Curling of the Concrete Slab The amount of curling of several different slabs has been measured over 24-hour periods The method employed was to drive stakes along the slab at 2-foot intervals, and on these to mount dials which showed the vertical motion of the slab in thousandths of an inch In Figure 5 the maximum curling is plotted for several periods of 24 hours From these curves it can be seen that the maximum movement occurs at the end of the slab, that the maximum bending moment occurs from six to eight feet from the end, and that the curling of the slab is restrained as the distance from the end is increased

The curling curves shown indicate that a fiber stress of at least 100 pounds per square inch is possible It is evident that secondary transverse cracking may occur from this cause, before the concrete has attained high strength due to the dead-weight of the slab and at a later period from traffic loads

ANALYSIS OF STRESSES IN CONCRETE PAVEMENTS DUE TO VARIATIONS OF TEMPERATURE

By H M Westergaard

University of Illinois, Urbana, Ill

This paper supplements a previous paper by the writer, published in *Public Roads*, April, 1926, under the title "Stresses in Concrete Pavements Computed by Theoretical Analysis," and in the Proceedings of the Fifth Annual Meeting of the Highway Research Board, held at Washington, D C, December 3–4, 1925, Part I, 1926, pp 90–118, under the title "Computation of Stresses in Concrete Roads" The present paper, like the previous paper, rests upon the assumption that the concrete pavement acts as a homogeneous elastic solid As in the previous paper, the method is that of the theory of elasticity, conclusions being drawn from a few simple physical laws by mathematical analysis In regard to the general assumptions and concepts the reader is referred to the previous paper The previous paper dealt with stresses and deflections produced by wheel-loads The present paper deals with stresses and deflections produced by variations of temperature The former paper, on account of the complexity of the mathematical processes involved, was limited to a statement of the assumptions and general principles and to the presentation of the results with illustration of how to use them Since the mathematical processes involved in the present paper are much simpler, it was considered to be expedient not to omit them Results are given in tables and diagrams, and the use of the results is illustrated by numerical examples

SYNOPSIS

The fact that cracks sometimes develop in a new pavement before any load is put on it shows the importance of the stresses which ordinarily are considered to be secondary stresses due to changes of temperature, to setting of the concrete, and to changes of moisture content of the concrete In a piece of concrete free to expand and contract the setting will produce a shrinkage equal to that due to a certain drop of temperature, and the absorption of moisture will produce a swelling equal to that produced by a certain rise of temperature In each case the essential feature is a tendency to change of volume Thus, the study of stresses due to variations of temperature will supply information also about the other secondary stresses With this condition in view, the discussion is limited to the case of variations of temperature

The stresses due to variations of temperature should be considered from two points of view One has reference to the early life of the pavement, before the pavement has obtained its full strength, and before it has been opened to traffic The other has reference to the later life of the pavement, after it has gained strength, and after it has been opened to traffic According to the first point of view, the stresses due to variations of temperature are considered as an independent cause of cracks, and are treated by themselves According to the other point of view, the stresses due to variations of temperature are to be combined with the stresses due to the loads This combination, in most cases, is a simple matter of addition

Whatever particular effect is studied, three places on the panel of pavement should be examined the corners, the interior area, and the edges

Two major cases will be investigated The first arises from a consideration of the slow seasonal changes of temperature, the second from a consideration of quick changes of temperature, occurring, for example, by the change from a cool night to a hot day and vice versa In the first case the final temperature is assumed to be uniform throughout the thickness of the pavement This case is treated under the heading "Uniform Decrease of Temperature" The second case is dealt with by assuming a definite temperature gradient through the thickness of the pavement with the temperature unchanged at the middle plane The stresses in this case are dependent upon the manner in which the slab deflects or curls This case is treated under the heading "Curling" Obviously, the stresses found in these two major cases may have to be combined by addition or subtraction

The case of a uniform change of temperature may be investigated by simple computations, involving the amount of change of temperature, the coefficient of temperature expansion, the modulus of elasticity, and Poisson's ratio of lateral expansion to longitudinal shortening as found in direct compression A numerical example shows the significance of the equations

The study of curling begins naturally with a statement of the relations between the amount of variation of temperature, the bending moments, and the curvatures of the deflected middle surface of the pavement Such a statement is found in equations (4) When the panel under consideration is very long, so that the essential curling is in the transverse direction, the direction of y, these relations lead to a simple differential equation, equation (12), which governs the flexure of the slab Solutions of this equation are given for a special case in equation (13) and for a more general case in equation (18). Corresponding values of stresses are given in equations (15) and (20) Numerical results computed from these equations are shown in the tables and diagrams The headings of the tables and the titles of the figures explain the nature of the results

The quantity l, numerical values of which are given in Table I, serves in the analysis as a sort of unit for the horizontal distances The significance of this unit is illustrated by the scales in feet at the bottom of the diagrams

