Shear Strength of Prestressed Concrete I-Beams

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• THE WORK discussed in this paper was done as a part of a research project on prestressed concrete being conducted at the Iowa Engineering Experiment Station. The first phase of the project was a study of the flexural behavior of a prestressed concrete bridge beam. It appears from this study and other investigations that the flexural characteristics of prestressed beams are well understood and can be predicted with reasonable accuracy.

The second phase was a study of the shear strength of prestressed concrete I-beams. The term "shear strength" is subject to varied definitions. In this paper shear strength is defined as the load at which a sudden diagonal crack completely traverses the web of the beam. The region in which the crack occurs is subject to direct and shearing stresses on its vertical planes, producing tensile stresses on the inclined failure plane.

A theory of failure for concrete under combined stresses is necessary for the rational solution of the problem of shear strength. Of the many theories advanced for failure of concrete under combined stress only one was considered. Grassam (*1*) has suggested a modified maximum tensile stress theory in which account is taken of some plasticity within the concrete. Arbitrarily taking as the failure stress the mean of the stresses which would cause failure under the assumption of elasticity and plasticity, his test results showed satisfactory agreement with the theory. He also suggests that an arbitrary value of critical tensile stress equal to 1.2 times the tensile strength will fit the results for a combined stress condition.

The results in this paper should be considered preliminary in nature, because the information from only four test beams is included. Three I-beams in which reinforcement was omitted from the shear spans were tested. However, 2-ft overhangs with web reinforcement were included on each beam so that the full prestress force would be developed outside the shear span. The only variable introduced in the three was the length of the shear span. These three beams were fabricated in a commercial prestress plant. The fourth beam with web reinforcement throughout its entire length was fabricated in the Iowa Engineering Experiment Station Laboratory. The depth and the cross-section of the fourth beam varied slightly from the first three beams.

EXPERIMENTAL INVESTIGATION

Test Specimens

To vary the shear with the direct stress, or combined stress, in the three beams without web reinforcement in the loaded span, a shear span of different length was adopted for each beam. All other properties of the three beams except concrete strength, which varied with time, were the same. The beams, designated as Beam I, Beam II, and Beam III, had 3-ft, 3-ft 6-in., and 4-ft shear span lengths, respectively. Each of these beams had a 2-ft overhang at each support in which web reinforcement was placed in order to prevent a horizontal shear failure of the beam during its fabrication. The overhang would also permit the full prestress force to be developed at the beginning of the shear span. The beam with web reinforcement, designated as Beam IV, had 4-ft shear spans.

A length of beam with zero shear and constant moment was necessary to determine the flexural cracking load and to follow the movement of the neutral axis. For this purpose a length of 3 ft between the load points was adopted for all four beams.

The cross-sectional dimensions of the beams are shown in Figure 1, and the web reinforcement for Beam IV is shown in Figure 2. The span dimensions for the test beams are shown in Figure 3.

Manufacture

In the usual plant production process the initial prestressing force is determined by putting a required elongation in the cables. For more accurate measurement of this force in the 3 commercially fabricated test beams, three A-12 SR-4 strain gages were placed on each of four cables in Beam I. These gages were read before and after tensioning, before and after cable release, and during the testing of the beam. The concrete for the three beams was placed from one batch, and an internal vibrator was used to consolidate it. Simultaneously, 4½- by 9-in. cylinders and 6- by 6- by 36-in. beams were cast. All the samples and the prestressed beams were steam cured. The prestress force was released when the

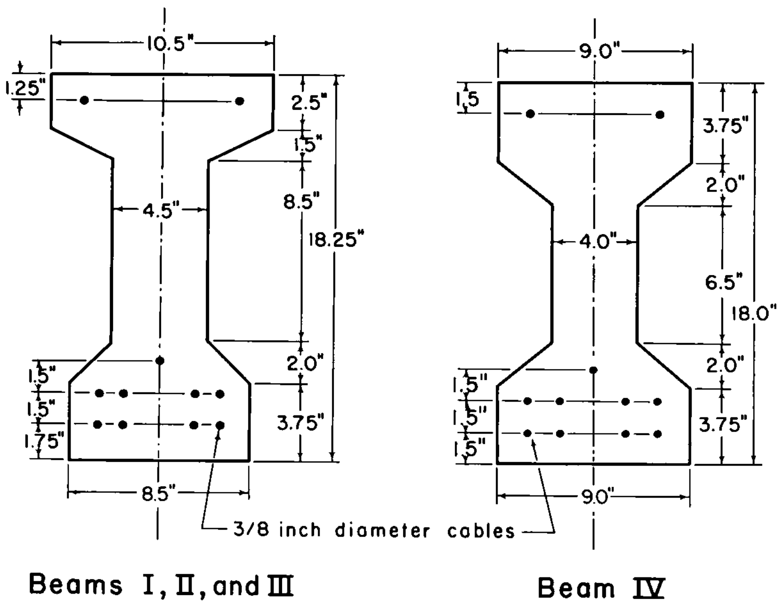

Figure 1. Cross-sectional dimensions of the test beams.

