DEPARTMENT OF SOILS, GEOLOGY AND FOUNDATIONS

Structural and Textural Influences on Thermal Conductivity of Soils

MARTINUS VAN ROOYEN AND HANS F. WINTERKORN Department of Civil Engineering, Princeton University

This paper treats briefly the common methods used for determining thermal conductivities of materials. A critical review of the test methods for soils leads to the adoption of a recently developed procedure for the laboratory investigation. The equipment employed in the research is described and evaluated.

The data obtained from the experimental investigation are interpreted and an equation is proposed showing the relationship between thermal resistivity, density, and moisture content. The constants in this equation are modified by parameters such as solid composition, grain size, temperature, and structure development.

Available theoretical and experimental knowledge on water and its movement in soils is utilized in an attempt to clarify questions which arise in heating of underground electrical cables. Practical considerations concerning soil surveying and trench design for the installation of underground cable are suggested.

IN 1957, the authors presented a paper (102) on theoretical and practical aspects of the thermal conductivity of soils and similar granular systems. This paper represented an exhaustive and critical survey of knowledge available at that time and pointed out the discrepancies existing between actually measured data on one hand and values calculated on the basis of theoretical concepts and of semi-empirical relationships, on the other hand. The importance of structural and textural relationships, in addition to the commonly used density and moisture factors, were indicated. The present paper gives the result of an experimental study designed to bridge, or at least to narrow, the gap between theory and reality.

The following factors were deemed to deserve closer consideration: volume relationships between solid, liquid, and gaseous phases; interfacial phenomena; particle size, shape, and other granulometric features; degree of structure formation and type of structure; and mineralogical composition of the soil solids. The considerable number of these factors and the existing coupling between them did not permit their completely separate treatment; but the experiments were so designed that they yielded as much pertinent information on these factors as could be hoped for within the time and financial limitations of this investigation.

EXPERIMENTAL DETERMINATION OF THERMAL CONDUCTIVITY

The phenomenological law for thermal conduction is

$$q = k \frac{A}{L} \Delta \theta t \tag{1}$$

in which q is the amount of heat that flows in the time t through a crosssectional area A if normal to this area a thermal gradient $\Delta\theta/L$ exists; k is a proportionality factor called the coefficient of thermal conductivity and 1/k is called the coefficient of thermal resistivity. In differential form Eq. 1 reads:

$$dq = -k A \frac{d\theta}{dL} dt \tag{2}$$

Considering a small-volume element of dimensions dx, dy, dz, of a homogeneous isotropic solid, and assuming that k is independent of temperature, time and direction, and that no heat is generated or absorbed within the element, one obtains, by the law of the conservation of energy, the general equation for heat transmission in a body:

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} = \zeta \frac{c_p}{k} \frac{\partial \theta}{\partial t} \qquad (3a)$$

in which ζ is the density of the solid and c_p is the specific heat of the solid.

For one-dimensional flow, Eq. 3 reduces to

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{a} \frac{\partial \theta}{\partial t} \tag{3b}$$

where $a = \frac{k}{c_p \zeta}$ = thermal diffusivity coefficient.

For the experimental determination of the coefficient k, two main groups of methods are available. These employ, respectively: (a) Steady-state conditions for which Eq. 1 and 2 apply; and (b) Non-steady conditions for which Eq. 3, or others derived from it, apply.

Steady-State Methods

The steady-state methods consist chiefly either of measuring by calorimetric methods the amount of heat withdrawn from the system, or of using electric heating elements and determining the amount of electric energy changed by the elements into heat and supplied to the system per unit time after the steady-state temperatures have been reached. The temperature difference at the steady state across the length of path L is measured by means of thermometers, thermocouples, thermistors, or other thermometric devices. The geometric shapes which lend themselves best to mathematical evaluation are spheres, infinitely long cylinders, and infinitely wide plates.

For concentric spheres of radii r_1 and $r_2(r_2>r_1)$ the applicable modification of Eq. 1 is

$$q_s = k \; 4\pi \; \frac{\Delta \theta}{(r_2 - r_1) / r_1 r_2}$$
 (4)

in which q_s is the heat flow per unit time. For infinitely long coaxial cylinders of radii r_1 and $r_2(r_2 > r_1)$

$$q_{\rm c} = k \ 2\pi \ \frac{\Delta\theta}{\ln r_2/r_1} \tag{5}$$

For the plate methods, Eq. 1 is used in its original form. Practical application of these methods are given elsewhere (8,9, 10, 16, 32, 43, 70, 88, 89, 90).

Methods Involving Variation of Temperature with Time

Solutions of Eq. 3a for different boundary conditions give rise to as many different experimental methods for determining the coefficients a and k. Three types of temperature variations with time have been employed: periodic, aperiodic, and transient. Periodic tem-perature variation was first used by Angström (5, 6, 7); the procedure was later modified by King (62). Neumann (75) was the first to employ aperiodic temperature variations. His method has been modified and improved as described by Jakob (56, 57). Tanasawa (95) used a periodic heat flow to measure the thermal diffusivity of moist foundry sand in order to eliminate moisture migration during the experiment. A similar method was developed by Albrecht (1)and applied to soils by Bracht (22).

Transient methods are applied preferentially for cases in which the possibility of convection exists, as in liquids, gases and systems composed of two or more phases (such as moist soils). A method using transient and steady-state conditions is given by Forbes (46, 47) for determining the thermal conductivity of fairly conductive materials such as metals. Transient methods in use for liquids and gases are the cooling thermometer method of Kundt and Warburg (66) and the warming up method of Stålhane and Pyk (93), which was developed further by other investigators (39, 100, 101). The pioneering work of Patten (78) on soils also employed transient conditions.

Beatty, Armstrong and Schoenborn (14) described an unsteady-state method to be used for materials such as plastics. The method is based on a solution of Eq. 3a with suitable boundary conditions and the simplifying assumption in the mathematical analysis that only the first term in an infinite convergent series is significant. A block of pure copper is sandwiched between identical plates of the material of which the conductivity is to be determined. The outer ends of these plates are suddenly raised to a higher constant temperature. The temperature rise of the copper block is measured as a function of time. The slope of the straight line obtained by plotting the time against the logarithm of the temperature rise easily yields the coefficient of thermal conductivity of the tested material.

In addition to these more or less absolute methods, a number of relative methods have been developed. Of special interest are those described by Ingenhouse (54), Biot (15), Depretz (36, 37,38), De Senarmont (33, 34, 35), Thelen (96), Voigt (104, 105), Bosworth (20), Keyes (61), and De Vries (40).

Methods Used for Soils

Almost all the methods mentioned have been applied to soil. The heated sphere method (23, 88), the coaxial cylinder method (70, 88), and the hot plate method (8, 89, 90) have become standard methods for testing porous materials and have been applied to soils as well. Soil scientists have used the periodic heat flow methods (22, 95), as well as the aperiodic method of Neumann (75). There is, of course, also the method of Patten (78).

The recent trend in measuring soil thermal conductivity is in the direction of employing transient methods which give results within a short period of time and with small temperature differentials. The most suitable of these is the warming up method of Ståhlane and Pyk (93), which has been applied to soils and other moist porous systems by a number of investigators (40, 52, 53, 69). The size of the probes ranges from 5 to 8 ft long with a $\frac{3}{8}$ -in. diameter (108), which is now widely used for field determinations. to needles with a diameter of 0.03 in. used for liquids (39) and of 0.055 in. with a length of 4.3 in. for measurements on soil in situ (40). The ratio of length to diameter should not be less than 100 (68).

The theory of this method is based on the assumptions that the heating element is a straight line of infinite length and infinitely small diameter; it is homogeneous and isotropic, and is embedded in a homogeneous and isotropic medium of infinite extent; and it possesses the same thermal properties as the surrounding medium. Because compliance with these theoretical assumptions is impossible, errors are introduced because of (a) the finite length of the probe, (b) its finite diameter, (c) the difference of its thermal properties from those of the surrounding medium, and (d) the limited radial dimensions of the medium. These errors have been considered by van der Held and van Drunen (101), Mason and Kurtz (69), and Hooper and Chang (52).

For a needle of finite length, there will be heat loss from the ends, which in turn will cause longitudinal heat flow in the needle. The error due to this effect has been found to be negligible for needles possessing length-to-diameter ratios larger than 100 (68). De Vries (40) estimated a maximum error of 1 percent for his setup, in which the heater wire was 0.02 cm in diameter and 15 cm long (L/D=750). Mason and Kurtz studied the effect of the finite diameter of the soil sample on the thermal flow lines using an electric analog. They concluded that, for at least the first 15 minutes, the behavior of the needle $(r=\frac{1}{8} \text{ in.})$ is determined only by the material within a 2- to 3-in. radius around it. After long time intervals, the finite length of the probe and the finite radius of the soil sample enter into the picture and affect the measurements to an extent that they are practically useless. If the soil sample is of cylindrical form and is well insulated at the ends, no error is to be expected from its finite length for the measuring times normally employed.

A correction that depends mainly on the thickness of the heater was developed by van der Held and van Drunen. This correction is taken as equivalent to a heat production prior to zero time; that is, the time when the heating current is actually switched on. Because of the finite diameter of the needle and the difference of its thermal diffusivity from that of the soil, it is reasoned (69) that at the point of temperature measurement (which is not at r=0) and after a definite time interval after switching on, the temperature will be the same as in the ideal case if the heating current had been turned on before or after the actual zero time, depending on whether the thermal diffusivity of the probe is greater or smaller than that of the surrounding material. After the initial warm-up of the needle, the rate of temperature rise at the measuring point in the probe should be the same as in the ideal case. A radius correction in accordance with this concept has been described by Hooper and Chang (52).

The effect of the difference in thermal properties of the probe material and surrounding soil was discussed in the preceding paragraph. Nonhomogeneity of the probe itself cannot be treated mathematically, but must be evaluated by the actual behavior of the probe (52). Mason and Kurtz (69) considered the fact that the temperature rise of the probe is measured rather than that of the surrounding material. The effect of contact resistance between probe and surrounding medium has also been investigated (17, 18, 19). The effect of the limited radial extent

of the soil sample was investigated by

Mason and Kurtz (69), who calculated theoretical temperature rise curves for different values of radial distance from the center line of the heater, using typical soil constants. From expressions given by van der Held and van Drunen (101), De Vries (40) calculated that the correction due to the finite dimensions of the soil sample was less than 0.1 percent for his apparatus, which consisted of a rectangular copper container 10 cm by 15 cm high and a central heating wire of 0.02-cm diameter.

A number of other factors influence the values obtained from needle probe measurements. These, however, depend more on the characteristics of the surrounding material than on the theory and construction of the probe. Soil conditions that give rise to errors, especially in the field, are layered structure, nonhomogeneity with respect to density and moisture content, uneven temperature distribution, and variability of compo-sition and structure from location to location. A great source of error may be deficient contact between needle and soil, especially in the case of dry soils, and also the "wall effect" of the geometry of the needle surface on the arrangement of the neighboring particles of granular soils. This effect sets a lower limit to the practically permissible diameter of the thermal probe.

Despite the described limitations, the thermal needle method possesses definite advantages over others previously developed and used. As a laboratory method, its value has been established beyond any justified doubt (40). The same is true in the field in the case of uniform soil deposits of appreciable thickness. However, when long probes are used in markedly stratified non-uniform soils, the values obtained must be handled with caution, not because of the inadequacy of the method but because of the variability of the soil.

COMPARISON OF NEEDLE AND STATIONARY METHODS

The various stationary methods require considerable time to come to equilibrium and often necessitate an elaborate setup in order to control temperatures and make measurements with sufficient accuracy. Due care must be taken for insulating or guarding the test portion of the sample. Stationary methods possess the advantage that the thermal conductivity can be calculated directly from the definitive equation and that the formulas used do not involve assumptions that cannot be realized closely in practice. If carefully employed, these methods are the most reliable ones available for certain materials, including soils in such conditions of moisture content and porosity as preclude convection from becoming an important factor (70, 88, 89). When convection becomes important and thermal moisture migration enters the picture, the stationary state methods become obsolete and data obtained with them must be mistrusted.

In the case of methods yielding thermal diffusivity values from which the conductivity coefficients must be calculated (22, 78, 101), the density and heat capacity of the soil system also must be determined. The density of a moist soil may be taken as total mass divided by total volume; the heat capacity of a moist soil, however, is not necessarily equal to the sum of the heat capacitics of its components in the uncombined state and should, therefore, be determined experimentally. This means considerable additional work.

The method of Stålhane and Pyk possesses the advantage of being an absolute one for determining the thermal conductivity of a material. By choosing sufficiently short times of run and small temperature rises, errors induced by possible convection and thermal moisture migration can be minimized, if not completely eliminated. Because of its proven advantages, this method was adopted for the work reported here.