The quantities z_0 and σ_0 defined by equations (14) and (7) are used as standard values or units in terms of which relative values of the deflections and stresses, respectively, are expressed Table III shows examples of these standard values Relative values of deflections and stresses are given in Table II, IV, and V and in the diagrams

Further numerical illustrations are given in the text

At the end of the paper a rectangular panel is investigated which is short enough to make it necessary to consider the curling in two directions at the same time This case leads to a numerical study of the influence of the variation of temperature upon the corner break

UNIFORM DECREASE OF TEMPERATURE

Let it be assumed that the temperature has decreased the same amount all through the depth of the pavement

Let

x, y = horizontal rectangular coordinates,
σ_x, σ_y = normal tensile stresses in the directions of x or y, respectively;
ϵ_x, ϵ_y = elongations per unit of length in the directions of x or y, respectively,
E = modulus of elasticity of the concrete;
μ = Poisson's ratio for the concrete;
ϵ_t = coefficient of temperature expansion;
t = decrease of temperature.

Then the following equations will apply,

$$\epsilon_x = \frac{1}{E}(\sigma_x - \mu\sigma_y) - \epsilon_t t, \quad \epsilon_y = \frac{1}{E}(\sigma_y - \mu\sigma_x) - \epsilon_t t \tag{1}$$

At a corner of an unloaded panel of pavement, obviously, there can be no stresses σ_x and σ_y At a small distance from the corner, stresses may develop if the friction offered by the subgrade is considerable In that case the loads are the least likely to produce a corner break The uniform drop of temperature, therefore, is not likely to be important as a contributory cause of a corner break

Consider next the central of a large panel Assume that the friction is sufficient to make $\epsilon_x = 0$ and $\epsilon_y = 0$ Then by solving equations (1), one finds

$$\sigma_x = \sigma_y = \frac{E\epsilon_t t}{1-\mu} \tag{2}$$

Consider finally an edge of the panel, parallel to the axis of x Consider a point of this edge at an appreciable distance from any corner, and assume the friction to be sufficient to make $\epsilon_x = 0$ Since, obviously, $\sigma_y = 0$, one finds by equations (1), at the edge,

$$\sigma_x = E\epsilon_t t \tag{3}$$

The stresses given by equations (2) and (3) may have to be added to those caused by the loads [1]

In a numerical example assume $E = 3{,}000{,}000$ pounds per square inch, $\mu = 0\,15$, $\epsilon_t = 0\,0000060$ per degree Fahrenheit, and $t = 50$ degrees Fahrenheit Then equations (2) and (3) give $\sigma_x = \sigma_y = 1059$ pounds per square inch in the central area, and $\sigma_x = 900$ pounds per square inch at the edge One may note that a shrinkage of 0 0003, an entirely possible value, would produce the same stresses

The possibility exists both in the case of low temperature in the cold season and in the case of shrinkage that a plastic flow of the concrete may relieve these stresses appreciably Otherwise the large stresses computed can be relieved only by the presence of joints or the forming of cracks, combined with a sufficient amount of sliding of the pavement on the subgrade

CURLING

Assume next that the temperature remains normal at the middle plane of the pavement, but is t degrees lower at the top of the pavement than at the bottom Assume a uniform temperature gradient

[1] See the paper by the writer, Stresses in Concrete Pavements Computed by Theoretical Analysis, *Public Roads*, April, 1926, or the same paper, with the title Computation of Stresses in Concrete Roads, Proceedings of the Fifth Annual Meeting of The Highway Research Board, December 3-4, 1925, Part I, 1926, p 90 Tables III and IV in this paper contain values of the stresses produced by a wheel load of 10,000 pounds in the central area and at the edge, respectively

through the thickness, h, of the slab Assume that the reactions of the subgrade are vertical only, and equal to kz per unit of area, where z is the deflection (positive downward), and k is a constant "modulus of subgrade reaction"[2] Denote by M_x and M_y the bending moments in the directions of x and y, respectively, per unit of width of cross-section, and by M_z the twisting moment in the directions of x and y, per unit of width of cross-section[3]

Then the curvatures of the elastic surface (the bent middle plane) will be, in the directions of x and y, respectively,

$$\left.\begin{aligned} -\frac{\partial^2 z}{\partial x^2} &= \frac{12}{Eh^3}\left(M_x - \mu M_y\right) + \frac{\epsilon_t t}{h}, \\ -\frac{\partial^2 z}{\partial y^2} &= \frac{12}{Eh^3}\left(M_y - \mu M_x\right) + \frac{\epsilon_t t}{h}, \end{aligned}\right\} \quad (4)$$

and the twist in the directions of x and y will be

$$-\frac{\partial^2 z}{\partial x\, \partial y} = \frac{12\,(1 + \mu)}{Eh^3} M_z \quad (5)$$

The thickness, h, and the difference of temperature, t, will be assumed from here on to be the same at all points of the slab

