coefficient Higher temperature ranges apparently produce higher coefficients

While wide differences in these coefficients can occur under extreme conditions, for the usual conditions to which pavement concrete is subjected the thermal coefficient of expansion may be expected to have an approximate value of from 0 0000055 to 0.0000060 per degree Fahrenheit.

A temperature differential between the upper and lower surfaces of a slab will cause a warping of the surface This may be sufficient to actually lift the edges or corners from the subgrade

TENSILE STRENGTH OF CONCRETE

There has been comparatively little research on the resistance of concrete to pure tension. Such data as are available indicate this property to be influenced by the same factors which affect the compressive strength, and further, that the effect of the type of aggregate is more marked

It is indicated by existing data that the tensile strength ranges from about 8 per cent to about 14 per cent of the compressive strength It is believed that for pavement concrete a fair assumption for tensile strength would be 10 to 12 per cent of the compressive strength.

COEFFICIENT OF FRICTION

Tests have shown that the coefficient of friction developed between the pavement and the subgrade varies with the smoothness, type of subgrade material and its moisture condition Its value ranges from less than 0 5 to 2 5 or more Also because of the nature of the subgrade material the coefficient tends to become larger under greater movement. Subgrades of the more usual types damp but firm may readily develop coefficients of 1 5 or 2.0.

CONTRACTION AND EXPANSION JOINTS

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Cracks in concrete pavements are objectionable and they should be avoided to an extent commensurate with economy in construction and maintenance.

TRANSVERSE JOINTS

It has been shown in the discussion on Control of Cracking that concrete varies in length with changing moisture and temperature conditions and that in the process of its resulting motion over the subgrade, frictional forces are set up as shown in Figure 1 which induce either tensile or compressive stresses in the slab in a direction parallel with the direction of traffic. If a decrease in temperature is accompanied by drying of the concrete, or if either of these phenomena takes place separately, tensile stresses are produced which will cause transverse cracks to form if the tensile strength of the concrete is exceeded On the other hand, during an increase of temperature or during a period of high moisture, expansion of the concrete results and the resistance to sliding offered by the subgrade produces compressive stresses The slab then is subjected to direct compression and acts much like a long column having a very large "length ratio " Finally when the stress becomes high enough the slab buckles or "blows up," the same phenomenon which occurs

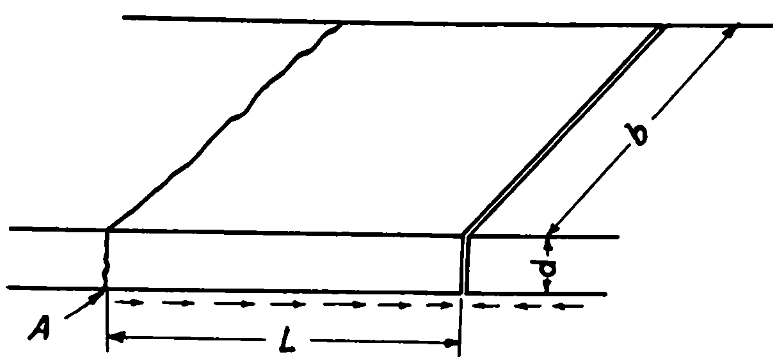


Figure 1

when a long column fails The question of joints must therefore be discussed from the standpoint of both contraction and expansion conditions

CONTRACTION JOINTS

In its simplest form, the theory applicable for conditions which obtain during a period of contraction may be expressed as follows:

Let L = Length in feet
S = tensile strength of concrete in pounds per square inch
f = coefficient of subgrade friction
w = weight of slab per square foot
b = width in feet
d = depth in inches
150 lbs = assumed weight of concrete per cubic foot

At failure when a transverse cracks forms at A, the total subgrade friction over the length L must equal the total tensile resistance of the slab Therefore·

$$fwLb = S12bd, w = \frac{d}{12} \times 150 = 12\ 5\ d$$

$$L = \frac{Sd\ 12}{fw} = \frac{Sd\ 12}{f \times 12\ 5d} = \frac{S \times 0\ 96}{f}$$

or

$$L = \frac{S}{f}$$ (Approximate but sufficiently exact formula)

It is seen from the above formula that the greater the tensile strength (S) of the concrete when contraction takes place, the greater will be the value for L· also the smoother the subgrade and hence the smaller the coefficient of friction (f), the greater the value for L that is, the further apart will be the transverse cracks Efficient curing to keep the concrete expanded and also to develop high tensile strength in the early period of its hardening are thus seen to be desirable, as is also a smooth subgrade rendered as frictionless as economically possible

