THE THEORY OF EXTERNAL LOADS ON CLOSED CONDUITS IN THE LIGHT OF THE LATEST EXPERIMENTS

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CONTENTS

I SCOPE OF APPLICATION AND DEFINITIONS

Working over a period of about twenty-one years, and assisted at different times by several persons,* the author of this paper has developed, and from time to time published,† in successive stages, a complete mathematical theory of external loads on closed conduits and of the supporting strengths of pipe conduits This theory applies to all kinds of closed conduits and to all classes of field conditions of conduit construction

The theory is supported strongly, and with remarkable unanimity: First, by all actual weighings on record of loads on closed conduits determined in experimental researches; second, by extensive field data obtained by several series of widespread examinations of actual conduits in use.

The term " conduits " includes culverts, drains, sewers, aqueducts, water pipes, gas mains, telephone conduits and underground steam mains; of all shapes, materials, degrees of rigidity, field construction conditions affecting loads, and field construction conditions affecting supporting strengths.

As to shapes, closed conduits may be rectangular, arched, oval, circular, or of any other shape

As to materials, closed conduits may be of masonry (brick, plain or reinforced concrete, or stone) constructed in place or otherwise or of *pipes or conduit sections,* of burnt clay, or plain or reinforced concrete, for culverts, sewers, tile drains, or telephone or other conduits, or of *metal,* including cast iron, steel or wrought iron and corrugated metal, or of *wood,* including box sections and wood stave pipe, and bored wood pipes.

As to degrees of rigidity, closed conduits may be classified as:

1. *Rigid conduits,* whose cross sectional shapes cannot be distorted sufficiently to change their vertical or horizontal dimensions more than 0 1 per cent without causing materially injurious cracks; including all rectangular conduits, and all cylindrical conduits made of plain or reinforced concrete masonry or pipes, or of burnt clay pipes
2. *Semi-rigid conduits,* whose cross sectional shapes can be distorted sufficiently to change their vertical or horizontal dimensions

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* See Acknowledgments, Appendix A

† See List of Publications of the Theory of External Loads on Closed Conduits, Appendix B

more than 0 1 per cent, but not more than 3 0 per cent, without causing materially injurious cracks; including segmental block conduits, and those made of cast iron pipe, together with some brick or stone block masonry cylindrical conduits.

3. *Flexible conduits,* whose cross sectional shapes can be distorted sufficiently to change their vertical or horizontal dimensions more than 3 0 per cent before causing materially injurious cracks; including those made of corrugated pipe, thin steel or wrought iron pipe, and probably some cylindrical conduits made of brick or stone block masonry.

As to field construction conditions affecting external loads, closed conduits may be classified as:

1. *Ditch conduits,* completely buried in ditches excavated in comparatively solid and inert materials
2. *Projecting conduits,* under embankments but projecting above the embankment sub-grades

The field construction conditions affecting external loads on projecting conduits may be classified as:

1. *The ditch condition,* a typical case of which is a very flexible culvert projecting above the sub-grade under a low embankment
2. *The incomplete ditch condition,* a typical case of which is a very flexible culvert projecting above the sub-grade under an embankment of considerable height
3. *The incomplete projection condition,* a typical case of which is a rigid or semi-rigid culvert projecting above the sub-grade under an embankment of considerable height
4. *The projection condition,* a typical case of which is a rigid or semi-rigid culvert projecting above the sub-grade under a low embankment

NOTE—There are some minor modifications of the above four main classes

Field construction conditions affecting supporting strengths of closed conduits may be classified as follows

1. *Impermissible bedding,* is that method of bedding conduits in which materially insufficient care is exercised to shape the foundation surface to fit the lower part of the conduit exterior and to refill all spaces under and around the conduit with granular materials at least partially compacted.
2. *Ordinary bedding,* is that method of bedding conduits completely buried in ditches in which the conduit is bedded, with "ordi-

nary" care, in an earth foundation shaped to fit the lower part of the conduit exterior with reasonable closeness for a width of at least 50 per cent of the conduit breadth; and in which the remainder of the conduit is surrounded to a height of at least 0 5 feet above its top by granular materials, shovel placed and shovel tamped to completely fill all spaces under and adjacent to the conduit, all under the general direction of a competent engineer.