Consider first the central area of a long and broad unloaded panel At a sufficient distance from the edges the deflections z must be zero Then the equations just stated give

$$M_x = M_y = -\frac{Eh^2\epsilon_t t}{12\,(1 - \mu)}, \quad M_z = 0 \quad (6)$$

The corresponding tensile stress at the top is found by dividing $-M_x$ by the section modulus per unit of width, $\frac{h^2}{6}$ The tensile stress is the same in all horizontal directions, and is

$$\sigma_o = \frac{E\epsilon_t t}{2\,(1 - \mu)} \quad (7)$$

With $E = 3{,}000{,}000$ pounds per square inch, $\mu = 0\,15$, and $\epsilon_t = 0\,0000060$ per degree Fahrenheit, as before, and $t = 10$ degrees Fahrenheit, one finds the stress at the top,

$$\sigma_o = 106 \text{ lb per sq in}$$

With $t = -10$ degrees Fahrenheit, that is, with the temperature 10 degrees higher at the top than at the bottom, the same tensile stress

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[2] A discussion of the assumptions involved here, especially in regard to the reactions of the subgrade, may be found in the paper referred to in the previous footnote

[3] Concerning the character and action of these moments, see, for example, the paper by W A Slater and the writer, Moments and Stresses in Slabs, Proc Am Concrete Institute, v 17, 1921, p 415, especially, p 424, or, National Research Council, Reprint and Circular Series, No 32, especially p 10

will be found at the bottom of the pavement This stress may have to be added to the stress produced by wheel loads [4]

Consider next a slab which has an edge along the axis of x, and which extends infinitely far in the directions of positive and negative x and positive y With the difference of temperature between top and bottom the same all over the slab, the deflection, z, will be a function of y only, independent of x

One finds, then, by eliminating M_x in equations (4)

$$M_y = \frac{Eh^3}{12\,(1 - \mu^2)}\left(-\frac{d^2z}{dy^2} - \frac{(1 + \mu)\,\epsilon_t t}{h}\right) \qquad (8)$$

The reactions of the subgrade, kz, are the only external forces to be taken into consideration The expression kz may be applied when z is negative, provided that there is still contact between the slab and the subgrade This contact may be maintained by the weight of the pavement or by this weight in conjunction with loads the effects of which are to be added afterward to the effects of the variation of temperature

The equilibrium of any small element of the slab requires, accordingly,

$$\frac{d^2 M_y}{dy^2} = kz \qquad (9)$$

By substituting M_y from equation (8) one finds:

$$\frac{Eh^3}{12\,(1 - \mu^2)}\frac{d^4z}{dy^4} + kz = 0 \qquad (10)$$

Here, as in other investigations of slabs on elastic subgrade, it is expedient to introduce the linear distance

$$l = \sqrt[4]{\frac{Eh^3}{12\,(1 - \mu^2)\,k}}, \qquad (11)$$

which is called the *radius of relative stiffness* This distance is a measure of the stiffness of the slab relative to the stiffness of the subgrade Table I gives values of the radius of relative stiffness for different values of h and k [5]

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[4] See Table III in the paper, Stresses in Concrete Pavements, etc , l c

[5] Table I is the same as Table I in the paper, Stresses in Concrete Pavements, etc , l c

TABLE I

VALUES OF THE RADIUS OF RELATIVE STIFFNESS, l, FOR DIFFERENT VALUES OF THE SLAB THICKNESS, h, AND OF THE MODULUS OF SUBGRADE REACTION, k, COMPUTED FROM EQUATION (11)

E = 3,000,000 pounds per square inch, μ = 0 15

Thickness of slab in inches h	Radius of relative stiffness, l in inches		
	k = 50 lb /in ³	k = 100 lb /in ³	k = 200 lb /in ³
4	23 91	20 11	16 92
5	28 28	23 78	20 00
6	32 40	27 26	22 92
7	36 40	30 60	25 73
8	40 23	33 83	28 44
9	43 94	36 95	31 07
10	47 55	40 00	33 62
11	51 08	42 94	36 11
12	54 52	45.84	38 56

With l introduced, equation (10) becomes.

$$l^4 \frac{d^4z}{dy^4} + z = 0. \qquad (12)$$

In the present case the functions z and $\frac{dz}{dy}$ must converge toward zero when y increases indefinitely Furthermore, at the edge, $y = O$, the bending moment M_y and the vertical shear $\frac{dM_y}{dy}$ must be zero. The following solution is correct because it satisfies these end conditions as well as equation (12).

$$z = - z_o \sqrt{2} \cos\left(\frac{y}{l\sqrt{2}} + \frac{\pi}{4}\right) e^{-\frac{y}{l\sqrt{2}}}, \qquad (13)$$

where

$$z_o = \frac{(1 + \mu)\, \epsilon_t t}{h} l^2 = \epsilon_t t \sqrt{\frac{(1 + \mu)\, Eh}{12\,(1 - \mu)\, k}} \qquad (14)$$

is the reversed (upward) deflection at the eyge, $y = 0$.

