DEPARTMENT OF SOILS, GEOLOGY AND FOUNDATIONS

The Static and Vibratory Cutting and Penetration of Soils

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This paper reports the recent results of an investigation of which preliminary results have been previously reported (*2*).

The basic purpose of the investigation is the development of a working hypothesis relating the various physical quantities influencing the efficiency of soil cutting and penetration by a combined static-vibratory loading. This development is primarily based on the methods of dimensional analysis in conjunction with extensive small-scale laboratory experiments. The physical quantities included in the study are the composition, moisture-density-unconfined compressive strength interrelationships, static and dynamic moduli in compression and shear, energy dissipation characteristics, and the frequency dependence of properties of the soil; the geometry of the penetrator; the properties of the combined soil-penetrator system, including resonance; and the nature of the static and dynamic loading, including the effects of frequency and amplitude of vibration. The experimental results of the investigation are all presented in non-dimensional form. This greatly enhances the transformation from model studies to prototype response.

• INVESTIGATIONS concerned with driving penetrators (piles) into soil and with the action of cutting blades (scrapers, buckets, etc.) moving through soil have been conducted by civil and mechanical engineers interested in construction methods and by agricultural engineers interested in soil tillage.

More than 25 years ago Nichols (*14–19*) did extensive full-scale investigations on soil reactions to tillage. Klein (*9*) conducted experiments on the drawing of various shaped blades through granular materials and solids subjected to plastic deformation and attempted to establish similarity between flow lines in sands and flow lines in fluids. Recent experimental results by Gunn and Tramontini (*8*) are in agreement with those of the authors that an oscillating implement reduces the draft and permits the use of lighter equipment.

The concept of using eccentric weights mounted on counter-rotating shafts to induce forced oscillations of foundation elements dates back to the early 1930's, when machinery of this type was used in both Germany and the Soviet Union. The use of such vibratory machinery for pile driving was described by Barkan (*3, 4*) almost 25 years ago.

Russian engineers have developed by continuous experimentation a series of rugged vibrators to accomplish the sinking of various pile types under differing field conditions. Only Soviet engineers have utilized this method in actual construction (*21*), but application of the vibratory method in the Soviet Union has progressed rapidly in the past ten years.

The technique of sheet pile driving by vibrators was developed in 1949 at the Gorky Hydroelectric Development. A total of 3,700 sheet piles was driven into saturated sand to a depth of 30 to 40 ft, the driving time for each pile being only 2 to 3 min. Vibrator efficiency reached 31 sheet piles per 8-hr shift, while the efficiency of a pneumatic pile driver did not exceed 18 sheet piles. In 1954 refinements in the design of the vibrators used at the Stalingrad Hydroelectric Station led to a vibrator efficiency of 53 sheet piles per shift to depths up to 50 ft (*4*).

Vibrators were used for the construction of sand piles in 1953. All of the principal operations for construction of the piles (that is, sinking of the steel pipe, compaction of the sand-gravel mixture into the pipe to form the pile, and extraction of the pipe from the soil) were carried out using the same vibrator. Pipe of 12-in. diameter penetrated saturated silty sands to a depth of 23 ft in 30 to 60 sec (*4*).

A vibratory method of advancing exploratory bore holes to depths up to 80 ft has been found very efficient by the Russians. They have also found that the use of vibration in conjunction with rotary drilling not only greatly increases the drilling speed but also markedly increases the life of the drilling bit (*4*).

During construction in 1955 of the piers for the Yangtze River Bridge at Hankow, China, Russian-designed vibrators were used in conjunction with water-jetting to sink 5-ft-diameter, hollow, thin-shelled, reinforced concrete columns to bedrock through 90 ft of overburden consisting of unstable fine sand underlain by coarse sand and gravel. Two to six tubular columns were sunk every 24 hr by means of the vibratory pile driver and 5 jet pipes. Larger capacity vibrators of this type have been used successfully to sink hollow reinforced concrete cylinders 16.5 ft in diameter to a depth of 115 ft (*22*).

Barkan (*4*) points out that the vibratory method also promises to give interesting results from the practical point of view in other constructional operations, particularly in mechanical earth excavation. Experimental research has shown that it is possible to decrease considerably the shearing resistance of soil if vibrating tools are used (*12*, *20*). Tests have indicated that maximum efficiency is achieved by using high-frequency vibrations in the direction of shear. Soil shear resistance can be reduced by 50 to 60 percent in comparison with shear resistance without vibration.

PURPOSE AND SCOPE

The basic purpose of this paper is to report the results obtained from a small-scale laboratory investigation conducted to relate the various physical quantities influencing the efficiency of soil cutting and penetration by a combined static-vibratory loading.

In addition to use in the development of efficient, lightweight, portable excavation equipment, the results of the investigation may be of practical importance in the following:

1. The dynamic compaction of both cohesive and non-cohesive soils.
2. The development of efficient, lightweight, portable pile-driving equipment.
3. The formulation of more rational dynamic pile driving formulas.
4. Problems relating to vehicle mobility.
5. The determination of the bearing capacity and the settlement characteristics of footings of buildings under vibratory and blast loadings.
6. The design of airfield and highway pavements.

The study is based on the methods of dimensional analysis and the results are presented in non-dimensional form. Because of the nondimensional presentation, the results could be expected to hold for similar full-scale studies provided all of the important variables have been included in the dimensional analysis and there is similitude between the corresponding non-dimensional parameters of the model and prototype. Thus, the seemingly impossible task of modeling is avoided. It must be pointed out that dimensional analysis and model analysis are quite different, with dimensional analysis being a much more powerful and fundamental tool. Because of the complexity of soil as a structural material and the difficulty of soil problems in general, it is felt that a more extensive use of the methods of dimensional analysis will contribute greatly to the field of soil mechanics. To demonstrate the power of dimensional analysis to those in soil mechanics who may be unfamiliar with the method, and to encourage its use in future investigations, the following section is developed in a detailed manner.

DIMENSIONAL ANALYSIS

The method of dimensional analysis as used to determine relationships between physical quantities may be briefly summarized as follows. If one has m physical quantities, containing n fundamental units, which can be related by an equation, then there are $(m-n)$ and only $(m-n)$ independent non-dimensional parameters (called π-terms) such that the π-terms are arguments of some indeterminate homogeneous function F. An important part of the dimensional analysis is the wise choice of the physical quantities involved. Once this is accomplished a methodical process is used to get the π-terms involved. The exact nature of the functional relationship is then experimentally determined.

To keep the interrelationships among the various variables as simple as possible, only the more important factors should be considered. If the number of factors considered is too large, the practical usefulness of the results will be greatly impaired. In addition, the difficulties involved in the separation of the non-dimensional parameters in the experimental determination of the exact nature of the functional relationship will become quite formidable. Conversely, if important variables are omitted that logically may influence the phenomenon, the results may be incomplete or erroneous.

The physical quantities included in this study are the magnitude of the static and dynamic forces, the perimeter, cross-sectional area and tip angle of the penetrator, the amplitude and frequency of vibratory loading, mechanical resonance, the duration of loading, and the soil properties. Because of the complexity of soil as a structural material in general and its viscoelastic nature in particular, the following simplifications have been made regarding the soil properties. The material constants needed to describe the deformation characteristics of the soil are implicit in the shearing strength, as expressed by the maximum unconfined compressive strength, and the viscosity. The term viscosity is used herein as the tangent to the curve of strain rate versus the stress in the soil. For the soil used in these experiments the viscous response has been found to be non-Newtonian (*13*, Fig. 13). It may very well be that the soil moduli in compression and shear should be used instead of the shearing strength, but with regard to practical applications these quantities are not as easily obtainable as the unconfined compressive strength. Such factors as moisture content, grain size, cohesion, angle of internal friction, and mineralogy are included only so far as they affect the shearing strength and energy dissipation properties of the soil.

The following physical quantities using the force-length-time system of fundamental units have been selected for use in the dimensional analysis of the vibratory cutting and penetration of soils:

Physical Quantity	*Symbol*	*Fundamental Units*
1. Penetration	x	L
2. Time	t	T
3. Total applied force	F_T	F

Physical Quantity	*Symbol*	*Fundamental Units*
4. Static force	F_S	F
5. Forcing frequency	ω	T^{-1}
6. Natural frequency	p	T^{-1}
7. Maximum unconfined compressive strength of soil	τ	FL^{-2}
8. Viscosity of soil	η	$FL^{-2}T$
9. Mass density of soil	ρ	$FL^{-4}T^2$
10. Acceleration of gravity	g	LT^{-2}
11. Cross-sectional area of penetrator	A_c	L^2
12. Perimeter of penetrator	C	L
13. Tip angle of penetrator	θ	$F^\circ L^\circ T^\circ$
14. Amplitude of vibration	a	L

Inasmuch as there are 14 physical quantities and three fundamental units, there must be 11 independent nondimensional π-terms. These π-terms can be methodically obtained by choosing three physical quantities, which contain all three fundamental units and cannot be formed into a π-term by themselves—for example, F_T, ω, and τ—and combining them with each of the remaining quantities, one at a time. As an example, combining them with the viscosity gives

$$F_T^\alpha\, \omega^\beta\, \tau^\gamma\, \eta^\delta = \pi = F^\circ L^\circ T^\circ \tag{1}$$

Substituting the fundamental units of each of the physical quantities involved into Eq. 1 gives

$$F^\alpha\, T^{-\beta}\, F^\gamma L^{-2\gamma}\, F^\delta L^{-2\delta} T^\delta = F^\circ L^\circ T^\circ \tag{2}$$

Equating exponents and solving by letting $\beta = 1$ gives $\alpha = 0$, $\gamma = -1$, $\delta = +1$. Therefore, the π-term is

$$\pi = \frac{\omega\eta}{\tau} \tag{3}$$

which is proportional to the familiar loss tangent of viscoelasticity, inasmuch as τ is functionally related to the elastic modulus of the soil.