Theory of Thermal Probe

The derivation of the equation used in connection with the thermal probe is given in a summarized form. Solution of Eq. 3b in an infinite medium is given by the Fourier integral solution. If the initial temperature distribution function, $\theta(x, 0)$, is integrable and piecewise regular, the solution may be given (24) in the form:

$$\theta(x,t) = \frac{1}{2\sqrt{\pi a t}} \int_{-\infty}^{\infty} \theta(x^{1'},0) e^{(x'-x)M} dx'$$
(6)

in which M is a substitution factor and

$$M = -\frac{1}{4at} \tag{7}$$

For symmetrical radial heat flow in which the temperature is a function of radius r and time t, the pertinent equation is:

$$\frac{\partial\theta}{\partial t} = \frac{a}{t} \frac{\partial^2(r\theta)}{\partial r^2} \tag{8a}$$

or

$$\frac{\partial(r\theta)}{\partial t} = a \frac{\partial^2(r\theta)}{\partial r^2} \tag{8b}$$

The solution of Eqs. 8a and 8b is a special case of Eq. 6 arrived at by considering that for every positive temperature at x there is an equal negative temperature at -x. The solution is then (55):

$$\theta(r,t) = \frac{1}{2r\sqrt{\pi at}} \left[\int_{0}^{\infty} \theta(\lambda,0) e^{(\lambda-r)^{2}M} d\lambda - \int_{0}^{\infty} \theta(\lambda,0) e^{(\lambda+r)^{2}M} d\lambda \right]$$
(9)

For initial conditions,

$$\theta(r, 0) = \text{constant} = \theta_0 \text{ for } 0 < r < R$$
$$= 0 \text{ for } r > R$$

Eq. 9 becomes

$$\theta(r,t) = \frac{\theta_0}{2r\sqrt{\pi at}} \left[\int_0^R \lambda e^{(\lambda-r)^2 M} d\lambda - \int_0^R \lambda e^{(\lambda+r)^2 M} d\lambda \right]$$
(10)

For an instantaneous point source at r=0 of strength q^1 while the rest of the medium is at 0 temperature, the temperature rise θ_0 of the small sphere of radius R (in the limit $R \rightarrow 0$) may be found from

$$q^1 = \theta_0 c \zeta \frac{4}{3} \pi R^3 \tag{11}$$

Substituting θ in Eq. 10, expanding the exponential in a power series, and neglecting all terms except the first, yields

$$\theta(r,t) = q^{1} \frac{a}{k} (4\pi a t)^{-3/2} e^{r^{2}M} \quad (12)$$

If instead of an instantaneous source, a constant-strength source liberating qunits of heat per unit time started at time t=0, then at a later time t the total effect will be given by the summation of each effect $q^1=q d\tau$, which started at $t-\tau$ previously, $0 < \tau < t$. By summing up a continuous series of point sources, a line source is obtained. The summation of the effects of the point sources constituting the line source yields the net effect of the line source.

Performing the indicated operations on Eq. 12, the expression for the temperature distribution after time t around a continuous line source generating qunits of heat per unit time and unit length of the source is given by

$$\theta(r,t) = \int_{-\infty}^{t} \int_{-\infty}^{-\infty} \frac{q_a}{k\sqrt{4\pi a^3}} \frac{1}{\sqrt{(t-\tau)^3}} e^{(r^2 + z^2)N} dz d\tau \quad (13a)$$

or

$$\theta(r,t) = \frac{q}{4\pi k} \int_0^t \frac{e^{r^2 N}}{(t-\tau)} d\tau \qquad (13b)$$

in which N is a substitution factor and

$$N = -\frac{1}{4a(t-\tau)} \tag{14}$$

This integral is tabulated in Appendix F, p. 253 of reference (55).

Eq. 13b also may be written

$$\theta(r,t) = \frac{q}{4\pi k} \int_{-0}^{t} \frac{e^{r^2 M}}{t} dt \quad (13c)$$

Expanding the exponential in an infinite series and integrating gives

$$\theta(r,t) = \frac{q}{4\pi k} \left[-c - \ln \frac{r^2}{4at} + \frac{r^2}{4at} \right] \quad (13d)$$

For small r (in the limit $r \rightarrow 0$) or large t, the expression for difference in temperature $\Delta \theta$ between time t_1 and t_2 becomes

$$\Delta\theta = \frac{q}{4\pi k} \ln \frac{t_2}{t_1} \tag{15}$$

Van der Held and van Drunen (101) differentiated Eq. 13c and got, after inversion,

$$\frac{dt}{d\theta} = \frac{4\pi k}{q} t e^{\frac{r^2}{(4at)}}$$
(16a)
$$\frac{dt}{d\theta} = \frac{4\pi k}{q} \left[1 + \frac{r^2}{4at} + \frac{1}{2!} \left(\frac{r^2}{4at}\right)^2 + \frac{1}{3!} \left(\frac{r^2}{4at}\right)^3 + \dots \right]$$
(16b)

For small r and large t values, only the first two terms need be considered. Setting

$$\frac{r^2}{4a} = t_0$$

they obtained

$$\frac{dt}{d\theta} = \frac{4\pi k}{q} \left[t + t_0 \right] \tag{16c}$$

The correction t_0 can be evaluated by plotting $\frac{dt}{d\theta}$ against t. The intercept of the linear portion on the t axis yields t_0 . The time in Eq. 15 is then corrected by the amount t_0 ; that is, (t_1+t_0) and (t_2+t_0) are used instead of t_1 and t_2 .

In accordance with Eq. 15, the coefficient of thermal conductivity is

$$k = \frac{q}{4\pi} \frac{\ln t_2/t_1}{\Delta\theta} \tag{17}$$

Therefore, if the temperature of the needle is plotted against the logarithm of the corresponding time during the course of the experiment, a straight line will be obtained. The slope of this line will be proportional to the coefficient of thermal conductivity of the surrounding material.

Design and Construction of Probes

The design and construction of the thermal probes followed the pattern set by other investigators and the relative sizes of the various components agreed well with those employed by De Vries (40).

To approximate the theoretical infinitely long, linear heat source, stainless steel Hypoflex tubing was employed; it had the following dimensions: 0.035-in. outside diameter, 0.006-in. wall thick-ness, 6-in. length. Two types of needles were constructed, differing only in minor aspects. The first type (Fig. 1) was intended for testing soil samples into which the probe could be pushed and afterwards withdrawn. The second type (Fig. 2B) can be inserted in the cylinder before compaction of the soil sample and left in place during subsequent treatment, the main objective being to cause as little disturbance as possible of the established soil structural arrangement. For easy differentiation in this report probes of the first type are called needles and those of the second type are called probes.

The thermocouples were made of 40 B & S gage (0.004-in. diameter) teflon-

insulated copper and constantan wires. The pertinent properties were:

Copper:

Resistance=1.0748 ohm per ft at 72 F Guaranteed accuracy= ± 0 F from -75 F to +75 F Constantan: Resistance=29.86 ohm per ft at

72 F Guaranteed accuracy = $\pm 1\frac{1}{2}$ F from -75 to +200 F

In constructing the three needles employed, the stainless steel tube was soldered to a brass piece, which was fitted onto an AN-3102-165-1B panelmounted Amphenol connector, as shown in Figure 1. The flange on the panelmounted connector was trimmed off in a



Figure 1. Thermal conductivity needle.



Figure 2. Tension cap and thermal probe.

lathe. The needle could then be connected to an AN-3106A-165-1R cablemounted Amphenol connector, which is fastened to the sliding rod of the needle stand. For constructing the 20 probes, the Hypoflex tubing was soldered to a brass fitting $\frac{3}{4}$ in. in diameter and $\frac{3}{4}$ in. high with a hole $\frac{7}{16}$ in. in diameter and $\frac{1}{2}$ in. deep, as shown in Figure 2B. The thermocouple and heating wire connections were cast into this fitting using Bakelite epoxide plastic.

In making the thermocouples, the ends of a copper and a constantan wire were bared. These ends were pushed through a short capillary tube so that the bared ends touched each other. The other ends of the two wires were connected to the positive terminal of a 50-v dc source. The negative terminal was connected to a small puddle of mercury. By just touching the surface of the mercury with the bared ends of the copper and constantan wires, the two were welded together.

A length of constantan wire was folded double and pushed into a glass capillary (0.014-in. outside and 0.008-in. inside diameter) to form the heating element. Care was taken that the glass capillary did not strip the insulation off the constantan wire. This and the other electric circuits were checked by measuring their resistances. As pointed out by De Vries (40), the resistance between the thermocouple circuit and the heater wire should be in the order of 10^6 ohms to give good results. This necessitates enclosing the heater wire in a glass capillary.

The heating element and the thermo-

couple wires were then pushed through the Hypoflex tubing; the thermocouple was pulled up until it was in the center of the working part of the probe or needle (that is, 3 in. from the end), whereas the heating element filled the entire length of the tube. The stainless steel tube and, as far as possible, the glass capillary were then filled with molten paraffin to insure watertightness and provide good heat conduction from the heating wires to the outside of the tube. In the needles, the wires were then soldered to the sockets of the Amphenol connector and the cavities were filled with molten paraffin. A special, thermalfree solder (Leeds & Northrup Co. Part No. 107-1-0-1) having a low thermal emf against copper was used in connecting the thermocouple wires to the Amphenol connector sockets. These connections were made in accordance with the manufacturer's recommendations.

For the probes, special sockets $\frac{1}{4}$ in. long and $\frac{1}{8}$ -in. outside diameter were made from 0.008-in. thick copper sheeting. The special thermal-free solder was used in making the connections; after the Hypoflex tubing had been filled with paraffin, the sockets were set in a Bakelite epoxide plastic. A cross-section of a probe is shown in Figure 2B. Probes N8 and N14, and needle C, were altered in such a way that the circuit contained only copper and constantan wires connected by means of a thermal-free solder.

Circuit and Equipment for Thermal Probe Tests

The arrangement for the heater circuit and the thermocouple circuit for one probe is shown schematically in Figure 3. Five probe circuits were used, requiring five ammeters and five rheostats in the heater circuitry, and five thermocouple circuits.

In the thermocouple circuits, the wires from the thermocouples were led to the reference junction. The five constantan wires were all welded to one common copper connecting wire (size 18, enamel insulation); this junction was kept at the temperature of the cold junction. Each copper wire from the thermocouple was similarly welded to a separate copper connecting wire (size 18, enamel insulation). These junctions were also kept at the temperature of the reference junction. From the reference junction, the wires from the copper wires of the thermocouples lead to a thermocouple selector switch (T.C.C.S.) and are then connected to one lead (positive) of the potentiometer through a thermocouple circuit switch (T.C.C.S.) The wire from the constantan thermocouple wires was connected directly to the negative lead of the potentiometer. All connections were made with the special, thermal-free



Figure 3. Circuit diagram of thermal resistivity apparatus.

solder. The T.C.C.S. either switches the potentiometer into the thermocouple circuit or shorts the leads of the potentiometer.

The reference junction was placed in a 1-qt wide-mouth vacuum bottle filled with water at as close to the initial temperature of the soil sample around the probe as practical. The thermocouple wires are enclosed in glass tubes filled with paraffin. The temperature of the reference junction is measured by one junction of a copper-constantan thermocouple. The other junction of this thermocouple is kept in melting ice inside a 1-pt thermos flask. The wires leading from the ice-bath are connected to the potentiometer in the same way as described for those from the reference innetion.

The dc heating current is supplied by two 12-v heavy-duty car batteries. Coarse selection of 2-v increments is made by including a varying number of cells in the circuit. The current through the heating element is measured by means of a 0-200-scale dc milliammeter connected in series with a 0-10-ohm 25-w rheostat and a circuit switch.

Potentiometer

The potentiometer used is a stabilized de microvolt amplifier having the following characteristics:

Limit of error:

- (a) Of amplifier response, $\pm 0.4\%$ of reading.
- (b) Of selfcontained meter, $\pm 1\%$ of full scale.
- (c) Of zero maximum offset, ± 0.5 microvolt.
- Indicated sensitivity: $\pm 0.5\%$ of scale span.

Response time (approx. values to within 1% of balance):

- (a) 2 sec for 2,000-ohm max source resistance.
- (b) 3 sec for 10,000-ohm max source resistance.

Power supply:

115 v, 50/60 cycles, approx. 50 watts. Special input connector with

goldplated contacts to eliminate thermal emfs.

Timer

A Gralab synchronous electric timer is used as a stopwatch to measure the time intervals. The range of this timer is 15 minutes, with $\frac{1}{2}$ -see dial division. The duration of the experiment is at most eight minutes (480 sec). The timing error must include the reaction time of the observer. The total error in reading the instrument depends on the rate of increase in temperature, which varies as the test progresses.

Needle Stand

To accommodate the needles and to insert them precisely and with the least possible disturbance into the soil samples, a needle stand (Fig. 4) was constructed. It consists mainly of a base to which a $1\frac{7}{8}$ -in. diameter rod is welded.



Figure 4. Needle stand.

Two connecting pieces fit into this rod and position the 1-in. diameter sliding rod to slide vertically and perpendicular to the plane of the base plate. The soil cylinder fits into a ring on the base plate with its center in line with the center of the sliding rod.

The sliding rod was provided with a cross stud near the lower end to facilitate its handling. An AN 3106 A 16S 1P cable-mounted Amphenol connector was screwed into the tapped end of the sliding rod, forming the connection between the thermal needle and the sliding rod. The heater wire leads and the thermocouple wires were soldered to the lugs of the cable-mounted Amphenol connector, using the special solder and technique previously described. The cavity above the Amphenol connector was filled with paraffin.

Operating Procedure and Calibration

First, the 1-pt vacuum flask is filled with chipped ice, frozen from distilled water in polyethylene plastic trays. Distilled water is added to the chips until all voids are filled. The ice-water system will then be at 0 C at normal barometric pressure. The cold junction of the thermocouple from the reference bath is then replaced and time is allowed for the setup to come to equilibrium.

The temperature of the soil sample to be tested is measured either with the -1 to 51 C Hg-thermometer, or by means of the thermocouple in a probe. In the latter case, the temperature differences between soil and reference junction, and reference junction and ice bath, are added to obtain the soil temperature.