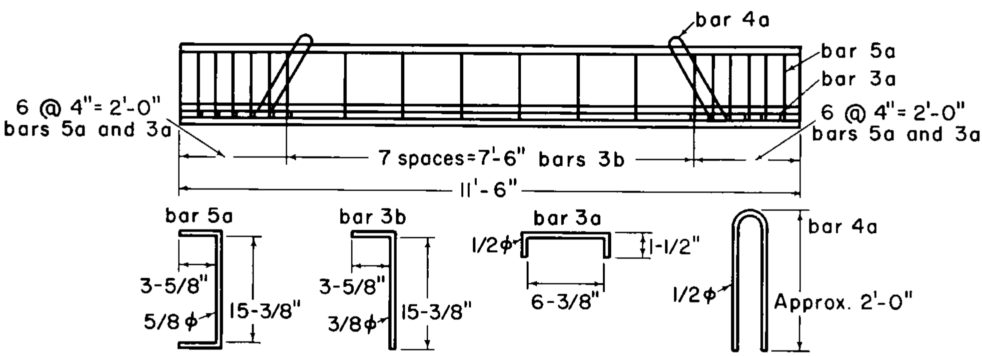

Figure 2. Web reinforcement in Beam IV.

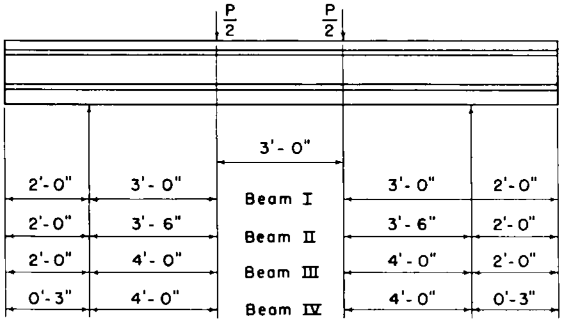

Figure 3. Span dimensions of the test beams.

concrete had attained an ultimate strength of 4000 psi. To release the prestress force, the cables were tightened further so as to loosen the end fastenings. As this process took place, the end beam, which was Beam I, cracked across a section near the middle. The crack was marked and no cracks were found on Beams II and III. After the cables were released the crack in Beam I closed up completely.

The usefulness of the beam was not impaired, because the crack appeared near the middle of the beam and would incur only flexural stresses. This crack would open when zero stress occurred in the bottom fibers of the beam enabling the final prestressing force to be calculated, when the beam was tested, without depending on experimentally determined values of the modulus of rupture.

Beam IV was fabricated in a prestress bed in the laboratory. The initial prestress force was measured by SR-4 strain gages placed on steel cylinder load cells fitted over the cables between the anchorage plate of the prestress bed and the Strandvise anchorages. These load cells were used on the four corner cables in the beam. A gage on the jack provided an approximate check on the determined load. The concrete was placed and was consolidated with an internal vibrator. Simultaneously 4½- by 9-in. cyclinders, 6- by 6- by 36-in. beams, and 4- by 4-in. cross-section tension specimens were cast. The concrete was covered with burlap and was moist cured for seven days.

MATERIALS

Concrete

The water-cement ratio used for the mix was 4 gal per sack of cement, and the concrete had a slump of 2½ in. The maximum size of the coarse aggregate was 1 in.

Steel

The prestressing steel used was ⅜-in. diameter stress relieved 7-wire strand. It had a minimum specified ultimate strength of 250,000 psi. The web reinforcement was fabricated from structural grade steel bars.

EXPERIMENTAL EQUIPMENT

SR-4 Electrical Strain Gages

Three A-12 gages were mounted on each of four cables in Beam I and were waterproofed with a wax coating. The gages were placed on individual wires of the cables. These gages were used to determine the initial prestress force for Beams I, II, and III. The initial prestress force for Beam IV was determined with load cells as described.

A-9 gages were used on the midspan section of each beam to locate the neutral axis throughout the tests. A-9 gages were also placed in a continuous line on the bottom fibers of the central 3 ft of the beams; these gages were used to determine the initial flexural cracking of the beams.

A number of AR-1 gages were placed on the webs of the shear spans in the regions where diagonal cracking was expected to occur. The data obtained from the AR-1 gages were not too satisfactory. This was probably because of the small gage length in comparison with the maximum size of the coarse aggregate used in the concrete.

Testing Machines

Hydraulic testing machines were used for all the testing. A 400,000-lb capacity machine was used to test the prestressed beams. Steel beams above and below the test beams provided a load frame for the testing machine. The tension specimens and compression cylinders were tested in a 300,000-lb capacity machine. The 6- by 6- by 36-in. concrete beams were tested in a 60,000-lb capacity machine.

Deflection Apparatus

The deflections were measured with 0.001-in. Ames dials. Two dials were set in place on the bearing plate at each end of the prestressed beam, and two dials were placed in contact with the bottom surface of the beam at midspan. Each pair of the dials was placed symmetrically with the longitudinal axis of the test beam. This arrangement of the dials eliminated the effect of any twisting of the beams and gave the midspan deflection on the longitudinal axis of the beams.

TESTS

Concrete

Several cylinders were tested to failure when the cables were released and when the prestressed beams were tested to failure. A compressometer was used to measure strain on all of the cylinders, and A-9 gages were used on a few as a check. The two measuring systems gave results which were in close agreement.

Two A-9 gages were placed on the bottom fibers at midspan of the 6- by 6- by 36-in. beams, and the beams were tested on a 30-in. span with third point loading. These specimens were tested when the prestressed beams were tested to failure.