By repeating the process for the other physical quantities involved, the following π-terms can be obtained:

$$\pi_1' = x\sqrt{\tau/F_T} \qquad \pi_2' = \frac{A_c\tau}{F_T}$$

$$\pi_3' = C\sqrt{\tau/F_T} \qquad \pi_4' = \theta$$

$$\pi_5' = \frac{\omega\eta}{\tau} \qquad \pi_6' = \rho\frac{F_T\omega^2}{\tau^2}$$

$$\pi_7' = \omega t \qquad \pi_8' = \frac{F_S}{F_T}$$

$$\pi_9' = \frac{g}{\omega^2}\sqrt{\tau/F_T} \qquad \pi_{10}' = \frac{F_T}{\tau a^2}$$

$$\pi_{11}' = \frac{\omega}{p} \tag{4}$$

Thus, the homogeneous functional relationship among the π-terms is

$$F(\pi_1', \pi_2', \pi_3', \cdots, \pi_{11}') = 0 \tag{5}$$

Because of the great difficulty in experimentally determining the exact nature of the function F and in relating the physical quantities involved, several major modifications must be made in the form of F. Inasmuch as the requirement of the function F is that it consist of independent nondimensional parameters, there is nothing unique about the foregoing forms of the π-terms. Therefore it is possible algebraically to transform the π-terms in any way desired as long as the eleven final π-terms are nondimensional and independent. For example,

$$\pi_1 = \frac{\pi_1'}{\pi_3'} = \frac{x\sqrt{\tau/F_T}}{C\sqrt{\tau/F_T}} = \frac{x}{C} \tag{6}$$

$$\pi_3 = \frac{(\pi_3')^2}{\pi_2'} = \frac{C^2(\tau/F_T)}{A_c(\tau/F_T)} = \frac{C^2}{A_c} \tag{7}$$

Because the penetration x is considered the dependent variable, it is desirable that it not occur in more than one π-term. By algebraic manipulation of the π-terms, the following non-dimensional parameters can be obtained:

$$\pi_1=\frac{x}{C} \qquad \pi_2=\frac{F_T}{A_c\tau}$$

$$\pi_3=\frac{C^2}{A_c} \qquad \pi_4=\theta$$

$$\pi_5=\frac{\omega\eta}{\tau} \text{ or } \frac{\tau t}{\eta} \text{ or } \frac{F_T t}{A_c\eta} \qquad \pi_6=\frac{g\rho}{\tau}\sqrt{F_T/\tau}$$

$$\pi_7=\omega t \qquad \pi_8=\frac{F_T}{F_S}$$

$$\pi_9=\frac{C\omega}{gt} \qquad \pi_{10}=\frac{a}{C}$$

$$\pi_{11}=\frac{\omega}{p} \tag{8}$$

The function F can be written as:

$$F\left(\frac{x}{C}, \frac{F_T}{A_c\tau}, \frac{C^2}{A_c}, \theta, \frac{\omega\eta}{\tau}, \frac{g\rho}{\tau}\sqrt{F_T/\tau}, \omega t, \frac{F_T}{F_S}, \frac{C\omega}{gt}, \frac{a}{C}, \frac{\omega}{p}\right)=0 \tag{9}$$

The functional relationship among the various physical quantities can be expressed as:

$$x=C\psi\left(\frac{F_T}{A_c\tau}, \frac{C^2}{A_c}, \theta, \frac{\omega\eta}{\tau}, \frac{g\rho}{\tau}\sqrt{F_T/\tau}, \omega t, \frac{F_T}{F_S}, \frac{C\omega}{gt}, \frac{a}{C}, \frac{\omega}{p}\right) \tag{10}$$

INTERPRETATION OF THE NON-DIMENSIONAL TERMS

Because the natural frequency of the soil-penetrator system is primarily a function of the magnitude and configuration of the equipment used (different for model and prototype) and not a fundamental soil property, it is not considered in detail in this study. This does not mean that the mechanical resonance of the equipment is not important, but only that it is not necessarily funda-

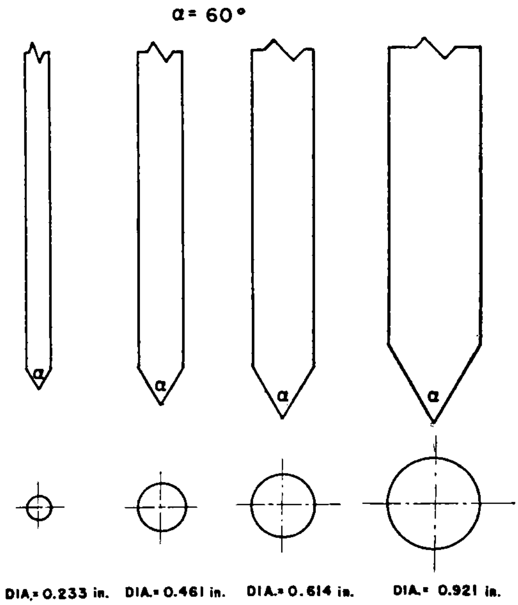


Figure 1. Circular penetrators.

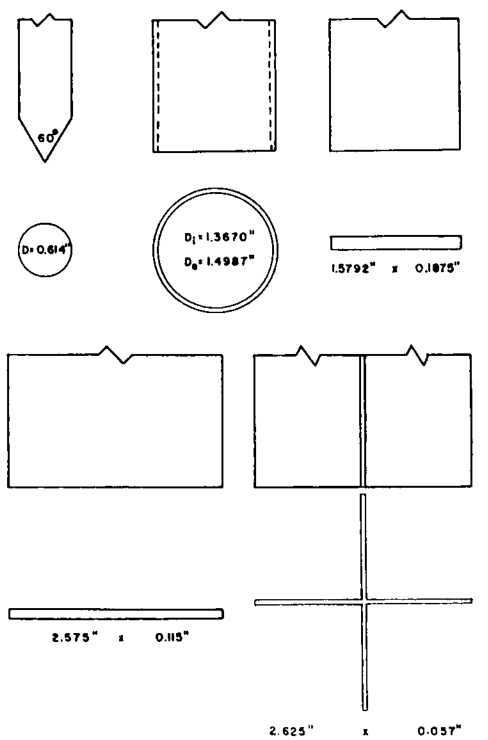


Figure 2. Penetrators of constant cross-sectional area and variable perimeter.

mental to the response of soils subjected to vibratory loading. Although the density of the soil may be important with regard to the propagation of shear and compression waves in soil, in the present study it is considered only so far as it affects the shearing strength of the soil. Therefore one is inclined to drop the π-terms containing p and ρ from the study. However, p will be discussed qualitatively later in the paper.

The remaining π-terms can be interpreted in the following manner. The number of dynamic stress applications per unit time is indicated by the term ωt and controls the rate of dynamic penetration or cutting. If τ is considered to be proportional to the elastic modulus of the soil, $\omega\eta/\tau$ is proportional to the

