REPORT OF COMMITTEE ON STRUCTURAL DESIGN OF ROADS

A T GOLDBECK, *Chairman*

Director, Bureau of Engineering, National Crushed Stone Association

SPACING OF DOWELS

H M WESTERGAARD

Professor of Theoretical and Applied Mechanics, University of Illinois

The dowels under discussion have the purpose of transmitting vertical forces across joints in concrete pavements Their effectiveness depends on their ability to relieve stresses in the case shown in Figure 1, where the wheel load, P, acts on one side of the joint, midway between two dowels Dowels at A, B, C, and D, with spacing s, transmit forces Q and R as shown Dowels farther away need not be considered as active in this case

The two panels are assumed to be fairly large Consider first the case of a load $P = 1$ acting alone at an edge of a slab, with no dowels active at this edge Denote by z_n, with $n = 0, 1, 2, 3$, the deflection produced in this case at the edge at the distance $\frac{ns}{2}$ from the load Then, by specifying that the two slabs, with their dowels acting, shall have the same deflection at A and B, one finds

$$2Q(z_0 + z_2) - 2R(z_2 + z_4) = Pz_1 \tag{1}$$

Similarly, by specifying that the slabs shall have the same deflection at C and D, one finds

$$-2Q(z_2 + z_4) + 2R(z_0 + z_6) = -Pz_3 \tag{2}$$

By solving the two equations, one finds

$$2Q = \frac{(z_0 + z_6)z_1 - (z_2 + z_4)z_3}{(z_0 + z_2)(z_0 + z_6) - (z_2 + z_4)^2} P \tag{3}$$

$$2R = \frac{(z_2 + z_4)z_1 - (z_0 + z_2)z_3}{(z_0 + z_2)(z_0 + z_6) - (z_2 + z_4)^2} P \tag{4}$$

The deflections depend on the stiffness of the slab and the stiffness of the subgrade The stiffness of the slab depends on the modulus of elasticity, E, and Poisson's ratio, μ, of the concrete, and on the thickness of the slab, h The stiffness of the subgrade is expressed in terms of an empirical constant, k, called the modulus of subgrade reaction, and representing the reaction per unit of area per unit of deflection From these quantities one may compute the distance, called the radius of relative stiffness,

$$l = \sqrt[4]{\frac{Eh^3}{12\,(1 - \mu^2)k}} \tag{5}$$

The upper part of Figure 2 shows a diagram of deflections, the lower part of Figure 2 a diagram of bending moments, produced at the edge of a slab by a single load at the edge These diagrams are obtained from a previous paper [1] The distance l appears as a unit of the horizontal scale of the diagrams The deflections z_n may be measured in the upper diagram

Horizontal distances representing one foot are obtained by computing values of the ratio $\frac{12\text{ in}}{l}$ The two sets of scales at the middle of

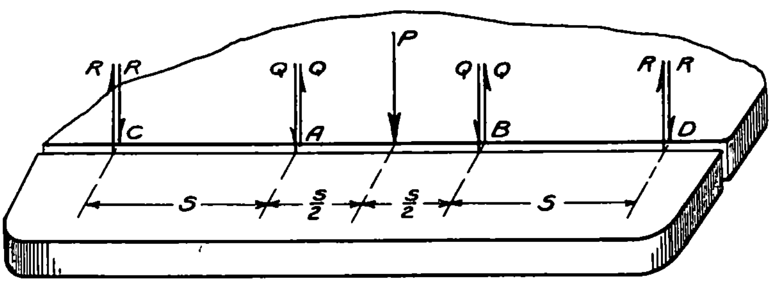

Figure 1. Forces Transmitted Through Dowels

Figure 2 were constructed in this manner In the case of a 7-inch pavement, for example, with $k = 50$ lb in^{-3}, distances are measured in feet on the fourth line from the top

The scale of the vertical distances in the diagram of deflections is immaterial so far as the present problem is concerned For the computation of the dowel reactions, Q and R, by equations 3 and 4, it is sufficient to know relative values of the deflections, z_n It is convenient to measure the deflections in the diagram using $2z_0$ as a unit (instead of inches or feet)