Tension specimens of the concrete used in Beam IV were tested for ultimate strength when the prestressed beam was tested to failure.

Beam I

Beam I was tested at an age of 29 days. During the tests of all the beams the strain gages and deflection dials were read at intervals up to the ultimate load.

A noticeable jump occurred in the strain reading of one of the bottom A-9 gages between 55 and 60 kips load. This gage was placed across the section which had cracked prior to cable release, and an examination showed that the crack had opened. Loading was continued, and flexural cracks appeared and extended into the web in the central 3 ft of the beam. However, it was not until a load of 80 kips was applied that a flexural crack appeared within the shear span.

At 95 kips load a diagonal crack occurred suddenly across the web of one shear span. A similar diagonal crack appeared in the other shear span at 100 kips load, followed by a second crack in the first shear span when the load reached 108 kips. Collapse of the beam occurred at 114 kips load (Fig. 4).

Beam II

Beam II was tested at an age of 44 days. Flexural cracking first occurred between 55 and 60 kips load, and the cracks progressed slowly up into the web between the load points as the load was increased. The first crack within the shear spans appeared at a load of 83 kips and progressed 7 in. from the bottom of the beam. It was a flexural crack, but when it reached the web it inclined from the vertical direction due to the shear at this section.

When the load was increased from 85 to 88 kips, a group of diagonal cracks appeared suddenly in one shear span. A similar series of cracks appeared in the other shear span at 95 kips load, and the

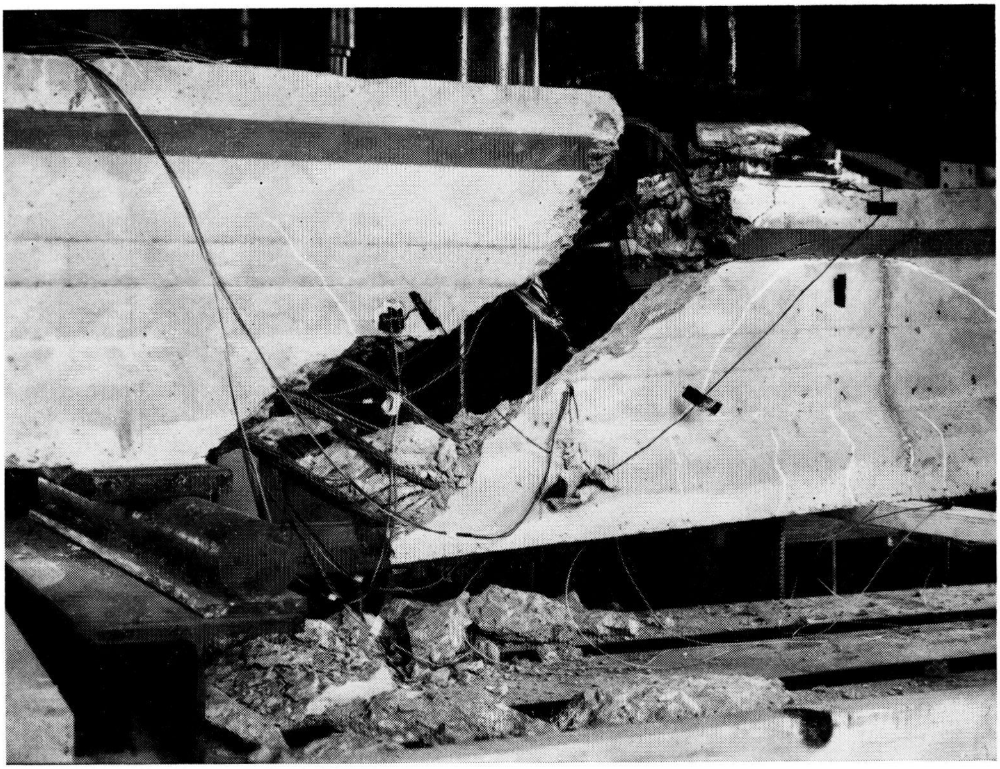

Figure 4. Beam I at ultimate load.

beam collapsed at a load of 96 kips. The failure is shown in Figure 5.

Beam III

Beam III was tested at an age of 52 days. The A-9 gages on the botton of the beam revealed flexural cracking between 50 and 55 kips load, and at a load of 65 kips flexural cracking occurred within the shear spans. One of these cracks appeared in each shear span and reached into the web at a load of 70 kips. At 75 kips load the flexural crack in one shear span had progressed 10 in. from the bottom of the beam.

Three diagonal cracks appeared in one shear span at a load of 80 kips. The crack nearest to the load point did not, however, completely cross the web. It was decided to note the effect of releasing the load at this stage. The load was reduced to 70 kips, and the crack nearest to the load point immediately spread along the web to the opposite load point.

Load was again applied, and a diagonal crack appeared in the other shear span at a load of 83 kips. At 86 kips load the longitudinal crack joined the diagonal cracks and the beam collapsed (Fig. 6).

Beam IV

Beam IV was tested at an age of 83 days. The first flexure crack was observed at the 60 kip load. This crack was 1 in. long in the bottom flange near the midspan of the beam. At 65 kips load, flexure cracks had progressed into the sloping portion of the bottom flange and were observed midway up the web at a load of 75 kips. When the load was released after the 100 kip load, all of these flexure cracks were noted to have completely closed.

