set down permit of solution only for the two dependent variables, T, the total annual transit time in hours, and C_2, the total of all other costs in dollars, since these are direct functions of the given parameters. But the equations will not be dollar-based unless the unit cost of time, S, can be evaluated. The author rejects the use of a prescribed value, or one derived elsewhere, on the ground that it would yield, at best, a defective solution. He elects to find the value of S in the data of the individual problem. He does this by invoking the principle of minimization of costs, under the terms of which an equation for S, the unit value of time, as a function of the parameters of the road, is derived. Since the decision-maker, or planner, is given his choice among all "admissible" policies, the value of S varies according to the choice. Thus the decision of the planner among admissible policies with respect to design features carries with it an implicit decision as to the unit value of time.

Doubts will arise as to the practicality of the proposed technique, but, in view of the widening usefulness of this general field of analysis, engineers should be open-minded. One criticism does occur. Highway design — the choice of the parameters of the road — is not as vague and fumbling a process, as dependent on personal judgment, as the author seems to imply. The designer is constrained in the first instance by the geometric and structural design standards, which, for the given conditions of traffic, soil type, and topography, dictate the dimensions of his cross-section, type of surface, base, and subbase, permissible curvature and gradient, and other features. He is further constrained by local conditions, which may make mandatory or desirable certain departures from the standards. The design standards are products of research and experience, always imperfect but subject to continual improvement. Even so, design standards themselves might well be a subject of operations research, as to whether they minimize total costs, including time and other subjective factors.

A pertinent technical question is that of whether design standards, which are in effect the parameters of the road, will lend themselves to expression as continuous variables. Those that vary numerically with traffic and other measurable conditions seem to satisfy that requirement, although such discontinuities as the shift from 2-lane to 4-lane divided may cause trouble. An allied difficulty is that of the assumed invariance of the traffic pattern. Although it will probably hold within a narrow range of values of the parameters, it is basically untrue and should yield to an assumption and a procedure that will acknowledge the influence of the type of facility on motorists' choices of alternative routes and alternative destinations. The fact that these strictures, if taken seriously might necessitate a more complex framework of analysis in no way detracts from the significance of this contribution.

Finally, there is the matter of independent measurements of the unit value of travel time. Campbell, as reported in the Yale Bureau bulletin, "Toll Bridge Influence on Highway Traffic Operations," made such evaluations by the use of origin-destination data for vehicles using West Virginia bridges. Cherniack of the Port of New York Authority has done useful work in this field, and there have been numerous others. Such work should continue. Even though appraisals of the unit value of time are inherent in the standards and decisions of highway design, they are not necessarily the correct appraisals. New and more accurate data regarding the value of time and other subjective factors might materially affect design standards and practices.

The value of an automobile-hour is a subjective one; indeed, in the mind of the average motorist, it is a subconscious one. It varies with the purpose of his trip, with the time of day, with his physical and mental state, with his income status, and with other, more obscure, influences. Any unit value of time, whether arrived at experimentally or analytically, must be an average of many widely-varying subjective values. The subjective evaluations of motorists can be studied statistically to learn something about the choices they make when confronted by alternatives.

In recent years measurements of the unit value of automobile transit time have been based on origin-destination studies under conditions which will reveal the amounts that motorists seem willing to pay — generally in tolls — to effect time savings. Unfortunately, the accuracy of such evaluations is impaired because the value of time cannot readily be factored out independently of that other benefit conferred by controlled-access roads — relief from the strains, annoyances, and inconveniences of congested driving. If the technique proposed and discussed here could be modified to provide a means of solution for these two principal factors of expressway advantage, it would be a definite step forward in the economics of highway transportation. For such work the analytical framework should be adapted for the utilization of research data, such as the results of origin-destination studies.

RAM VASWANI, *Closure.* — Mr. St. Clair has raised several important questions which deserve the special attention of any highway planner who intends to employ an analytic approach for improving his decisions. The use of an analytic approach necessitates abstracting from the real world. No abstraction ever purports to represent reality in all respects and the important question is not how precisely the abstraction duplicates the real world, but how useful it is. In designing an analytic approach to a problem, a researcher has to strike a balance between including all the variables and keeping the problem simple enough to make it amenable to attack.

In deciding to use an analytic approach devised by someone else, the highway planner should verify that the abstraction on which the analytic approach is based has its counterpart in the real world. If this fact is not recognized, much money and energy might be wasted. Therefore, an application of the proposed methodology to a highway problem must be preceded by a study to see if the abstraction used as a basis for the methodology is isomorphic with the highway problem under consideration.

The five basic assumptions stated in the paper define the abstract system. These assumptions are contingent and may not hold completely in the case of a real highway. To the extent that reality approaches these assumptions, the proposed methodology will be useful. Serious departures of these basic assumptions from reality may render the proposed methodology useless or lead to the development of techniques adequate to correct the solution for these departures.