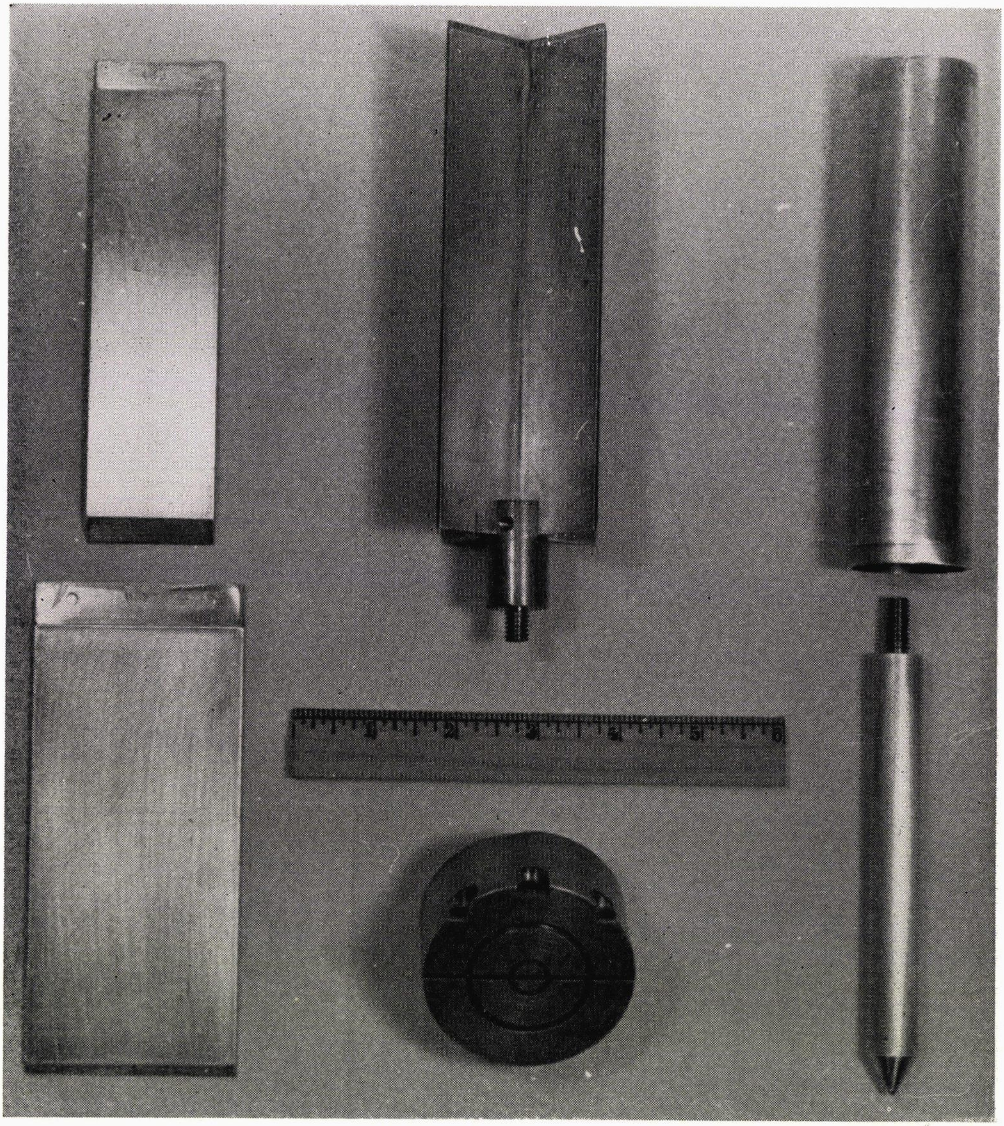


Figure 3. Penetrators of constant cross-sectional area and variable perimeter.

familiar loss tangent of viscoelasticity, which is the ratio of the energy dissipated to the energy elastically stored per loading cycle. The term $C\omega/gt$ is associated with the effect of the frequency of dynamic loading on the ultimate penetration. This does not necessarily include mechanical resonance.

The term F_T/F_S can be expressed as $1+F_D/F_S$ or $1+R$ and is essentially the effect of the force ratio R (that is, the ratio of the amplitude of the dynamic stress to the static stress). Work in the dynamic viscoelastic properties of cohesive soils, some of the preliminary results of which are given by Kondner (*12*), indicates that the magnitude of the static state of stress has a considerable effect on the response of soil when subjected to vibratory loads. This is analogous to saying that the dynamic stress-strain-time relations are a function of the static state of stress.

The ratio of the applied stress to the unconfined compressive strength of the soil is the term $F_T/A_c\tau$. The parameter C^2/A_c is a penetrator shape factor and θ is the tip effect. The effect of the amplitude of vibration is expressed by the term a/C.

Thus, Eq. 10 can be reduced to the form:

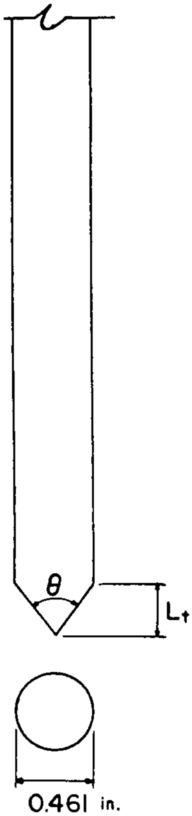
$$x=C\psi\left(\frac{F_T}{A_c\tau}, \frac{C^2}{A_c}, \theta, \frac{\omega\eta}{\tau}, \omega t, \frac{F_T}{F_S}, \frac{C\omega}{gt}, \frac{a}{C}\right) \quad (11)$$

which has a solution in a nine-dimensional space.

EXPERIMENTAL APPARATUS

Penetrators

The penetrators used in the study are shown in Figs. 1, 2, 3 and 4. They include a set of four circular blades of various diameters; a set of constant-area blades with varying perimeters which include a solid circular blade, three flat plate-like blades of various thicknesses, and a hollow cylindrical cutter; and a set of 9 solid rods of circular cross-section having various end profiles. All of the pene-



TIP ANGLE θ (degrees)	LENGTH OF TIP L_t (inches)
ROUNDED TIP	0.2305
180	0.0000
140	0.0839
120	0.1331
90	0.2305
60	0.3992
45	0.5565
30	0.8602
15	1.7508

Figure 4. Penetrator set, variable tip.

trators are made of brass and have a polished finish.

Static Test Apparatus

The static test apparatus is shown in Figure 5. The load is applied to the penetrator through a shaft by placing weights on the loading platform. The vertical motion of the shaft in the guides is measured with an indicator dial.

Vibratory Test Apparatus

The two types of vibratory test apparatus are shown in Figures 6 and 7.

Centrifugal Force Apparatus. The apparatus in Figure 6 consists of a penetrator and a variable-speed motor mounted on a slide assembly which is free to move vertically on guide rods.

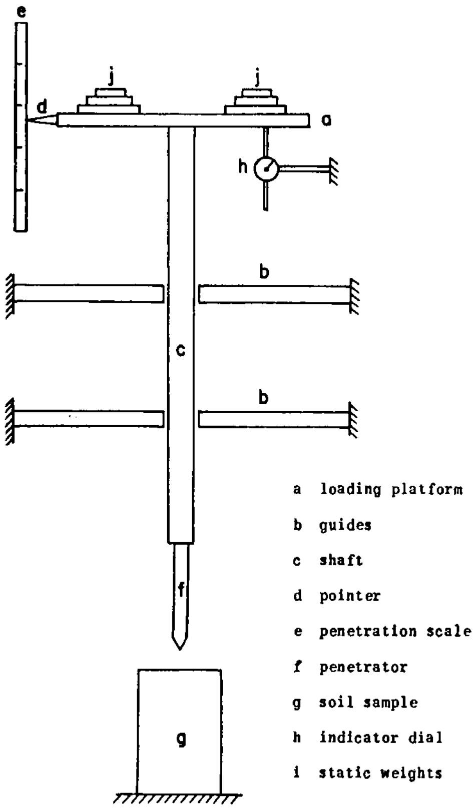


Figure 5. Schematic diagram of static test apparatus.

The motor operates a pair of counter-rotating shafts, to which is attached a set of four eccentric weights. The rotating weights impose a vertical sinusoidally varying force on the penetrator while the horizontal forces cancel. The static and dynamic forces are measured with a dynamometer in series with the penetrator. The dynamometer response is amplified and recorded on an oscillograph. The penetration is recorded continuously against time with a slide-type rheostat and a graphic recorder. The oscillations of the penetrator system are measured with the use of an accelerometer and vibration meter.

Electromagnetic Apparatus. The electromagnetic apparatus (Fig. 7) is similar to that described by Ayre and Kondner (*2*) except that the penetration is measured electrically with a slide-type rheostat and a graphic recorder.

MATERIALS TESTED

The soils used in this investigation are a natural clay, a Vicksburg loess, and a sand. Extensive penetration tests have been run on the clay both statically and dynamically. Preliminary static and dynamic penetration tests have been run on both the loess and the sand, and plans have been made for an extensive series of penetration tests to be run on these materials. Only the results obtained using clay are presented in this paper.

The clay is a Jordan Buff Natural Clay obtained from the United Clay Mines Corporation. It is mined from a natural deposit located on US 40 approximately 6 miles north of Baltimore at Poplar, Md. This deposit is a part of the Patapsco Formation of the Potomac Group, which is of the Lower Cretaceous Period. The loess was obtained from the U.S. Army Engineer Waterways Experiment Station at Vicksburg, Miss. The sand investigated is a typical bank sand sold commercially by the Arundel Corporation of Baltimore.

Specific Soil Properties

The characteristics of the clay and loess are as follows:

	Clay	*Loess*
Liquid limit.........	42%	28%
Plastic limit.........	21%	24%
Plasticity index......	21%	4%
Specific gravity......	2.68	2.69

The particle size distributions of the clay, loess and sand are shown in Figure 8.

Moisture-Density-Strength Relationships

The samples used for both static and dynamic penetration tests and the samples used in determining the unconfined compressive strengths and densities were compacted using the Modified AASHO energy of 56,250 ft-lb per cubic foot.