[1] H M Westergaard, "Computation of Stresses in Concrete Roads" Highway Research Board, Proc Meeting of Dec 3–4, 1925, Part I, p 90 (also in Public Roads, April, 1926) The assumptions of the analysis are discussed in that paper, especially those concerning the reactions of the subgrade Table 1 in the paper gives numerical values of l The diagrams in the present Figure 2 correspond to the case of Poisson's ratio, $\mu = 0\ 15$ The ordinates were obtained by interpolating between the corresponding values for $\mu = 0$ and $\mu = 0\ 25$ shown in Figures 8 to 11 in the paper referred to

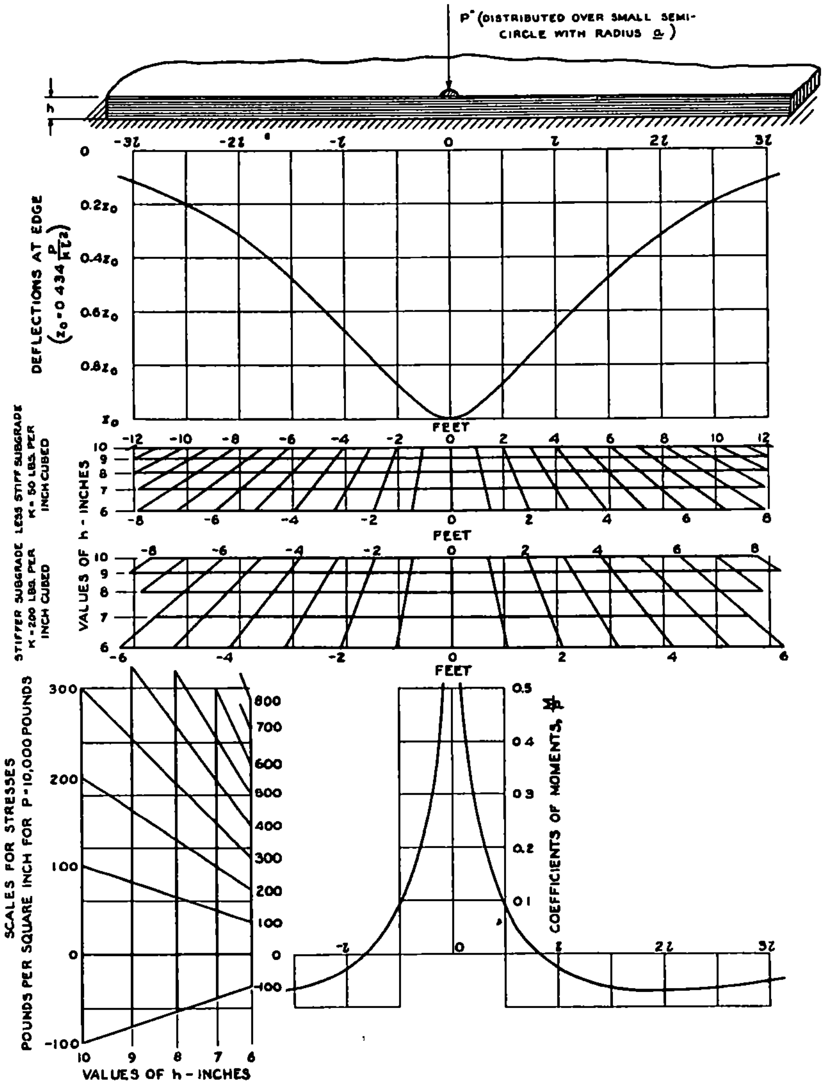

Figure 2. Deflections, Bending Moments, and Stresses at an Edge of a Slab Produced by a Single Load at the Edge. Numerical Values Assumed·

$E = 3{,}000{,}000$ lb. in.$^{-2}$, $\mu = 0.15$

Table I shows two numerical examples In one the spacing of the dowels is 2 feet in the other 3 feet

Let, σ = resultant tensile stress produced at the bottom of the slab directly under the wheel load by the joint action of the wheel load and the dowel reactions;

σ_P, σ_{2Q}, σ_{2R} = contribution to this stress resulting from the wheel load, P, alone, from the two dowel reactions Q alone, or from the two dowel reactions R alone, respectively

Then one may write

$$\sigma = \sigma_P + \sigma_{2Q} + \sigma_{2R} \qquad (6)$$

The stress σ_P may be taken from a formula or a table in the paper referred to in a previous footnote (Equation 12 or Table IV in the paper) [1] Assuming P to be distributed uniformly over the area of a semi-circle with the radius a, as indicated in Figure 2, and assuming $a = 4$ inches, one finds in both of the examples in Table I· $\sigma_R = 494$ lb in $^{-2}$

