3 That repeated impacts which stressed the concrete to about 55 per cent of its static modulus of rupture would eventually cause failure

4. That below this fatigue limit equivalent loads, without regard to their method of application (whether static or impact), produce the same stress, while beyond this limit it is indicated that the difference in behavior is due to the duration of the load application.

SUGGESTIONS FOR FURTHER RESEARCH

1. A study of the balance of various thickened-edge cross-sectional designs

2 A study of the efficiency of various forms of longitudinal joints with a view to determining what additional edge strengthening, if any, is needed by each

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3. The Six-Wheel Truck and the Pavement, L. W Teller, Public Roads, Vol. 6, No. 8, October, 1925

4. Static and Impact Strains in Concrete, J. T Thompson, Public Roads, Vol 7, No 5, July, 1926

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THEORY OF CONCRETE PAVEMENT DESIGN

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The structural design of a concrete pavement is based on an estimate of the stresses occurring under given circumstances Two papers dealing with computation of stresses in concrete pavements were published in the Proceedings of the preceding two annual meetings of the Highway Research Board The first of these papers deals with stresses and deflections due to loads, the second with stresses and deflections due to variations of temperature

The analysis rests on certain assumptions First, the wheel loads are dealt with as static loads—that is, the inertia action of the pavement and subgrade under a sudden or moving load is not considered Secondly, the subgrade is assumed to be uniform, without local soft or hard spots, and to be everywhere in contact with the slab. The stiffness of the subgrade is taken into account only by introducing the so-called "modulus of subgrade reaction," which is expressed as pounds of reaction per square inch of area per inch of deflection—that is, in pounds per cubic inch This modulus (which is denoted by k) may have different values in different cases, but in dealing with any particular case it is assumed to be a constant, applicable to the whole area of pavement which is under consideration With this assumption of a constant modulus, one cannot expect to find the true distribution of reactions of the subgrade. It is argued, however, that by a proper choice of the value of the modulus, as long as the loads remain of one type—that is, wheel-loads on top of the pavement—one may obtain a fairly accurate picture of the distribution of the deflections, the stresses, and the reactions Estimates of the values of the modulus may be obtained empirically by comparing results of the analysis with the deflections found by tests on the road In the tables given in the two papers, three values of the modulus of subgrade reaction are introduced: $k = 50$, 100, and 200 pounds per cubic inch. The two extreme values were intended to represent a poor subgrade and a good subgrade, respectively, both in wet condition.

The pavement itself is assumed to act as a homogeneous elastic slab—that is, the influence of reinforcement, or of cracks extending through a part of the thickness, is left out of consideration

After setting up these assumptions, the pavement is treated according to the mathematical theory of elasticity—as a slab capable of resisting bending and twisting in all directions and preserving its continuity at all points.

The first of the two papers contains formulas and tables for the stresses produced by a single wheel-load in a pavement of uniform thickness under the following circumstances.

Case I. The wheel-load is close to a rectangular corner of a panel of pavement, and tends to produce a corner break The center of the load is at the small distance a, from each of the two intersecting edges (distances $a^1 = a\sqrt{2}$ from the corner itself). Formula (7) and Table II in the paper give values of the stresses

Case II The wheel-load is in the interior of the area of the panel, at a considerable distance from any edge The load is assumed in the analysis to be distributed uniformly over the area of a small circle with radius a. Formula (11) and Table III in the paper give values of the stresses.

Case III. The wheel-load is at the edge of the panel, at a considerable distance from any corner The load is assumed to be distributed uniformly over the area of a small semi-circle with the center at the edge, and with radius a Formula (12) and Table IV give values of the stresses.

The dimension a introduced in each of the three cases is to be chosen according to the expected size of the area of contact between the tire and the pavement It need not be the same in the three cases.

The formulas were derived and the tables computed for a modulus of elasticity of the concrete of 3,000,000 pounds per square inch and a Poisson's ratio of 0.15.

The formulas, as they stand, are simplified transcriptions, which give approximately the same values as the more elaborate expressions to which the analysis leads directly.

The three tables give values of the stresses for different thicknesses of the pavement The values correspond to a wheel-load of 10,000 pounds With other values of the load the stresses should be changed in proportion.

The first of the two papers contains, furthermore, diagrams of deflections and bending moments showing the effects of a load at some distance from the load. By use of these diagrams in connection with the tables, one may determine, by superposition, the combined effect due to a group of wheel-loads (four wheels of one truck, or the wheels of two trucks passing each other)

The second of the two papers deals with the effects of two types of changes of temperature the slow seasonal change from summer to winter, and the quick change from a warm day to a cold night In the case of the slow change, the temperature may be assumed to be the same through the thickness of the pavement The quick change, on the other hand, produces a temperature gradient between the top and the bottom, resulting in a tendency of the pavement to curl In the discussion, emphasis is put upon the combined action of loads and changes of temperature

Numerical examples in both papers show how the formulas, tables, and diagrams may be used.

For comparison of the analysis with tests, reference is made especially to two papers which, like the two papers on theory, appeared as items of the reports of this committee for the two preceding years See the bibliography W3 and W4

FACTS BROUGHT OUT BY THE INVESTIGATIONS

1 A structural analysis of the concrete road according to the theory of elasticity is possible

2 The analysis which has been made is subject to limitations on account of the assumptions introduced, especially those concerning the subgrade The analysis, nevertheless, can be said to have furnished a truthful picture of the structural action of the concrete road.