By substituting these expressions in equation (8) one finds the bending moment M_y By dividing M_y by the section modulus per unit of width, $\frac{h^2}{6}$, one finds the tensile stress at the top of the slab in the direction of y,

$$\sigma_y = \sigma_o\left(1 - \sqrt{2} \sin\left(\frac{y}{l\sqrt{2}} + \frac{\pi}{4}\right) e^{-\frac{y}{l\sqrt{2}}}\right), \qquad (15)$$

where

$$\sigma_o = \frac{E \epsilon_t t}{2(1-\mu)} \qquad \text{(Same as equation (7))}$$

By solving the first of equations (4) for M_x with $\frac{\partial^2 z}{\partial x^2} = 0$, and dividing $-M_x$ by the section modulus, $\frac{h^2}{6}$, one finds the tensile stress at the top, in the direction of x,

$$\sigma_x = \frac{E \epsilon_t t}{2} + \mu \sigma_y = \sigma_o + \mu(\sigma_y - \sigma_o), \tag{16}$$

or, by substituting σ_y from equation (15),

$$\sigma_x = \sigma_o\left(1 - \mu\sqrt{2}\sin\left(\frac{y}{l\sqrt{2}} + \frac{\pi}{4}\right)e^{-\frac{y}{l\sqrt{2}}}\right) \tag{17}$$

The stresses σ_x and σ_y are principal stresses, one is the greatest, the other the smallest stress at the particular point, and there are no shearing stresses in the directions of x and y

Table II contains relative values of z, σ_y, and σ_x for different values of y, computed from equations (13), (15), and (17)

The full curve in Figure 1 (a) shows the same relative values of z_o The dotted curve shows the relative values of σ_y

The scales shown at the bottom of Figure 1 correspond to the values of l given in Table III By means of these scales one may measure the distances from the edge in feet

The next case to consider is that of an infinitely long strip of slab of finite width b Let the center line be along the axis of x Then the edges have the equations, $y = \pm\frac{b}{2}$

Assume again the difference of temperature between the top and the bottom of the slab to be constant, equal to t Then, as in the previous case, the deflection, z, will be a function of y only, independent of x The flexure will be governed again by equations (8) (9), (10), (12), and (16) The end conditions are in this case:

$$M = 0, \frac{dM}{dx} = 0, \text{at the edges } y = \pm\frac{b}{2}$$

The following solution is correct because it satisfies these end conditions and the differential equation (12) [6]

$$z = -z_o \frac{2\cos\lambda\cosh\lambda}{\sin 2\lambda + \sinh 2\lambda}\left[(-\tan\lambda + \tanh\lambda)\cos\frac{y}{l\sqrt{2}}\cosh\frac{y}{l\sqrt{2}} + (\tan\lambda + \tanh\lambda)\sin\frac{y}{l\sqrt{2}}\sinh\frac{y}{l\sqrt{2}}\right], \tag{18}$$

[6] Equations (13) and (18) and corresponding expressions for M_y and M_x were stated in a paper by the writer, Om Beregning af Plader pas elastisk Underlag med saerligt Henblik paa Spörgsmaalet om Spaendinger i Betonveje, Ingeniören (Copenhagen) v, 32, 1923, p 513, see p 524