The water from the reference bath is then heated or cooled (by adding ice) until the temperature as measured with the Hg-thermometer corresponds to within -0.2 C with that of the soil. The water is stirred continuously and thoroughly in order to insure the same temperature throughout. It is then poured back into the vacuum jar and the cold junction of the thermocouples is replaced.

A coarse selection of voltage is made

by including a certain number of cells in the circuit, depending on the estimated conductivity of the material under consideration. Higher voltages are used when the thermal conductivity of the material is expected to be large and when there is no danger of convection and moisture migration. Lower voltages (and therefore lower heating currents) are selected when the thermal conductivities are low and when disturbing convection and water movement features are present. The desired probe lead is selected and the plug inserted in the probe, which may be already in the soil or is inserted after connection has been made. By closing the heat circuit switch and adjusting the rheostat, the heating current may be set at any convenient reading of the milliammeter. This adjustment must be made as quickly as possible (should take less than 2 to 3 sec) to prevent any considerable heating up of the probe.

The potentiometer is switched on with the thermocouple leads short-circuited. The appropriate potentiometer scale is selected that will result in the maximum deviation during the run of the test without necessitating switching from scale to scale, if possible. Usually, the lower scales (0-50, 0-100) are used for highly conductive material and the higher scales (0-500, 0-1,000) for the least conductive materials.

The temperature of the reference junction is read (for convenience called icebath reading) usually on the 0- to 1,000microvolt scale for room temperatures, and the difference between reference bath and probe thermocouple is read on the scale selected. The thermocouple circuit is left open and the timer and heating current are switched on simultaneously. The potentiometer is read at 0, 5, 10, 15, 20, 25, 30, 35, 40, 50, 60, 75, 90, 105, 120, 150, 180 sec, 4 min, 5 min, 6 min, and 8 min. Care must be taken that the heating current remains constant during the test; usually no drift will be noticed.

The potentiometer readings are plotted on a natural scale against the logarithm of time passed after switching on of the heating current. The best representative straight line is drawn through the plotted points, preference being given to potentiometer readings taken after more than 15 sec from the start of heating. The change in potentiometer readings Δp for one logarithmic cycle (log $\frac{10x}{x}$) of time is determined from the straight line. A constant *C* divided by Δp gives the value of the coefficient of thermal conductivity of the material surrounding the probe. A sample run is recorded in Table 1, and the corresponding plot is shown in Figure 5.

As a standard for temperature measurements, the -1 C to 51 C Hgthermometer, 0.1 C scale divisions, was selected. The 0 C point on this thermometer was checked in an ice-water mixture as previously described.

$T\Lambda$	BLE	1
------------	-----	---

THERMAL TEST ON DRY OTTAWA SAND

N1; probe; Ottawa sand size #30 - #40 U. S. sieves; dry (0.09% moisture); density, 1.535 gr/cc (95.8 lb/cu ft); 6 v, 150 milliampere; No. 4 leads; ice bath reading, 892 (22.7 C); potentiometer scale, 500

Time	Readin	gs (°C)	
(see)	Heating	Cooling	Results
0	4.5		
5	41.0	63.0	From Fig. 5
10	49.5	53.0	-
15	54.0	47.0	$\Delta p \equiv 26.1$
20	58.5	43,5	-
25	61.0	41.0	
30	63.5	39.0	k = 2.437 milliwatt/C em
35	65,25	37.0	
40	67.0	35.5	
50	69,0	33.0	
60	71.0	31.0	
1.15	74.0	29.0	
1.30	76.0	27.0	
1.45	77.5	25.5	
2.00	79.5	24.5	
2.30	81.75	22.5	
3.00	84.0	20.75	
4.00	87.0		
5.00	89.5		
6.00	91.5	_	
8.00	95.0	<u> </u>	



Figure 5. Heating time vs potentiometer readings for Ottawa sand.

First, the potentiometer scales were checked against one another for various temperature values of the thermocouples. All data given by the manufacturers that could be checked with the available laboratory equipment were found to be satisfactory. Some trouble was encountered in calibrating the thermocouples, as running the stirrer in the constanttemperature bath employed caused deflections of the potentiometer indicator. The procedure finally adopted was as follows.

The reference bath was filled with icewater mixture and the test bath with water that had been cooled with ice. The thermometer and probes were placed into the latter, which was stirred thoroughly by hand. The time and thermometer readings were taken and then, as quickly as possible and compatible with the desired accuracy, the potentiometer was read for the nine thermocouples in succession, after which the thermometer was read again. Then the temperature of the water was raised and the procedure repeated. The temperatures were plotted against corresponding potentiometer readings for each of the nine probes (Fig. 6). The thermocouple (No. 6) for measuring the temperature of the reference bath relative to the ice bath was calibrated in a similar manner.

Calculation of Thermal Probe Constants

Resistance of heating wire: R=2,986 ohm per 100 ft=0.9797 ohm per cm. Heat supplied per cm length of probe:

$$\begin{array}{c} q = 2 \ I^2 \ R = 1.9594 \times i^2 \times 10^{-3} \\ \text{milliwatt per cm} \end{array}$$

where i = milliampere reading.

Thermal conductivity k of material is obtained from the theory as

$$\begin{aligned} k &= \frac{q}{4} \frac{1}{\Delta \theta} \ln \frac{t_2}{t_1} \\ &= \frac{1.9594}{4} i^2 \frac{100 \times 10^{-3}}{\text{scale factor}} \cdot \left(\frac{1}{p - p_1}\right) \end{aligned}$$



Figure 6. Calibration of thermocouples.

$$2.3026 \log \frac{t_2}{t_1}$$

$$= \frac{C}{(p-p_1)} \log \frac{t_2}{t_1}$$
milliwatt per °C cm² per cm (18a)

in which p and p_1 are measured as percentages of the table scale reading on the potentiometer, or

$$k = \frac{C}{\Delta p} \tag{18b}$$

where Δp is the difference over one decade of time. Values of *C* for various scale factors and heating currents are given in Table 2.

The probes were tested in dry Ottawa sand and were found to work satisfactorily. When saturated sand was used, serious trouble developed. Because of its high conductivity, high heating currents were necessary to produce a perceptible rise in temperature of the probes. This caused the brass connecting piece on the top of the probe to heat up, too, as some heating wire length is present in the top. Heating of the brass top causes a difference in temperature between the two ends of the copper connections and creates a secondary thermocouple working against the couple in the probe. The contacts of the thermocouple were then converted so that they would be as close as possible to the same temperature by constructing the leads in such a fashion that the thermocouple wires leading to the reference junction make contact inside the brass top of the probe. This resulted in marked improvement.

From tests in air and in water, it was found that in all cases (for needles as well as for probes) convection will occur invariably after 60 sec and usually after

 TABLE 2

 VALUES OF C FOR VARIOUS POTENTIOMETER

 SCALE FACTORS AND HEATING CURRENTS

Scale Factor (microv)	100 m-amp	140 m-amp	150 m-amp	160 m-amp	200 m-amp
100	141.193	276.74	317.68	361.85	564.8
200	70.597	138.37	158.80	180.93	282.4
500	28.239	55.35	63.54	72.37	112.96
1000	14.119	27.67	31.77	36.19	56.48
				_	

30 see of heating. From tests on dry Ottawa sand, it was observed that during the first 20 see the limitations of the instrument affect the readings considerably unless special correction factors (40, 52, 101) are applied. Experimental procedures were developed for obtaining dependable data (102a).

Apparatus for Preparing the Test Specimens

Brass cylinders of 4-in. outside diameter, $\frac{1}{16}$ -in. wall thickness, and 6-in. length were used as containers for the soil samples. Red copper caps (Fig. 2A) were provided for closing off the soil cylinders.

To obtain various densities, different compaction methods were used. Equipment employed on dry cohesionless soil included a 1/2-in. diameter steel rod 18 in. long for rodding (Fig. 7) and a mechanical sieve shaker to densify the material by vibration. For moist or cohesive soil, the compaction molds of a Proctor apparatus were modified (Fig. 8) to compact the soil in the sample containers. A compaction mold was made of a 41/2-in. outside diameter, 4-in. inside diameter brass cylinder 6 in. long, to accept the 4-in. outside diameter soil sample cylinder. Two 1- by 1/2- by 1/2-in. brass lugs were screwed to the mold to secure it to the base of the compaction apparatus. A steel spacer disk 4 in. in diameter and $\frac{3}{4}$ in. thick with a $\frac{3}{4}$ -in. diameter hole in the center is fitted into the bottom of the mold to allow the soil to be compacted in the cylinder around the thermal probe. In a modification of this, the soil is compacted around a steel wire of a slightly smaller diameter than that of the probes. The wire is withdrawn after compaction and the probe inserted.

Two sizes of compaction hammers weighing $5\frac{1}{2}$ and 4 lb, respectively, were used (Fig. 7). The samples were compacted in three layers with 25 blows per layer, using a drop of 10 in.

To obtain relatively undisturbed natural soil samples, two soil samplers (Fig. 9) were constructed. A 12-in. long steel tube of 4¹/₈-in. outside diameter and $\frac{1}{8}$ -in. wall thickness was bored to an inside diameter of 4 in. over a length of 11 in. The bottom inch of the sampler was beveled and contracted to an inside diameter of $3\frac{13}{16}$ in. The bored-out part received the 6-in. long brass sample cylinders and a 4-in. long brass spacer cylinder of the same diameter as the soil cylinder. The head of the soil sampler consisted of a steel disk $4\frac{1}{8}$ in. in diameter and 3 in. thick. Four 3/8-in. diameter holes were drilled radially into the head to allow air and water to escape from the sampler. A 7/8-in. diameter hole in the center of the disk was threaded to receive regular drilling rods. A $\frac{1}{2}$ -in. diameter steel ball seating on a 3%-in. diameter hole in the head shut off the space inside the soil sampler when the instrument was withdrawn from the soil. The partial vacuum thus created contributed to full recovery of the soil sample inside the cylinder. Four $\frac{1}{4}$ -in. screws secured the sampler tube to the head.

Operating Procedure

In cohesionless materials the desired range of densities was obtained by various means. For the lowest density the material was made to flow into the soil cylinder by means of a funnel without a drop. To eliminate subsequent disturbance, the probe was inserted before the cylinder was filled and the handling of the cylinder was minimized. A higher density was obtained by using a 6-in. drop. The next higher density resulted



Figure 7. Mold, rods, and hammers for compacting specimens.



Figure 8. Compaction mold, soil cylinder, and spacer.

from rodding the full cylinder 10 times with the $\frac{1}{4}$ -in. diameter steel rod. The next higher density was obtained with 25 roddings. Tapping the cylinder continuously on the outside, while it was being slowly flowed full, yielded a still higher density. The two highest density values were obtained by using the mechanical sieve shaker. Running the shaker at half speed for 30 sec yielded a slightly lower density than at full speed for 15 sec. For the higher densities, the material was densified with the probe in place in order to avoid subsequent disturbance of the packing by insertion of the needle.

With cohesive soils the standard Proctor procedure was used except for the previously described special hammers. The steel wire was strung along the centerline of the cylinder and the material was compacted around it in three layers. Then the wire was withdrawn, the sample was smoothed off, and the probe was inserted. In addition to providing samples for thermal conductivity testing, the optimum moisture of compaction may be determined at the same time.

For natural soil samples the soil cylinders are fitted into the sampler and the latter is pushed hydraulically into the ground. Subsequently, the sampler is withdrawn and the soil cylinder with sample and spacer cylinder is retrieved from the sampler. The sample is trimmed to size, capped, and taped to prevent loss of moisture. The regular tests may then be run on the sample in the laboratory. The sampler performed satisfactorily in cohesive soil that was not too dry and dense and did not contain large pieces of stone or gravel.

Auxiliary Equipment

Constant Temperature Cabinet. To run the conductivity tests at definite and constant temperatures, a constant-



Figure 9. Cross-section of soil sampler.

temperature cabinet was constructed. For greatest ease of control, the desired constant temperature was obtained by thoroughly mixing cold and warm air and pumping it through the constanttemperature cabinet. The complete unit consisted of a refrigeration cabinet, a heating and mixing cabinet, and a tension table cabinet.

The arrangement is shown in Figure 10A. The refrigeration cabinet consisted of a used refrigerator left unaltered except for two 2-in. diameter holes drilled into the back panel, one at the top and one at the bottom. This refrigerator supplied the ice and the cold air needed. The mixing cabinet, constructed from an old ice box, contained the air blowers, heating coils, probe stand, air pycnometer, reference temperature and ice baths, and the needle and probe connections. The two air blowers were driven by the same 30-w. 110-v, 60-cycle, 1,500-rpm electric motor. Two 2-in. outside diameter 1/32-in. wall brass tubes led from the blowers through the top panel of the cabinet, one to the refrigerator and one to the tension table cabinet. A slot was made in the tube leading to the refrigerator opposite the heating coils through which air could be blown over the heating coils. By varying the opening in the tube above the slot, the amount of air which is circulated through the refrigerator could be controlled. For temperatures above room temperature, this tube is closed off and all the air is passed over the heaters. The return air from the tension table cabinet is led back to the mixing cabinet through a 2-in. brass tube in the top panel. The air from the refrigerator returns through a brass tube at the bot-





Figure 10. Constant-temperature control apparatus and detail of tension table.

tom of the rear panel. Connections between the various brass tubes were made by means of 2-in. inside diameter heater hose and hose clamps.