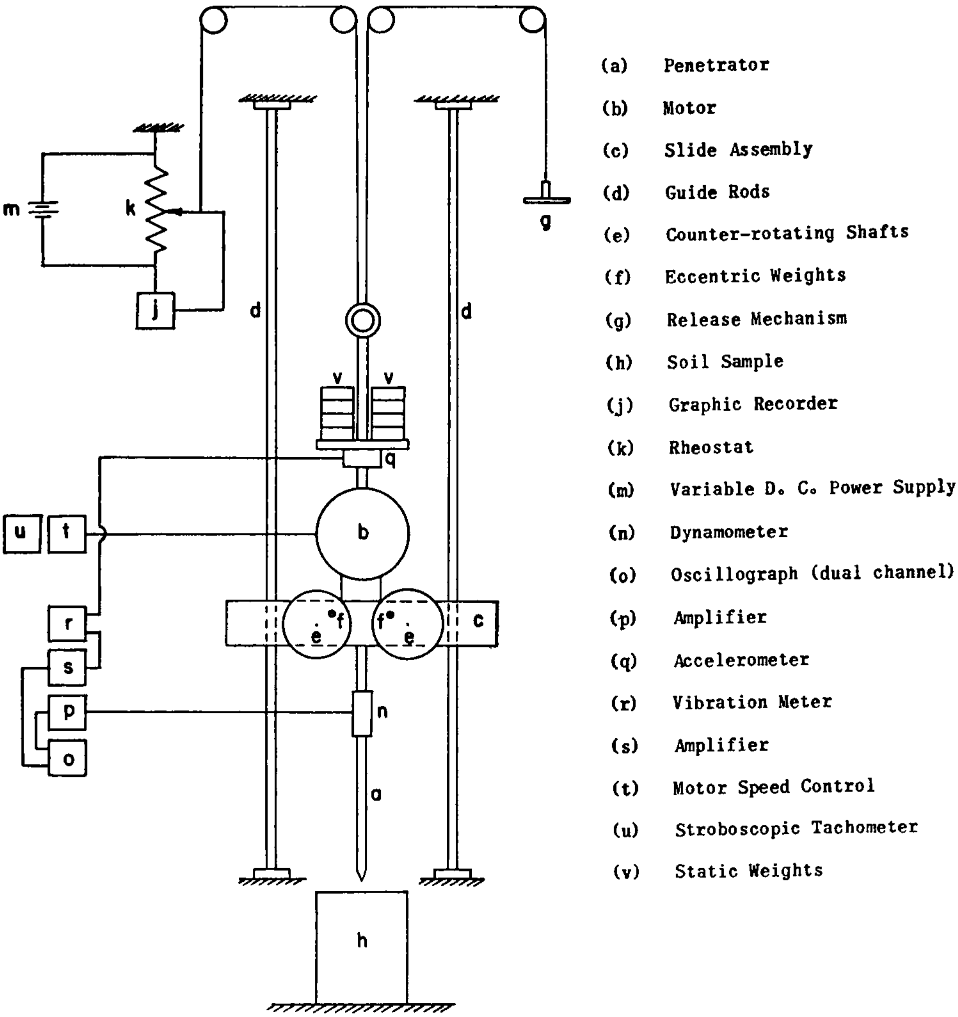


Figure 6. Schematic diagram of centrifugal force apparatus.

The curves of moisture content versus dry density for clay, loess and sand are shown in Figure 9. The variation of the maximum unconfined compressive strength as a function of moisture content and the unconfined compression modulus, E, for the clay are given in Figures 10 and 11.

EXPERIMENTAL RESULTS

The following are the results to date of the experimental determination of the importance of the various non-dimensional parameters on the cutting or penetration. Because of the complexity of the viscous response of the soil and the difficulties involved in the dynamic testing, no attempt is made to cross-correlate the various π-terms in this paper.

Static Tests

Because of the many variables included in the study, the experiments

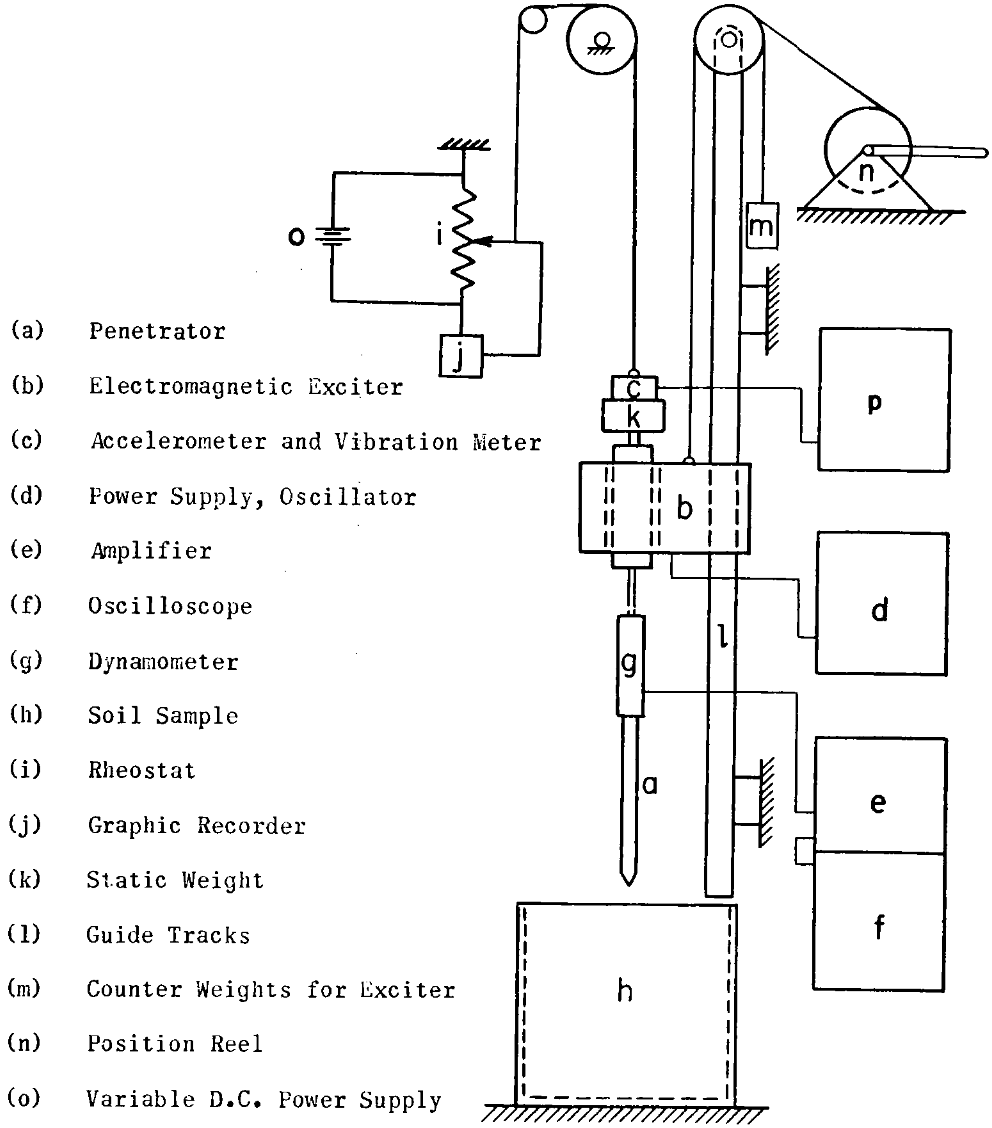


Figure 7. Schematic diagram of electromagnetic apparatus.

first conducted were for static loadings. The static penetration relation as given by Eq. 10 can be expressed as

$$\frac{x}{C}=\psi\left(\frac{F_T}{A_c\tau},\frac{C^2}{A_c},\frac{\tau t}{\eta},\theta\right) \qquad (12)$$

The time or viscous effects are included in the $\tau t/\eta$ term. Those familiar with the use of rheological models will recognize the term η/τ as being functionally related to the relaxation time for a Maxwell material and proportional to the retardation time of a Kelvin material. Therefore, $\tau t/\eta$ controls the rate of penetration in a static test. Very little in a quantitative manner can be done with this term at present. Kondner

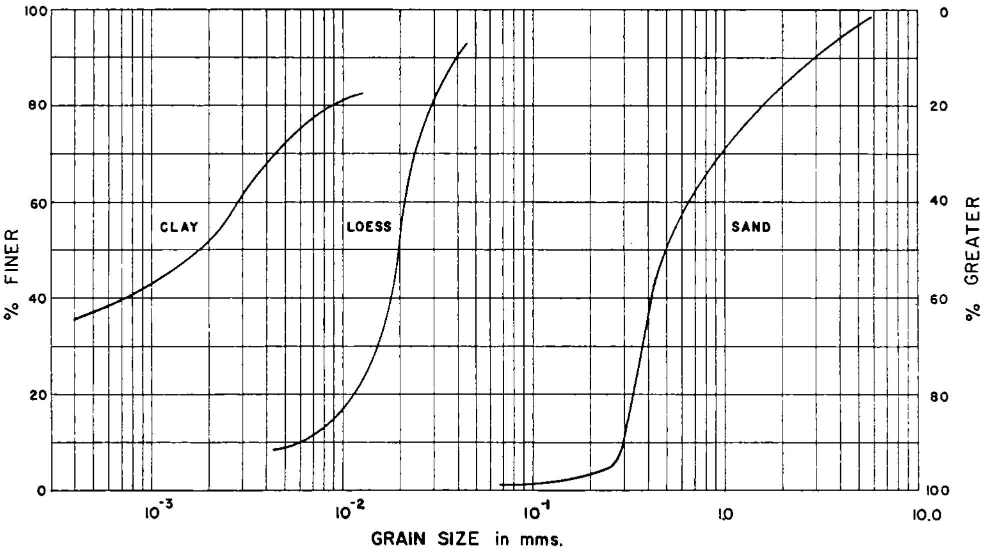


Figure 8. Grain size distribution.

(*12*) has recently initiated an extensive research program into the static and dynamic viscoelastic properties of cohesive soils and is hopeful that considerable progress can soon be made in stress-strain-time phenomena in soils.

Figure 12 is a typical plot of penetration as a function of force for several of the circular penetrators ($\theta=60°$) of Figure 1 obtained using samples of clay with different unconfined compressive strengths. The same test results are plotted in the non-dimensional form of x/C *versus* $F_T/A_c\tau$ in Figure 13, which also includes the results obtained for all of the circular blades using samples of clay and loess with unconfined compressive strengths ranging from 0.3 to 4.75 tons per square foot. Figure 13 clearly illustrates the convergent power of the non-dimensional form of presenting experimental data and the advantages of dimensional analysis as an experimental guide. The initial curved portion of the non-dimensional curve is greatly influenced by the tip angle of the penetrator. It must be remembered that Figure 13 holds only for $C^2/A_c=4\pi$ (a geometric minimum for penetrators), a tip angle of 60°, and a constant value of $\tau t/\eta$. Unfortunately, the field of soil mechanics has not yet reached the state of development where the latter term can be numerically evaluated. In general, the variation of x/C as a function of $F_T/A_c\tau$ is influenced by time effects, such as the loading sequence, the duration of the load interval, and the loading rate. The tests reported were for load interval durations large enough to allow for completion of most of the creep phenomena.