Sometimes the temperature falls considerably within a few hours after the concrete is placed At such an early period the tensile strength of the concrete is low and the theoretical result is frequent transverse cracking

To arrive at an estimate of the frequency of transverse cracking it is essential to have numerical values for S, the tensile strength of the concrete, at the particular period when considerable contraction or shrinkage is taking place and also the value for the coefficient of friction f Assuming the tensile strength of concrete to be approximately $\frac{1}{12}$ of its compressive strength, 3000 pound concrete at 28 days would have the following tensile values

7 days	100 pounds
15 days	194 pounds
28 days	250 pounds
3 months	400 pounds

While the tensile strength of concrete at early periods is problematic and no extensive data are available, the above values are in accord with such data as do exist. Tests for the coefficient of friction made by the Bureau of Public Roads indicate values, depending upon the roughness of the subgrade, extending as high as 2 3 with a dry clay subgrade

Merely for illustrative purposes, let it be assumed that reasonable care is taken in the preparation of the subgrade and that the coefficient of friction is 2 0, further, that a sudden decrease in temperature ensues immediately after the concrete has hardened, when the tensile strength is only 75 pounds per square inch The distance between the first crack and an expansion joint would then be calculated as follows.

$$L = \frac{75}{2\ 0} = 37\ 5 \text{ feet}$$

If it should happen that the temperature is more or less uniform during the curing period of 15 days and that shrinkage does not begin until the expiration of this period, when the concrete has attained a tensile strength of 194 pounds per square inch, the distance between cracks will be 194 ÷ 2 0 = 97 feet

Should the subgrade be exceptionally smooth with a coefficient of friction of only 1 0 instead of 2 0 the distance between cracks will be 194 ÷ 1 0 = 194 feet If contraction joints are to be provided so as to prevent transverse cracks forming due to contraction and shrinkage, obviously the conditions during the construction will have to be considered Thus, if the road is constructed at a time of the year when the night temperatures are considerably lower than the day temperatures, a small value of S must be assumed because the tensile strength of the concrete may be exceeded during the first night of hardening when the tensile resistance is very low [1]

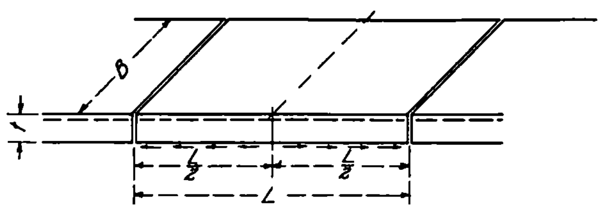
Full data do not exist which will permit of the theoretically correct spacing of contraction joints to prevent additional contraction cracks from forming but in the absence of such data it is best to assume the possibility of shrinkage and of contraction due to temperature during the very early stages of hardening, for it has often been observed that contraction cracks do form during the 24-hour period following the placing of the concrete A tensile strength of 75 pounds per square inch for the concrete during this period is not unreasonably low and a coefficient of friction of 2 0 may be quite usual A minimum spacing of cracks, and therefore of contraction joints to prevent such cracks from forming, may therefore be placed at 75 — 2 0 or 37 5 feet The uncertainty of the tensile stress of concrete during the early period of hardening coupled with the uncertainty of the frictional resistance and combined also with the effect of bending stresses due to settlement of the subgrade and warping of the slab due to temperature change may also create additional stress which will still further reduce the spacing of cracks Even though contraction cracks are prevented during the initial stages of hardening by the spacing of contraction joints at intervals of 30 to 40 feet it is not unlikely that the action of traffic, by the creation of high bending stresses will produce intermediate cracks, thus further dividing these 30 to 40 feet intervals

[1] See "Public Roads" July, 1924, p 19, "Friction Tests of Concrete on Various Sub-bases," by A T Goldbeck

THE INTER-RELATION OF TRANSVERSE JOINT SPACING AND LONGITUDINAL STEEL

If all transverse cracking is to be prevented, unquestionably the transverse joints would have to be very closely spaced when adverse subgrade and climatic conditions obtain. And it is here that longitudinal reinforcing steel plays an important rôle in preventing these intermediate cracks from spreading There is a distinct relationship between the amount of longitudinal steel to be used and the spacing of transverse

SPACING OF TRANSVERSE JOINTS AS INFLUENCED BY LONGITUDINAL STEEL



Let L = Spacing of Transverse Joints
a = Area of Steel
B = Width of Pavement
S = Allowable Tension in Concrete, S_1 in Steel
f = Coefficient of Friction at Subgrade
W = Weight of Concrete per Square Foot
t = Thickness of Pavement
E_s = Modulus of Elasticity of Steel E_c of Concrete