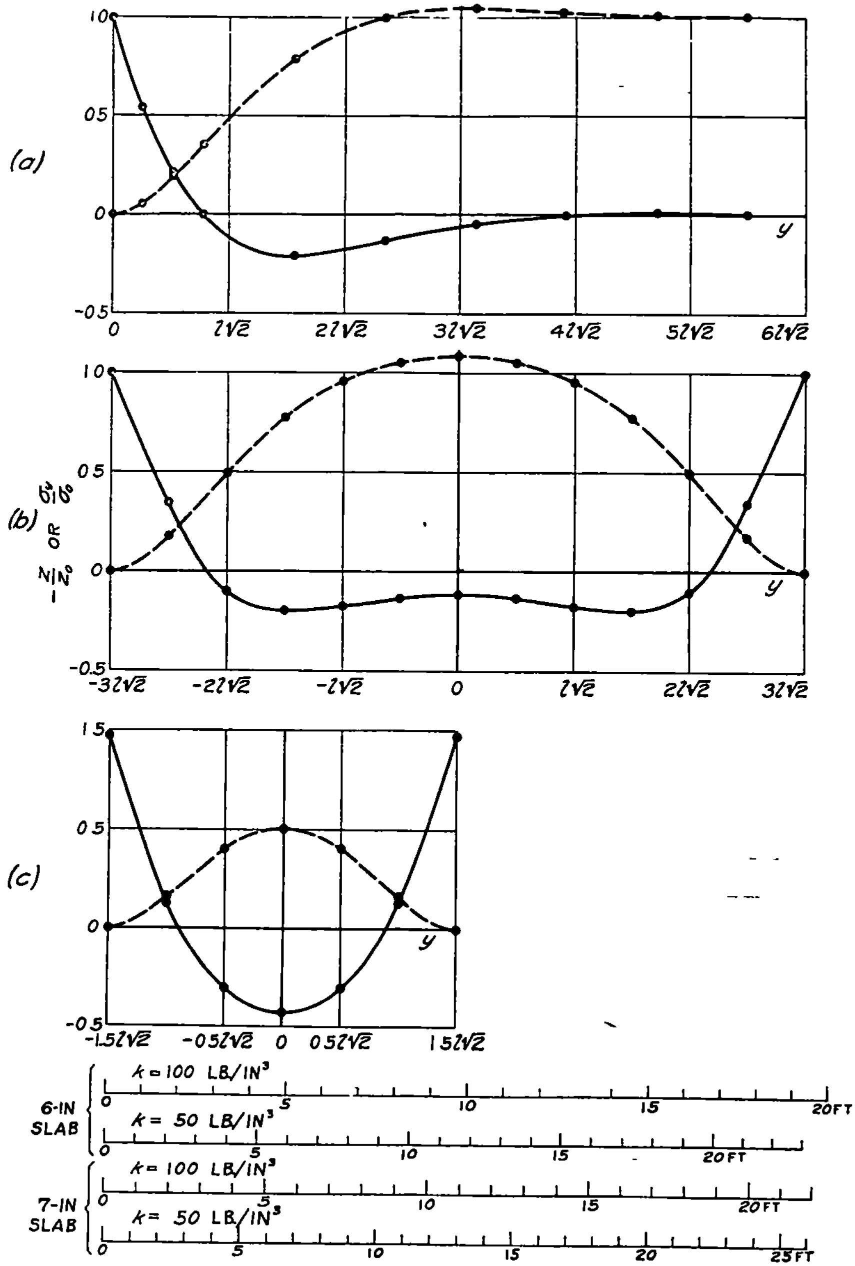

FIGURE 1.—Curling of pavements when the temperature is normal at the middle plane, and t degrees lower at the top than at the bottom. The horizontal distances y are measured in case (a) from the edge, and in cases (b) and (c) from the center line. The pavement covers, in case (a), the whole positive side of the axis of x, and in cases (b) and (c) the strip enclosed between the lines $y = \pm\frac{b}{2}$. The ordinates of the full curves represent relative values of the deflections, z, and those of the dotted curves represent relative values of the tensile stresses, σ_y, at the top of the pavement in the direction perpendicular to the edges. The curves are plotted from the values in Tables II and IV. Examples showing the significance of z_0 and σ_0 are given in Table III. The scales for y at the bottom of the figure apply for the particular values of l stated in Table III.

TABLE II

RELATIVE VALUES OF THE DEFLECTIONS, z, AND THE STRESSES σ_x AND σ_y, AT DIFFERENT DISTANCES, y, FROM THE EDGE, IN THE CASE OF CURLING DUE TO A TEMPERATURE t DEGREES LOWER AT THE TOP THAN AT THE BOTTOM OF THE PAVEMENT THE SLAB IS ASSUMED TO COVER THE WHOLE POSITIVE SIDE OF THE AXIS OF x THE VALUES HAVE BEEN COMPUTED FROM EQUATIONS (13), (15), AND (17) GRAPHICAL REPRESENTATION IN FIGURE 1 (a)

$\frac{y}{l\sqrt{2}}$	$\frac{z}{z_o}$	$\frac{\sigma_y}{\sigma_o}$	$\frac{\sigma_x}{\sigma_o}$ for $\mu = 0\ 15$	Remarks
0	−1 000	0	0 850	Edge
$\frac{\pi}{12}$	−0 544	0 057	0 859	
$\frac{\pi}{6}$	−0 217	0 191	0 879	
$\frac{\pi}{4}$	0	0 355	0 903	
$\frac{\pi}{2}$	0 208	0 792	0 969	Maximum deflection
$3/4\pi$	0 134	1 000	1 000	
π	0 043	1 043	1 006	Maximum stresses
$5/4\pi$	0	1 028	1 004	
$3/2\pi$	−0 009	1 009	1 001	
$7/4\pi$	−0 006	1 000	1 000	
2π	−0 002	0 998	1 000	
$9/4\pi$	0	0 999	1 000	
∞	0	1 000	1 000	