It should be noted that the relationship between x/C and $F_T/A_c\tau$ is unique (subject to constant values of θ and $\tau t/\eta$) for circular penetrators of any diameter regardless of the strength of the cohesive soil and the magnitude of the applied force. Practical illustrative examples have been worked by Kondner (*13*) for his study on footings. His work on the sinkage of footings also makes use of the parameters x/C and $F_T/A_c\tau$. The range of these parameters applicable to footing studies is on the curved portion of Figure 13. For such a limited range of parameters viscous effects can be considerable. Therefore, Figure 13 should not be considered for footing studies, but the original work (*13*) should be consulted.

The variation of the penetration

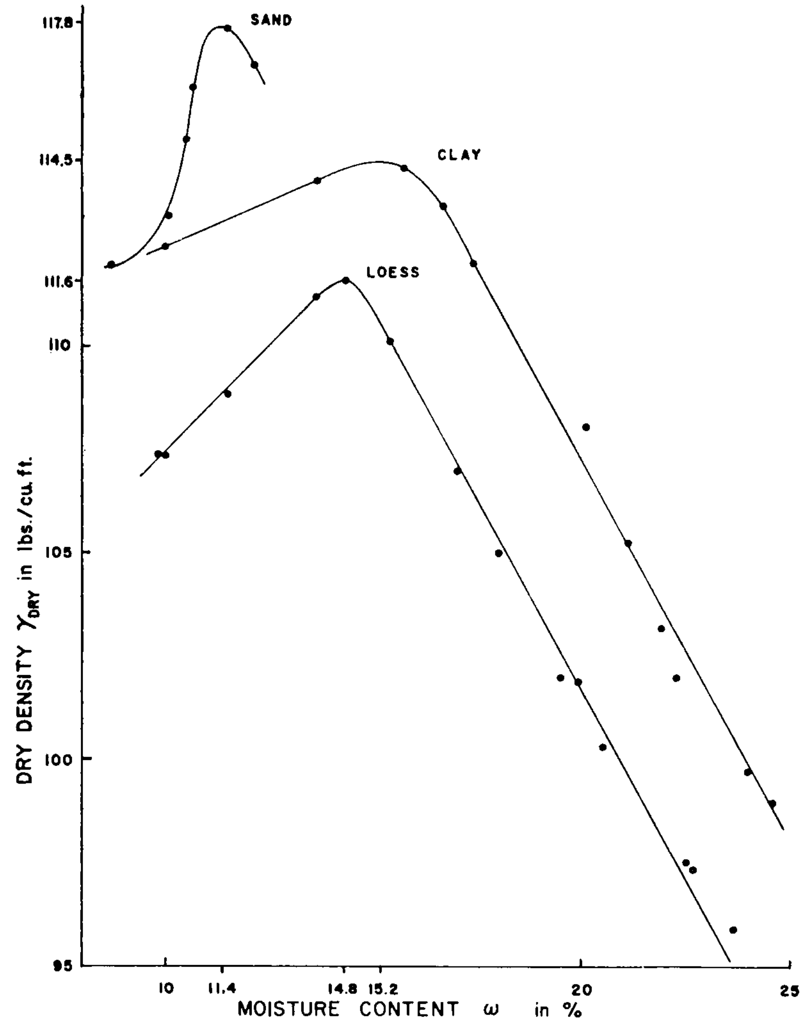


Figure 9. Dry density vs moisture content.

parameter x/C as a function of C^2/A_c was obtained by conducting a series of load-penetration tests with a set of five penetrators of constant cross-sectional area but with values of C^2/A_c equal to 4π, 42.1, 97.6, 273, and 371 for soil samples having approximately the same unconfined compressive strengths. The penetrators used are shown in Figures 2 and 3. A plot of x/C versus F_T/A_{cT} was obtained for each test and was found to be similar in shape to the curve given in Figure 13 except that the slope and intercept of the straightline portion varied with C^2/A_c. Figure 14 is a plot of x/C versus C^2/A_c for various constant values of F_T/A_{cT}. Thus, for a constant value of F_T/A_{cT} the cutting or penetration parameter x/C increases approximately exponentially with a decrease in C^2/A_c. The circular-shaped penetrator has the greatest penetration characteristics.

When $\log x/C$ is plotted against $\log(C^2/A_c)$ for constant values of F_T/A_{cT}, the resultant curves can be approximated

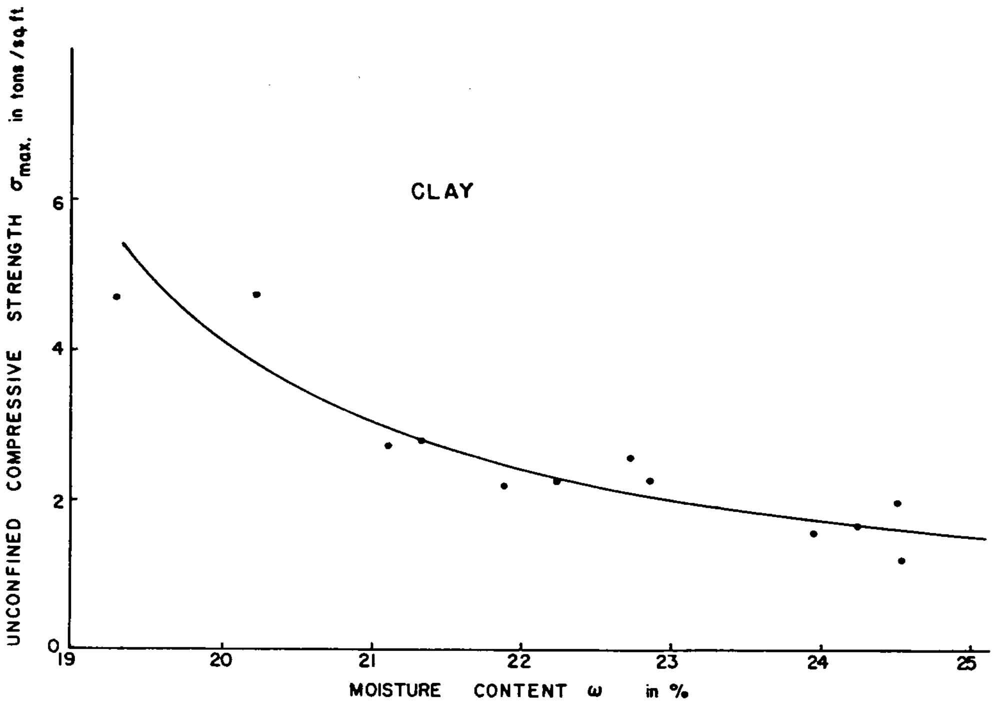


Figure 10. Maximum unconfined compressive strength vs moisture content; clay.

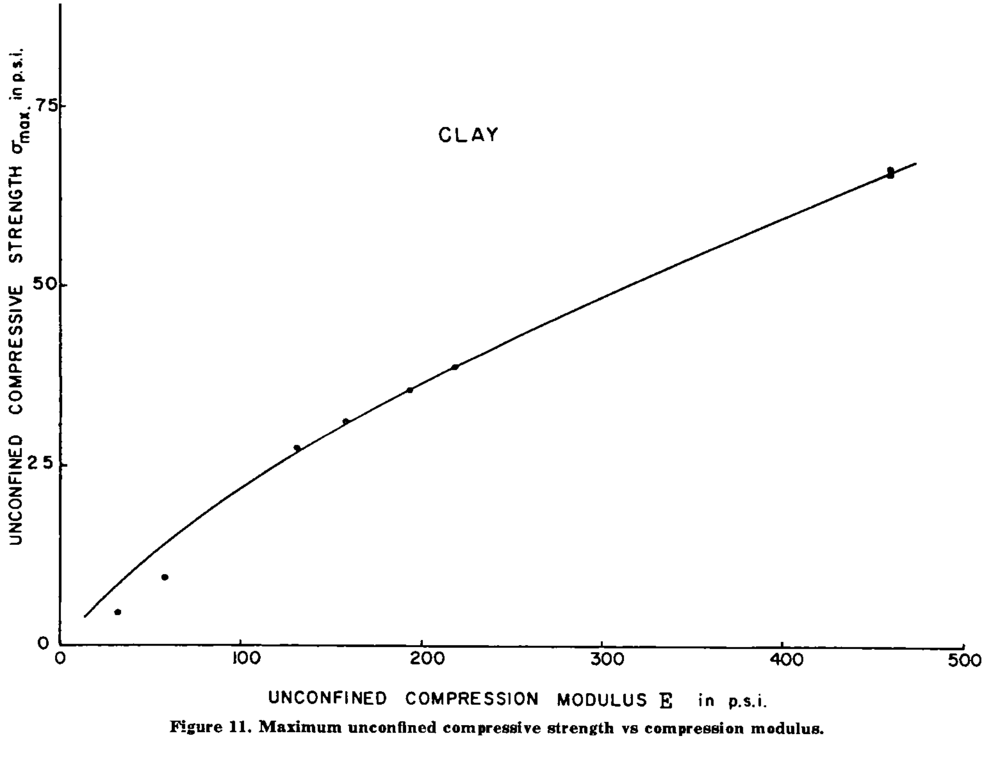


Figure 11. Maximum unconfined compressive strength vs compression modulus.

