# Application of Digital Simulation Techniques to Freeway On-Ramp Traffic Operations

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This paper reports a study on a digital computer application to the problem of freeway on-ramp operations, giving design answers to that knotty problem at a small fraction of the cost involved in actual construction methods. With the techniques described it is possible to determine the effects of changes in traffic volume, velocity, geometric design, etc.

• HIGHWAY TRAFFIC is composed of a multitude of individual drivers, each free to choose his speed and course, but for analysis it is often treated by engineeers as a homogeneous entity. By cataloging the highway and driver characteristics, the engineer attempts to predict the traffic flow for a given set of conditions. The accuracy of the predictions is tested by the construction of roadway networks. Presently, cases occur where the flow is smooth and free; that is, the accuracy was sufficient. If major congestion occurs, the poor predictions result in expensive modification requirements.

A number of attempts have been made to alleviate this design problem using a variety of mathematical models and/or simulation schemes. These attempts have met with varying degrees of success owing to either oversimplification of the real process in order that the model may be handled mathematically or a deficiency or want of data from the observed traffic process.

Driver behavior at access areas greatly affects the efficiency of high-speed freeways. As such, a simulation model has been devised for use by highway design engineers to determine ramp and acceleration area configurations for given traffic conditions. The basis for the simulation is the statistical analysis of data from a number of interchange locations which describe flow and driver behavior in the merging process.

Through use of Monte Carlo techniques and a general purpose digital computer, each vehicle in the portion of roadway under study is allowed to maneuver through the model access area with the same freedom of decision as do their real-life counterparts. The effects of changes in traffic volume, velocity, geometric design, etc., may be determined by noting changes in such measures of effectiveness as traverse time and waiting time on the ramp.

The computer thus represents the real situation at a fraction of the expense which would be incurred by sampling the actual traffic process. Furthermore, new ramp designs and traffic control devices may be evaluated without their actual construction.

Two sets of test runs on an IBM 709 computer show the effects that changes in the length of acceleration lane have on traffic flow. Although the test conditions for the simulation had low input volumes and did not create merging difficulty, the results are indicative as to how the merging complex is simulated and analyzed.

Under the given traffic conditions, extending the acceleration lane from 595 ft to 765 ft reduced the delay per 1,000 ramp vehicles from 26 sec to 16 sec. This magnitude of delay is insignificant, but it does indicate an advantage of the longer lane at high traffic volumes. Traverse times for both through and ramp vehicles are increased by the shorter acceleration lane.

Thus far, the effects of several design and behavioral factors have not been considered. The continuing research program includes study of the effects of elevated ramps, roadway grade, commercial traffic, and signing procedures. It is plausible, however, that some of these or other factors have negligible effects on traffic flow. In that case, inclusion of such factors in a simulation model only for realism would necessitate additional computer storage and would contribute little to analysis of interchange design.

This investigation shows that simulation methods can aid the design engineer by supplying information on added service to the driver by length of on-ramp, etc., and thereby allow him to weigh these factors in determining the most favorable design for given traffic conditions.

## LITERATURE SEARCH

In an extensive literature search it soon became apparent that, in most cases, the descriptive theories concerning vehicular traffic were inadequate or restricted to a very limited situation.

In an attempt to classify these theories, Haight (*14*) proposed three basic types. The first, an analytic and deterministic model, considered the characteristics of the vehicle and assumed driver behavior. This type of model is characterized by the work of Pipes (*24*), Chandler, Herman, and Montroll (*6*), and Herman, Montroll, Potts and Rothery (*16*).

A second class of solutions involved queue theory treatments of a stochastic model. Queue theory necessitates that all vehicles enter at one point, a major simplification of the problem. However, in very restrictive situations the theory of queues is beneficial. Reasonable results may be obtained when traffic is actually queued, velocity is uniform, and the driver has few free decisions, as typified by Tanner (*26*) and Newell (*22*).

A third approach, which describes traffic flow in a continuum, has been treated by Newell (*21*), Newell and Bick (*22*), Tse-Sun Chow (*28*), Lighthill and Whitham (*20*), and Greenberg (*12*). The individual vehicles are treated analogously to molecules of a semicompressible fluid; traffic flow must obey appropriate differential equations of fluid flow.

A fourth method of solution is the Monte Carlo integration or simulation model and has been neglected by Haight. Although the previous three methods have application in very restrictive situations, the simulation method is completely general. Simulation has been defined by Harling (*15*) as "the technique of setting up a stochastic model of a real system which neither oversimplifies the system to the point where the model becomes trivial, nor incorporates so many features of the real system that the model becomes untractable or prohibitively clumsy."

Highway traffic simulation has been investigated by Wong (*30*), Trautman *et al.* (*27*), and Goode (*11*) in recent years. In each of these, oversimplification (generally no passing and fixed velocity) resulted in nonconformance with the environment they attempted to simulate. The present simulation was preceded by a thorough analysis of observed traffic data in order to understand and simulate the real traffic complex.

## ANALYSIS OF DATA

Data gathered at four Chicago interchanges by the Bureau of Public Roads provided the basic information on which this simulation model was constructed. Complementary information was acquired from the Bureau of Public Roads (*4, 5, 17*), Yale University (*3*), AASHO (*1*), and the ENO Foundation (*8*), for example. The Chicago data were coded and punched into 20,000 IBM cards, each card representing a vehicle crossing one of the four recording mechanisms at each site (Fig. 1). A card contains the location, type, velocity and lateral placement of a vehicle, the time (to the nearest 0.0001 of an hour) that the vehicle passed the recorder, and if the vehicle made an unusual maneuver, the maneuver was coded and punched into the card. Time gaps, the time distance between

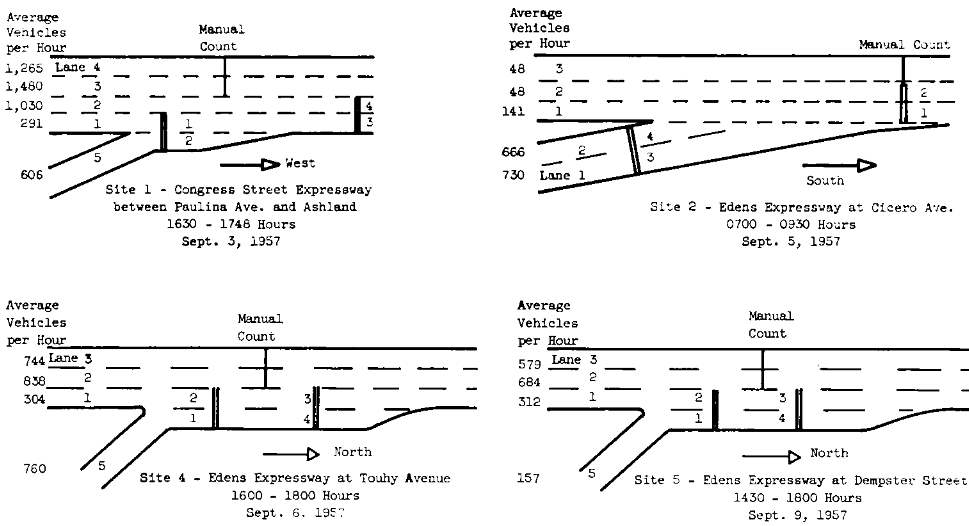



**Figure 1. Data collection sites, Chicago, Ill., September 1957.**

successive vehicles, were calculated and punched into the card representing the trail vehicle.