For Condition of no Cracking $f\frac{L}{2}WB = B12tS + a\frac{E_s}{E_c}S$ ---(1)

For Condition of no Wide Cracking $f\frac{L}{2}WB = aS_1$ ----------------(2)

Results shown in Table are obtained from above formulas, assuming
$S = 30$ #□" at 10-15 days
$\frac{E_s}{E_c} = 10$
$f = 2.0$ $t = 6"$
$W = 75$ $B = 18'$
$S_1 = 25000$

Amount of Longitudinal Steel	Required Spacing of Transverse Joints	
	For no Intermediate Crack	For no Wide Cracks
Plain Concrete	288	288
4-¾"⌀=1.76□"=a	292	326
8-¾"⌀=3.52□"	296	652
12-¾"⌀=5.28□"	300	978

Figure 2

joints, for steel of a given cross-sectional area is capable of dragging only a given length of slab over the subgrade before its tensile resistance is exceeded. It becomes necessary, therefore, to space the joints in a reinforced concrete road in such a manner that the allowable tensile stress in the steel is not exceeded

Two conditions of cracking must be considered when longitudinal steel is used.

(1) The condition of no cracking because of the use of large amount of steel and a given joint spacing

(2) The condition of no wide cracking (See Figure 2)

In Figure 2 the value $S = 30$ pounds at 10 to 15 days has been assumed as a safe unit stress at this period. This value gives the concrete a factor of safety of approximately 6 or 7 against direct tension The values above are used only for illustrative purposes and are not necessarily figures which should be used in design They are very interesting however in showing a number of points In the first place, it is evident that the use of a large amount of steel is not warranted if its purpose is to extend the possible distance between joints, for as will be seen from the above table there is an increase of only 1 2 feet between the joint spacing when no steel is used as compared with the use of 12 three-quarter-inch longitudinal bars. In this case the joint spacing has been extended from 28 8 up to only 30 feet. On the other hand, it is seen that the larger the amount of steel the greater can be the distance between joints without danger of intermediate wide cracks forming The above relation as expressed in formula No 1, Figure 2, is a very useful relation for arriving at the proper spacing of joints when given amounts of steel are used The proper value for the coefficient of friction (f) should however, be obtained for the particular conditions of subgrade known to exist There is no reason why this determination should not be made for the proper design of a concrete highway just as a determination for bearing value should be made for the proper design of a footing in building construction The spacing of concealed joints or so-called planes of weakness may be obtained in a manner identical with the above

EFFECT OF SUBGRADE SETTLEMENT

The above theory and calculations neglect bending effects due to subgrade settlement and also the stress effects from traffic. Both of these effects may be so severe that cracks may form between the joints where spacing is calculated as above To provide against cracking under the most severe action of bending would be very uneconomical

A very simple case of this sort may exist when there is excessive settlement in spots due to no subgrade support. One such case is illustrated in Figure 3.

$$M = S\frac{I}{c}$$

$$20{,}000 \times \mathrm{L} \times 12 = \frac{Sbd^2}{6} = \frac{600 \times 10 \times 12 \times 8 \times 8}{6}$$

$$L = \frac{600 \times 10 \times 12 \times 64}{20{,}000 \times 6} = 38\ 4 \text{ in.} = \text{say 3 feet}$$

If there is a soft spot 5 feet long the thickness of the pavement required to prevent cracking due to bending would be.

$$PL = \frac{Sbd^2}{6}, d = \sqrt{\frac{6PL}{Sb}} = \sqrt{\frac{6 \times 20{,}000 \times 5 \times 12}{300 \times 10 \times 12}} = 14.3 \text{ in}$$

Obviously, concrete pavements cannot economically be made thick enough to avoid cracking due to bending stresses when conditions permitting such stresses exist It appears that very frequent joint spacing would be necessary to eliminate all cracking and the economy of such a procedure is at least open to question

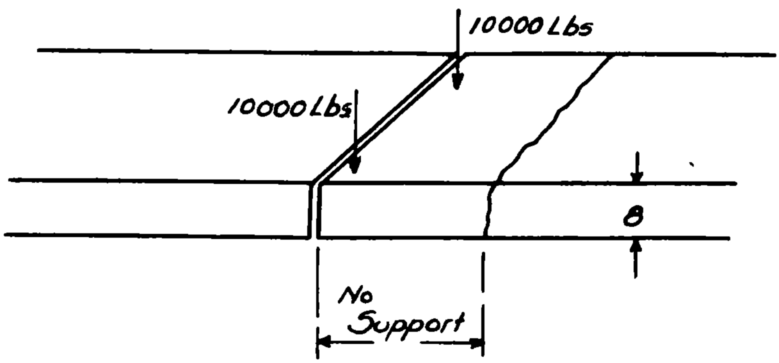


Figure 3

THICKNESS OF EXPANSION JOINTS

The thickness of expansion joints is dependent upon.