2a *Ordinary projection bedding,* is that method of bedding projecting conduits under embankments in which the conduit is bedded, with "ordinary" care in an earth foundation shaped to fit the lower part of the conduit exterior with reasonable closeness for a width of at least 50 per cent of the conduit breadth; and in which the remainder of the conduit is surrounded by granular materials, shovel placed and shovel tamped to completely fill all spaces under and adjacent to the conduit; all under the general direction of a competent engineer

3\. *First class bedding,* is that method of bedding conduits completely buried in ditches in which the conduit is carefully bedded on fine granular materials in an earth foundation carefully shaped to fit the lower part of the conduit exterior for a width of at least 60 per cent of the conduit breadth, and in which the remainder of the conduit is entirely surrounded to a height of at least 1 0 foot above its top by granular materials carefully placed by hand to completely fill all spaces under and adjacent to the conduit, and thoroughly tamped on each side and under the conduit so far as practicable, in layers not exceeding 0 5 feet in thickness; all under the direction of a competent engineer, represented by a competent inspector constantly present during the operation

3a *First class projection bedding,* not yet defined

4 *Concrete-cradle bedding,* is that method of bedding conduits in which the lower part of the conduit exterior is bedded in plain or reinforced concrete, of suitable thickness under the lowest part of the conduit exterior and extending upward each side of the conduit for a greater or less proportion of its height.

II EXTERNAL LOADS ON CLOSED CONDUITS

The most important external loads on closed conduits result from the action of gravity. Their components parallel to the conduit axis are so relatively unimportant that, for practical purposes, the load at any point on the exterior surface is sufficiently expressed by its vertical and horizontal components acting in the perpendicular cross sectional plane at the point The vertical components of such loads are usually much greater and more important than the horizontal components, and the latter are most conveniently expressed in terms of their ratios to the vertical components.

Hence, the mathematical formulas for computing external loads on closed conduits most conveniently express the vertical load components only, leaving the horizontal pressures to be calculated from their ratios to the vertical loads

The external loads on closed conduits comprise: First, the downward loads, applied usually to the upper portions of the conduit exteriors, second, the resultant upward foundation pressures, applied usually to the lower portions of the conduit exteriors The total foundation pressure upward is equal to the total load downward

The external loads on closed conduits are of two classes:

First, the *loads due to the filling* materials placed over and around the conduits.

Second, *the loads transmitted through the filling materials but due to extraneous super-loads,* applied at the upper surface of the fill or at some lower level higher than the top of the conduit.

Super-loads may be concentrated as in the case of truck wheel loads, or they may be distributed, as in the case of piles of construction materials at the fill surface.

III THE MATHEMATICAL THEORY OF LOADS ON CLOSED CONDUITS DUE TO FILL MATERIALS

A. NOTATION

Let B = horizontal breadth of a conduit or ditch, feet
B_c = greatest horizontal breadth of a conduit, feet
B_d = horizontal breadth of a ditch at top of conduit, feet
C = conduit load calculation coefficient, abstract number
C_c = load calculation coefficient for conduits projecting above the adjacent embankment sub-grade, abstract number
C_d = load calculation coefficient for conduits completely buried in ditches, abstract number
H = vertical height from top of conduit to the upper surface of the fill, feet

$H_e =$ "height of equal settlement," $=$ vertical height from the top of the conduit to the level at and above which the fill materials directly over the conduit settle equally with the adjacent fill materials, feet

$p =$ the projection ratio, $=$ the ratio of the *vertical height of the top of the conduit above the embankment sub-grade level* to B_c, abstract number

NOTE—The conduit projection $= pB_c$.

$d_c =$ *the increment of deflection of the conduit* (i. e., the shortening of its vertical height), feet

$s_f =$ *the increment of settlement of the conduit foundation,* feet

$s_g =$ *the increment of settlement of the embankment sub-grade,* feet

$s_m =$ *the increment of settlement of materials adjacent to the conduit between the levels of the conduit top and the adjacent embankment sub-grade,* feet

NOTE—d_c, s_f, s_g and s_m are the increments due to the addition at or above the height of equal settlement of any (the same for all) incremental, uniform layer of fill materials

$r_{sd} =$ the "settlement deflection ratio," abstract number

$$r_{sd} = \frac{(s_m + s_g) - (d_c + s_f)}{s_m}$$

NOTE—r_{sd} is *negative* for the "complete" and the "incomplete ditch condition"

r_{sd} is *positive* for the "complete" and the "incomplete projection condition."

$W_c =$ the vertical external load on a closed conduit due to fill materials, pounds per foot length

$w =$ the unit weight of fill materials, pounds per cubic foot

$K =$ the ratio of *active horizontal pressure* at any point in the fill to the *vertical pressure which causes the active horizontal pressure,* abstract number

$$K = \frac{\sqrt{\mu^2 + 1} - \mu}{\sqrt{\mu^2 + 1} + \mu} \text{ (Rankine's formula)}$$

$\mu =$ the "*coefficient of internal friction*" in the fill materials, abstract number

$\mu' =$ the "*coefficient of sliding friction*" between the fill materials and the sides of the ditch, abstract number