The primary unit of measure in the study of merging vehicles is the time gap. For the purposes of this investigation, a gap is defined as the time space between the front axle of a vehicle and the front axle of the succeeding vehicle. In previous literature (*3, 4*) the occurrence or arrival distribution of gaps was found to be exponential and is given by

$$P(t) = 1 - \exp(-Vt/3{,}600)$$

for

$$t \leq 1 \text{ and } V \leq 275$$

and where

$P(t)$ = percent of gaps equal to or less than $t$ sec;
$t$ = length of time gap, in sec;
$V$ = number of vehicles per hour through a lane;
3,600 = empirically derived constant (sec in an hour).

A chi-square goodness-of-fit test (*31*) was applied to determine how well gap size in the Chicago data agreed with this exponential distribution. In this test for significance there were 65 degrees of freedom; therefore, the expression $\sqrt{2\chi^2} - \sqrt{2n'} - 1$ was used as a normal deviate with unit variance where $n'$ is the number of degrees of freedom (*32*). For the Chicago data the standard normal variate is $t = -3.95$, which infers an extremely close agreement between the data and the exponential distribution. If the gaps were not distributed exponentially, this close agreement would occur with a probability of 4/1,000,000. It may be stated with a high degree of confidence that $1 - \exp(-Vt/3{,}600)$ describes the probability of the arrival of a gap equal to or less than $t$ sec (Fig. 2).

Contingency tables (*31*) and the chi-square test for statistical independence (*31*) were used to evaluate the randomness of successive gaps between vehicles traveling at approximately the same speed. The test results clearly indicated no dependence between gap $n$ and gap $n + 1$ in the same traffic lane. These same tests were performed to test the dependency between a gap and the velocity of the lead vehicle and also between a gap and the velocity of the trail vehicle. At hourly volumes of 950 and 1,400 vehicles, in neither of these tests was there any indication of dependence between the variables.

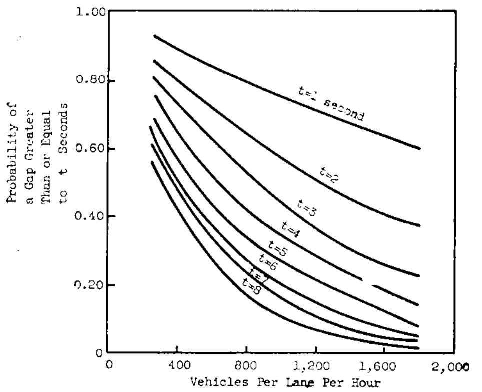



**Figure 2. Probability of a time gap of a given length or longer for various hourly traffic volumes.**

The distribution of gaps that is acceptable to the merging driver is not well defined. The Chicago data were not coded for this purpose; however, the results of a small sample taken by the Yale University Traffic Bureau (*3*) are reasonable and coincide with inferences that may be drawn from the Chicago data. Until further study is made, the distribution of gap acceptance will be assumed as follows (Fig. 3):

1. If merging vehicles are forced to stop before merging, the probability that a driver will accept a gap of $t$ sec is

$$P_A = 0.333\ (t - 2.5) \qquad 2.5 \leq t \leq 4$$
$$= 0.125\ t \qquad 4 \leq t \leq 8$$

2. When the merging vehicle is traveling 15 to 30 mph, the probability of acceptance is

$$P_A = 1.00\ (t - 1.5) \qquad 1.5 \leq t \leq 2$$
$$= 0.25\ t \qquad 2 \leq t \leq 4$$

3. If, however, the merging stream is able to travel parallel to the main stream at 15 to 30 mph

$$P_A = 0.50\ (t - 1) \qquad 1.5 \leq t \leq 2$$
$$= 0.25\ t \qquad 2 \leq t \leq 4$$

The tails of the distribution of acceptance will be neglected until the whole distribution is more precisely defined. No measure of driver impatience will be included in the model at this time.

The distributions of vehicle velocities (Fig. 4) are fairly normally distributed and rather peaked; that is, with the exception of site 2, where the volume on the ramp was very heavy and through volume was extremely light. The mean velocity $\bar{v}$ decreases as traffic volume increases; the standard deviation $\sigma$ of the velocity distributions also decreases linearly with increases in volume. The coefficient of variation $\sigma/v$ decreases along a mild hyperbolic arc (*7*). In the Chicago study data, the coefficient of variation is 0.3 for the acceleration lane at site 1 and 0.2 for the same lane at sites 4 and 5. The coefficients for lanes 1 and 2 are slightly greater than 0.3 because of the negatively skewed distributions. The volume of traffic during the time of study never exceeded 1,200 vehicles per lane per hour. Because of the driver's reduced freedom to travel at his desired velocity in high volume conditions, the coefficient of variation is reduced to approximately 0.1 when the traffic flow approaches capacity.