TABLE III

EXAMPLES OF VALUES OF THE REVERSED DEFLECTION, z_o, AT THE EDGE IN THE CASE OF FIGURE 1(a), COMPUTED FROM EQUATION (14), FOR $E = 3{,}000{,}000$ POUNDS PER SQUARE INCH, $\mu = 0\ 15$, $\epsilon_t = 0\ 0000060$ PER DEGREE FAHRENHEIT, $t = 10$ DEGREES FAHRENHEIT, FOR TYPICAL VALUES OF THE THICKNESS OF THE SLAB, h, AND THE MODULUS OF SUBGRADE REACTION k CORRESPONDING VALUE OF THE STRESS σ_o ACCORDING TO EQUATION (7) $\sigma_o = 106$ POUNDS PER SQUARE INCH THE VALUES OF l ARE TAKEN FROM TABLE I

h *inches*	k *lb /in* [3]	l *inches*	z_o *inches*
6	100	27 26	0 0085
6	50	32 40	0 0121
7	100	30 60	0 0092
7	50	36 40	0 0131

wheie

$$\lambda = \frac{b}{l\sqrt{8}}, \tag{19}$$

and, as before,

$$z_o = \frac{(1+\mu)\,\epsilon_t\,t}{h} l^2 = \epsilon_t\,t\sqrt{\frac{(1+\mu)\,Eh}{12\,(1-\mu)\,k}}$$

(Same as equation (14)).

By the process applied in the preceding case, one finds the corresponding principal stresses at the top of the slab

$$\sigma_y = \sigma_o\left[1 - \frac{2\cos\lambda\cosh\lambda}{\sin 2\lambda + \sinh 2\lambda}\left((\tan\lambda + \tanh\lambda)\cos\frac{y}{l\sqrt{2}}\cosh\frac{y}{l\sqrt{2}} + (\tan\lambda - \tanh\lambda)\sin\frac{y}{l\sqrt{2}}\sinh\frac{y}{l\sqrt{2}}\right)\right], \tag{20}$$

$$\sigma_x = \sigma_o + \mu\,(\sigma_y - \sigma_o), \tag{21}$$

where, as before,

$$\sigma_o = \frac{E\,\epsilon_t\,t}{2\,(1-\mu)}.$$ (Same as equation (7))

Numerical computations by equations (18) and (20) become a simple matter by use of the noteworthy book of tables published by *Hayashi* in 1921[7] These tables give the values of the hyperbolic and natural sines and cosines in a convenient form Results of computations of this sort are given in Tables IV and V The nature of the results is explained in the headings of the tables

Figure 1, (b) and (c), and Figure 2 represent the same material graphically

According to the scales at the bottom of Figure 1 the diagram (b) may represent, for example, a pavement, 20 feet wide, without center joint and still uncracked The maximum stress, σ_y, at the center (see also Table IV), is 1 084 σ_o If this pavement cracks along the center line, the left half of it will be represented by the diagram (c) The maximum value of σ_y becomes 0 508 σ_o. With $\mu = 0\,15$ the corresponding stresses, σ_x, in the direction of the road become, according to equation (21). in case (b),

$$\sigma_x = \sigma_o(1 + 0\,15 \quad 0\,084) = 1\,013\,\sigma_o$$

in case (c),

$$\sigma_x = \sigma_o(1 - 0\,15 \quad 0\,492) = 0\,926\,\sigma_o.$$

That is, in order that the stress σ_x shall be relieved in a manner comparable to that of σ_y, transverse cracks or joints are required At the edge, without transverse cracks or joints, the stresses are the same before and after the longitudinal crack has formed,

$$\sigma_y = 0, \quad \sigma_x = 0.85\,\sigma_o.$$

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[7] *K Hayashi*, Funfstellige Tafeln der Kreis—und Hyperbelfunktionen sowie der Funktionen e_x und e^{-x} mit den naturlichen Zahlen als Argument, Berlin and Leipzig, 1921 (Walter de Gruyter & Co)