St. Clair has pointed out that the lack of assumptions of continuity and invariance of traffic pattern can cause difficulties and this warning should be heeded by all highway planners who intend to apply the technique to their problems. The specific case pointed out, where the assumption of continuity fails to hold, is when the number of lanes is used as a design parameter. It might be possible to get around this difficulty by substituting pavement width as a single parameter for two parameters — the number of lanes and width of lanes.

In spite of such devices in which engineers usually are adept, discontinuities will still occur. A case in point is the parameter of median separation. A discontinuity occasioned by median separation will necessitate dividing the problem into (a) highway without a median separation and (b) highway with a median separation. A detailed consideration of such problems will lead into the area of design of optimum highways which is not the subject of the present paper.

The basic assumption of invariance of traffic pattern will not hold when the road user demand is elastic. When the elasticity is so great as to make the objective of minimum transportation cost inappropriate, the complexity of the problem at one increases. In such a case, an appropriate criterion is the "gain" to the society from the highway system, where "gain" is defined as "worth" minus cost. By "worth" is meant the effect of different highway designs on the land use pattern, commerce and industry, etc. In fact, in the broader sense of the term, worth will involve weighing the impact of each possible design of a highway on the whole national economy.

The use of the criterion of gain is an impossible task, since, quantitatively speaking, little or nothing is known about the worth of a highway system to the community, through which the highway passes, and to the nation at large. Studies of benefits derived from new highways are still in their infancy (*12*). Because of these difficulties, the attack is made on the less ambitious sub-optimization problem, using transportation cost as the criterion and assuming the traffic pattern to remain constant.

It might be pointed out that the terminal use of a dollar value for transit time is the design of highway projects. In such a case, apart from taking into consideration the effect of elasticity of demand for highway use, the highway planner also has to grapple with the problem of designing a highway for an uncertain traffic demand. This probably is as serious a problem as the assumption of invariance of traffic pattern.

By an uncertain traffic demand is meant the condition in which the highway planner is not all clear about the specific traffic pattern which will prevail after a highway project is completed. He may have some idea about some of the possible traffic patterns but he may not know with certainty which one of these possible traffic patterns will prevail after the completion of the project. In such a case, the highway planner has more or less to play a "game" with nature and a theory that is entirely satisfactory does not exist which will enable the highway planner to make a good decision. Some rules for guidance can be provided by the use of statistical decision theory. However, these rules can only aid the highway planner, rather than provide definitive decision procedures, in the choice of an optimum design. This same theory can be used to relax slightly the basic assumption of invariance of traffic pattern.

St. Clair has mentioned the restrictions which soil, topography, local conditions, etc., impose on the highway planner's choice of the geometric design. Restrictions exist in most decision problems. The restrictions merely provide bounds or limits on the parameters and define the policy and input spaces (see Figs. 1 — 5) within which the planner has the freedom of choice. Given these restrictions or bounds on the policy and input spaces, the planner attempts to find a design which is optimum according to an appropriate criterion, for example, the total transportation cost.

The most troublesome assumption at the present stage of development of highway engineering is the basic assumption of technology. If highways are to be understood, an early beginning must be made in expressing highway transportation costs as algebraic functions of highway and traffic parameters. Since the task of finding cost functions which are universally applicable appears to be impossible to attain, a beginning should be made with some specific projects and the experience gained from work on these projects transferred to later projects. This essentially is the area in which highway planners and engineers have a great contribution to make. Until some applications in appropriate situations are tried, no definitive statements can be made about the practicality or usefulness of the proposed methodology.

A highway planner must continue to secure information about the preferences of highway users from the traditional origin-destination (O-D) surveys. Hence, the O-D surveys will continue to be useful and will furnish the planner the feedback information about the conditions which highway users prefer. In addition to O-D surveys, techniques of direct evaluation, such as those proposed by Breuning and Bone (*13*), also will be useful. In his actual decision of what constitutes satisfactory highway conditions for users, the planner will take into consideration the preferences of users as well as various other factors, for example, budgetary restrictions (the input restrictions). In fact, if the types of factors on the basis of which highway planners prefer one highway design over another can be revealed by actual case studies, this will be an achievement.

It would be desirable to devise a methodology for evaluating the two principal factors of expressway advantage pointed out by St. Clair — driving comfort and transit time. As the technique stands at present, driving comfort is not taken explicitly into consideration. However, it is taken implicitly into consideration while an appropriate value is being assigned to transit time. This is accomplished by a suitable choice by the highway planner of a satisfactory highway design which is used as a basis for assigning an appropriate value to transit time. Evidently, more work needs to be done to ascertain the effectiveness of such an implicit consideration.

Obviously the proposed methodology does not solve all the problems which highway decision-makers would like to see solved. It is the author's hope that the methodology has advanced a little farther beyond the AASHO's current procedure for economic analyses of highway projects. Furthermore, it is hoped that researchers will raise several questions in connection with the proposed methodology and develop useful improvements and modifications.