At the 75 kip load the first shear crack was observed in one of the shear spans. At the 80 kip load several more shear cracks appeared, one of which completely traversed the web of the beam. As the load was further increased, other diagonal cracks appeared. A close-up of the cracks in one of the shear spans at the ultimate load is shown in Figure 7. When the load was released after the 100 kip load, the flexure cracks disappeared but the shear cracks did not.

As the load was re-applied and increased to 112 kips, a large chip of concrete spalled off the top flange of the beam at the midspan and failure occurred.

RESULTS AND DISCUSSION

Concrete Properties

Compressive Strength. The concrete in Beams I, II, and III had an ultimate strength of 4000 psi when the prestress force was released and an ultimate strength of 5420 psi, 6080 psi, and 6320 psi, respectively, when the beams were loaded to failure. The ultimate strength of the concrete in Beam IV was 5000 psi at eight days, 7370 psi when the prestress force was released, and 7400 psi when the beam was loaded to failure.

Modulus of Elasticity. Modulus of elasticity values were found from the data of the compression tests, the modulus of rupture tests, the deflections of the prestressed beams, and the A-9 gages on the bottom surface of the prestressed beams.

The values of the modulus of elasticity adopted at the time of prestress release were $3.77(10)^6$ psi for Beams I, II, and III, and $4.3(10)^6$ psi for Beam IV. A value of $4.5(10)^6$ psi was adopted for all the beams when they were loaded to failure.

Modulus of Rupture. The modulus of

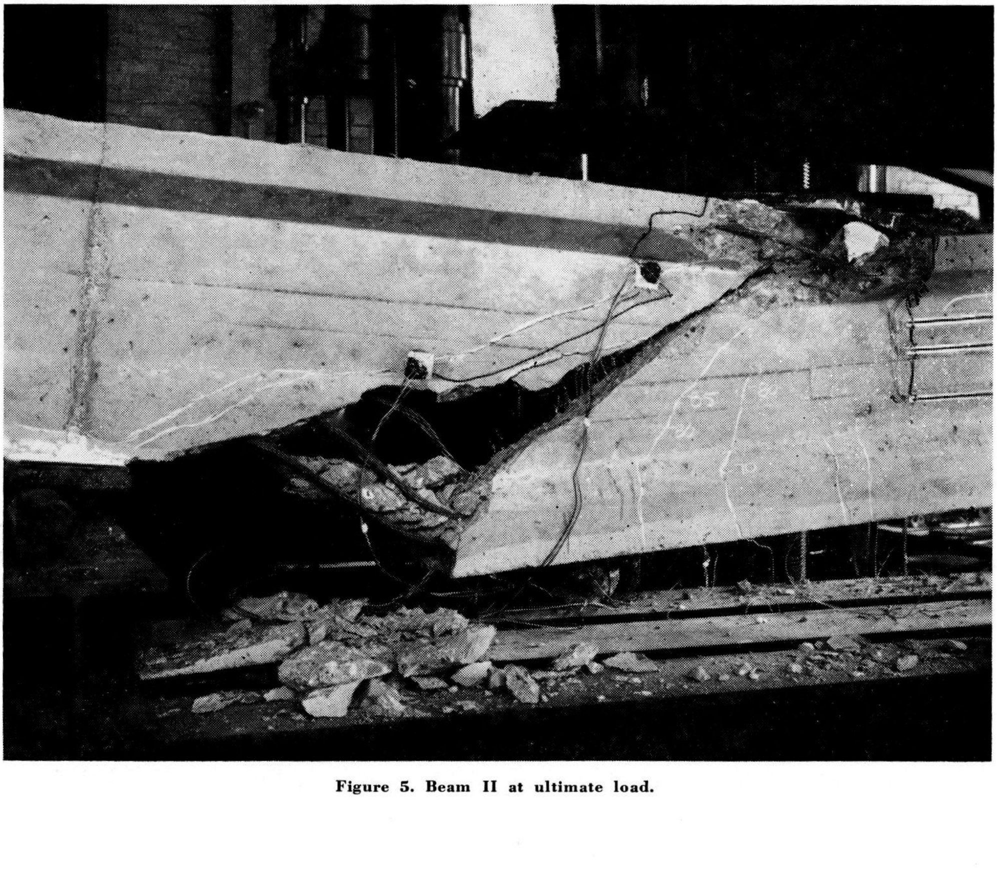

Figure 5. Beam II at ultimate load.