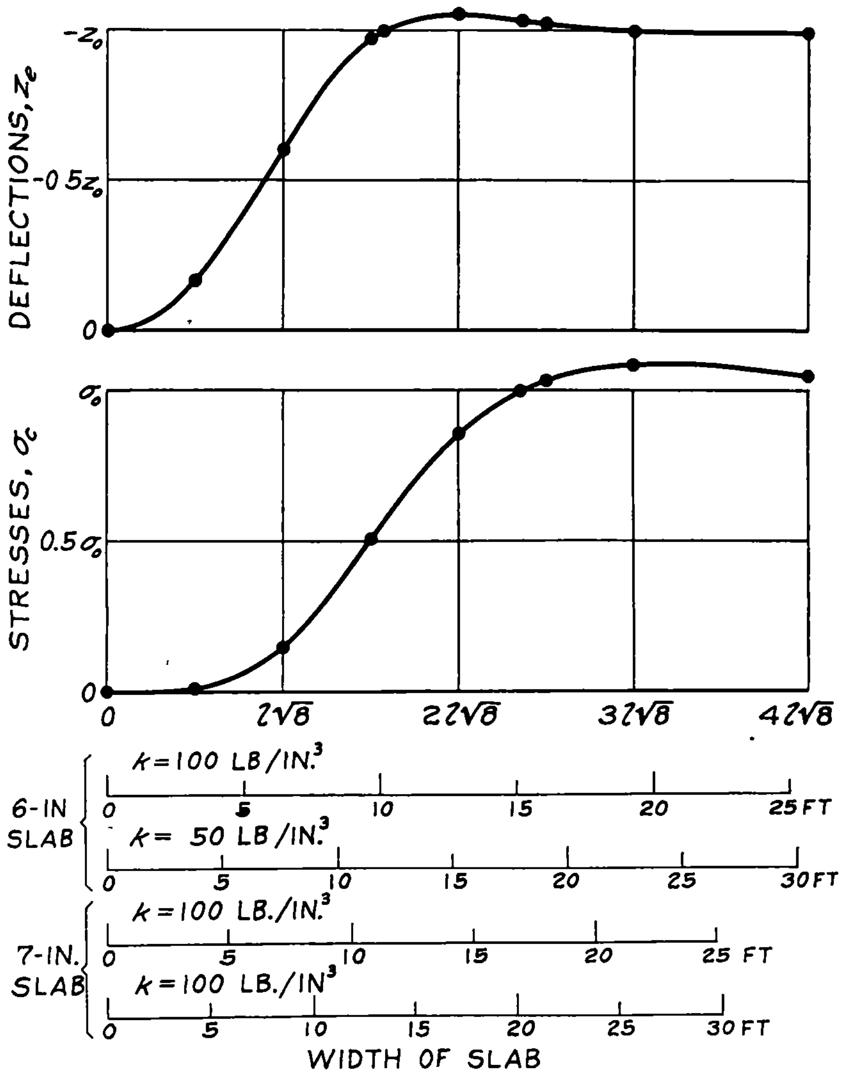

FIGURE 2—Curling of a strip of pavement of width b, when the temperature is normal at the middle plane, and t degrees lower at the top than at the bottom Values of the deflections, z_0, at the edge, and the stresses, σ_c, across the center line, for different values of b, are plotted from Table V The scales for b at the bottom of the figure apply for the particular values of l stated in Table III

TABLE IV

TWO EXAMPLES OF CURLING OF A STRIP OF PAVEMENT OF WIDTH b, WHEN THE TEMPERATURE AT THE TOP OF THE PAVEMENT IS t DEGREES LOWER THAN AT THE BOTTOM RELATIVE VALUES OF THE DEFLECTIONS, z, AND THE STRESSES, σ_y, ACROSS THE STRIP, AT DIFFERENT DISTANCES, y FROM THE CENTER LINE, FROM EQUATIONS (18) AND (20) GRAPHICAL REPRESENTATION IN FIGURE 1 (b) AND (C) EXAMPLES OF NUMERICAL VALUES OF z_o AND σ_o ARE GIVEN IN TABLE III

$\frac{y}{l\sqrt{2}}$	$b = 6l\sqrt{2}$		$b = 3l\sqrt{2}$	
	$\frac{z}{z_o}$	$\frac{\sigma_y}{\sigma_o}$	$\frac{z}{z_o}$	$\frac{\sigma_y}{\sigma_o}$
0	0 1126	1 084	0 4320	0 508
0 5	0 1326	1 056	0 3049	0 405
1	0 1773	0 959	−0 1260	0 163
1 5	0 1980	0 775	−0 972	0
2	0 1021	0 496		
2 5	−0 3476	0 178		
3	−1 003	0		

TABLE V

CURLING OF A STRIP OF PAVEMENT OF WIDTH b, WHEN THE TEMPERATURE AT THE TOP OF THE PAVEMENT IS t DEGREES LOWER THAN AT THE BOTTOM RELATIVE VALUES OF THE DEFLECTIONS, z_e, AT THE EDGE, AND OF THE STRESSES, σ_c, ACROSS THE CENTER LINE, FOR DIFFERENT VALUES OF b, COMPUTED FROM EQUATIONS (18) WITH $y = \frac{b}{2}$, AND (20) WITH $y = 0$ GRAPHICAL REPRESENTATION IN FIGURE 2 EXAMPLES OF NUMERICAL VALUES OF z_o AND σ_o ARE GIVEN IN TABLE III

$\frac{b}{l\sqrt{8}} (= \lambda)$	$\frac{b}{l}$	$\frac{z_e}{z_o}$	$\frac{\sigma_c}{\sigma_o}$
∞	∞	−1 000	1 000
4	11 31	−0 999	1 052
3	8 49	−1 003	1 084
2 5	7 07	−1 026	1 032
2 365	6 69	−1 036	1 000
2	5 66	−1 057	0 856
1 571	4 44	−1 000	
1 5	4 24	−0 972	0 508
1	2 828	−0 599	0 148
0 5	1 414	−0 165	0 010