The tension table cabinet was constructed out of an old (6 cu ft) Frigidaire cabinet with the cooling system removed and the shelves discarded. Lugs were installed in the walls to support four tension tables. The air from the mixing cabinet was circulated through 2-in. outside diameter $\frac{1}{32}$ -in. wall thickness brass tubes, one at the top and one at the bottom, installed through the side panel. The tension table cabinet housed all soil samples to be kept at constant temperature. To prevent evaporation of soil moisture in the circulating air, the sample containers remained capped while reaching and maintaining constant temperature.

The heating system consisted of two circuits, each having its own heater and thermoregulator. A Cenco DeKotinsky bimetallic thermoregulator was installed in each of the 2-in. brass tubes leading from the tension table cabinet and the mixing cabinet. The thermoregulator in the outflow air from the tension table cabinet actuated a 220-w heater element; that in the efflux of the mixing cabinet was connected in series to a 100-w light bulb. This arrangement could be improved by placing the latter heat source in the air stream leading from the mixing cabinet, before it reaches the thermoregulator.

For work above room temperatures, the refrigerator circuit is closed off and the air is circulated through the mixing and tension table cabinets only. The temperature of the air from and to the mixing cabinet is determined with a dry Hg-thermometer reading from -1 C to 51 C with divisions of $\frac{1}{10}$ C, placed in the air stream in the 2-in. diameter brass tubes. The temperature is read when the thermoregulators switch on and off regularly. The setting of the regulator is then adjusted, if necessary, until the average of the temperature readings of the inflow and outflow air from the mixing cabinet corresponds to the desired

temperature. For temperatures not too different from the ambient room temperature, the system takes about 3 to 4 hr to reach equilibrium.

When working below room temperatures, it is necessary to draw in cold air from the refrigerator. The opening in the tube leading to the refrigerator is increased slowly without the heater working The air is circulated through the system and the temperature measured as described previously in the influx and efflux of the mixing cabinet. When the average readings correspond to or are slightly below the desired working temperature, the refrigerator tube opening is kept at that setting and finer adjustments are made by incorporating the thermoregulators in the heating circuit and following the previously described procedure.

Measurements by means of an 0.08in. diameter copper-constant an thermocouple in the air stream showed that the temperature in the working part of the system was constant within 0.5 to 1 C.

Moisture Tension System

For determination of the pore size distribution in the soil samples, and also for reducing the moisture content of samples in a natural manner, it was decided to use a moisture tension system. After considerable experimentation with the previously mentioned tension tables, individual tension caps were constructed for each specimen mold. The caps (Fig. 2A) had a permeability in the order of 10^{-6} cm per sec and fitted directly on the cylindrical soil specimen molds. They drained the soil within one day to about 5 cm from the bottom of the sample.

Air Pycnometer

For the measurement of the air volume within the soil specimens, an air pycnometer (Fig. 11) was employed (42, 65, 77, 86, 98, 103, 106). The volume percent of air within a soil specimen can be measured by this instrument with an accuracy of about 1 percent.



Figure 11. Air pycnometer.

EXPERIMENTAL

Materials and Sustems

The materials used in this investigation, together with their properties, are given in Tables 3, 4, and 5. The experiments with these materials were designed in a manner that would furnish, within the available time and funds, data that would throw light on such factors as density, moisture content, granulometry, temperature, mineral composition, secondary structure, inter-particle binding. and composition of the aqueous phase.

For the thermal resistivity-densitygranulometry relationships, it was thought profitable to run the tests in the dry state on six different sizes of glass beads, crushed quartz, white quartz sand, and Ottawa sand, respectively. The dry state was selected because moisture migration and convection would be completely eliminated. It also permitted rapid compaction and determination of the thermal conductivity by means of the needle. The compaction procedures

TABLE 3A

DESCRIPTION OF M	ATERIALS	EMPLOYED
------------------	----------	----------

No. or	Size	Dia.	Sp.	%
Name	(U. S. Sieve)	(mm)	Gr.	Finer
	(a) GL	ASS PEARLS		
G 107	35-40	0.47	2.496	
G 110	60-80	0.20		
G 112	120-140	0,12	2.500	
G 114	170 - 200	0.082		
G 116	230 - 270	0.057	2.468	
G 119	< 325	0.028	2.413	
	(b) Crus	HED QUARTS	2	
CQ 40-50	40-50		<u>ا</u>	
CQ 60-100	60-100			
CQ 100-200	100 - 200		> 2.638	
$C\bar{Q} < 200$		0.074-0.007		
CQ < 400		0.050-0.000	8)	
	(c) Wh	IITE SAND 1		
WS 10-20	10-20		2.644	81.5 ²
WS 20-30	20-30			66.0 ²
WS 30-40	30 - 40			51.0 ²
WS 40-50	40-50		2.643	35.0 2
WS 50-60	50-60			9.0 2
WS 60-100	60-100			1.0 2
	(d) Отт	AWA SAND ⁸		
OS 20	20)	99.6
OS 40	40		≥ 2.64	2.6
OS 60	60		J	0

194+ percent quartz, by visual inspection.

² Smaller sieve. ³ 99+ percent quartz by visual inspection.

TARLE 3B

DESCRIPTION OF MATERIALS EMPLOYED (Continued)

No. or Na	ame Identification or Description
	(a) Soils Supplied by Companies
CE 1	Bank sand, Indiana.
CE 2	No. 1 top sand, Material Service Corp. Yard 18, Morris No. 1.
CE 3	Soil, 119th Street sub.
CE 4	Sand, 106th Street beach.
CE 5	Sand, pit at Rt. 45, 107th St., south end of pit.
PD 1	No. 1 excavated soil.
PD 2	No. 2 excavated soil.
PD 3	No. 3 backfill soil.
PD 4	No. 4 backfill soil.
DE 1	Backfill soil around 138-kv pipe-type cable lines.
DE 2	Backfill soil around 138-kv pipe-type cable lines.
Dor	Dorchester special blend sand, clay mineral is kaolinite.
Wh	Thermal sand 1; backfill sand around 138-kv pipe-type cable; clay min eral is kaolinite.
Pt	Thermal sand 2; proposed backfil sand; clay mineral is mixture o about 3 parts illite to 1 part kaoli nite.

(b) UNDISTURBED NATURAL SOIL FROM PRINCETON

	VICINITI
NS 2-8 Sod P.	Sampling depth 2-8 in.; sod patch.
NS 2-8 Proj.	Sampling depth 2-8 in.; grass cover.
NS 10-16 Proj.	Sampling depth 10-16 in.; grass cover.
NS 18-24 Proj.	Sampling depth 18-24 in.; grass cover.
NS 2-8 SF	Sampling depth 2-8 in.; side of cult. field, no veg. cover.
NS 12-18 SF	Sampling depth 12-18 in.; side of cult. field. no veg. cover.
NS 1-7 MF	Sampling depth 1-7 in.; higher cen- tral part, cult, field.
NS 8-14 MF	Sampling depth 8-14 in.; higher cen- tral part, cult, field.
NS 15-21 MF	Sampling depth 15-21 in.; higher cen- tral part, cult, field.
NS 1-7 MLF	Sampling depth 1-7 in.; lower cen- tral part, cult, field.
NS 8-14 MLF	Sampling depth 8-14 in.; lower cen- tral part, cult, field.
NS 6-12 MLF	Sampling depth 6-12 in.; lower cen- tral part, cult, field.
NS 15-21 MLF	Sampling depth 15-21 in.; lower cen- tral part, cult. field.
	(c) Additives
Acid kaolinite	Prepared from Ga. kaolin; sp gr, 2.50- 2.625; liq. limit, 41.1%; plastic limit. 29.7%.
Sodium bentonite	Sp gr, 2.4; liq. limit, 490.0%; plastic limit. 46.6%.
Portland cement Asphalt Sodium chloride Calcium chloride	Normal-hardening. RC-2.

were as described previously; the various methods yielded densities that varied from the loosest possible to the densest dry condition that could be obtained by vibration.

The influence on thermal resistivity of inter-particle binding or cementing and of the composition of the soil-water solution was investigated by running tests on

		Mechanical Analysis (%)						
	Sa		nd			Consistency Limits		mints
Soil No. Gravel ¹	Gravel 1	Coarse ²	Fine ³	Silt 4	Clay ⁵	Liquid Limit (%)	Plastic Index ⁶	Field Moist. Equiv. (%)
White sand	0	91	9	0	0		NP	
CE 1	0	20	80	0	0	15.2	NP	27.0
CE 2	1	82	16	1	0	12.7	NP	25.2
CE 3	5	20	55	10	10	28.5	12.0	21.5
CE 4	1	35	63	1	-0	16.0	NP	25.7
CE 5	40	26	6	15	13	26.5	8.1	29.5
PD 1	7	65	21	õ	2	13.4	NP	17.9
PD 2	19	59	15	4	3	78 1	ŇP	18.2
PD 3	12	34	41	9	4	12.0	NP	11.4
PD 4	13	33	40	11	3	14 2	NP	18.0
CQ < 200	-0	0	30	$\overline{70}$	õ		NP	
čů 2 400	ŏ	ŏ	Ő	71	29	-		
Dor	7.6	46.4	370	'â 0	6.0	16.4	1.9	15.0
Wh.	7.6	46.4	37.0	3.0	6.0	16.4	1.9	15.0
Pt	1.0	58.0	33 0	4 7	3 2		ŇP	
Nat soil	1.4	4.6	28.0	43.5	22.3			

 $^1\,2$ mm. $^2\,2.0$ to 0.25 mm. $^3\,0.25$ to 0.05 mm. $^4\,0.05$ to 0.005 mm. $^5\,{\rm Less}$ than 0.005 mm. $^6\,{\rm NP}$ \equiv non-plastic.

graded white quartz sand (WS) without and with the addition of the following materials (on a dry-weight basis) at the indicated percentages:

(a) Sodium chloride at $\frac{1}{4}$, $\frac{1}{2}$, 1, and 2 percent.

(b) Calcium chloride at $\frac{1}{4}$, $\frac{1}{2}$, 1, and 2 percent.

(c) Asphalt at $\frac{1}{4}$, $\frac{1}{2}$, 1, and 2 percent. (Percentages based on the weight of cutback asphalt used per unit weight of dry soil.)

(d) Portland cement at 1, 2, 3, and 5 percent.

(c) Acid kaolinite at 1, 3, 6, and 10 percent.

(f) Sodium bentonite at 1, 3, 6, and 10 percent. Percentages determined on a dry weight basis.

TABLE 5

MAXIMUM DENSITIES AND OPTIMUM MOISTURE CONTENTS¹

Soil No.	Optimum Moisture (%)	Max. Dry Density (lb/cu ft)	Method of Compaction
ws	8	100.5	Altered Proctor
ČE 1	8.8	97.4	Altered Proctor
ČE 2	11.5	102.3	Altered Proctor
ČE 3	13.5	113.6	Altered Proctor
ČE 4	8.0	96.1	Altered Proctor
ČĒ 5	14.2	115.0	Altered Proctor
PD 1	7.5	104.5	Altered Proctor
PD 2	12.5	107.8	Altered Proctor
PD 3	9.2	120.8	Altered Proctor
PD 4	10.0	117.3	Altered Proctor
Dor	9.7	120.2	Dietert

¹ Physical properties determined according to applicable ASTM specifications. Clay mineral types determined by X-ray diffraction.

The sodium chloride and calcium chloride were dissolved in the mixing water before it was added to the dry soil. The asphalt was added to the dry sand and thoroughly mixed. The mixture was then left overnight. Subsequently, the mixing water was added and the samples were compacted. The portland cement, kaolinite and bentonite were mixed with the sand in the dry condition and the mixing water was added afterward. Allowance was made in the amount of water added to obtain the optimum moisture content and to provide enough water for the hydration of the portland cement, also to bring the kaolinite and bentonite admixtures to their respective plastic limits.

Each composition was run at four different moisture conditions, as follows:

1. Moisture at which compacted.

2. Oven-dried for 24 hr.

3. Soaked for 4 days.

4. Oven-dried for 5 days.

In the case of 1 and 2 percent asphalt, and 3, 6 and 10 percent bentonite, the material was still moist after 5 days oven-drying. This was attributed to the low permeability of the materials. The materials with 1 and 2 percent asphalt exhibited shrinking during the moisture change cycle, whereas the sand containing 3, 6 and 10 percent bentonite swelled considerably during wetting and shrank

TABLE 4 MECHANICAL ANALYSIS AND CONSISTENCY DEODEDTIES OF CRANIILAR MATERIALS

during oven-drying. The largest volume change was noticed in the 6 percent bentonite material, mainly because sand with 10 percent bentonite was more impervious to water penetration than sand with 6 percent and therefore could not satisfy its absorption capacity within the time allotted for soaking. The data for the obtained thermal conductivity are somewhat erratic, which reflects the disturbance of packing and sand grain contact during swelling and shrinking of the material.

In the case of sodium and calcium chloride mixture, two phenomena were observed — high moisture content of the material still present after 24-hr drying in the oven, and crusts formed in the completely dry material. The crusts formed by calcium chloride could be described as follows:

Ca	Cl_2

Conc.