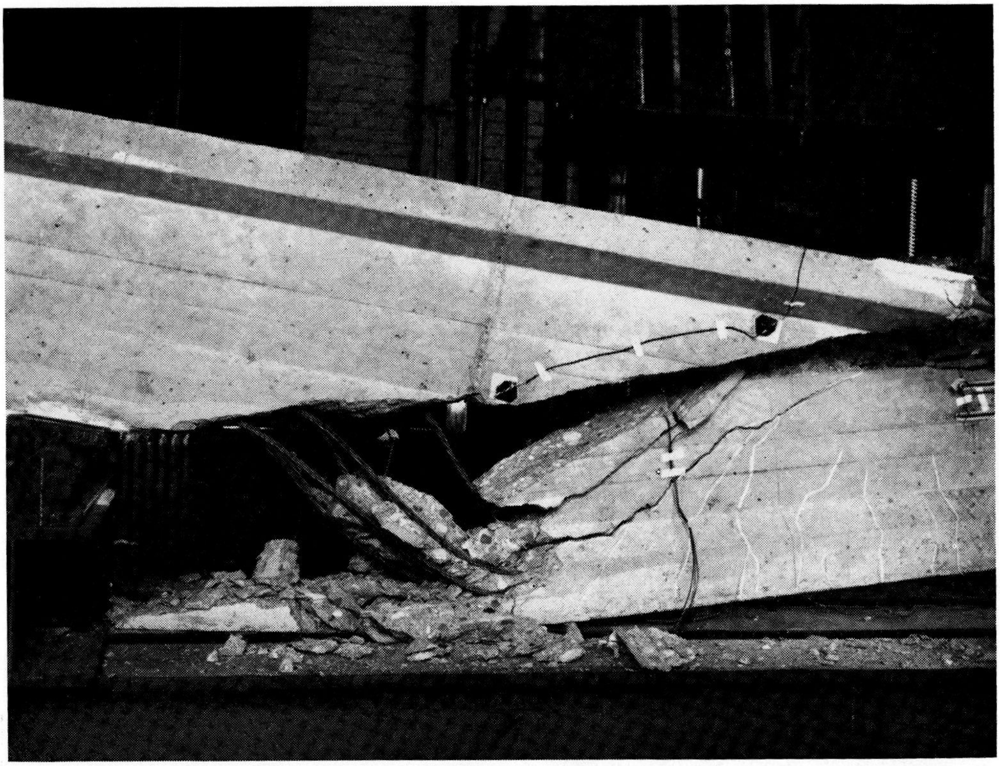

Figure 6. Beam III at ultimate load.

rupture tests were made at the time of the ultimate load tests of the prestressed beams. The average values found were 560 psi, 630 psi, 630 psi, and 740 psi, respectively, for Beams I, II, III, and IV.

Tensile Strength. There are no standard tests for direct tension on concrete, and no tests for ultimate tensile strength were made of the concrete in Beams I, II, and III. In the analysis of the data, however, it became evident that the values of the tensile strength would have been useful, and so recourse was made to the results of tests carried out by Johnson (*2*).

Johnson made 100 modulus of rupture and 350 direct tension tests with concrete strengths ranging nearly up to the values obtained in this investigation, and found the modulus of rupture of concrete to be 1.8 to 2.3 times the tensile strength. Using the lower value, the corresponding tensile strengths of the concrete in Beams I, II, and III could be estimated as 310 psi, 350 psi and 350 psi, respectively.

Direct tension tests were made for the concrete in Beam IV, and the tensile strength was found to be 500 psi. Based on this information the modulus of rupture was 1.48 times the tensile strength.

Final Prestressing Force

The final prestress force was based on the load which produced a crack at the bottom fibers of the beam in the constant moment region. The cracking load was taken as the average value obtained from the three procedures.

The strain gages placed in a continuous line on the bottom fibers of the beam were read at intervals of load; and, when a larger than normal increase in strain was recorded for one of the gages, the formation of a crack was indicated. This was usually accompanied by a smaller than normal increase in strain for adjacent gages.

The midspan deflection data also indicated when a crack had formed. The load deflection curves were straight lines until flexural cracks occurred in the bottom of the beam. The cracks caused a change in the effective cross-section of the beam and resulted in a deviation of the curve from a straight line.

The movement of the neutral axis during the loading was determined from the strain distribution indicated by the strain gages placed at the midspan section. The neutral axis progressed upward after cracks had formed in the bottom fibers of the beam.

The load which produced a crack in the bottom of Beam I was one which produced zero flexural stress in the bottom of the beam because the beam had been cracked during fabrication. The final prestressing force was calculated from the load which gave a zero stress in the bottom of the beam and was found to be 129.3 kips resulting in a prestress loss of 26.1 percent.

The final prestressing force for Beams II, III, and IV was found by calculating the force necessary to give a flexural tensile stress at the bottom of the beam equal to the modulus of rupture; it is assumed that the concrete would crack at this stress. The final prestress forces in Beams II, III, and IV were 126.0 kips with 28.1 percent loss, 127.7 kips with 27.0 percent loss, and 125.5 kips with 15.5 percent loss, respectively.

Shear Strength of the Beams

Concept of Failure. The cracks in the prestressed beams may form in two general ways as the beams are loaded to failure. The crack may start at the bottom fibers of the beam and progress up-