Table VI shows numerical values obtained by this procedure on the basis of the values assumed in connection with Table III Stresses like those in Table VI may have to be combined with the stresses resulting from a superimposed uniform drop of temperature (drop of temperature in the middle plane of the slab), with forces of friction active between the slab and the subgrade

TABLE VI

EXAMPLE OF STRESSES, IN POUNDS PER SQUARE INCH, OCCURRING IN A 20 FOOT PAVEMENT WITHOUT TRANSVERSE CRACKS OR JOINTS, BEFORE AND AFTER A LONGITUDINAL CRACK HAS FORMED ALONG THE CENTER LINE THE TEMPERATURE AT THE TOP OF THE PAVEMENT IS ASSUMED TO BE 10 DEGREES FAHRENHEIT LOWER THAN AT THE BOTTOM $E = 3{,}000{,}000$ POUNDS PER SQUARE INCH; $\mu = 0\ 15$, $\epsilon_t = 0\ 0000060$ PER DEGREE FAHRENHEIT, $\sigma_o = 106$ POUNDS PER SQUARE INCH (COMPARE TABLE III)

Stress	Before cracking longitudinally		After cracking longitudinally	
	Center of 20 ft strip	Edge	Center of 10 ft strip	Edge
Transverse stress, σ_y	115	0	54	0
Longitudinal stress, σ_x	107	90	97	90

By assuming $t = -10$ degrees Fahrenheit instead of $+10$ degrees, that is, by assuming the top of the pavement to be 10 degrees warmer than the bottom, the tensile stresses given in Table VI will still exist, but will be at the bottom of the pavement instead of at the top These tensions at the bottom are significant in that they may have to be combined with the tensions due to wheel loads, both in the central area and at the edge[8]

It remains to discuss the case of a rectangular panel Let the edges have the equations $x = \pm\frac{B}{2}$ and $y = \pm\frac{b}{2}$ Let $z = f(y)$ represent the solution for $B = \infty$ and b finite, and let $z = F(x)$ represent the analogous solution for B finite and $b = \infty$ Assume for the time being that Poisson's ratio is zero Then, with the finite values of both B and b, the following solution will satisfy all the conditions

$$z = f(y) + F(x) \tag{22}$$

[8]Values of these stresses due to wheel loads are given in Tables III and IV in the paper Stresses in Concrete Pavements, l c

With this solution the stresses σ_y may be found as if B were infinite, and the stresses σ_x as if b were infinite

It is a fact, however, that Poisson's ratio, μ, is not equal to zero With $\mu = 0\ 15$ the influence of this ratio is exemplified in Table VI, where the stresses σ_x vary from 90 to 107 pounds per square inch If Poisson's ratio were zero, these stresses would all be 90 pounds per square inch The variation is a matter of 18 per cent If errors of this magnitude can be tolerated in a first approximation, then the solution indicated by equation (22) may be accepted as a first approximation It may be noted that the radius of relative stiffness, l [see equation (11)] is affected only slightly by a variation of Poisson's ratio, and the values of l given in Table I, therefore, may still be used

This consideration will be applied for the purpose of estimating the influence of curling due to a difference of temperature upon the corner break The stresses due to the variation of temperature must be combined, in this case, with the stresses due to a wheel load acting close to the corner Assume that the slab is 7 inches thick and that the resultant of a wheel load equal to 5000 pounds is applied 4 inches from each of two edges which form the rectangular corner Assume that the modulus of subgrade reaction is $k = 100$ lb /in [3] Then Table I gives $l = 30\ 60$ inches According to a previous paper[9] the load produces in this case a tensile stress at the top equal to $\sigma_c = 0\ 5 \quad 390 = 195$ pounds per square inch This stress occurs in a section which intersects the edges at a distance from the corner approximately equal to

$$x_1\sqrt{2} = 2\sqrt{2}\sqrt[4]{4\sqrt{2}\quad 30\ 60} = 37\ 2 \text{ inches} = 0\ 860\, l\sqrt{2}.$$

Both Figure 1 (a) and Figure 1 (b) indicate at this distance from the left of the diagram a temperature stress $\sigma = 0\ 40\,\sigma_o$ Figure 1 (c) gives $\sigma = 0\ 34\,\sigma_o$ Since these temperature stresses are not maximum values, a somewhat larger value, for example, $\sigma = 0\ 5\,\sigma_o$, should be assumed to combine with the stress due to the load, $\sigma_c = 195$ pounds per square inch With $t = 10$ degrees Fahrenheit, and the other numerical values as in the previous examples, except that $\mu = 0$, one finds $\sigma_o = 90$ pounds per square inch, that is, $\sigma = 45$ pounds per square inch Since the stresses σ_c and σ belong in two different sections, the combined maximum stress will be, in this case, approximately

$$\sigma_c + \frac{1}{2}\sigma = 195 + \frac{1}{2}\ 45 = 218 \text{ pounds per square inch}$$

[9] Stresses in Concrete Pavements, l c, Table II and equation (4)