(%) Description of Crust

- ¹/₄ Layer about ¹/₂-in. thick adjacent to brass cylinder; easily disintegrated by fingers.
- ^{1/2} About 1 in. on top of sample, ^{3/4} in. on bottom, ^{3/4} in. on outside; could be disintegrated by fingers.
- About 1¼ in. on top of sample, ½ in. on bottom, ¾ in. on outside; top crust hard to disintegrate by fingers.
- 2 About 13/4 in. on top of sample, 3/4 in. on outside; outside crust hard to disintegrate by fingers; top crust could not be disintegrated by fingers.

In all cases the material inside the crusts flowed out as if it contained no additive.

The crusts formed by sodium chloride could be described as follows:

$NaCl_2$		
Conc.		
(%)	Description of Crust	

¹/₄ Very soft, about ¹/₄-in. thick adjacent to brass cylinder; soil inside flowed out.

- 1/2 Soft (easily disintegrated by fingers), about 1/2 in. on top, 1/4 in. on bottom, 1/2 in. on outside; material inside flowed out.
- 1 About 1¼ in. on top, ¼ in. on bottom, ¾ in. on outside; crust could be disintegrated by fingers.
- 2 Soil crusted on outside, leaving an inside cone of somewhat coherent material; cone base 4-in. dia., height to about 1 in. from top; outside material could be disintegrated by fingers, inside cone loosened by light scratching.

The sodium chloride-sand adhered to the metal cylinder much more than the calcium chloride-sand in the completely dry state.

To evaluate the thermal resistivitymoisture content-secondary structure relationships, tests were run on systems with varying moisture contents. Instead of running the tests at selected moisture contents, the thermal resistivity was determined at definite moisture tensions, the motive being that the moisture retention, pore size distribution, and thermal resistivity could be interrelated. The samples were each run at 0 (saturated), 30, 60 and 100 cm, respectively, of moisture tension. In the granular materials with large grain size, a reasonable variation in moisture content was possible over the range of moisture tensions. However, with the smaller size grains, the variation in moisture tension was not sufficient to obtain the desired range of moisture content values. Determinations at different densities were not possible because of difficulties encountered in retaining the same density throughout the course of the experiment. The greatest trouble was encountered when a lowdensity saturated granular material was drained.

The materials used for this phase were the six different sizes of glass beads, crushed quartz, and white sand, respectively, and the undisturbed natural soil from the vicinity of Princeton. The undisturbed soil samples were also tested in the dry state. Some of these undisturbed samples underwent considerable shrinking during the drying-out process.

The thermal resistivities of the soils supplied by the various sponsoring companies were determined in the moist and dry states. The Dorehester special blend sand, thermal sand 1, and thermal sand 2, were investigated more thoroughly because of the known low resistivities of these materials. The last three sands were compacted to different densities and each density was run at five different moisture contents (one being at zero moisture content).

Analysis of Experimental Data

From the experimental data, Figures 12, 13, 14, 15, 17, 18, and 19 were constructed. Supplementing the authors' data, some reported by other workers (40, 60, 63) were employed. It was regarded as advantageous to express the moisture content on a volume basis rather than on a weight basis. This treatment aids in visualizing the soil system and permits comparison of soils in which the specific gravity of the solid particles varies. As a further simplification, the moisture content is represented as the fraction m of voids filled by water. In Figures 12, 13, 14, 15, 16, 17, 18, and 19, the resistivity ρ of the different materials has been plotted against m.

Of all the simple functions attempted, the type of equation that fitted the experimental data best, was found to be of the form

$$\rho = A \ 10^{-B \ m} + s \tag{19}$$

The constants A, B and s are functions of density, mineral type, granulometry, and the other respective factors mentioned in the preceding section. Generally, the constant s may also be a function of moisture content m, but as a simplification this is considered not to



Figure 12. Thermal resistivity vs moisture content, crushed quartz.





be the case. A better fit may be obtained by using some series which will converge within the given interval to the curves shown. However, to express all the constants in the series as functions of density, granulometry, mineral type, and the other variables would be a formidable task.

It was noticed from the curves that for a value of m higher than 0.50 the variation in resistivity was less than 10 percent, or within the range of experimental error. The value of s for each density and each soil was thus determined by averaging the values of resistivity for which m was larger than 0.50. When these asymptotic values (for m >0.50) of resistivity s were plotted against density d a scatter plot as shown in Figures 20 and 21 was obtained. Lacking a large variation in density at these moisture contents for each grain size and to simplify the analysis, it was assumed that s was a linear function of density d and independent of the grain size, yielding the expression

$$s = s_1 - s_2 d$$
 (20*a*)

where s_1 and s_2 are functions of the type of material. Using the method of least squares, the following straight-line expressions were obtained:

Crushed quartz, s=107.08-47.91d (20b) Quartz sand,

$$s = 116.14 - 49.26d$$
 (20c)

Thermal sand 1,
$$s=118.8-48.58d$$
 (20*d*)

Thermal sand 2,
$$s=109.18-42.08d$$
 (20*e*)

Natural soil,

$$s = 128.14 - 45.22d$$
 (20f)

The values of s_2 do not differ much for these materials. As a weighted av-



Figure 14. Thermal resistivity vs moisture content, thermal sand 1.

erage, the value for s_2 was established as 47.5, thus giving for s_1 the values given in Table 6.

For the glass beads, there was no apparent variation in s for variation in density. In this case, s_2 was taken as zero and s_1 was averaged as 137.3 for all sizes.

From the data on the dry material, the values $(\rho - s)$ were calculated for each dry density. The term A in Eq. 19 was then evaluated. It was noticed that A will be an exponential function of the type (Figs. 22 and 23):

$$A = (\rho - s)_{m=0} = 10^{a_1 + a_2 d} + a_3 d^2 \quad (21)$$

By trial and error, it was found that a_2 is very close to zero. For all the materials, a_2 was taken as zero; as a further simplification, a_3 was taken to be the same for all materials. According to these assumptions, only a_1 will be a function of granulometry and type of material. An average value was found to be $a_3 = -0.44$. The a_1 -values are tabulated for each grain size and material in Table 6.

To determine B in Eq. 19 as a function of density, the A term was calculated for Eq. 21 and the $(\rho - s)$ -values were plotted against the moisture fraction m on semi-logarithmic paper (Fig. 24). As stated before, only the first power of m was used. A higher power of m against log $(\rho - s)$ would give a better fit of the points, but this refinement was not justified in view of the limited range of the experimental data. Only data for which m was less than 0.40 gave acceptable values for the slope. Errors are present because the shape of the experimental curves does not follow a true exponential.

Variation of B with density could be obtained only on the thermal sand 1, thermal sand 2, Russia black earth, crushed quartz P4703 and P4714, and Ottawa sand P4701 and P4702. An approximate linear variation of B with density d was found when the values were plotted in Fig. 25. The equations for the straight lines were of the form

$$B = b_1 - b_2 d \tag{22}$$

The b_1 and b_2 coefficients will be functions of granulometry and solid composition. Values for b_2 in Eq. 22 were found for the following materials:

Soil	b_2
Thermal sand 1	4.213
Thermal sand 2	6.628
Russia black earth	5.640
Crushed quartz	22.873
Fine crushed quartz	23.404
Graded Ottawa sand	14.933
20-30 Ottawa sand	20.541

For the first three soils, the deviation of experimental points from the selected curve was much less than for the last four; also a larger density range was covered in the case of the thermal sands and the Russia black earth. Consequently, the exceptionally large values obtained for b_2 from the crushed quartz and Ottawa sands were discarded in favor of those obtained from the thermal sands and black earth. An average value of $b_2 = +5.5$ was established. Values of b_1 for all the other materials were calculated and tabulated in Table 6, based on the same value ($b_2 = 5.5$) for all soils.

To account for variation of s_1 , s_2 , a_1 , and b_1 with different materials and soils, the additional parameters evoked are percentage of quartz q, percentage of clay c, and r the surface area per unit volume of material larger than the clay size. The percentage quartz q was em-



Figure 15. Thermal resistivity vs moisture content, thermal sand 2.



Figure 16. Thermal resistivity vs moisture content, Southern Russia black earth, Chernozen Curves from data by Kolyasev and Gupalo, (63).

ployed to express the resistivity of the solids and the clay percentage c to account for the cementing of the particles in the dry state. Both parameters q and c contain the additional property of expressing the state and amount of water absorbed on the solid surfaces. The percentage quartz was determined by visual examination of the larger fraction under a microscope and the clay type by means of X-ray diffraction. The percentage clay was taken as the percentage material smaller than 0.005 mm determined by sedimentation method according to ASTM specifications. It is evident that this method of expressing so complicated a characteristic is fundamentally inadequate. As an example, consider the glass beads or crushed quartz CQ < 400. In the glass beads, there is no quartz crystal: in CQ < 400 there is 100

percent quartz, but 29 percent of the material is smaller than 0.005 mm. For soils, however, there will be no glass beads and a situation like in CQ < 400 is very unlikely to exist. For the mentioned sample of crushed quartz, the material was treated as 100 percent quartz and the clay size fraction ignored.

The factor r is determined from the grain-size accumulation curve. If W_i signifies the weight of material retained on one sieve, the most probable diameter d_i will bear a certain relation to the opening size of the sieve, the material passed, and the opening size of the sieve on which it is retained. The volume of a particle with linear dimension d_i will be $f_1(d_i^3)$. If n_i particles are retained, the total volume retained on the *i*th sieve will be $W_i f_1(d_i^3)$. If G represents the density of the solid, then W/G will also

1

represent the volume of the material retained on the *i*th sieve.

$$\frac{W_i}{G} = n_i f_1(d_i^{\,3}) \tag{23a}$$

 \mathbf{or}

$$\frac{W_i}{G} = n_i d_i f_2(d_i^2) \qquad (23b)$$

 $\frac{W_i}{d_i} = G n_i C \times \text{Surface area of one par-ticle}$

 $= k \times \text{Total surface area of par$ ticles retained on the*i*th sieve(A_i) (23c)

Over the whole range,

$$\Sigma A_i = \Sigma \frac{1}{k} \frac{W_i}{d_i} \tag{23d}$$

The factor k may vary for different grain sizes. As an average, it may be taken as that for a sphere; namely,

$$k = \frac{G}{6}$$
. The A, will be in dimensions of

surface area per unit weight. If the right-hand side is multiplied by the density of the solids, the surface area is obtained per unit volume of solid material,

$$r = \Sigma A_i G = \Sigma \frac{G}{k} \frac{W_i}{d_i} \qquad (24a)$$

$$r = 6 \Sigma \frac{W_i}{d_i} \tag{24b}$$

 W_i and d_i are obtained from the grainsize accumulation curve for grain sizes larger than 0.005 mm.

Employing these additional parameters, the following was deduced:

s-factor:

For
$$q > 75\%$$
,
 $s_2 = -47.5$; $s_1 = 200 - 0.94q$ (25a)



Figure 17. Thermal resistivity vs fraction of voids in white quartz sand containing saline (NaCl, CaCl₂) solutions of different concentrations.



Figure 18. Thermal resistivity vs fraction of voids filled with water for white sand containing various percentages of kaolinite and asphalt, respectively.

For
$$75\% > q > 20\%$$
,
 $s_2 = 2q - 200; \ s_1 = 435 - 4.07q$ (25b)

A-factor:

$$a_1 = 16r^{-0.9} + a_i^{1} \tag{26a}$$

and

 $a_i^{1} = 3.4 \times 10^{-0.008c} \tag{26b}$

B-factor:

Most of the values of b_1 and b_2 for the cohesionless materials were obtained from only two points. There seemed to be a trend that the larger the *r*-factor, the smaller the b_1 -values. The trend was inconsistent and closer inspection showed that this trend was only because the intervals of *m*-values were different for the various materials. Averaging over all b_1 -values for cohesionless materials the value of $b_1 = 16.18$ was obtained. Ascertaining the dependence of b_1 on the clay content, the following relationship was obtained:

$$b_1 = 5.6 \times 10^{-0.04c} + 9.58 \tag{27}$$

DISCUSSION

Test Data

In the following, a qualitative account is given of what it has been attempted to describe quantitatively in the preceding section. Concentrating mainly on the figures, rather than on the equations developed, the following points are evident:

1. The relationship between density, moisture content, and thermal resistivity can be stated as follows. If the same material is taken (Figs. 14, 15, and 16), it will be noticed that increase in density causes a downward shift of the curves toward lower resistivity. However, at high moisture contents, the reduction in resistivity with the same variation in density is not so marked as it is at low or zero moisture content. The result is that the slope of the resistivity-moisture content curve tends to flatten out with increase in density. This means that at high soil densities additional water will have a lesser effect on the resistivity than at low densities. At high densities, the permeability of the material is decreased, with the result that moisture movement will be slowed down; the water will be held more tightly because of reduction in pore diameters and also because the fraction of water close to solid interfaces per unit pore space will be increased.

The same picture for moisture-densityresistivity relationship holds whether the material is sand, crushed quartz, clay, or natural soil. The relative values will differ for the various materials.

2. With respect to the effect of the clay (< 0.005 mm) fraction on the thermal resistivity, inspection of Figures 18 and 19 indicates the following. In the cases of kaolinite, bentonite, and even asphalt addition, it seems that increasing amounts of additive will tend to increase the resistivity a small amount at the high moisture contents, but cause a large reduction in resistivity at very low moisture contents. The same tendency was found for all clavey and natural soils; namely, a fairly flat curve for resistivity ρ against moisture content m(taken at a specific density). This is evident from Figures 14, 15, and 16. If these curves are compared with those for crushed quartz and sand (Figs. 12 and 13), it will be noticed that the granular cohesionless soils have a higher dry thermal resistivity and a lower resistivity at saturation than the clavey soils of the same density. Also noticeable is



Figure 19. Thermal resistivity vs fraction of voids filled with water for white sand containing various percentages of bentonite and portland cement, respectively.