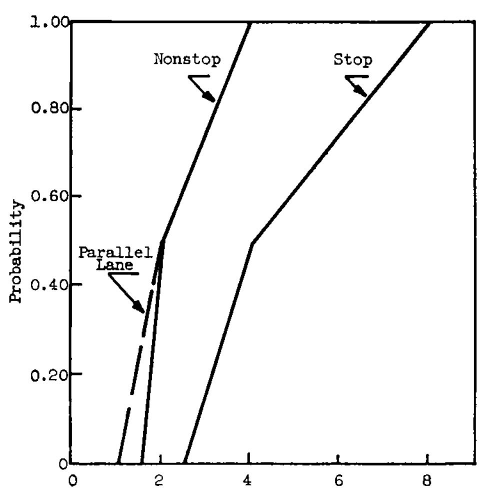



**Figure 3. Probability of a driver accepting a gap of t seconds.**

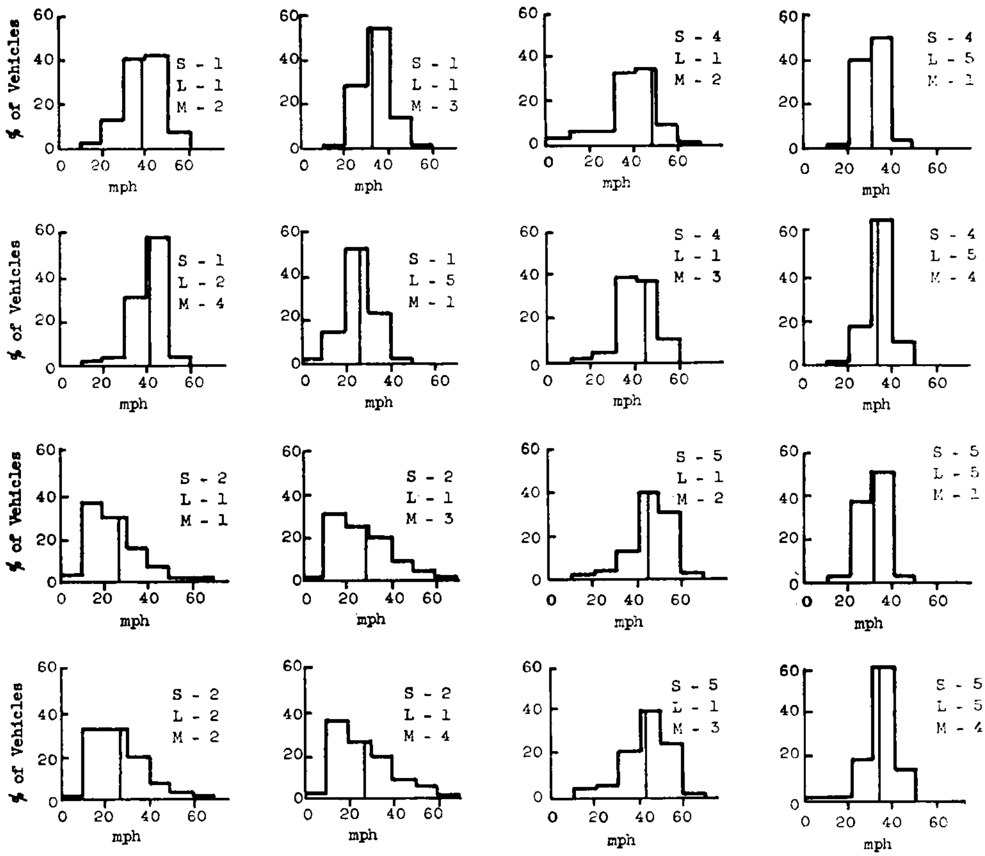


**Figure 4. Distribution of vehicle velocities at site S, lane L, and machine M.**

Velocities in the acceleration lanes were tested for correlations to the volumes and velocities in lane 1. No significant dependence was found between the volume in lane 1 and the velocities on the ramp, or between the velocities in lane 1 and the velocities on the ramp. Furthermore, changes of volume on the ramp caused no correlated change of velocity in lane 1. As shown in Figure 1, volumes in lane 1 were generally rather light. At higher volumes the dependence is obvious.

The distribution of the total traffic volume to the lanes of an expressway is dependent on the total volume. This distribution is also dependent on roadway design criteria; for example, the proximity of on- or off-ramps. Third-degree least squares curves were fitted (Fig. 5) to lane utilization data from the Chicago study and from a study of California freeways (*17,* p. 600). These three curves express the fraction of the total traffic volume in each of the three lanes of a freeway as the traffic approaches an on-ramp on the right. These data were collected under heterogeneous conditions: varying ramp volumes, physical design, signing procedures, etc. These conditions are probably the sources of variation seen in the scattered data points. The plotted points to the left of the broken line are from the Chicago data, the points to the right are from the California studies. Although the data are from two sources, the fitted curves are fairly smooth. The curves are described by:

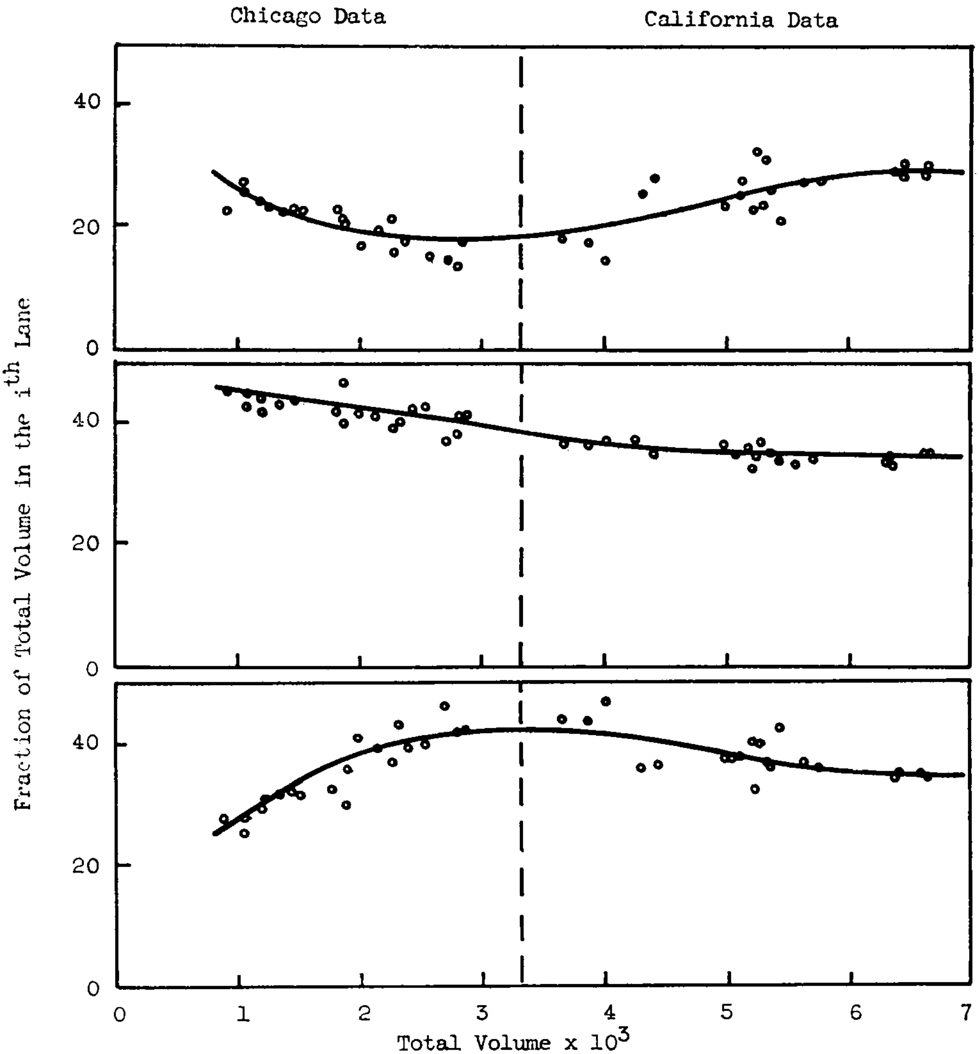



**Figure 5. Distribution of traffic volume across three lanes in one direction at approach to ramp on right.**

$$P_1 = 0.43693 - 0.22183\alpha + 0.05730\alpha^2 - 0.00406\alpha^3 \qquad 1 \leq \alpha \leq 7$$

$$P_2 = 0.48820 - 0.03136\alpha + 0.00006\alpha^2 + 0.00024\alpha^3$$

$$P_3 = 0.07487 + 0.25319\alpha - 0.05736\alpha^2 + 0.00382\alpha^3$$

where $P_i$ is the proportion of total volume in the $i$th lane, $\alpha$ is the total freeway volume in thousands of vehicles per hour, and $\Sigma P_i = 1$.