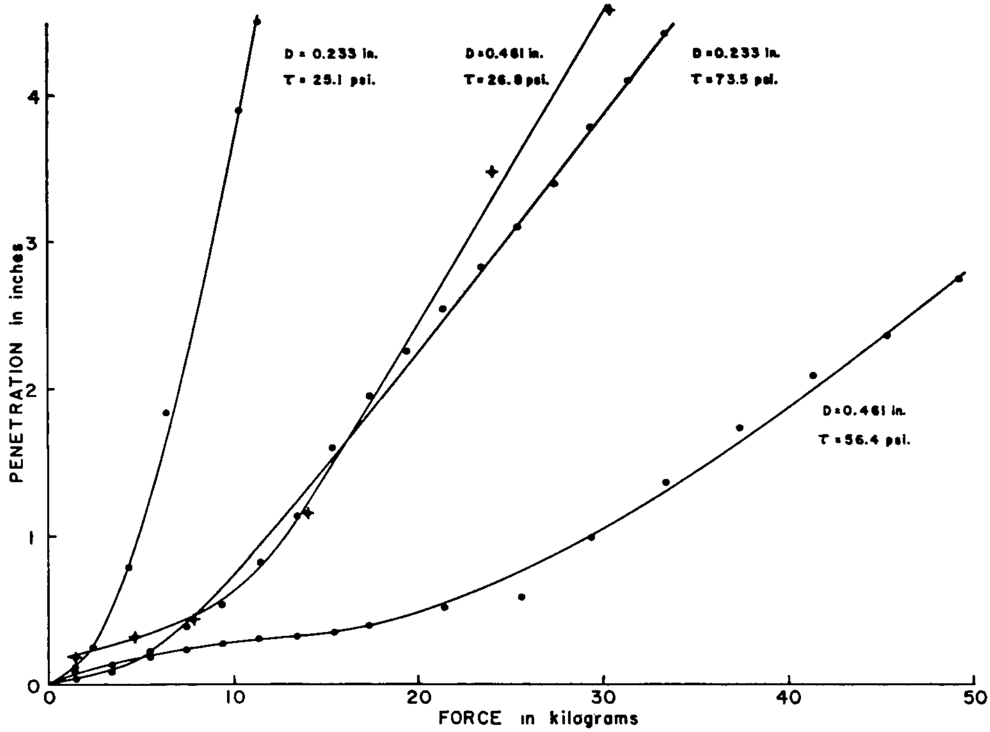


Figure 12. Penetration vs load; circular penetrators.

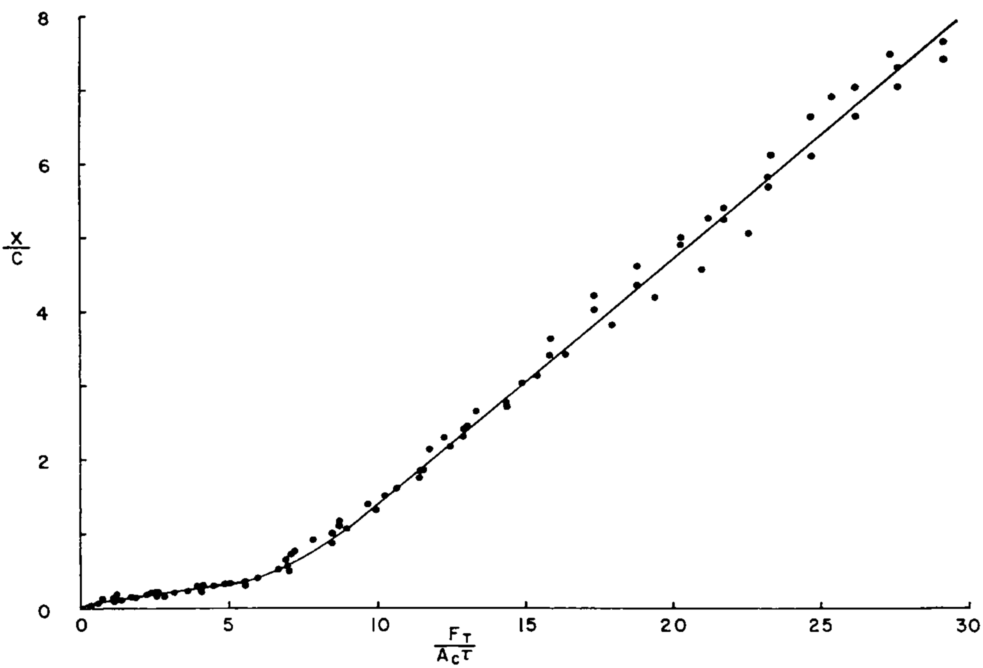


Figure 13. Non-dimensional plot of penetration parameter x/C vs force, area, and soil strength parameter $F_T/A_C\tau$.

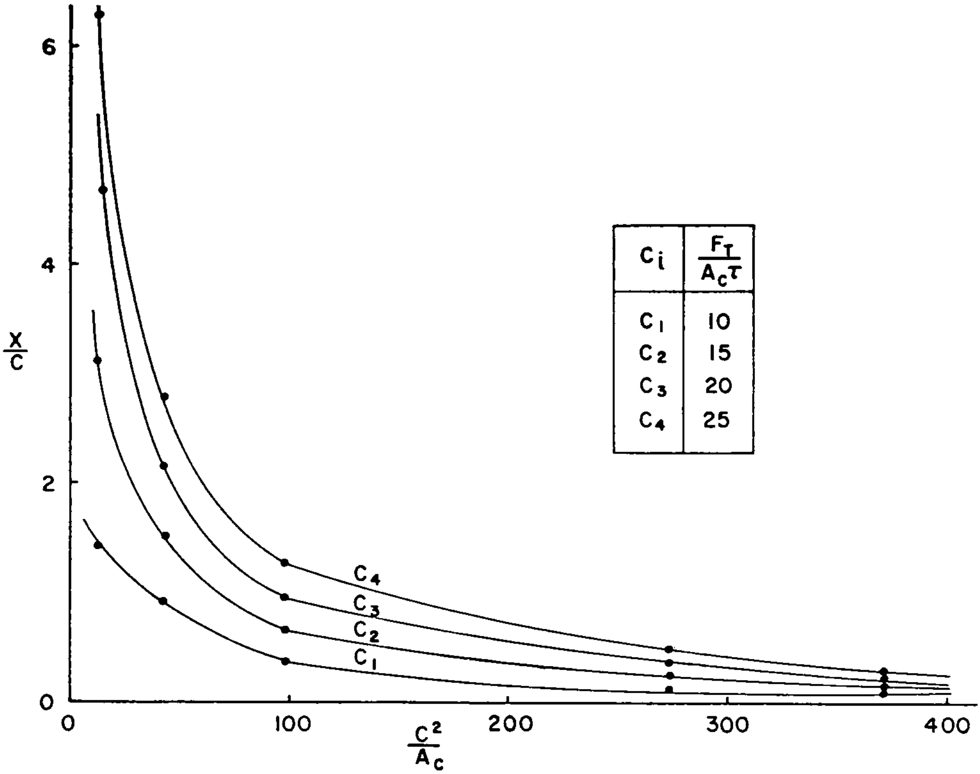


Figure 14. Non-dimensional plot of x/C vs C^2/A_c for constant values of $F_T/A_c\tau$.

by straight lines whose intercepts and slopes are functions of $F_T/A_c\tau$. By plotting the slopes and intercepts as functions of $F_T/A_c\tau$, it was found possible to express x/C in the form

$$\frac{x}{C}=\left[b+d\left(\frac{F_T}{A_c\tau}\right)\right]\left(\frac{A_c}{C^2}\right)^S \qquad (13)$$

where b and d are related to the variation of the intercepts with $F_T/A_c\tau$ and S is the slope as a function of $F_T/A_c\tau$. Although little can be said about the term $\tau t/\eta$, some qualitative indications of its effect on x/C can be obtained by conducting creep tests with one penetrator in soil of constant strength for different magnitudes of a single load application. The results of such tests will undoubtedly be similar to the creep tests reported in the study on footings (*13*).

To study the effect of the end profile on the penetration, a series of load-penetration tests was conducted on nine rod-type blades (Fig. 4) of constant diameter but with tip angles of 15°, 30°, 45°, 60°, 90°, 120°, 140°, 180°, and a hemispherical tip. The curves of x/C versus $F_T/A_c\tau$ are similar in shape to Figure 13. The slopes of the straight-line portions are approximately constant, but the intercepts vary inversely with the tip angle. Thus, the sharpest tip gives the greatest penetration, but once the blade has sufficiently penetrated (approximately $x/C=1$ for all the blades tested) the change in x/C is constant for a unit change in $F_T/A_c\tau$. Figure 15 is a plot of $\log x/C$ against $\log\theta$ for constant values of $F_T/A_c\tau$. Straight lines having various slopes and intercepts are obtained. The variation of these slopes, m, and intercepts, I, as functions of $F_T/A_c\tau$

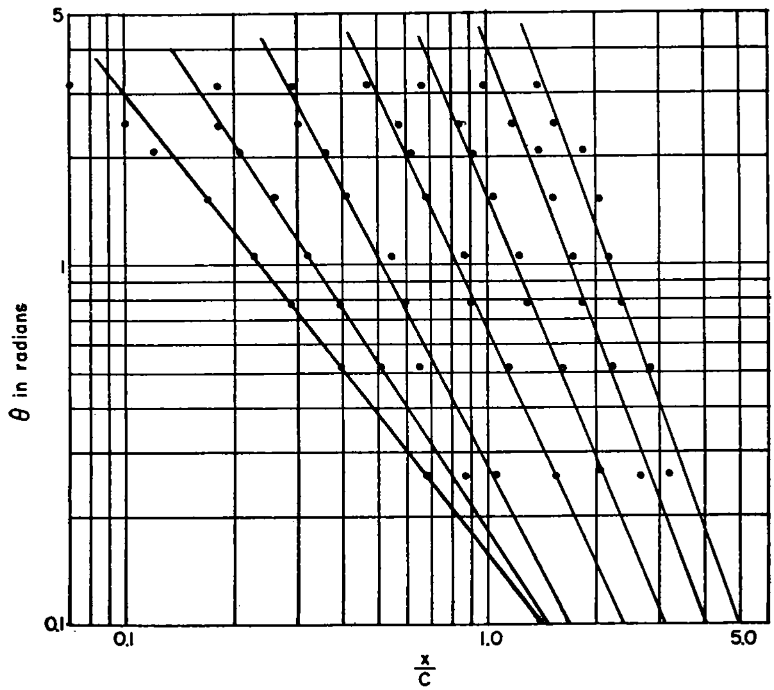


Figure 15. Tip effect: log x/C vs logθ.

for $C^2/A_c = 4\pi$ gives

$$\frac{x}{C} = I[10\theta]^{-1/m} \tag{14}$$

where θ is in radians.