The lower part of Figure 2 shows the bending moments produced at the edge at some distance from the load when a single load is applied at the edge. The corresponding stresses, except in the immediate vicinity of the load, are found by dividing the moments by the section-modulus per unit of width, that is, by $\frac{h^2}{6}$ This diagram becomes a stress

TABLE I

Two Numerical Examples

Dowel Reactions, Q and R, determined by equations 3 and 4

Numerical values assumed in both examples $E = 3{,}000{,}000$ lb in $^{-2}$, $\mu = 0\ 15$, $k = 50$ lb in $^{-3}$, $h = 7$ in , $P = 10{,}000$ lb

Example number	s ft	Relative values of z_0	z_1	z_2	z_3	z_4	z_6	$2Q$ lb	$2R$ lb
1	2	0 500	0 466	0 404	0 335	0 270	0 160	5,750	795
2	3	0 500	0 436	0 335	0 240	0 160	0 060	5,630	692

diagram when the proper vertical scales are introduced Such scales are shown on the left of the diagram Stresses produced by a 10,000 pound load in a 7-inch slab, for example, may be measured on the second line from the right The horizontal scales are the same as those applying to the diagram of deflections Using the curve at the bottom of Figure 2 either as moment diagram or as stress diagram one may determine the influences of the two forces Q and the two forces R at the distances $\frac{s}{2}$ and $\frac{3s}{2}$, respectively

In the two numerical examples in Table I the section modulus per unit of width is $\frac{1}{6}\,7^2 = 8\ 167$ in ^{2} In the first example the two forces Q act at the distance $\frac{s}{2} = 1$ ft from P, and the forces R at the distance $\frac{3s}{2} = 3$ ft The corresponding coefficients of bending moments are, according to the diagram, 0 186 and $-0\ 023$, respectively Thus, by taking the values of $2Q$ and $2R$ from Table I, one finds, in the first example

$$\sigma_{2Q} = \frac{0\ 186\ (-5750\ \text{lb})}{8\ 167\ \text{in}^{2}} = -131\ \text{lb}\ \ \text{in}^{-2}$$

and

$$\sigma_{2R} = \frac{-0\ 023 \times 795\ \text{lb}}{8\ 167\ \text{in}} = -2\ \text{lb}\ \ \text{in}^{-2}$$

Thus the resultant stress becomes

$$\sigma = 494 - 131 - 2 = 361\ \text{lb}\ \ \text{in}^{-2}$$

That is, the effect of the dowels is to reduce the critical stress from 494 by 133 to 361 lb in $^{-2}$ In the second example one finds in a similar manner the coefficients of moments at the distances 1 5 ft and 4 5 ft to be 0 099 and −0 064, respectively The corresponding stresses then are computed to be $\sigma_{2Q} = -68$, $\sigma_{2R} = -5$ lb in $^{-2}$ That is, the dowels spaced 3 ft apart reduce the critical stress from 494 by 73 to 421 lb in.$^{-2}$

In both examples the values of R and σ_{2R} are small By ignoring the reactions R altogether, one finds from equation 1,

$$2\,Q = \frac{z_1}{z_0 + z_2}\,P \tag{7}$$

which gives, in the first example, $2Q = 5150$ lb , and in the second, $2Q = 5220$ lb The corresponding contributions to the critical stress are −117 and −63 lb in $^{-2}$, respectively These values are fairly close to the corresponding values, −133 and −73 lb in $^{-2}$, which were computed by the more accurate method.

One may conclude that it will be sufficiently accurate to consider as active only the two dowels nearest to the load Then, from equation 7 and by inspection of the deflection diagram in Figure 2 it is seen that roughly $2Q = 0\ 5P$, that is, if the dowels are strong and stiff enough to cause equal deflections of the two slabs at the points of connection, the dowels transmit about one-half of the total load The corresponding changes of stress due to the action of the dowels may be read quickly by use of the moment diagram in connection with the horizontal scales measuring feet and the vertical scales measuring stresses in pounds per square inch per 10,000 pound load

The significant result of the computations is that in the case of the seven inch slab and the subgrade specified the dowels spaced 2 feet apart bring about a material reduction of the critical stress On the other hand, reduction produced by dowels spaced 3 feet apart does not appear to be worth the cost of the dowels That is, if dowels are to be counted on, the spacing should be fairly close, probably not more than about 2 feet