## CONSTRUCTION OF SIMULATION MODEL

The first step in simulating the merging process is the reduction of the data analysis to logical flow diagrams. Particular care is necessary to prevent oversimulating the real process; that is, it

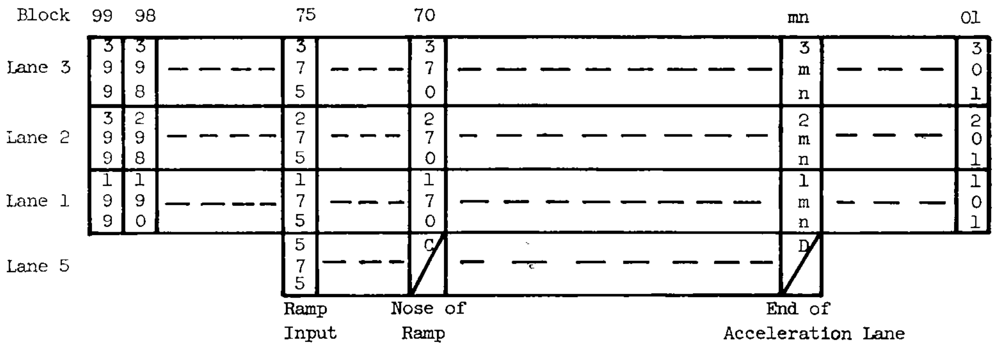


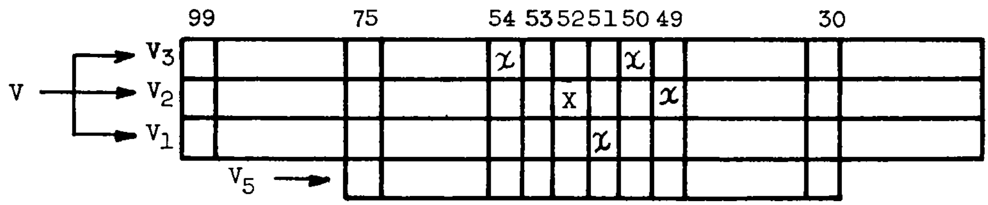

**Figure 6. Study area matrix for computer simulation at an interchange access area.**

would be unwise to include factors which contribute little to the experiment just for the sake of realism. Only those factors which have been analyzed and appear to have an effect on this process are included in the model.

The study area is set up in a 4 x 100 matrix (Fig. 6), the four rows representing the three through lanes, 1, 2 and 3, and the ramp and acceleration lane, 5. The 100 columns are blocks of 17-ft length, approximately the length of an automobile. The acceleration lane is of variable length; point D may be changed by changing its block number. Point C is stationary, so that the initial 30 blocks on the through lanes are a zone of stabilization, as are the initial 5 ramp blocks.

Each second of real time, each block in the study area is inspected for occupancy. If the block is occupied the vehicle is allowed forward movement and other possible maneuvers. After the entire area is inspected, each of the input locations is evaluated. Inspection starts at block 01, lanes 3, 2, 1 and 5, and then by increasing block number. Vehicles enter the system in blocks 399, 299, 199 and 575 and leave at 301, 201, and 101.

The following parameters are used in the flow diagrams (Figs. 7, 8, 9, 10):




Let X = vehicle under inspection

and X = other occupied blocks

Then

$B_n$ = 252  v = velocity of vehicle in 252

$_2B_{n-1}$ = 249  $_3v_n$ = velocity of vehicle in 354

$_1B_{n-1}$ = 151  C = 575

$_3B_{n-1}$ = 350  D = 530

$_3B_n$ = 354

**Figure 7. Example of symbols used in flow diagrams.**

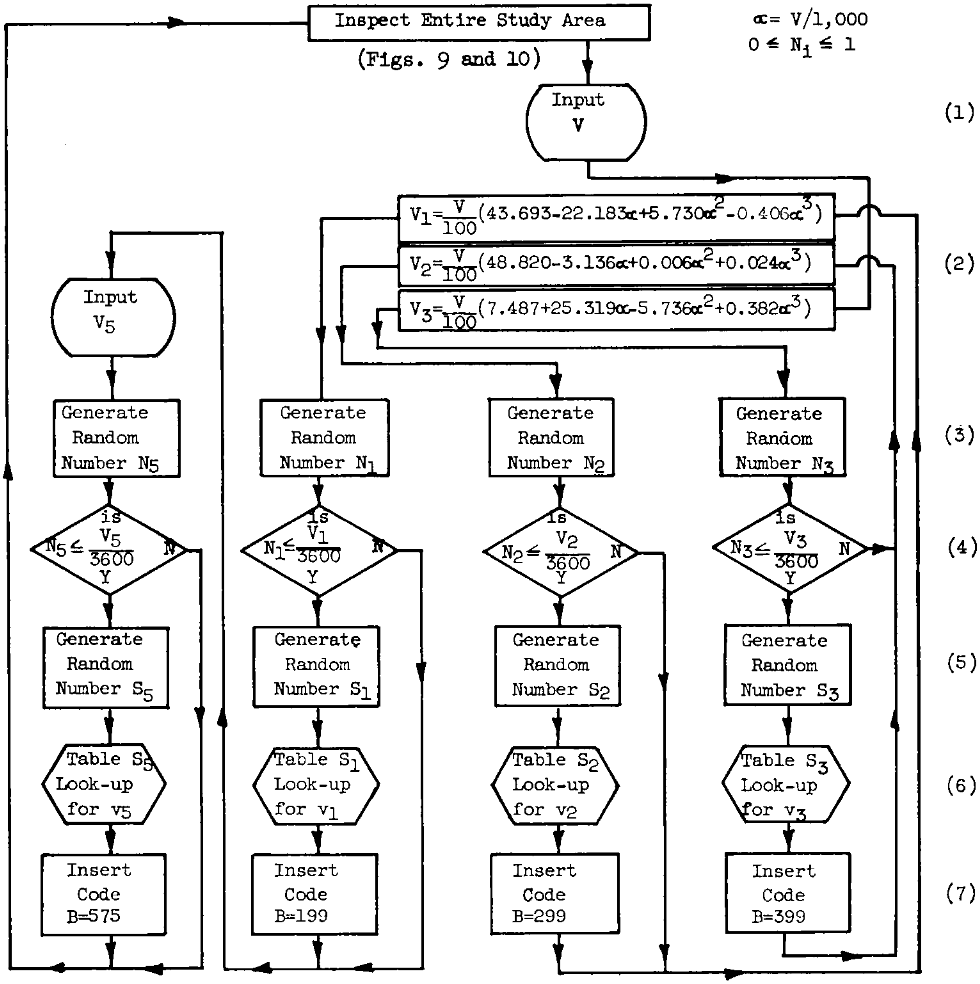



**Figure 8. Flow diagram, input routine.**

$V$ = total volume in lanes 1, 2, and 3, vehicles per hour;

$V_i$ = vehicles per hour in the $i$th lane,

$$\sum_{i=1}^{3} V_i = V;$$

$\alpha = V/1{,}000$;

$B$ = block number under inspection;

$B_n$ = block number of vehicle under inspection;

${}_iB_{n-1}$ = block number of vehicle in $i$th lane preceding vehicle under inspection;

${}_iB_n$ = block number of vehicle in $i$th lane parallel to or behind vehicle under inspection;

$v$ = velocity of vehicle under inspection;

${}_iv_n$ = velocity of vehicle in $i$th lane, parallel to or behind vehicle under inspection;