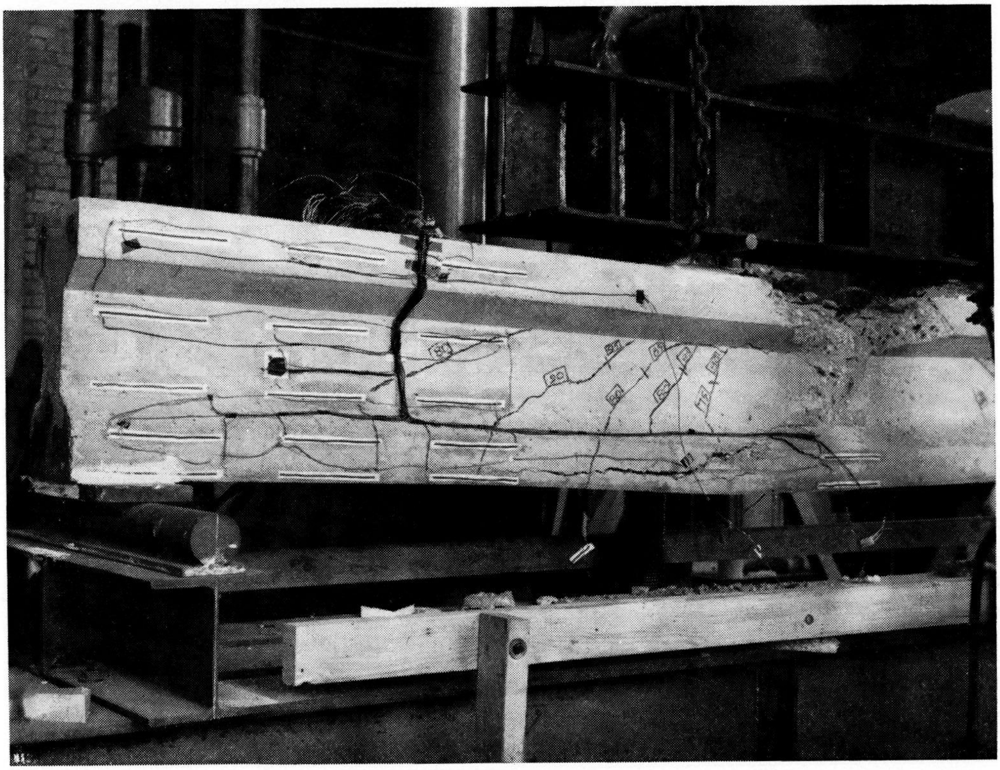

Figure 7. Beam IV at ultimate load.

ward. These cracks are caused entirely by flexural stresses, because no shear is present at the outside fibers of the beam. If these cracks start in a region of constant moment, the cracks will progress vertically upward as the load is increased. If these cracks start in a region where shear is present, the crack will progress vertically upward and then progress diagonally as the shearing stresses become larger. Within reasonable limits of load, these cracks will close completely when the load is removed from the beam.

In contrast to the cracks which progress slowly as the load is increased, diagonal cracks may form suddenly and completely traverse the web of the beam without an increase in load. These cracks may form only in the web or they may extend to the bottom of the beam and may even extend into the top flange. If the load is removed immediately after the formation of these diagonal cracks, it is found that they will not close up as did the cracks which were caused initially by flexural stresses.

The shear strength of a beam has been defined as the load at which a sudden diagonal crack completely traverses the web of the beam. Shear strength as defined in this manner is dependent on the type of loading as well as the properties of the beam.

Location of Critically Stressed Points. The stress trajectories for the beams were drawn for the loads which caused the initial shear cracks in each case. These trajectories were obtained from the principal stresses and their directions which were calculated for an elastic stress distribution. The stress trajectories and superimposed shear cracks are shown in Figure 8, and they almost coincide.

Several shear cracks occurred at once in each beam, but in each case the crack nearest to the load point had been subjected to greater stresses than the others. It is probable that this crack occurred first and initiated the others. The principal stresses are indicated (Fig. 8) at a number of points in the shear span. It appears that the most critical stresses occur between the neutral axis and the flange web junction. However, it is recognized that, at the loads being considered, flexural cracking has already occurred near the load point; and this must result in stresses different from those indicated at the upper ends of the trajectories.

Figure 8 shows that the most critical web tensile stresses occur at point Z in each beam, and that they decrease at every point further into the shear span. Theoretically, therefore, the first shear crack should occur at point Z, and the absence of these cracks could only be explained by the fact that flexural cracking has already occurred on the outer fibers, and these cracks have relieved the tensile stresses at the base of the web. The nominal tensile stresses on the outer fibers under point Z are 1465 psi, 1752 psi, 1914 psi, and 2064 psi for Beams I, II, III, and IV, respectively; and flexural cracks have, of course, already appeared. It is proposed that shear cracks are initiated in the web only when the section considered has not obtained stress relief due to flexural cracking. It follows that for a smaller web thickness to section modulus ratio the web stresses would become critical nearer to the load point and for a smaller load. Trajectories close to the load point, however, curve and do not reach high into the web so that cracking there may not be serious.

The shear cracks are dangerous because in occurring suddenly they sweep to the top of the web and lead to a compression or shear compression failure. If a crack is initiated in flexure, however, and its trajectory has a path which leads high into the web where large tensile stresses are already present, this flexure crack could occur just as suddenly and travel as far as a crack initiated in the web. Thus crack D in Beam III (Fig. 8) was apparently a flexure crack, since the flexural stress exceeded the modulus of rupture of 630 psi. Yet it occurred together with the shear cracks, and because of the high tensile stresses in the web it progressed up into the beam as suddenly as the shear cracks.