NOTE—μ' may be less than μ, but cannot be greater

$\epsilon = 2.7182818 =$ base of natural logarithms, abstract number

B. MATHEMATICAL FORMULAS FOR CALCULATING VERTICAL EXTERNAL LOADS ON CLOSED CONDUITS

MATHEMATICAL FORMULAS for CALCULATING VERTICAL EXTERNAL LOADS on CLOSED CONDUITS

DUE TO FILL MATERIALS

The following general formula applies to all commonly encountered conditions

$$W_c = CwB^2 \text{ ------------------- } (1)$$

Formulas (2) and (3) apply to "ditch conduits" (completely buried in ditches excavated in comparatively "passive" and solid materials)

$$W_c = C_d w B_d^2 \text{ ------------------- } (2)$$

$$C_d = \frac{1-\varepsilon^{-2k\mu'\frac{H}{B_d}}}{2k\mu'} \text{ ------------------- } (3)$$

Formulas (4), (5), (6), and (7) apply to "projecting conduits" (under embankments and projecting above the embankment sub-grades).

$$W_c = C_c w B_c^2 \text{ ------------------- } (4)$$

For the "complete" ditch and projection conditions

$$C_c = \frac{\varepsilon^{\pm 2k\mu\frac{H}{B_c}}-1}{\pm 2k\mu} \text{ ------------------- } (5)$$

For the "incomplete" ditch and projection conditions

$$C_c = \frac{\varepsilon^{\pm 2k\mu\frac{H_e}{B_c}}-1}{\pm 2k\mu} + \left(\frac{H}{B_c}-\frac{H_e}{B_c}\right)\varepsilon^{\pm 2k\mu\frac{H_e}{B_c}} \text{ ------- } (6)$$

Note – In formulas (5) and (6), use the – sign for the ditch conditions and the + sign for the projection conditions.

For calculating H_e in equation (6)

$$\varepsilon^{\pm 2k\mu\frac{H_e}{B_c}} \mp 2k\mu\frac{H_e}{B_c} = \pm 2k\mu r_{sd}p + 1 \text{ ----------- } (7)$$

Note – In equation (7),
use the – , + and – signs for the ditch condition
use the + , – and + signs for the projection condition

FOR THE "COMPLETE" and "INCOMPLETE" DITCH CONDITIONS WITH "ACTIVE" MATERIALS ADJACENT TO THE CONDUIT

For use in equation (4)

in place of (5) --- $C_c = \frac{H}{B_c} - k\mu\left(\frac{H}{B_c}\right)^2$, "complete" ditch condition (8)

in place of (6) --- $C_c = \frac{H}{B_c}\left(1-2k\mu\frac{H_e}{B_c}\right) + k\mu\left(\frac{H_e}{B_c}\right)^2$, "incomplete" ditch condition (9)

in place of (7) --- $\frac{H_e}{B_c} = \sqrt{-\frac{r_{sd}p}{k\mu}}$, "incomplete" ditch condition (10)

Figure 1

C. COMPUTATION DIAGRAMS FOR EXTERNAL VERTICAL LOADS ON CLOSED CONDUITS DUE TO FILL MATERIALS

The computation of vertical external loads on closed conduits by formulas (2) and (4), (Figure 1), is made very simple and easy by computation diagrams, from which the values of the calculation coefficient C_d and C_c can be obtained directly instead of by special computations.

For "Ditch Conduits" Completely Buried in Ditches

$$W_c = C_d w B_d^2 \tag{2}$$

The values of C_d may be read directly on Figure 2, herewith, reproduced from Bulletin 47, Iowa Engineering Experiment Station, without special computations by equation (3).

For "Projecting Conduits" (Under Embankments and Projecting Above the Embankment Sub-Grade)

$$W_c = C_c w B c^2 \tag{4}$$

The values of C_c may be read directly on Figure 3, herewith, without special computation by equations (5), (6), (7), or (8), (9). (10), (Figure 1).

D PHYSICAL FACTORS REQUIRED FOR COMPUTATIONS OF EXTERNAL LOADS ON CLOSED CONDUITS, DUE TO FILL MATERIALS

Three special physical factors required in using equations (1) to (10), (Figure 1), are:

1. The coefficient, μ, of internal friction in the fill material.
2. The projection, pB_c, of the conduit above the adjacent embankment sub-grade
3. The settlement-deflection ratio, r_{sd}, of the *relative* settlements of the embankment sub-grade, the fill materials adjacent to the conduit, and the conduit foundation, together with the deflection of the conduit top.