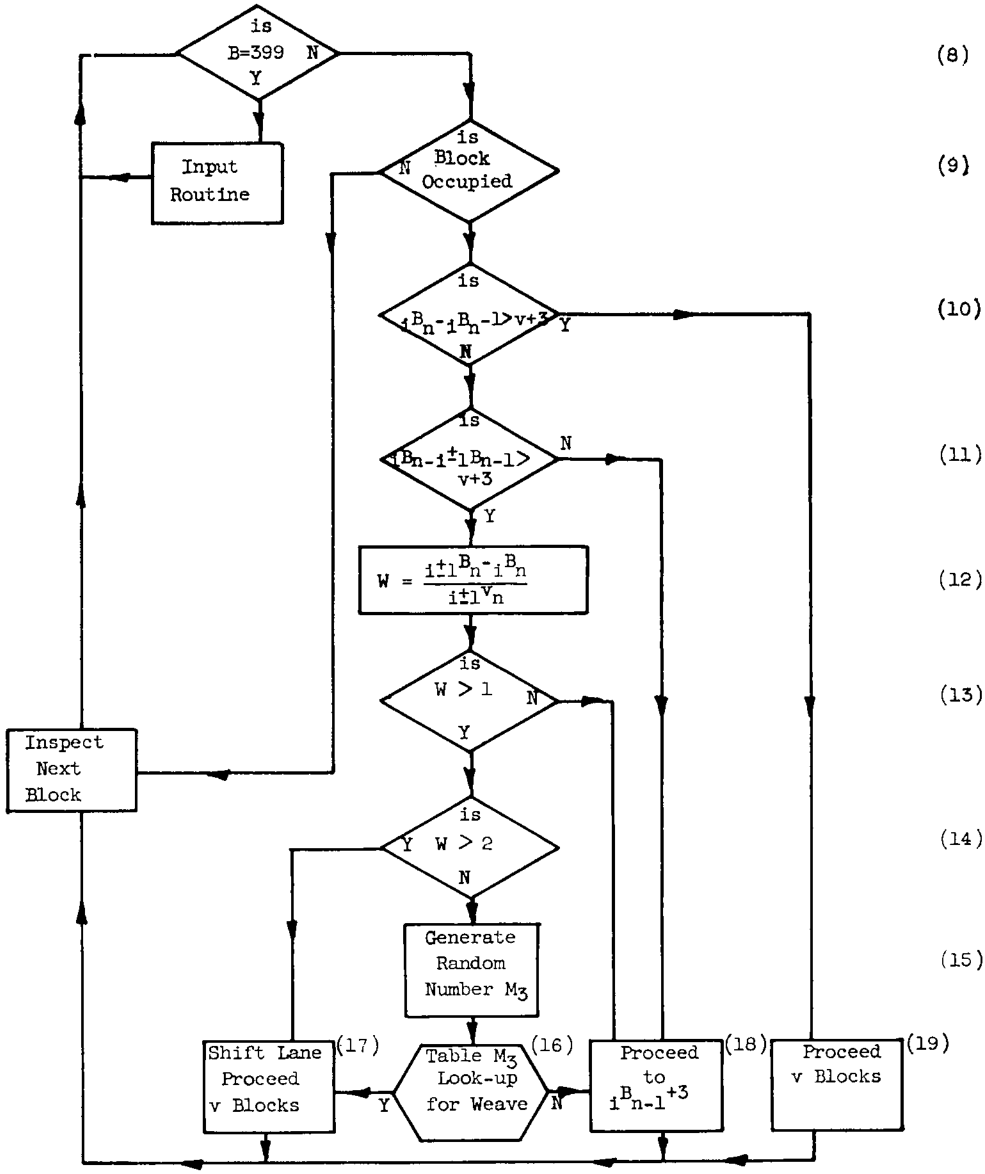



**Figure 9. Flow diagram, inspection routine lanes 1, 2, 3.**

$C$ = block number at nose of ramp; and

$D$ = block number at end of acceleration lane.

The flow diagrams are "talked through" for further explanation in Appendix A. An example of the use of the foregoing symbols is given in Figure 7.

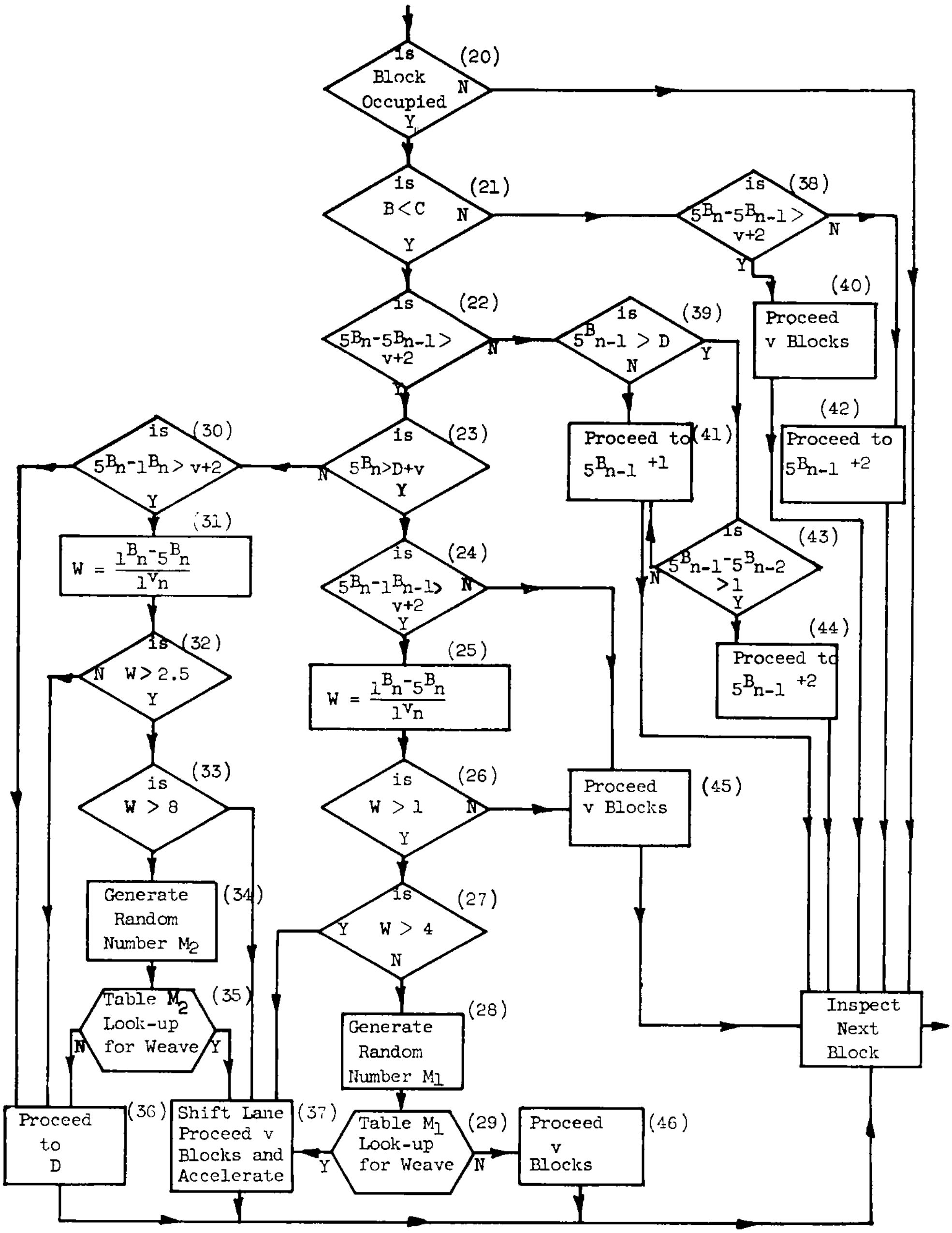



**Figure 10. Flow diagram, inspection routine lane 5.**

COMPUTER PROGRAM AND TEST RUNS

The simulation model as described in the flow diagrams was programmed for the IBM 709 digital computer.