Figure 20. Variation with density of asymptotic value of thermal resistivity at saturation.

the steep slope in the resistivity-moisture curve of the sands at low moisture content, compared to that of a clayey soil over the same moisture interval, both being taken at the same density. Comparing Figures 12 and 16, the following is evident.

From m=0 to m=0.10, the slope for the sand is much steeper than that for the clay. Between m = 0.10 and m = 0.50. the slope for the clay becomes steeper than that for the sand, whereas for m >0.50 all curves flatten out. Depending on which moisture interval the resistivity change is taken in, the sands will show a larger or smaller variation in thermal resistivity than the clays. Over the low moisture content range, the thermal resistivity varies considerably more for sand than for clayey soils. Under the same climatic conditions, clayey soils will retain moisture better than sands and will not show as great a variation in moisture. It may therefore be quite possible to observe a great seasonal variation in thermal resistivity in sands and not in clavs, as has indeed been observed in practice (27).

3. To investigate completely the effect of grain size, the clav-size particles should be included in the considerations. However, in the clay size range, other phenomena are encountered which do not appear in inorganic material of sizes larger than 0.005 mm. Some of these are the cementing property of the colloids. their interaction with water, as revealed in the consistency limits and swelling and shrinking properties, and their reaction with ions or organic matter. It should thus be more appropriate to consider the effect of the grain size on thermal resistivity for sizes larger than 0.005 mm apart from that of the clay fraction. That the separation of clay from the rest of the materials on the basis of size alone is not sufficient is clearly obvious from the data on quartz samples that contained large percentages of the fraction smaller than 0.005 mm.

The moist systems were not so thoroughly investigated with respect to grain sizes as were the dry materials. In the development of the equations, the grain size was only considered in the dry resistivity values. As a representation of

the effect of the grain size, the total surface area per unit volume of solids was employed. The glass beads show an increase in thermal resistivity with increase in total surface area, whereas for all granular quartz materials the resistivity decreases with increase in surface area, as is evident from Eqs. 19, 21, and 26a. The reason may be that for larger surface areas there will be larger areas of particle-to-particle content. It may also be that with smaller grain sizes, more moisture will be retained if the material is not absolutely dry. In addition, the smaller quartz particles had been crushed to size and were, therefore, not of spherical shape.

4. The effect of additives to the white quartz sands is shown in Figures 17, 18, and 19. The influence of addition of kaolinite, bentonite, and asphalt has been discussed previously to some ex-

tent. The sodium chloride and calcium chloride salts do not have any noticeable effect on the thermal resistivity at high moisture contents, but caused a significant decrease of resistivity in the dry state. A complete evaluation of the beneficial effect of interparticle binding unfortunately could not be accomplished because the cemented crust formed on the outer surfaces of the samples, but the thermal probe was inserted along the center line. It is believed that if the salt had been deposited homogeneously throughout the material, a much larger reduction in thermal resistivity of the dry material would have been obtained. The capacity of the sand to retain moisture during drying, when the soil water contains the mentioned salts, is of importance in inhibiting the loss of moisture when the temperature of the soil is raised. Mixing of sodium chloride and



Figure 21. Variation with density of asymptotic value of thermal resistivity at saturation.

Soil No.	81	82	<i>a</i> 1	<i>a</i> 3	<i>b</i> 1	b2	$r (\text{cm}^2/\text{ec})$	Clay (%)	Quartz (%)
			(a) C	RUSHED QUA	ARTZ .				
CQ 40-50 CQ 50-60 CQ 50-100 CQ 100-200 CQ 200 CQ 200 CQ 400 P 4703 P 4714	106.5	47.5	3.558 3.520 3.496 3.449 3.397 3.448 3.611 3.349	-0.44	$\begin{array}{c} 16.953\\ 16.624\\ 12.975\\ 10.232\\ 9.018\\ 15.265\\ 14.522\\ 14.523\\ \end{array}$	5.5	168.2203005401,9003,088169.300	29.0	$100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 96 \\ 96$
			(b)	QUARTZ SAU	ND				
WS 10-20 WS 20-30 WS 30-40 WS 40-50 WS 60-60 WS 60-100 WS Graded Wageningen	113.3	47.5 47.5	$\begin{array}{c} 3.572 \\ 3.544 \\ 3.541 \\ 3.563 \\ 3.523 \\ 3.514 \\ 3.634 \\ 3.556 \end{array}$	-0.44	$18.713 \\ 20.846 \\ 19.396 \\ 19.854 \\ 18.274 \\ 17.998 \\ 16.180$	5.5 5.5	$\begin{array}{c} 61.6\\ 83.4\\ 119.2\\ 175.5\\ 213.5\\ 244.0\\ 147.7\\ 171.3\end{array}$		95 95 95 95 95 95 95 95 89
<u> </u>			(e)	OTTAWA SAN	с				
Graded 20-30 30-40	113.3	47.5	$3.660 \\ 3.697 \\ 3.712$	-0.44	$\frac{16.400}{20.925}$	5,5	$97.3 \\ 84.0 \\ 135.7$		99 99 99
			(<i>d</i>)	NATURAL SO	-IL				
Nat. soil Th. sand 1 Th. sand 2 Russia black earth Fairbanks silty loam Silty clay loam Healy clay	$131.5 \\ 116.8 \\ 118.8 \\ 326.2 \\ 216 \\ 187.2 \\ 307.8 \\$	$\begin{array}{r} 47.5\\ 47.5\\ 47.5\\ 157.9\\ 134.6\\ 134.6\\ 152.7\end{array}$	$3.213 \\ 3.303 \\ 3.475$	$-0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.44 \\ -0.4$	9.595	5.5 5.5 5.5 5.5 5.5 5.5	2,030 239 266 1,770 1,398 420	$ \begin{array}{r} 22.5 \\ 6.0 \\ 3.2 \\ 11.5 \\ 27 \\ 78 \\ \end{array} $	$ \begin{array}{r} 60 \\ 88 \\ 78 \\ \overline{54} \\ 64 \\ 23 \\ \overline{54} \\ 23 \\ \end{array} $
			(e)	GLASS BEAR	08				
G 107 G 110 G 112 G 114 G 114 G 116 G 119	$137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.3 \\ 137.$	0	3.663 3.667 3.682 3.663 3.687 3.703	0.44	$\begin{array}{c} 16.859 \\ 13.086 \\ 13.774 \end{array}$	5.5	1283005007321,0532,145		Glass Glass Glass Glass Glass Glass

 TABLE 6

 CONSTANTS IN RESISTIVITY — DENSITY-MOISTURE CONTENT RELATIONSHIP

calcium chloride with clayey soils may prove beneficial in reducing the thermal resistivity; however, the sodium ion may cause the clay to swell and shrink during wetting and drying. A disadvantage of these salts is that they tend to leach out and present corrosion problems to buried pipes or metallic structures.

The addition of asphalt generally causes an increase in resistivity over that for the plain white quartz sand, except at low moisture contents. Increase in asphalt percentage increased the thermal resistivity. Some reduction in resistivity was found for 0.5 percent asphalt. Unfortunately, the significance of this could not be established. Additional tests might have indicated whether this was to be expected or was just due to errors in measuring. When the asphalt content was increased, the density obtainable by the same compaction method was decreased. The low permeability of asphalt-treated soils obstructs the flow of water through the material. It will dry out more slowly than plain sand, but, when dry, it will also wet much more slowly.

The effect of kaolinite as additive has been discussed. Another feature which will amplify the desirability of kaolinitic clay as binder in the sand is that it does not swell and shrink as much as other types of clay minerals. With a proper gradation of coarser material (>0.005 mm), a clay content of between 6 and 10 percent will result in a material of low water permeability. A highdensity material is most desirable and enough clay should be present to obtain this without causing swelling and thus separation of the coarser grains.

The unwanted effect of swelling was noticed in the case of the bentonite admixture. The 6 and 10 percent bentonite clay additions caused erratic results. These were probably due to changes in density during the drying-wetting-drying cycle: it is also possible that during these cycles the quartz-to-quartz contacts are replaced by bentonite clay bridges. Judging from the position of the dashed curves of Figure 19, bentonite has a much higher thermal resistivity than either quartz or kaolinite. The thermal resistivities of wet bentonitic materials are about of the same order as was found for the asphalt admixture. Unfortunately, the dry resistivity was not determined for all the percentages of bentonite addition and it was thus not possible to establish the influence of bentonite in the dry state.

Portland cement constituted a somewhat different type of binding agent in that it does not soften with wetting after having set up. Figure 19 shows the effect of increasing amounts of portland cement on the white quartz sand. The addition of cement decreases the thermal resistivity both in the moist and the dry conditions. However, certain points must be remembered, such as that the tests were run within a period of about 6 days; hence the cement had not reached its full binding strength. Aging of the material would probably further decrease the thermal resistivity of cement-bonded sands. Higher percentages of portland cement facilitated compaction of the material, with resulting higher densities. using the same compaction effort. The portland cement addition gave the greatest over-all reduction in thermal resistivity per percentage added, but has the drawback that cement-bonded sands are



Figure 22. Dry thermal resistivity values (p - s) vs dry density.







Figure 24. Thermal resistivity $(\rho - s)$ vs fraction of voids filled with water.



Figure 25. Exponential B factor vs dry density.

not so easily worked and placed, and once hardened cannot be handled as easily as clay-bonded sands.

Influence of Structure

From the experimental investigation, little could be learned about the influence of soil structure on thermal resistivity. The tests on the natural soils proved that structure formation can cause large deviations in thermal resistivities from one location to another.

The main manifestations of structure formation in a soil are the formation of clods or aggregates and the subsequent deviation of pore size distribution and water and air permeabilities from the values that would be expected if the particles were uniformly dispersed. As discussed previously, pore size distribution, permeability and aggregate sizes can be interrelated. It was thought advisable and faster to determine the pore size distribution rather than the grain size distribution of a structured soil. If it is assumed that all the clods or aggregates have the same density, grain and pore size distribution, the aggregates themselves may be considered as primary particles.

The knowledge gained on granular materials, without or with interparticle binding (whichever is applicable) may then be applied to structured soil. The thermal resistivity of the primary particles will be different from that of quartz and will vary with variation in moisture content inside the aggregate. It is clear that the density of the aggregates must be more than the over-all density of the total soil sample. The clods will therefore be saturated with water before appreciable moisture will collect between the aggregates. The density of the clods may be determined if the pore size between the secondary particles (as one measure, the noncapillary porosity may be used) has been established. The porosity inside the aggregates (internal porosity) will be the difference between the total pore volume and the porosity outside the aggregates. From knowledge of the moisture content, the fraction of internal voids (inside the aggregates) filled can be calA

culated. Having the density and fraction of voids filled of the aggregate, as well as its solid composition (which will be the same as for the total soil) the thermal resistivity of the secondary particles may be evaluated, using the formulas previously developed. Using this value as the thermal resistivity of a primary particle, the system will consist of granular particles, using as porosity value the porosity between the aggregates. Calculation of the total thermal resistivity may then be attempted using this equivalent system and employing the established formulas.

There will be a number of difficulties in employing this method. The effect of variation in thermal resistivity of the individual particles on the total resistivity was not uniquely determined, but was implied in factors such as quartz percentage and clay percentage. Possibly equations developed for the glass beads may be more applicable.

In identifying different shapes and types of structure formation, the aggregates may be described by three dimensions — 2a, 2b, 2c — representing the three principal axes of the ellipsoid approximating the form of the soil clod. The theory developed by Maxwell (70), Burger (26), Eucken (44), and applied to soils by De Vries (40), may be used to determine the effective thermal conductivity of the entire sample. As previously given (102), the over-all resistivity ρ equals 1/k, where k is the overall thermal conductivity and is expressed as

$$k = \frac{\sum_{i=0}^{n} k_{i} X_{i} K_{i}}{\sum_{i=0}^{n} X_{i} K_{i}}$$
(28)

in which

- k_i = thermal conductivity of the *i*th type of particles with volume fraction X_i ;
- k_{o} = the thermal conductivity of the medium of volume fraction X_{o} ; and
- n = number of different types of particles.

 K_i is a factor dependent on the geometry of the inclusions and the ratio of thermal conductivity of the *i*th-type particle to that of the medium,

$$K_{i} = \frac{1}{3} \left\{ \frac{1}{1 + \left(\frac{k_{i}}{k_{o}} - 1\right)A_{1}} + \frac{1}{1 + \left(\frac{k_{i}}{k_{o}} - 1\right)A_{2}} + \frac{1}{1 + \left(\frac{k_{i}}{k_{o}} - 1\right)A_{3}} \right\}$$
(29)

For ellipsoidal particles the values of A_1 , A_2 and A_3 are given in Stratton (94) as

$$A_{1} = \frac{a \ b \ c}{2} \int_{0}^{\infty} \frac{ds}{(s+a^{2})^{3/2} \ (s+b^{2})^{\frac{1}{2}} \ (s+c^{2})^{\frac{1}{2}}}$$
(30a)

$$A_{2} = \frac{a \ b \ c}{2} \int_{0}^{\infty} \frac{ds}{(s+c^{2})^{\frac{3}{2}}} \frac{ds}{(s+b^{2})^{\frac{3}{2}} (s+c^{2})^{\frac{3}{4}}}$$
(30b)

$${}_{3} = \frac{a \ b \ c}{2} \int_{0}^{\infty} \frac{ds}{(s+a^{2})^{\frac{1}{2}} \ (s+b^{2})^{\frac{1}{2}} \ (s+c^{2})^{\frac{3}{2}}} \quad (30c)$$

For a spherical particle $A_1 = A_2 = A_3 = \frac{1}{3}$.