3. A change of the modulus of subgrade reaction by doubling it or cutting it in two does not produce large changes of the critical stresses It should be pointed out in this connection that the stiffness of the subgrade varies in fact within a very wide range.

4. A wheel-load at the corner of a panel of pavement produces the greatest tensile stress at some distance from the corner. The average stress at the top of the section of failure in case of corner break is appreciably smaller than the value computed by the cantilever formula (that is, the formula by which the load is considered to be a concentrated force at the corner).

5. A wheel-load at some distance from any corner, either at the edge or in the interior area of the panel, produces the greatest tensile stress at the bottom of the pavement, directly under the load

6 These maximum tensile stresses at the bottom are highly localized stresses, occurring only within a small region, and falling off rapidly in all directions A localized stress of a given magnitude is less dangerous than a stress of the same magnitude distributed over a greater area

7 With a constant thickness of the pavement a given wheel-load produces a greater stress at the edge than in the interior area Balanced design requires a strengthened edge.

8. The size of the area of contact between the tire and the pavement has some influence upon the maximum stress. With a given total pressure, an increase of the area of contact leads to a decrease of the critical stress It should be pointed out that a change to a larger tire may result in the case of impact in a

greater hardness of the tire and a greater total wheel pressure, whereby the beneficial effect of the larger area of contact is counteracted

9 At the edge the critical stress is unidirectional tension, while in the interior there is a state of stress with tension in all horizontal directions Poisson's ratio of lateral contraction has some influence in this connection The result is that a stress of a given magnitude, when found in the interior, is accompanied by a smaller strain than is the same stress when found at the edge Conversely, a given strain represents a greater stress when observed in the interior area than when observed at the edge. These considerations are of some importance when the significance of the stresses is to be judged.

10. The essential portion of any critical stress produced by loads is due to the nearest wheel Other wheels have some influence, but it is not large

11 The preceding statement applies to the six-wheel truck In typical cases which were examined, the adjacent rear wheels were found partly to counteract each other in their influence upon the critical stresses, with the result that the critical stress is accounted for practically by the action of one wheel only.

12 The stresses computed by the formulas applying to the seasonal changes of temperature may be relieved by plastic flow of the concrete or by sliding of the pavement on the subgrade Sliding is made possible by transverse and longitudinal joints or by the forming of cracks.

13. Changes of temperature between day and night produce a tendency of the pavement to curl Bending stresses are set up, depending upon the extent to which the pavement is prevented from following this natural tendency In typical cases the analysis showed the transverse bending stress cut in two by introduction of a longitudinal center joint

STRONG INDICATIONS

1. In view of the assumptions introduced and the corresponding limitations of the analysis, it would be improper to look upon the definite stresses computed and stated in the tables as absolute values It is proper, however, to look upon them as relative values

2 It is proper to use the analysis for the purpose of design in the following way Compute the stresses which should occur according to the analysis in certain pavements which have proved to be satisfactory, use these stresses as working stresses in deciding on the thickness of other pavements

3 It is reasonable that by this method one should arrive at different working stresses for the corner, the interior, and the edge, and that all of these working stresses should be rather high

4 It is proper to use the formulas and tables in deciding what modification of design would be permissible or necessary if the maximum wheel-load were reduced or increased by a certain ratio Thus the analysis may be used for the purpose of apportioning the cost of the pavement to the different kinds of traffic, or deciding what can be saved by eliminating some of the heaviest vehicles, or estimating the additional cost of providing for a heavier traffic

5 The assumption that the pavement remains in contact with the subgrade is farther from being justifiable in the case of curling due to a difference of temperature between the top and the bottom than it is in the case of deformation due to wheel-loads. The analysis gives a fair general idea of the phenomenon of the curling of the pavement due to temperature variation alone, but on account of this assumption of contact, it leads, on the whole, to too small upward deflections. On the other hand, if contact between the pavement and the subgrade is brought about within the area where the important stresses occur by the action of wheel-loads at the same time, the analysis is likely to lead to a fair estimate of the combined stresses due to wheel loads and the temperature difference

6 It is reasonable that two different values of the modulus of subgrade reaction should be used in analyzing the effects of loads and temperature variations, respectively

SUGGESTIONS FOR FURTHER WORK

1 The tables of stresses were computed for three values of the modulus of subgrade reaction: $k = 50$, 100, and 200 pounds per cubic inch It is a simple task to extend the tables so as to include a wider range of values of the modulus The following set is suggested: $k = 25$, 50, 100, 200, 400, 800 pounds per

cubic inch It may be observed in this connection (see formulas (11) and (12) and Tables III and IV in the first paper) that in changing the modulus k from 100 to 200, the stress at the edge or in the interior area is reduced by the same amount as by the change from 50 to 100 The same reduction occurs within the range of applicability of the formulas, each time k is doubled That is, it applies in changing k from 200 to 400, and again in changing k from 400 to 800 pounds per cubic inch This scheme is not applicable to the stresses producing corner breaks

2 The blow-up of pavements should be accounted for.

3 It is desirable that the following influences upon the stresses be studied theoretically.

- a The gradual diminishing of the thickness from the edge of the pavement toward the center.
- b Local soft or hard spots in the subgrade
- c Horizontal components of the reactions of the subgrade.
- d The dynamic action of the pavement and subgrade under impact and under a load moving rapidly

4. Experiments to determine values of k

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