The balance of machine time left after "debugging" the program was used for a controlled experiment. Five trial runs were made to check the repeatability of the measures of effectiveness and to test the effect of a change of length of acceleration lane. Each trial run was 200 sec of real time with an initial 100 real seconds used to fill the system and allow for stabilization. The ratio of real time to machine time was approximately 10 to 1.

Three trials were run with an acceleration lane of 595 ft and two runs at 765 ft. The hourly input volumes on the freeway and ramp were held constant at 3,600 and 470 vph, respectively. The distributions of velocities were as shown in Table 1, which correspond to step (6)

TABLE 1

VELOCITY DISTRIBUTIONS FOR TRIAL RUNS

| Velocity | | Distribution, % | | |
|---|---|---|---|---|
| MPH | Blocks/ Sec. | Lane 5 | Lane 1 | Lanes 2 and 3 |
| 18 | 1.55 | 01–02 | — | — |
| 23 | 2.00 | 03–09 | 01–02 | — |
| 28 | 2.40 | 10–27 | 03–09 | 01–02 |
| 33 | 2.85 | 28–49 | 10–27 | 03–07 |
| 38 | 3.30 | 50–71 | 28–49 | 08–17 |
| 43 | 3.70 | 72–89 | 50–71 | 18–33 |
| 48 | 4.15 | 90–96 | 72–89 | 34–49 |
| 53 | 4.60 | 97–98 | 90–96 | 50–65 |
| 58 | 5.00 | 99 | 97–98 | 66–81 |
| 63 | 5.45 | — | 99 | 82–91 |
| 68 | 5.90 | — | — | 92–96 |
| 73 | 6.30 | — | — | 97–98 |
| 78 | 6.75 | — | — | 99 |

Tables $S_i$ in the input routine flow diagram (Fig. 8). When a two-digit random number is generated in step (5) of that routine, this number is located in the proper column of Table 1 and the corresponding velocity is assigned to the vehicle as it enters the system.

Table 2 is the distribution of gap acceptance for the various lanes corresponding to Tables $M_i$. When a two-digit random number $M_i$ is generated in step (15), (28), or (35), the computer looks in the tables for $W$ as calculated in the previous step. If random number $M_i$ is less than the tabled value, the vehicle under inspection is allowed to merge.

TABLE 2

GAP ACCEPTANCE DISTRIBUTIONS FOR TRIAL RUNS

| W | Lane 5, Stop Sequence | W | Lane 5, Nonstop Sequence | W | Lanes 1, 2, 3, Weave Sequence |
|---|---|---|---|---|---|
| 0 to 1 | — | 0.0–0.5 | — | 0.00–0.25 | — |
| 1 to 2 | — | 0.6–1.0 | — | 0.26–0.50 | — |
| 2 to 3 | −04 | 1.1–1.5 | −10 | 0.51–0.75 | — |
| 3 to 4 | −32 | 1.6–2.0 | −38 | 0.76–1.00 | — |
| 4 to 5 | −55 | 2.1–2.5 | −56 | 1.01–1.25 | −10 |
| 5 to 6 | −69 | 2.6–3.0 | −69 | 1.26–1.50 | −30 |
| 6 to 7 | −81 | 3.1–3.5 | −81 | 1.51–1.75 | −60 |
| 7 to 8 | −94 | 3.6–4.0 | −94 | 1.76–2.00 | −99 |

The only other input information were rules for acceleration by merging vehicles. Ramp vehicles in the nonstop sequence were accelerated 0.2 block per second for 4 sec and the vehicles in the stop sequence accelerated 0.3 block per second for 10 sec. Acceleration began as a vehicle found an acceptable gap and was additive to the vehicle's desired ramp speed in the first case or from one-half its desired velocity in the second.

The distributions of traverse times for through vehicles are practically undisturbed by the extension of the acceleration lane (Fig. 11), although the mean

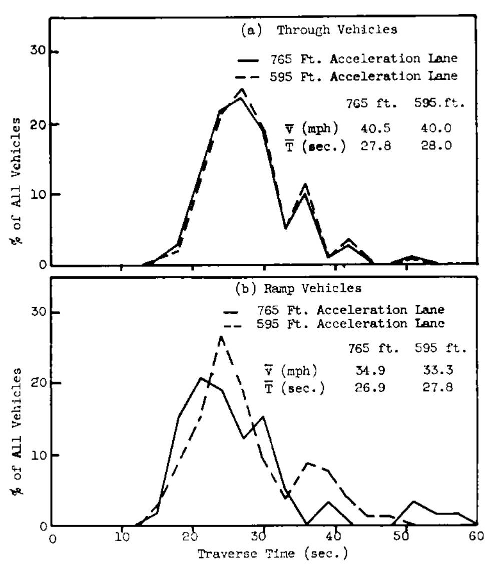



**Figure 11. Distribution of traverse time.**

traverse time and velocity output are markedly different than the input. Table 3 shows the mean output values for which the distributions are plotted in Fig. 11. The 3-sec periodicity, which is obvious in the distribution of traverse times as tabulated by the computer, is due to the incremental velocity input. If, for example, $v = 2.85$ blocks per sec, then traverse time, distance $/v$ or 99/2.85, is 34.7 sec, which in discrete units accounts for the peak at 35-sec traverse in Fig. 11. The plotted points are grouped in 3-sec intervals, which smooth the curve somewhat. The same periodicity occurs in the plot of ramp traverse time. Figure 12 is a plot of the number of vehicles waiting to merge at the end of the acceleration lane during each second of real time.

TABLE 3

CHANGES IN MEAN VALUES OF VELOCITY AND TRAVERSE TIME FOR VEHICLE IN SIMULATION SYSTEM DUE TO EXTENSION OF ACCELERATION LANE FROM 595 TO 765 FEET

| Traffic | Accel. Lane (ft) | $\bar{v}$(mph) | | $\bar{T}$(sec)[1] | |
|---|---|---|---|---|---|
| | | Input | Output | Input | Output |
| Through | 765 | 48.1 | 40.5 | 23.0 | 27.8 |
| | 595 | 48.1 | 40.0 | 23.0 | 28.0 |
| Ramp | 765 | 35.5 | 34.9 | 24.7 | 26.9 |
| | 595 | 35.5 | 33.3 | 24.7 | 27.8 |

[1] Through vehicles travel a fixed distance of 1,683 ft while ramp vehicles travel 1,275 ft.

It is possible that under the given conditions this problem is trivial. It is obvious from the results of these trial runs that an added 170 ft of acceleration lane is unnecessary for the light load it is to carry. The results do show, however, that the variation in traverse time is fairly small from run to run (Table 4).