Vibratory Tests

Natural Frequency. It was previously noted that the natural frequency of the soil-penetrator system is primarily a function of the magnitude and configuration of the equipment used and may be entirely different for model and prototype. Although the mechanical resonance of the system is important, the present study is more concerned with the response of vibratory-loaded penetrators as a function of the frequency-dependent properties of the soil. Kondner's current research on the dynamic viscoelastic properties of the soil in question shows that the soil properties are definitely frequency-dependent and that their strengths can be greatly reduced by vibrations. As a qualitative indication of the possible effects on the natural frequency of soil strength, applied stress, and cross-sectional area, reference is made to the natural frequency considerations of the study on the vibratory loading of footings (*13*).

Time Effects. Figure 16 is a typical curve of x/C versus ωt. This curve is actually the result of a dynamic creep test and is similar in shape to the curves obtained for static creep tests. The most consistent fit of such data was obtained by plotting $\log(A - x/X)$ versus ωt, as shown in Figure 17, where X is the ultimate penetration and A is a constant. Because the resultant curve is approximately a straight line, the penetration can be expressed over a wide range of time in the form

$$x = X[A - B\exp(-2.3S\omega t)] \tag{15}$$

where B and S are the intercept and slope, respectively. For a controlled set of tests it may be possible to get

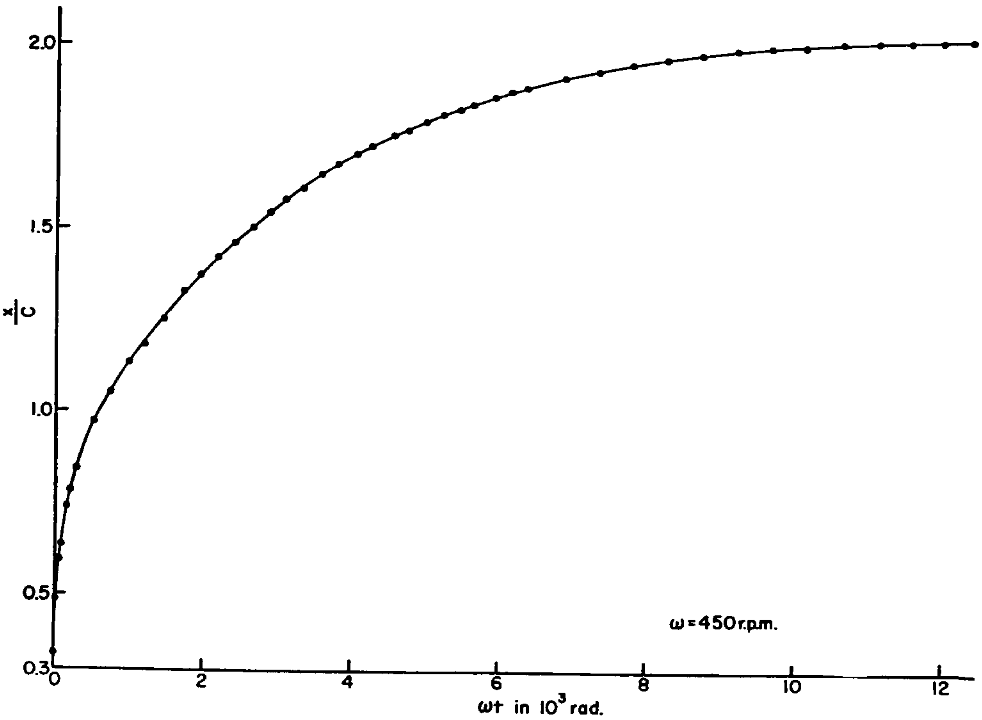


Figure 16. Dynamic creep test: x/C vs ωt.

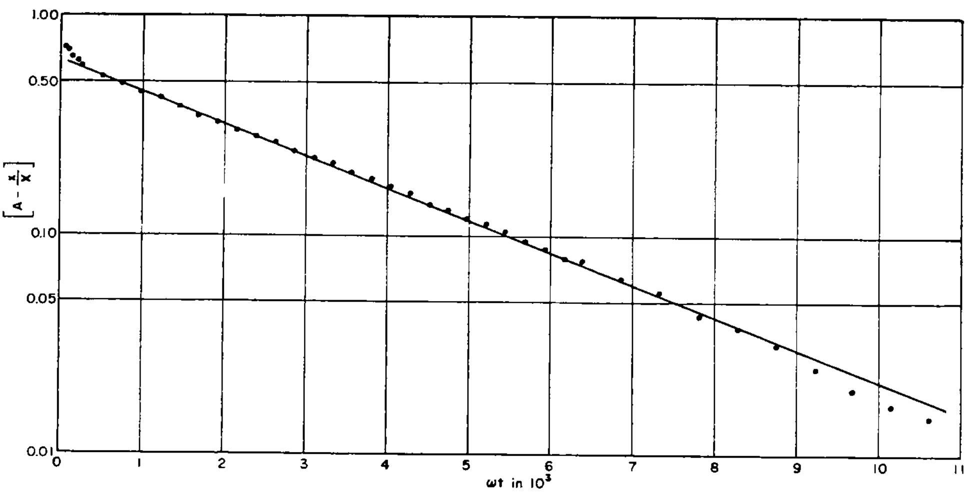


Figure 17. Dynamic creep test: ωt vs log $[A-x/X]$.

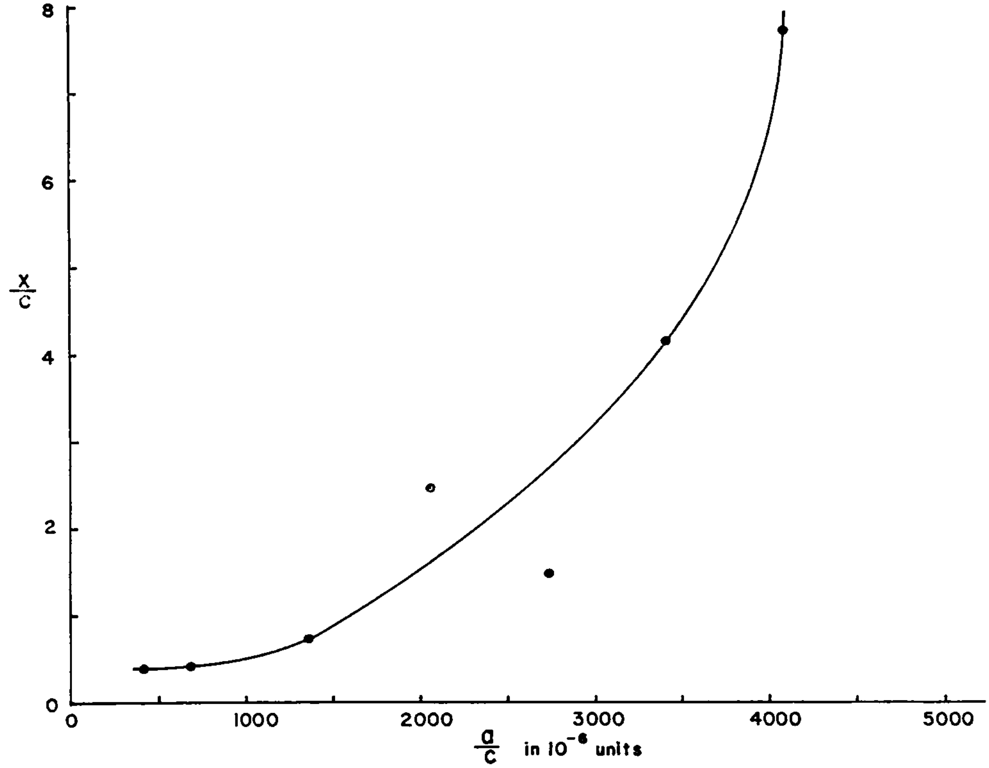


Figure 18. Penetration parameter vs amplitude-of-vibration term.

A, B, S and X as functions of the other π-terms. To date such an ambitious test program has not been conducted because of difficulties involved in controlling the many nondimensional parameters.