(1) Amount of temperature expansion to be expected
(2) Amount of moisture expansion to be expected
(3) Compressive deformation created in the concrete due to the frictional resistance

The coefficient of expansion of concrete is approximately 0 0000055 The expansion of concrete above its 24 hour length due to moisture is 0 0001 inch per inch of length The expansion of concrete from a thoroughly dry condition to a saturated condition is approximately 0 0006 inch per inch of length The maximum temperature to be expected in a concrete pavement is approximately 135°F If the pavement is constructed in warm weather at a temperature of 90°F, the expansion to be allowed for is that due to 135° − 90°F = 45°F On the other hand, if constructed in cold weather at 50°F, the increase in temperature to be allowed for is 135° − 50° = 85°F Obviously, one condition will require a greater thickness of expansion joints for a given spacing of joints than another The following method of calculating the necessary thickness of expansion joints for a given spacing is proposed

Let L = spacing of joints
J = thickness of joint
t = increased temperature of concrete to be expected
f = coefficient of subgrade friction
S = compressive stress in concrete
E = modulus of elasticity of concrete
W = weight of concrete per square foot of surface
d = depth of slab

Total free expansion due to temperature $= L \times t \times$ 0 0000055

Total free expansion due to moisture $= L \times$ 0 0005 (from dry to wet condition) Combined free expansion $= L$ (0 0005 $+ t \times$ 0 0000055) equals decrease in thickness of joints if there were no subgrade frictional resistance

Subgrade resistance $= \left(fw \frac{L}{2}\right)$ for each half slab

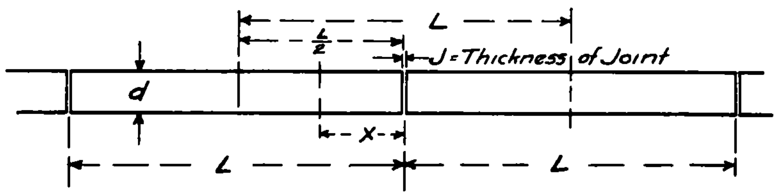


Figure 4

Compressive stress produced at any section A by this frictional resistance $= \frac{fwx}{d \times 12}$.

Average stress produced over the length $\frac{L}{2} = \frac{1}{2}\left[\frac{fw\frac{L}{2}}{d \times 12}\right]$

Deformation produced in concrete over the length $\frac{L}{2} = \frac{S}{E} =$

$$\frac{1}{2}\left[\frac{fw\frac{L}{2}}{d \times 12}\right] - E$$

Deformation produced in concrete over the length $2\frac{L}{2} = \frac{1}{E}\left[\frac{fwL}{24\,d}\right]$

$$w = \frac{d}{12} \times 150 = 12\ 25\ d$$

$$\frac{1}{E}\left[\frac{f12\ 25d\ L}{24\ d}\right] = \frac{1}{E} fL \times 0\ 51$$

If L = 100 feet

t = maximum temperature of 135° — minimum temperature of 35° = 100°

Combined free expansion due to temperature and moisture = 100 × 12 (0 0005 + 100 × 0 0000055) = 1200 × 0 00105 = 1 26 inches

Deformation in concrete $= \frac{1}{E} fL \times 0\ 51$

If f = 2 0 and E = 4,000,000 $= \frac{1}{4{,}000{,}000} \times 2 \times 1200'' \times 0\ 51 = 0\ 0003$ inches.

TABLE I

JOINT THICKNESS

Joint spacing feet	Free expansion of concrete inches	Joint thickness inches
100	1 25	1 87
75	0 94	1 40
50	0 62	0 94
40	0 50	0 75
30	0 37	0 56
20	0 25	0 37

Numerical values assumed in calculating above table

The decreased movement of the free ends of the slab due to subgrade friction is seen to be negligible and the thickness of the joint should therefore be made sufficient to provide for the free expansion of the concrete with some additional thickness to allow for the material not squeezed out by the pressure.

Assuming that the joint material should not be squeezed to less than one-half of its original thickness and that a maximum expansion under extreme conditions of $1\frac{1}{4}$ inches per 100 feet may be obtained, the joint thicknesses shown in Table I are necessary

When expansion joints are installed at long intervals contraction joints or planes of weakness should be installed between them to control the position of transverse cracking The location of such planes of weakness may be determined in the manner previously explained in connection with contraction joints