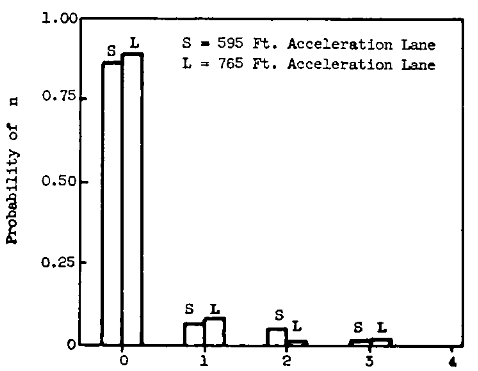



**Figure 12. Distribution of waiting line length.**

TABLE 4

MEAN AND VARIANCE OF TRAVERSE TIMES

| Vehicles | Case | Accel. Lane (ft) | $\bar{T}$ | $\sigma_T{}^2$ |
|---|---|---|---|---|
| Through | 1 | 595 | 27.29 | 24.14 |
| | | | 28.86 | 32.34 |
| | | | 27.76 | 29.80 |
| | 2 | 765 | 27.45 | 32.37 |
| | | | 28.08 | 32.93 |
| Ramp | 3 | 595 | 26.50 | 34.75 |
| | | | 28.82 | 60.53 |
| | | | 28.00 | 67.41 |
| | 4 | 765 | 24.79 | 32.35 |
| | | | 28.97 | 117.29 |

If it were desirable to establish confidence limits on the greatest mean traverse time for a given run, $\bar{\sigma}_T$, the standard deviation of the mean would be a function of the vehicular volume where

$$\sigma_{\bar{T}} = \sigma_T/\sqrt{N}$$

in which $N$ is the number of vehicles generated.

For example, if the mean ramp traverse time were to be known within 1 sec at a 95 percent confidence level, $\sigma_T = 8.85$ as in Case 4, Table 4, and the ramp volume was to be 800 vph,

$$\sigma_{\bar{T}} = 1.96\sigma_T/\sqrt{N}$$

$$1 = 1.96\ (8.85)/\sqrt{N}$$

$$N = 300.88$$

which is 0.38 of the hourly volume. This, in turn, means that in order to establish $\bar{T} \pm 1$ sec with 95 percent confidence the run must last 22.8 min, or about 2.28 min of computer time.

The 95 percent confidence limits on the mean traverse time in the trial runs are: Case 1, 27.99 ± 0.44; Case 2, 27.76 ± 0.56; Case 3, 27.80 ± 1.68; Case 4, 27.87 ± 2.32. The greater variances in

Cases 3 and 4 are because of the smaller number of ramp vehicles than through vehicles in Cases 1 and 2.

## CONCLUSIONS

Merging areas are constructed to move traffic quickly, smoothly, and safely. It is necessary that these merging areas handle expected volumes of traffic without excessive costs of construction and yet perform functionally. The simulation model described herein will give the design engineer a tool to determine the efficiency of a design before it is actually constructed. Future studies will show applications to other freeway areas.

The results of simulation have yet to be compared to the actual traffic process. This comparison will specify the degree of realism necessary to accomplish a specific amount of accuracy. Another aspect yet to be realized is the education of the design engineer insofar as the use of and benefts provided by this tool. This problem will be alleviated when the simulation has been validated for several physical designs under various traffic conditions. Then analysis and simulation together can aid the traffic engineer to understand more fully the science of traffic control and highway design.

## ACKNOWLEDGMENTS

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# APPENDIX A

## FURTHER EXPLANATION OF FLOW DIAGRAM

*Input Routine*

(1) Input total hourly volume to lanes 3, 2 and 1 in turn.

(2) Calculate $V_3$, the percent total volume in lane 3.

(3) Generate random number $N_3$.

(4) Calculate $P(t)$, $t = 1$, the probability of an arrival. If $P(1)$ is less than $N_3$, no vehicle has arrived during this interval, proceed to inspect lane 2. If $P(1)$ is greater than $N_3$, proceed to (5).

(5) Generate random number $S_3$.

(6) Look up table of velocities for lane 3. $S_3$ determines the desired speed of the vehicle which is entering the study area.

(7) Insert vehicle code in block 399 and then repeat routine for remaining lanes. The ramp routine is identical except for the volume input.

*Inspection Routine, Lanes 1, 2 and 3*

Vehicles in lanes 1 and 2 may weave left, vehicles in lane 3 weave right. All other steps are identical.

(8) Is the next block to be inspected block 399?

(9) Is the next block occupied?

(10) Can vehicle under inspection proceed at his desired velocity without interference from the preceding vehicle?

(11) Can vehicle weave to adjacent lane without interference from the preceding vehicle in that lane?

(12) Calculate $W$, the effective time gap.

(13) Is $W$ greater than 1 sec?

(14) Is $W$ greater than 2 sec?

(15) Generate random number $M_3$.

(16) Look up probability of weaving into various sizes of gaps.

(17) Shift lane, proceed at desired velocity.

(18) Decelerate and proceed to two car lengths behind preceding vehicle.

(19) Proceed at desired velocity.

*Inspection Routine, Lane 5*

(20) Is the next block occupied?

(21) Is the vehicle beyond the nose of the ramp (Point c)?

(22) Same as (10) except because of slower velocities a vehicle is allowed to approach closer to the preceding vehicle.

(23) Is vehicle within 1 sec of the end of the acceleration lane?

(24) Can vehicle weave to adjacent lane without interference from the preceding vehicle in that lane?

(25) Calculate $W$.

(26) Is $W$ greater than 1 sec?

(27) Is $W$ greater than 4 sec?

(28) Generate random number $M$.

(29) Look up probability of weaving into various sizes of gaps.

(30) Can vehicle weave to adjacent lane without interference from the preceding vehicle in that lane?

(31) Calculate $W$.

(32) Is $W$ greater than 2½ sec?

(33) Is $W$ greater than 8 sec?

(34) Generate random number $M_2$.

(35) Look up probability of weaving into various sizes of gaps.

(36) Proceed to end of acceleration lane.

(37) Merge, proceed at desired velocity and accelerate.

(38) Same as (10) except because of slower velocities a vehicle is allowed to approach closer to the preceding vehicle.

(39) Is vehicle at end of acceleration lane (Point D)?

(40) Proceed at desired velocity.

(41) Approach preceding vehicle, one block behind.

(42) Approach preceding vehicle, two blocks behind.

(43) Is preceding vehicle in line at end of acceleration lane?

(44) Approach preceding vehicle, two blocks behind.

(45) Proceed at desired velocity.

(46) Proceed at desired velocity.

# APPENDIX B

## Organization of the Problem for the Computer

An efficient computer program requires the handling of vehicular data within the computer slightly different from the preceding discussion; within the computer, a list of the cars in each lane was maintained. The first car on the list was the front car in each lane, and separate lists were maintained for lanes 1, 2, 3 and 5.

The data for each vehicle were contained in two computer storage locations. In the first storage location, position data and the time of entry by the vehicle into the system were contained; in the second, velocity data and acceleration characteristics were contained. Acceleration data were present only for vehicles which entered the system from the ramp.

As each vehicle enters the system, the position information for that vehicle is set as block 100 (75 for lane 5). When a vehicle leaves the system, all remaining vehicles in the list are moved up in the list, so that the list contains only those vehicles within the system and the front vehicle is the first vehicle on the list.

The "moving" of each car, at each second of the analysis, consists of subtracting the value of the velocity from the position of the vehicle. When the position value becomes zero or negative, the car has passed out of the system, and the various transit data taken.