Amplitude Effects. Figure 18 shows the effect of the amplitude-of-vibration term on the penetration parameter for one penetrator, constant static contact pressure, and the same soil, with a constant frequency of vibration, at various values of ωt. The increase in penetration with an increase in amplitude is to be expected. It must be remembered that the results of such a non-dimensional plot will change with variations in soil type, soil strength, frequency of vibration and magnitude of the static and dynamic stress.

Frequency and Force Ratio Effects. The variation of the penetration parameter x/C as a function of the frequency term $C\omega/gt$ is presented in Figure 19 for different values of the force ratio, R. These results were obtained for a stiff sample using a 0.23-in. diameter circular blade with a 60° tip. The total force (static plus maximum dynamic force amplitude) was maintained constant for all of these tests. The natural frequency values determined by the free vibration method using impulsive forces varied considerably. Although numerous harmonics were excited, a fundamental frequency of approximately 20 cycles per second was fairly consistent. Figure 19 shows a decrease in the frequency of the peak penetration for an increase in the force ratio. Because the total force was maintained constant, the static weight of the system was decreased with an increase in force ratio. This indicates that the frequency of the peak penetration should have increased with an

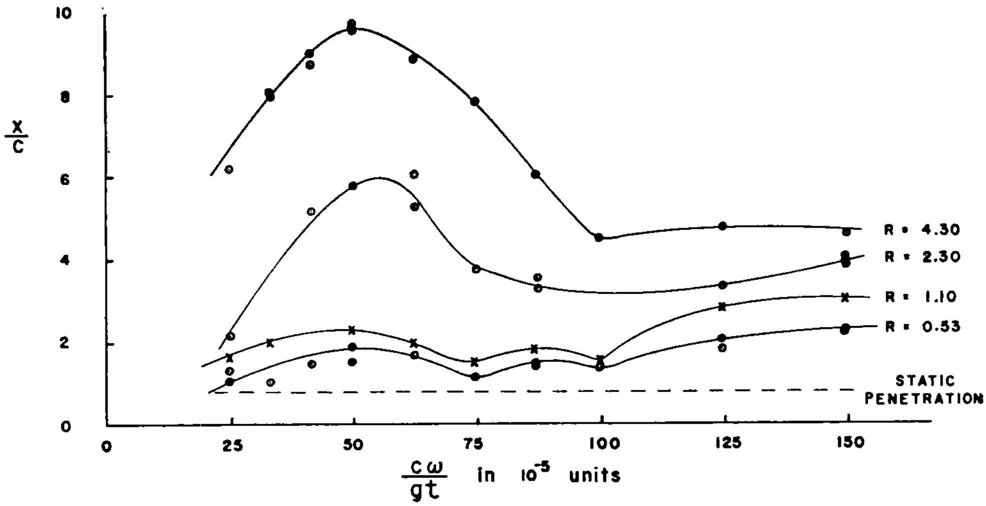


Figure 19. Penetration parameter vs frequency term.

increase in force ratio. Therefore, the influence of the amplitude of the dynamic force on the peak penetration frequency is an indication of non-linear spring and damping characteristics. The non-linearity increases with an increase in the force ratio. When related to spring characteristics, this indicates a "soft" spring response. This agrees with Kondner's recent work on the dynamic viscoelastic properties of the soil used (*12*).

The dashed straight line in Figure 19 is the static value of x/C for an equivalent static force equal to the total force (static plus dynamic) used for these tests. A comparison of the static x/C with the x/C-values for various frequencies and force ratios gives an indication of the variation of the penetration ratio x_D/x_S, which is the ratio of the dynamic to static penetration, for a constant applied total force.

Figure 20 is a cross plot of the data of Figure 19 showing x/C versus $1+R$ for various frequencies of vibration. The effect of the force ratio on x/C is influenced by the frequency. Thus, the penetration parameter, frequency term, and force ratio, are all interrelated.

The consistently high value of the penetration ratio indicates that the dynamic penetration is not necessarily a mechanical resonance phenomenon but may also be a function of the frequency and stress level dependence of the soil properties. Because of the limited power output of the vibratory apparatus currently being used, a wider range of force ratios and frequencies could not be investigated.

FAILURE MECHANISM

Kondner's recent investigations of the electro-osmotic characteristics, the vibratory penetration and cutting, and the dynamic viscoelastic characteristics of cohesive soils leads him to believe that the failure mechanism involved in the vibratory cutting and penetration of cohesive soils is a rate process in which the vibratory energy is a form of activation energy that causes progressive alterations in metastable states of the soil structure into more stable states. This process takes place in the diffuse layers of non-Newtonian water surrounding the soil particles and is a function not only of the electro-chemical properties of the soil grains and the pore water but also of the static stress level that may be applied prior to the vibratory loading. It is to be noted,

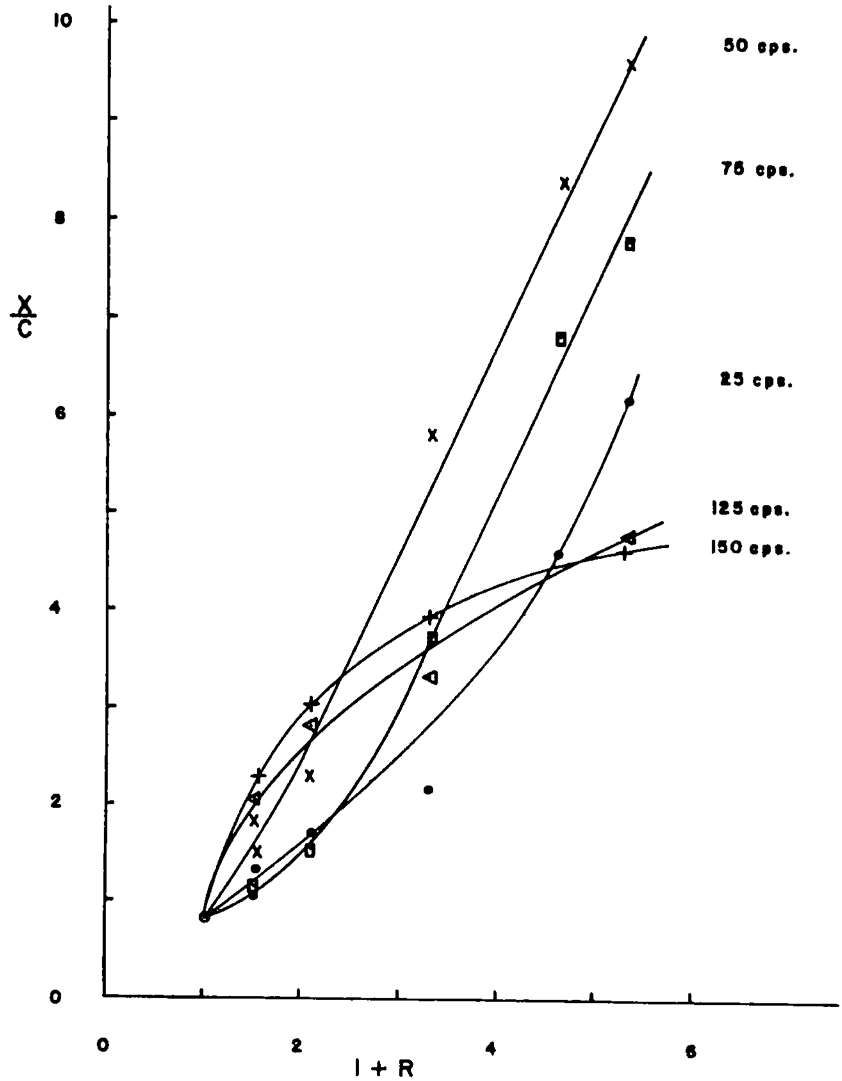


Figure 20. Penetration parameter vs force ratio.

with regard to future practical importance, that the activation energy involved in the changing of the internal structure of a soil could possibly be mechanical, electrical, thermal or magnetic in its nature.

CONCLUSIONS

The method of dimensional analysis forms a rational basis for investigating the static and vibratory cutting and penetration of cohesive soils. Such a basis greatly enhances the transformation from model studies to prototype phenomena.

Depending on the evaluation of the importance of viscous effects, there are indications that the static penetration of piles, footings, and cutters can be accurately predicted from the non-dimensional presentation of the results of model studies. At present little is known about the viscous characteristics of cohesive soils. Practical illustrative examples have been worked by Kondner (*13*) for his study on footings. An extension of some of the tests reported herein, in conjunction with other rheological experiments, might possibly be used to obtain quantitative information about such phenomena.

For cutters or penetrators having the same cross-sectional area under the same loading the one with the smallest value of C^2/A_c will have the greatest penetration. A solid circular penetrator has the geometric minimum value of C^2/A_c.

With all other factors constant the cutter or penetrator with the greatest tip angle will have the greatest penetration. For values of x/C approximately greater than one, the change in penetration per unit force is approximately constant. Therefore, for penetrations greater than the circumference, the slip field in the soil does not seem to be affected by the shape of the tip.

In the cutting and penetration of soils the use of vibratory loading in conjunction with a static load is definitely more advantageous than static loading alone. The cutting and penetration increase rapidly with increases in the amplitude of vibration and the force ratio. Although the natural frequency of the soil-penetrator system is important, large values of cutting and penetration can be obtained at frequencies other than the mechanical resonance. Such response is believed to be due to the frequency and stress level dependence of the soil properties. The penetration and cutting of cohesive soils is a highly non-linear problem and considerable basic research is needed. Although most of the present study is for cohesive soils, limited field studies reported in the literature indicate the possibility of even greater advantages for cohesionless materials.

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