It would appear that these sudden shear cracks occur along the stress tra-

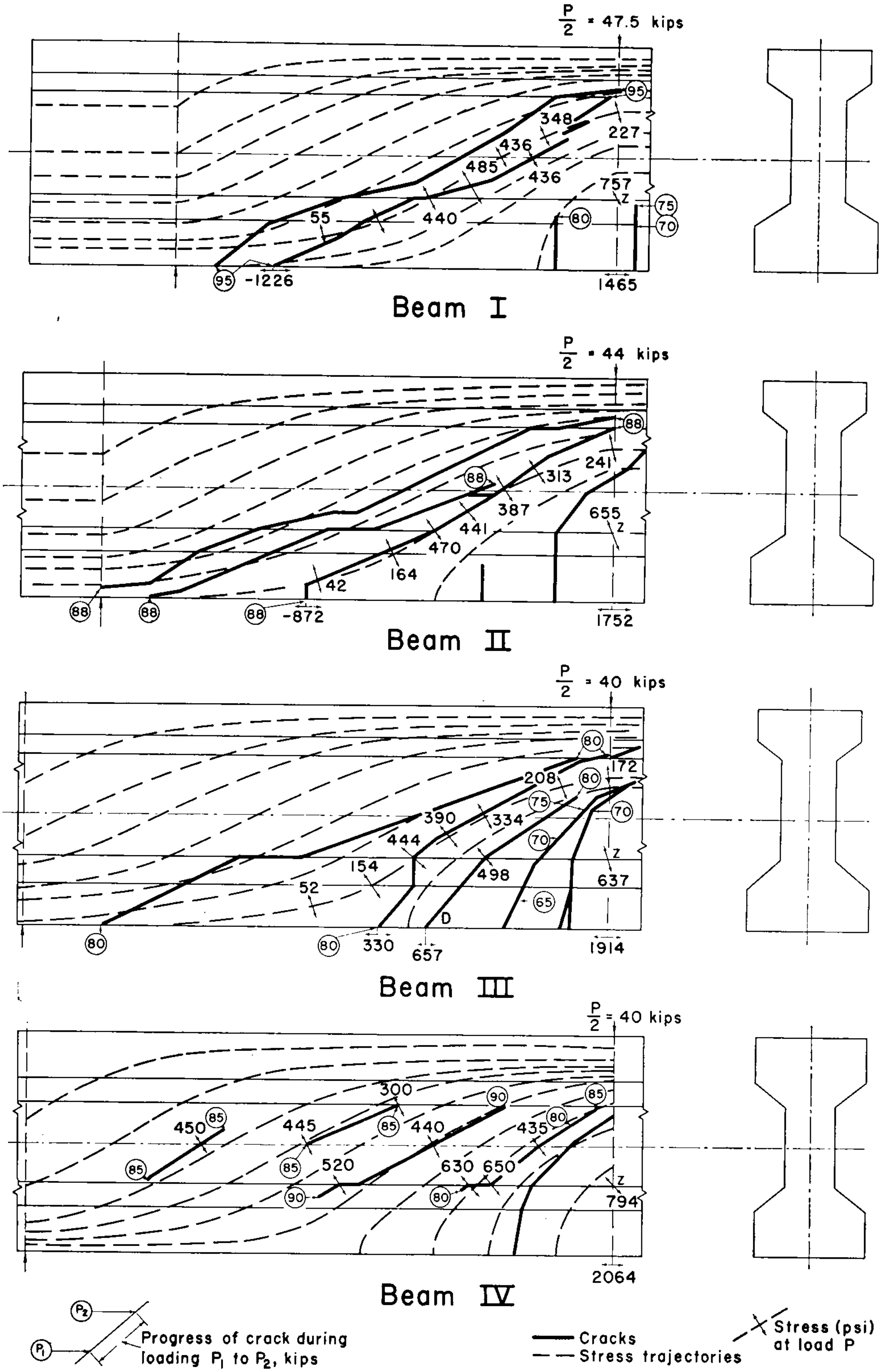

Figure 8. Stress trajectories and shear span cracks.

jectories which have paths leading high into the web. The cracks originate between the neutral axis and the flange web junction if the critical stresses there have not been relieved by flexural cracking when they had a much smaller value. The flexural cracks may also initiate the shear cracks if the web stresses are large enough for the flexural crack to continue. The critically stressed points occur at a distance from the load point which depends on the inclined angle of the trajectories leading to the top of the web.

Effect of Web Reinforcement. The beams without web reinforcement carried a small additional load after the shear cracks had formed in the beam and the final failure occurred in the shear span. The beam with web reinforcement had shear cracks which formed at about the same stress condition as should produce cracks in a beam without web reinforcement. However, the web reinforcement caused the beam to carry a much larger ultimate load, and failure occurred as a compression failure at the midspan of the beam.

Modified Maximum Tensile Stress Theory of Failure. The fact that the modulus of rupture of concrete is greater than the tensile strength indicates redistribution of stress in concrete under conditions of non-uniform stress distribution.

It has been described how Grassam took a mean of the two stress values which cause failure under the assumptions of elasticity and plasticity and with this modified maximum tensile stress criterion obtained satisfactory results with his predictions. The modulus of rupture and tensile strength of concrete are two examples of failure stresses with and without stress redistribution, and for the beams tested the tensile strengths were taken as 310 psi, 350 psi, 350 psi, and 500 psi; and the values of modulus of rupture were 560 psi, 630 psi, 630 psi, and 740 psi for the four successive beams. The critical stresses at shear failure for the four beams in the same rotation were 485 psi, 470 psi, 444 psi, and 650 psi (Fig. 8). A mean value of tensile strength and modulus of rupture would thus have given a fairly accurate prediction of the failure stress.