In the processing of the vehicles, the front car in the system is first examined, moved, etc. Then the second car in the system, irrespective of its lane, is examined. This process is continued until every car has been processed. Decisions on weaving, blocked cars, etc., are made as previously described.

If a car reaches the end of the acceleration lane (point D in Fig. 6) without being able to weave into lane 1, its desired velocity value is divided by 2, and the vehicle halted. New acceleration data are entered and used to bring the vehicle up to speed, starting at the new desired velocity value.

### *Computer Operating Notes*

To operate the programs, an IBM 704 computer is needed which has at least 4,000 words of core storage. Only one tape unit is required, that one being used to produce an output tape to be printed by an off-line printer.

To run the program, assemble the program deck with a series of control cards immediately following the deck. The number and type of control cards will control the analyses to be run.

There are eight different control cards to be entered which insert the necessary setup data describing the ramp configuration and velocity and weaving distributions. Immediately following the program deck, there should be one of each of these eight control cards. A ninth card, called the Problem Card, containing data peculiar to each analysis then follows and, after this ninth card, change cards for any of the eight control cards and other problem cards may be entered to control the running of a series of traffic problems.

### *Control Card Format*

The eight control cards are divided into three groups. Four cards provide the distribution of velocity for lanes 1, 2, 3 and 5; a second group of three cards contains data for the weaving probability and the eighth card contains data concerning the initial point of the ramp, and the final point of the ramp in the configuration of the problem. The Problem Card contains a test or identification number, the volume of the through traffic, the volume of the ramp traffic, and the length of time for which the analysis is to be run.

The first group of control cards is identified by punching 1, 2, 3, or 4 in Col. 6 of the card. Cols. 7-72, inclusive, contain 11 items of data describing the distribution of velocities for cars entering the lanes. Each item of data is six decimal digits. The first two digits are that proportional amount of the possible 99 percentiles of the accumulative distribution curve below the velocity value which is entered into the last four digits of the item. The last four digits contain the velocity value with a decimal point as illustrated in Table 5. For an accumulative velocity distribution, the data would be punched into a control card according to the values shown in Table 6.

The second group of control cards contained data for the weave probability and are identified by the numbers 5, 6, or 7

TABLE 5

CARD FORMAT FOR VELOCITY DISTRIBUTION CONTROL CARDS

| Card column | Data Punched |
|---|---|
| 1–5 | Blank or zeros |
| 6 | 1 for lane 1<br>2 for lane 2<br>3 for lane 3<br>4 for lane 5 |
| 7–8 | Percentile value of first distribution point |
| 9–12 | Velocity value for first distribution point (decimal point between Cols. 10 and 11) |
| 13–14<br>15–18 | Percentile and velocity values for second distribution point |
| 19–20<br>21–24 | Percentile and velocity values for third distribution point |
| 67–68<br>69–72 | Percentile and velocity values for 11 distribution point (if needed)[1] |
| 73–80 | Irrelevant |

[1] Any number of distribution points from 1 to 11 can be used.

TABLE 6

ILLUSTRATIVE VELOCITY DISTRIBUTION CONTROL CARD

| Card Column | Punched Data |
|---|---|
| 1–6 | 000003 |
| 7–12 | 100200 |
| 13–18 | 250350 |
| 19–24 | 480450 |
| 25–30 | 750500 |
| 31–36 | 990640 |
| 37–80 | Blank |

punched into Col. 6 of the card. Cols. 7 to 54, inclusive, are used to provide eight items of distributional data giving the probability of weave from one lane to another. In this case only the first two digits of each item are used to indicate the relative proportion of 99 total percentiles in the accumulative distribution curve. In this distribution the space of "no weave" and "probable weave" (in units shown in problem flow diagram) is divided into eight equal intervals. The percentile value used is that value of the cumulative distribution curve at the midpoint of the interval (see Fig. 1, Sites 4 and 5).

The punching of these control cards is best illustrated by an example. Assume the probability of weaves is as shown in Fig. 1, Sites 4 and 5; the appropriate punching of the card is shown in Tables 7 and 8.

Control card 8 is identified by the number 8 in Col. 6 of the card. Cols. 7 through 30, inclusive, contain the data as shown in Table 9.

TABLE 7

CARD FORMAT FOR WEAVING DISTRIBUTION CONTROL CARDS

| Card Column | Data Punched |
|---|---|
| 1–5 | Blank or zeros |
| 6 | 4 for weaving from lane 1 to lane 2, from lane 3 or from lane 3 to lane 2<br>5 for weaving from lane 5 to lane 1, when car is not blocked at end of ramp<br>6 for weaving from lane 5 to lane 2, when car is blocked at end of ramp |
| 7–12 | First item in weave table (XX.XXXX) |
| 13–18 | Second item in weave table (XX.XXXX) |
| 24 | |
| 30 | |
| 36 | |
| 42 | |
| 48 | |
| 49–54 | Eighth item in weave table (XX.XXXX) |

TABLE 8

ILLUSTRATIVE PUNCHING OF CARD FOR DATA

| Card Column | Punched Data |
|---|---|
| 1–6 | 000004 |
| 7–12 | 000000 |
| 13–18 | 000000 |
| 19–24 | 000000 |
| 25–30 | 000000 |
| 31–36 | 120000 |
| 37–42 | 370000 |
| 43–48 | 630000 |
| 49–54 | 870000 |

TABLE 9

CARD FORMAT FOR RAMP DATA CONTROL CARD

| Card Column | Data Punched |
|---|---|
| 1–5 | Blank or zeros |
| 6 | 8 |
| 11–12 | Number of first block in ramp, point C in Fig. 6 |
| 17–18 | Number of last block in ramp, point D in Fig. 6 |
| 19–20 | Number of seconds car will accelerate after it leaves ramp (if not halted at D) |
| 21–24 | Rate of acceleration (XX.XX) for above number of seconds |
| 25–26 | Number of seconds car will accelerate after it leaves block at D |
| 27–30 | Rate of acceleration for above number of seconds |

The problem card contains an identifier number in the first six columns of the card; within these six columns must be punched at least two decimal digits and/or alphabetic characters. Cols. 6 through 24 then contained data according to the format in Table 10, where only integer data can be entered.

TABLE 10

CARD FORMAT FOR PROBLEM CARD

| Card Column | Data Punched |
|---|---|
| 1–6 | Test identifier (must contain at least two numbers and/or characters) |
| 7–12 | Time of analysis (sec) |
| 13–18 | Through traffic volume (cars per hr) |
| 19–24 | Ramp traffic volume (cars per hr) |

After a problem card, a blank card may be used to repeat the run specified on the preceding problem card, but with different starting values for the random number generator; hence, a difference solution will result. By this means, several runs of the same problem can be accomplished merely by placing a series of blank cards after each problem card.

After the first problem card (and any succeeding blank cards that may be used for repeat runs), changes in the control cards may be entered in any manner. That is, if control cards 1 through 7 are to be the same for another problem, but control card 8 would be different, only the appropriately changed control card 8 need be placed in the deck and then another problem card inserted to start the solution.

The program will continue to run, making the necessary repeat runs and changes according to the control cards, blank cards and problem cards inserted in the deck until there are no more cards in the card reader of the 704.

An end-of-file marker is not put on the output tape by this program. It must be placed on by a manual operator at the control console of the computer.