Either the air or water phase, whichever is continuous, may be used as the medium. Treating the air as the medium will give conductivity values that are too low, because touching and interparticle cementing is neglected. The best way will therefore be to treat the moisture as the medium and the aggregates and air as inclusions.

Interpretation of Equations

The equations developed suggested the type of surface shown in Figure 26, in which the density is expressed as the volume fraction of solids per unit volume of material on the x-axis, the moisture as the volume fraction of water per unit volume of material on the y-axis, and the thermal resistivity along the z-axis. The intersection c of the solids and moisture axes will be 100 percent air. In Figure 26 the point a on the



Figure 26. Three-dimensional surface depicting thermal resistivity as a function of solid, air, and water phases.

x-axis represents 100 percent solid; aa¹ is then the thermal resistivity of the solids alone. The point b on the y-axis represents 100 percent water and bb¹ the thermal resistivity of water. The length cc^{1} along the z-axis will constitute the thermal resistivity of air.

For different soils, the only limiting value in the sketch that will show a considerable change will be the effective thermal resistivity of solids alone, which is represented by the z-value aa¹. The value of bb¹ may be altered slightly by the amount and type of salts in solution, temperature, or presence of other liquids. Similarly, cc¹, the thermal resistivity of air may be changed a little by changes in its composition and temperature. Considering these assumptions, the curve b¹ c¹ will remain unchanged for all different soil types; however, curve b¹ c¹ will not be encountered in systems of practical engineering importance.

Curve $a^1 b^1$ represents the variation in thermal resistivity for saturated systems in which the solid content varies from 0 to 100 percent. It is not believed that this curve will be a straight line, but rather that it has a shape similar to that of curves $b^1 c^1$ or $a^1 c^1$. The curvature of $a^1 b^1$ will be less noticeable than that of $b^1 c^1$ or $a^1 c$, because the difference between aa^1 and bb^1 will be much less than between bb^1 and cc^1 or aa^1 and cc^1 .

In addition to the variable aa^1 (ρ solids) for different soils there will be other variables, the most important of which will be the slope \overline{n} of the normal to the surface $a^1 b^1 c^1$. The slope \overline{n} will be a function of the solid and moisture content, or

$$\overline{n} = f(d, m) \tag{31}$$

For different soils, this function will not be the same, but will depend on the resistivities of the solids, grain size, interaction of the solid surfaces with water and ions, temperature, and the cementing or cohesive properties of the solid constituents in the dry (m=0), moist, and saturated (m+d=1) states.

As an illustration of this variation, the dotted curves were drawn, which are

thought to be representative of a clay system with lower thermal resistivities in the dry state and somewhat higher resistivities in the saturated state, than would be expected for a pure quartz material (represented by the solid curves); also indicated is the variable range over which data have been accumulated during the present investigation.

The complex form of the surfaces shown will indicate to what extent the equations developed have described the relationship between thermal resistivity, density, and moisture content. Within the range of the experimental data, the equations may be used to predict the thermal resistivity of a soil for given values of the variables treated. It is believed that the accuracy to which this prediction is possible will be better than 20 percent.

CONCLUSIONS

From the study of theoretical expressions developed with a view toward obtaining the over-all thermal resistivity of granular systems, and interpretation of the experimental data accumulated, the following conclusions may be drawn.

As in most investigations of this nature, the results found are not as conclusive as desired and are found lacking in many respects. This is due mainly to the fact that although many experimental data were accumulated, they were not sufficient to establish the obtaining relationships with the desired degree of accuracy. In addition, because of practical problems, the tests could not be run at close enough intervals or over as wide a range of moisture and density as would have been advantageous for a good theoretical interpretation of the problem. The characteristics and properties of materials used were not varied over a range sufficient to determine accurately their effect on thermal conductivity. A formidable obstacle presented itself, insofar as there exists no simple basic numerical measure by which the properties of materials could be determined and compared with one another. In all materials, the engineering constants represent numerical attempts

to describe the behavior of the particular material under selected conditions over a limited range of the variables involved.

With all these limitations in mind, the following were observed.

1. Soil thermal resistivity is affected by as many variables as are operating in a soil. The most important of these were found to be moisture; density; mineral composition; amount, type, and composition of the binder fraction (smaller than 0.005 mm); and the grain size distribution of the material larger than 0.005 mm. Additional parameters will be temperature, effect of organic matter, structure formation in natural soils, thermally-induced moisture transport, etc.

2. Increase in moisture causes decrease in thermal resistivity. The rate of decrease of resistivity with increase in moisture is different for different density values and depends also on the type and amount of clay material. This rate of decrease of resistivity is also a function of the moisture content.

3. Increase in density causes a decrease in thermal resistivity. The rate of decrease of resistivity with increase in density is a function of the density, moisture content, amount and type of binder material (smaller than 0.005 mm), and grain size. The effect of density is more pronounced in the dry state than in either the moist or the saturated condition.

4. Increase in inorganic clay content (smaller than 0.005 mm) causes reduction in thermal resistivities in the dry state, but small increase in resistivities at saturation. The magnitude of reduction and increase depends on the type of binder material.

5. The lower the resistivity of the solids and the better their interaction with water, the lower will be the over-all thermal resistivity of the soil. In this investigation, the percentage of quartz was taken as a measure of the effect of the mineral composition of the material larger than 0.005 mm on the thermal resistivity.

6. The effect of granulometry of the material larger than 0.005 mm on the thermal resistivity of a soil was found to be of importance only in the dry state. For quartz materials, an increase in the surface area of particles per unit volume of solids caused a slight decrease in the dry thermal conductivity.

7. Effective binding or cementing of the soil grains causes a decrease in thermal resistivity. Portland cement proved to be the most effective additive, but kaolinitic clay may be more practical in application. Sodium and calcium chloride give increased moisture retention capacities to the soil and cement the particles together in the dry state. These salts tend to leach out and may not result in permanent lowering of thermal resistivity.

The following suggestions are made with reference to possible future work. It is desirable to:

1. Run additional experiments in order to establish the nature of the functions where assumptions had to be made because of lack of information within a certain range.

2. Try to correlate the cementing and binding effect of the fraction smaller than 0.005 mm with thermal resistivity in the dry condition. The dry cementing effect may be obtained by determining the strength of dried clay pats.

3. Obtain more specific relations between thermal resistivity and other soil properties, possible linking with such characteristics as water absorption, shrinkage, ion exchange capacity, and soil-water composition.

4. Exercise care that the sample from whose properties the prediction is made will be representative of the soil in that area before practical application is made of the established relationships. The variables which may change with respect to time must be established and correction allowed in the calculation.

5. Study the variation of thermal resistivity and all the other relevant soil properties with variation in temperature.

ACKNOWLEDGMENTS

This paper was prepared in conjunction with a research project dealing with the thermal characteristics of soils that has been carried out during 1956-58 at the Soil Physics Laboratory of the Department of Civil Engineering, Princeton University, under the direction of Dr. Hans F. Winterkorn, Director of the Laboratory. The project was initiated by the A.I.E.E. Committee on Insulated Conductors and was made possible by financial contributions of the Edison Electric Institute, the Association of Edison Illuminating Companies, the Engineering Foundation, the Insulated Power Cable Engineers Association, various cable manufacturers, and certain governmental and municipal power agencies in the United States and Canada

REFERENCES

- 1. ALBRECHT, F. Meteorologische Zeits., 49:294-299 (1932).
- 2. ALDERFER, R. B., "A Study of the Factors Affecting the Structure, Permeability and Erodibility of Soils." Ph.D. dissertation. Penn State Univ. (1947).
- 3. ALTEMÜLLER, H. J., "Mikroskop-ische Untersuchung Einiger Löss-Bodentypen mit Hilfe von Dünnschliffen." Zeit. für Pflanzener-nährung, Düngung, Bodenkunde, 72:2, 152-167 (1956).
- 4. ALTEMÜLLER, H. J., "Neue Möglichkeiten zur Herstellung von Bodendünnschliffen." Zeits. für Pflanzenernährung, Düngung, Bodenkunde, 72:1, 56-62 (1956).
- 5. ÅNGSTROM, A. J., "Neue Methode, das Wärmeleitungsvermögen der Körper zu Bestimmen." Ann. d. Physik u. Chemie., 114:513-21 (1861).
- 6. ÅNGSTROM, A. J., "Über das Wärmeleitungsvermögen des Kupfers und des Eisens bei Vershiedener Temperatur," Ann. d. Physik u. Chemie, 118:423-31 (1863).
- 7. ÅNGSTROM, A. J., "Nachtrag zu dem Aufsatz: Neue Methode das Wärmeleitungsvermögen der Kör-

Bestimmen." Ann. d. per zu Physik u. Chemie, 123:628-40 (1864).

- 8. A.S.T.M., "Thermal Conductivity of Materials by Means of the Guarded Hot Plate." ASTM C-177-45 (1955).
- 9. BATES, K. O., "Thermal Conductivity of Liquids." Ind. Eng. *Chem.*, 25:4, 431-7 (1933). 10. BATES, K. O., "Thermal Conduc-
- tivity of Liquids." Ind. Eng. Chem., 28:4, 494-8 (1936).
- 11. BAVER, L. D., "Factors Contributing to the Genesis of Soil Mi-crostructure." Proc. Am. Soil Survey Assn., 16:55-6 (1935).
- BAVER, L. D., "Soil Porosity as an Index of Soil Structure." Proc. Am. Soil Survey Assn., 14:83-5 (1933).
- 13. BAVER, L. D. "Soil Physics." John Wiley & Sons, New York (1948).
- 14. BEATTY, K. O., ARMSTRONG, A. A., and SCHOENBORN, E. M., "Thermal Conductivity of Homogeneous Materials." Ind. Eng. Chem., 42: 8, 1527-32 (1950). 15. Bior, J. B., "Traite de Physique."
- Vol. 4, p. 669 (1816).
- 16. BIRCH, F., AND CLARKE, H., "The Thermal Conductivity of Rocks and Its Dependence upon Temperature and Composition." Am. Jour. Sci., 238:529-58; 612-35 (1940).
- 17. BLACKWELL, J. H., "Radial-Axial Heat Flow in Region Bounded Internally by Circular Cylinders." Canad. Jour. Physics, 31:472-9 (1953).
- 18. BLACKWELL, J. H., "Transient Flow Methods for Determination of Thermal Constants of Insulating Materials in Bulk." Jour. Appl. Physics, 25:137-144 (1954).
- 19. BLACKWELL, J. H., AND MISENER, A. D., "Approximate Solution of a Transient Heat Flow Problem." Physical Soc. Proc., 64 A:1132-33 (1951).
- 20. Bosworth, R. C. L., "Adsorption of the Lower Fatty Acids." Trans. Faraday Soc., 28:903-912 (1932).

- BOSWORTH, R. C. L., "Heat Transfer Phenomena." John Wiley & Sons, New York (1952).
- BRACHT, J. Veröff., Geophys. Inst., Univ. Leipzig. Zweite Serie 14 Heft 3 (1949).
- 23. BRITISH AND ALLIED INDUS. RE-SEARCH ASSN., "Directions for the Determination of Thermal Resistivity of the Ground." Tech. Rep. F/s5, London (1937).
- BRONWELL, A., "Advanced Mathematics in Physics and Engineering." P. 263. McGraw-Hill, New York (1953).
- 25. BROOKES, A. S., AND STARRS, T. E., "Thermal and Mechanical Problems on 138-Kv Pipe-Type Cable in New Jersey." *AIEE Trans.*, Paper 57-652 (1957).
- Paper 57-652 (1957).
 26. BURGER, H. C., "Das Leitvermögen Verdünnter Mischkristallfreier Legierungen." Phys. Zeits., 20:73-76 (1919).
- 27. BURRELL, R. W., "Industry Turns to Soil Research." *Elec. World* (Apr. 2, 1956).
- CARSLAW, H. S., AND JAEGER, J. C., "Conduction of Heat in Solids." Oxford Press (1947).
- 29. "Chemical Engineer's Handbook." (Textbook Edition) McGraw-Hill, New York (1934).
- CHU, T. Y., DAVIDSON, D. T., AND WICKSTROM, A. E., "Permeability Tests for Sands." A.S.T.M. Spec. Tech. Publ. No. 163, p. 43-55 (1954).
- CZERATZKI, W., "Zur Wirkung des Frostes auf die Struktur des Bodens." Zeits. für Pflanzenernährung, Düngung, Bodenkunde, 72: 1, 15-32 (1956).
- 32. DE HAAS, W. J., AND BIERMASZ, T., "Die Wärmeleitfähigkeit von Kristallen bei Tiefen Temperaturen." *Physica*, 5:32-4 (1938).
- DE SENARMONT, . Annalen Chim. Phys. (1848, 1850, 1887).
- 34. DE SENARMONT, --- . Poggendorffs Annalen (1848, 1849, 1850).
- 35. DE SENARMONT, . Wiedemanns Annalen (1848, 1850).