The mean value assumption of stress at failure would not, however, fit all types of stress distribution. It seems that most stress redistribution can take place in pure bending, and the flexural cracks in the prestressed beams occurred at a nominal stress equal to the modulus of rupture. On the other hand, the tensile strength is the stress at failure for a uniform direct stress situation, although in a varying stress distribution, failure would take place at some intermediate value. This value would be closer to the modulus of rupture or tensile strength, depending on which of these two stress situations is more dominant in the region being considered. The mean value of the two extreme types of stress distribution is probably a good approximation in most cases.

It has already been noted, however, that the shear cracks occurred close to the stress trajectories, which were based on an elastic distribution of stress, so that the large stress redistributions are probably localized in the area in which the tensile stresses are critical; and the concrete fails before the plastic action can change the stress distribution over the complete section of the beam.

The modified maximum tensile stress theory of failure, therefore, is compatible with the shear failure obtained in the four beams. It is a limiting stress theory and thus very practicable for design purposes.

CONCLUSIONS

1. For the prestressed beams tested the cracks almost coincided with the paths of the stress trajectories, which were determined from an elastic analysis. This shows that the concrete failed on a plane perpendicular to the direction of principal tension.

2. The addition of web reinforcement to Beam IV did not materially change the shear strength of the beam, but the

ratio of ultimate strength to shear strength was greater than the ratio for the beams without web reinforcement.

3. A modified maximum tensile stress theory of failure, accounting for some plastic redistribution of stress within the concrete was compatible with the computed nominal critical stresses at failure. The test results indicate that this limiting stress varies between the tensile strength and the modulus of rupture.

4. The shear cracks are usually initiated in the web between the neutral axis and the flange-web junction when there are critical tensile stresses present. The shear cracks, however, may also be initiated at the bottom fibers when the tensile stresses there exceed the modulus of rupture and the tensile stresses in the web are close to the critical value.

ACKNOWLEDGMENT

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DISCUSSION

M. SCHUPACK, *Vice President and Chief Engineer, The Preload Company, Inc., New York, N. Y.* — This paper has added additional insight into shear behavior of prestressed concrete beams and is of particular interest since it estimates the tensile stresses at the time of diagonal cracking. For actual design application it is important to point out, however, that these tests do not take into account the stresses which are additive to shear effects. These stresses caused by interaction effects are not easily determined. This is another place where the practicing designer must try to understand the complete interaction of a structure so that he can properly place reinforcing steel to control cracking.

The interaction effects are as follows:

1. The transverse moment which the slab may introduce into the stringer because of the torsional stiffness available from the stringer. This may cause a crack at the juncture of the top flange and web (Fig. 9).

2. The distribution effects of the slab and diaphragms introducing torsional stresses and in some cases upward forces in the stringers (Fig. 10).

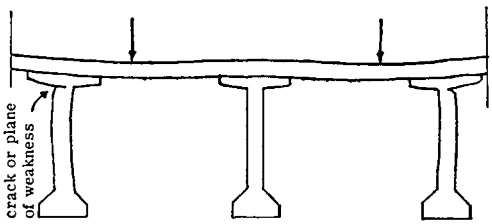

Figure 9. Bridge cross-section.

3. Localized effects at diaphragms from whatever distribution assistance they may offer.

4. Torsional stress on fascia beams during and after the cantilever slab is placed.

Furthermore, some methods used for supporting slab formwork may introduce

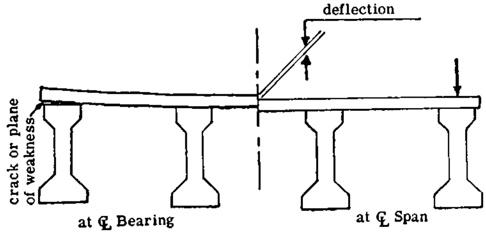

Figure 10. Skew bridge cross-section.

local vertical tension in the stringers and create incipient cracks which are future planes of weakness.

Even considering that one could possibly minimize the above effects, there is basically an inherent weak spot where the top flange meets the web because most stringers are concreted in one lift. In T-beam type construction it is advisable to permit the stem concrete to settle before the flange is concreted. This procedure is generally not followed in prestressed concrete I-beam fabrication since the beams are usually concreted in one lift.

All reports on the shear resistance of prestressed concrete beams are apparently treated as pertaining to an isolated structural member. Under these conditions the shear stresses are generally quite favorable, and the full significance of the detail requirements involved are not emphasized.

Most of the structural and construction behavior mentioned above applies to reinforced concrete and structural steel bridge decks. However, in the reinforced concrete case, a 12-in. web (or greater) and sufficient stirrups are generally required not to create any problems. In the steel beam bridge, the wide flange sections have all the characteristics required to cope with this situation. In the thin webs of lightly web-reinforced prestressed concrete beams, however, these "secondary" stresses may be an important consideration.

Even though tests on individual beams indicate that web reinforcement requirements are nominal, consideration must be given to actual structural integration of the member and construction procedures. It is suggested that the maximum stirrup spacing of ¾ depth be given further consideration. For shaped bridge-stringers, it is recommended that a maximum stirrup spacing should be 12 in. All stirrups that go into the bottom flange should be continuous around the bottom face of the flange.