- DESPRETZ, C. M., "Sur la Conductibilité de Plusiers Substances Solides." Ann. Chimie et Physique, 19:97-106 (1821).
- DESPRETZ, C. M., "Sur la Conductibilité des Principaux Métaux et de Quelques Substances Terreuses." Ann. Chimie et Physique, 36:422-26 (1827).
- 38. DESPRETZ, C. M., "Über des Wärmerleitungsvermögen der Hauptsachlichsten Metalle und Einiger Erdigen Substanzen." Ann. Physik und Chemie, 12:281 (1828).
- 39. D'EUSTACHIO, D., AND SCHREINER, R. E., "A Study of a Transient Heat Method of Measuring Thermal Conductivity." *Heating, Piping and Air Conditioning*, p. 113 (June 1952).
- 40. DE VRIES, D. A., "Het Wärmtegeleidingsvermögen von Grond." Doctoral Thesis. Rijksuniversiteit, Leiden (1952).
- DONAT, J., "Das Gefüge des Bodens und Dessen Kennzeichung." Trans. 6th Comm. Intern. Soc. Soil Science, Zürich, B:423-439 (1937).
- 42. EDLEFSEN, N. E., AND ANDERSON, A. B. C., "Thermodynamics of Soil Moisture." *Hilgardia*, 15:2, 31-298 (1943).
- EUCKEN, A., AND KUHN, G., "Ergebrisse Neuer Messungen der Wärmeleitfähigkeit Fester Krystallisierter Stoffe bei 0° und -190° C." Zeits. Phys. Chemie, 134:193-219 (1928).
- 44. EUCKEN, A. Forschungsheft No. 353, Verein Deutscher Ingenieure, Berlin (1932).
- FINK, L. H., "Control of the Thermal Environment of Buried Cable Systems." *AIEE Trans.*, 73:405-10 (1954).
- 46. FORBES, J. D., "Experimental Inquiry into the Laws of the Conduction of Heat in Bars and into the Conducting Power of Wrought Iron." Trans. Royal Soc. of Edinburgh, 23:133-46 (1864).
- 47. FORBES, J. D., "Experimental Inquiry into the Laws of Conduc-

tion of Heat in Bars. Part II. On the Conductivity of Wrought Iron, Deduced from the Experiments of 1851." Trans. Royal Soc. of Edinburah, 24:73 (1867).

- FOURIER, J. B. J., "Analytic Theory of Heat." University Press, Cambridge, England (1878).
- GEORGE, N. C., "A Note on the Pressure Plate-Membrane Apparatus." Soil Sci., 74:4, 315-322 (1952).
- HALLAIRE, M., "Soil Water Movement in the Vapor Phase Under the Influence of Evapotranspiration." HRB Special Rep. 40 (1958).
- 51. HONDA, K., AND SIMUDU, T., Science Reports, Tohoku Univ. (1) 6:219 (1917).
- 52. HOOPER, F. C., AND CHANG, S. C., "Development of the Thermal Conductivity Probe." *Heating*, *Piping and Air Conditioning*, p. 125-130 (Oct. 1952).
- 53. HOOPER, F. C., AND LEPPER, F. R., "Transient Heat Flow Apparatus for the Determination of Thermal Conductivities." *Heating, Piping* and Air Conditioning, p. 129-34 (Aug. 1950).
- 54. INGENHOUSS, I., "Sur les Métaux comme Conducteurs de la Chaleur." Jour. de Phys., 34:68, 380 (1789).
- 55. INGERSOLL, L. R., ZOBEL, O. J., AND INGERSOLL, A. C., "Heat Conduction." McGraw-Hill, New York (1948).
- 56. JAKOB, M., "Heat Transfer." Vol. 1. John Wiley & Sons, New York (1949).
- 57. JAKOB, M., "Wärmeleitung." Handbuch der Physik. Band XI (1926).
- JUNG, E., "Untersuchungen über die Einwirkung des Frostes auf den Erdboden." Kolloidchem. Beihefte, 32:320-73 (1931).
- 59. KENNELLY, A. E., "On the Carrying Capacity of Electric Cables Submerged, Buried or Suspended in Air. Minutes, 9th Ann. Mtg., Assn. of Edison Illum. Cos., New York (1893).

- KERSTEN, M. S., "Thermal Properties of Soils." Bull. No. 28, Eng. Exp. Station, University of Minnesota (1949).
- KEYES, F. G., "Experimental Determination of Heat Conductivity for Gases." Trans. Am. Soc. Mech. Eng., 71:939 (1949).
- KING, R. W., "A Method of Measuring Heat Conductivities." Phys. Rev. (2), 6:437-45 (1915).
- 63. KOLYASEV, F. E., AND GUPALO, A. I., "On the Correlation of Heat and Water Properties of Soils." International Symposium on Water and Its Conduction in Soil. HRB Special Rep. 40 (1958).
- 64. KUBIENA, W. L., "Micropedology." Collegiate Press, Ames, Iowa (1938).
- 65. KUMMER, F. A., AND COOPER, A. W., "Soil Porosity Determinations with Air Pressure Pycnometer as Compared with Tension Method." Agr. Eng., 26:21-23 (1945).
- 66. KUNDT, A., AND WARBURG, E., "Über Reibung und Wärmeleitung Verdünnter Gase." Ann. der Physik (Poggendorffs Ann.), 155:337, 525 (1875).
- LEAMER, R. W., AND SHAW, B. T., "A Simple Apparatus for Measuring Non-Capillary Porosity on an Extensive Scale." Am. Soc. Agron., 33:1003-8 (1941).
- MAKOWSKI, M. W., AND MOCHLIN-SKI, K., "An Evaluation of Two Rapid Methods of Assessing the Thermal Resistivity of Soil." *Proc. Inst. Elec. Eng.*, 103: Part A (1956).
- MASON, V. V., AND KURTZ, M., "Rapid Measurement of the Thermal Resistivity of Soil." Trans. Amer. Inst. Elec. Eng., Tech. Paper 52-158, 71:570-7 (1952).
- MAXWELL, G., "Treatise on Electricity and Magnetism." P. 365 Oxford Press (1873).
- 71. McAdams, W. H., "Heat Transmission." McGraw-Hill, New York (1933).

- 72. NEHER, J. H., "The Temperature Rise of Buried Cables and Pipes." Trans. AIEE, 68:9-17 (1949).
- NEHER, J. H.; "The Temperature Rise of Cables in a Duct Bank." Trans. AIEE, 68:540-6 (1949).
- 74. NEHER, J. H., AND MCGRATH, M. H., "The Calculation of the Temperature Rise and Load Capability of Cable Systems." Trans. AIEE, Paper 57-660, General Meeting (June 24-28) (1957).
- 75. NEUMANN, F., "Expériences sur la Conductibilité Calorifique des Solides." Ann. Chimie et Physik (3), 66:181-7 (1862).
- NUSSELT, W., "Forschungsarbeiten auf dem Gebiete des Ingenieruswesens." No. 63, 64 (1909).
- 77. PAGE, J. B., "Advantages of the Pressure Pycnometer for Measuring the Pore Space in Soils." Proc. Soil Soc. of Amer., 12:81-84 (1947).
- PATTEN, H. E., "Heat Transference in Soils." Bull. 59, U. S. Dept. of Agriculture, Bureau of Soils (1909).
- 79. PELE, T. C., BEALE, O. W., AND LESENNE, F. F., "Irrigation Requirements of South Carolina Soils." Agr. Eng., 29:157-161 (1948).
- REITMEIER, R. F., AND RICHARDS, L. A., "Reliability of the Pressure-Membrane Methods of Extraction of Soil Solution." Soil Sci., 57:119-136 (1943).
- 81. RICHARDS, L. A., "Porous Plate Apparatus for Measuring Moisture Retention and Transmission by Soil." Soil Sci., 66:105-110 (1948).
- RICHARDS, L. A., "Pressure-Membrane Apparatus Construction and Use." Agr. Eng., 28:441-455, 460 (1947).
- 83. RICHARDS, L. A., "Report of the Subcommittee on Permeability and Infiltration." Comm. on Terminology. Proc. Soil Sci. Soc. Amer., 16:85-8 (1952).
- 84. RICHARDS, L. A., AND WEAVER, L. R., "Moisture Retention by Some

Irrigated Soils as Related to Soil Moisture Tension." *Jour. Agr. Res.*, 69:215-235 (1944).

- 85. Roy, S. E., AND WINTERKORN, H. F., "Scintillation Methods for the Determination of Density and Moisture Content of Soils and Similar Granular Systems." HRB Bull, 159, p. 58 (1957).
- RUSSELL, M. B., "A Simplified Air-Pycnometer for Field Use." Proc. Soil Sci. Soc. Amer., 14:73-76 (1949).
- SCHOLTE, T. G., "A Contribution to the Theory of the Dielectic Constant of Polar Liquids." *Physica*, 15:437 (1942).
- ica, 15:437 (1942).
 88. SMITH, G. S., AND YAMAUCHI, T., "Thermal Conductivity of Soils for Design of Heat Pump Installations." *Heating, Piping and Air-Conditioning*, p. 129-35 (July 1950).
- SMITH, W. O., "Thermal Conductivities in Moist Soils." Proc. Soil Sci. Soc. Amer., 4:32-40 (1939).
- SMITH, W. O., "The Thermal Conductivity of Dry Soil." Soil Sci., 53:435-59 (1942).
- 91. SMITH, W. O., AND STALLMANN, R. W., "Measurement of Permeabilities in Ground-Water Investigations." ASTM Spec. Tech. Publ. No. 163, p. 98-122 (1954).
- Publ. No. 163, p. 98-122 (1954).
 92. SPANGLER, M. G., "Soil Engineering." International Textbook Co., Scranton, Pa. (1951).
- 93. STÅLHANE, B., AND PYK, S., "A New Method for Determining the Coefficients of Thermal Conductivity." Teknisk Tidskrift, 61: 383-93 (1931).
- 94. STRATTON, J. A., "Electromagnetic Theory." McGraw-Hill, New York (1941).
- 95. TANASAWA, Y. Trans. Soc. Mech. Eng. Japan, 1:217 (1935).
- 96. THELEN, P., "The Differential Thermal Conductivities of Certain Schists." Bull. of Dept. of Geology, Univ. of Calif., 4:11, 201-226 (1904-1906).
- 97. THORNTHWAITE, C. W., AND MATHER, J. R., "The Water Bal-

ance." The Laboratory of Climatology, Publications in Climatology, Vol. 8, No. 1 (1955).

- TORSTENSSON, G., AND ERICKSON, S., "A New Method for Deter-98. mining the Porosity of the Soil." Soil Sci., 42:405-17 (1936).
- 99. TSCHEBOTARIOFF, G. P., "Soil Mechanics, Foundations and Earth Structures." McGraw-Hill, New York (1951).
- 100. VAN DER HELD, E. F. M., HARDEBOL, J., AND KALSHOVEN, J., "On the Measurement of the Thermal Conductivity of Liquids by a Non-Stationary Method." Phusica, 19:208-16 (1953).
- 101. VAN DER HELD, E. F. M., AND VAN DRUNEN, F. G., "The Measurement of the Thermal Conductivity of Liquids Based on a Warming-Up Process." Proc. VII International Congress for Applied Mechanics, London, Section III, Paper 8 (1948).
- 102. VAN ROOYEN, M., AND WINTERKORN, H. F., "Theoretical and Practical Aspects of the Thermal Conductivity of Soils and Similar Granular Šystems." HRB Bull. 168, p.
- 143 (1957). 102a. VAN ROOYEN, M., Ph.D. Thesis, Princeton University, v. 958.
- 103. VISSER, W. C., "Pore Space Deter-mination as a Field Method." Soil Sci., 44:478-9 (1937).
- 104. VOIGT, W., "Eine Neue Methode zur Untersuchung der Wärmelei-tung in Krystallen." Ann. d. *Physik.*, 60:350-67 (1897). 105. VOIGT, W., "Bestimmung Relati-
- ver Wärmeleitfähigkeiten nach

der Isothermenmethode." Ann. d. Physik., 64:95-100 (1898).

- 106. VON NITZCH, W., "Der Porengehalt des Ackerbodens: Messverfahren und Ihre Brauchbarkeit." Zeits. für Pflanzenernährung, Düngung, Bodenkunde, 1:110-115 (1931).
- 107. WAIDELICH. W. C., "Physico-Chemical Factors Influencing the Consolidation of Soils." HRB Special Report No. 40 (1958).
- 108. WINTERKORN, H. F., "Tentative Method of Test for Thermal Re-sistivity of Soil by the Thermal Probe." AIEE Research Project on Soil Thermal Resistivity, First Ann. Rep., App. 3 (July 1956); ASTM Soil Testing Methods (1958).
- 109. WINTERKORN, H. F., "The Soil Environment." AIEE Research Project on Soil Thermal Resistivity, First Semi-Annual Report (Feb. 1956).
- WINTERKORN, H.F., "Water Move-ment Through Porous Hydro-110. philic Systems Under Capillary, Electrical and Thermal Potentials." ASTM Spec. Tech. Publ. No. 163, p. 27-35 (1954).
- 111. WINTERKORN, H. F., AND TSCHEBO-TARIOFF, G. P., "Sensitivity of Clays to Remolding and Its Possible Causes." Proc. HRB, 27: 435-43 (1947).
- 112. WINTERKORN, H. F., "Soil Water Interactions and Water Conduction in Soils." P. 43-83. Paper delivered at Princeton Univ. Conf. on Water Supply, (Jan. 29-30, 1958).