1. The Coefficient of Internal Friction, μ, in the Fill Material

The values of μ for use in the formula should be those corresponding to the pulls required to establish slow *continued* motion in the usual laboratory tests (in which bottomless boxes of the material are pulled over horizontal surfaces of the same material)

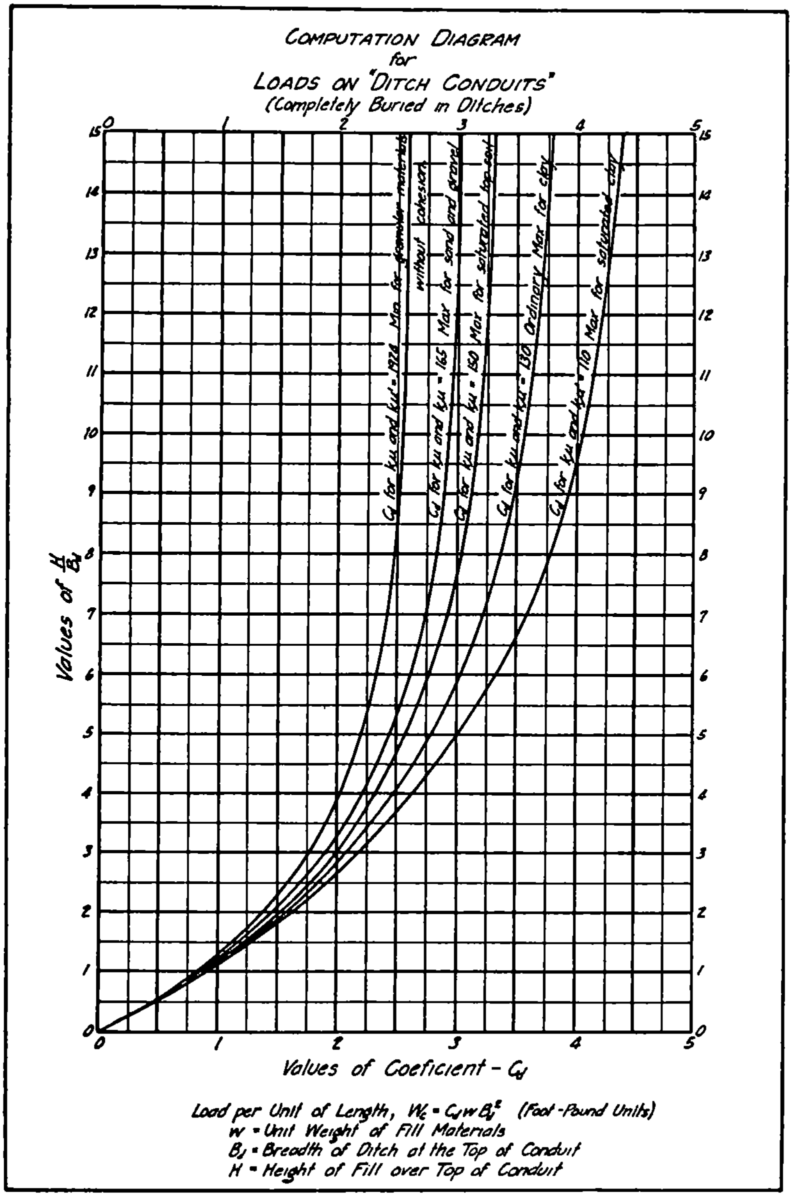

COMPUTATION DIAGRAM
for
LOADS ON "DITCH CONDUITS"
(Completely Buried in Ditches)
Cd for Kμ and Kμ' = .1924 Min for granular materials without cohesion
Cd for Kμ and Kμ' = .165 Max for sand and gravel
Cd for Kμ and Kμ' = .150 Max for saturated top soil
Cd for Kμ and Kμ' = .130 Ordinary Max for clay
Cd for Kμ and Kμ' = .110 Max for saturated clay
Values of H/Bd
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
Values of Coeficient - Cd
0
1
2
3
4
5
Load per Unit of Length, Wc = Cd w Bd² (Foot-Pound Units)
w = Unit Weight of Fill Materials
Bd = Breadth of Ditch at the Top of Conduit
H = Height of Fill over Top of Conduit

Figure 2

That these are the correct values to use has been clearly proven: First, by hundreds of comparisons of computed loads with actual loads on conduits, as determined by actual weighings in experiments; second, by extensive field data of actual conduits whose supporting strengths are known by laboratory tests

The values of μ obtained as above are somewhat of the nature of the semi-empiric physical factors, or coefficients, so common in engineering problems.

No laboratory methods for measuring μ have yet been devised in which the pressure conditions in actual fills are reproduced with near approximation In the actual fill, there is sufficient time for important slight movements of the grains of material into positions of comparatively stable equilibrium, and they apparently are held in comparatively stable equilibrium against rolling along the vertical plane of slipping by the comparatively large vertical pressures which cause the smaller horizontal pressures which cause the friction. In the laboratory tests (in which the plane of slipping is usually rotated 90°), the time adjustment of the grains in the actual fill is mainly absent, the major (vertical) pressures of the actual fill are entirely omitted, the grains along the plane of slipping are in comparatively unstable equilibrium against rolling, and an initial slipping occurs at a fraction of the true internal friction pull. This phenomenon has long been known, and was reannounced by the author in Bulletin 31, Iowa Engineering Experiment Station, 1913.

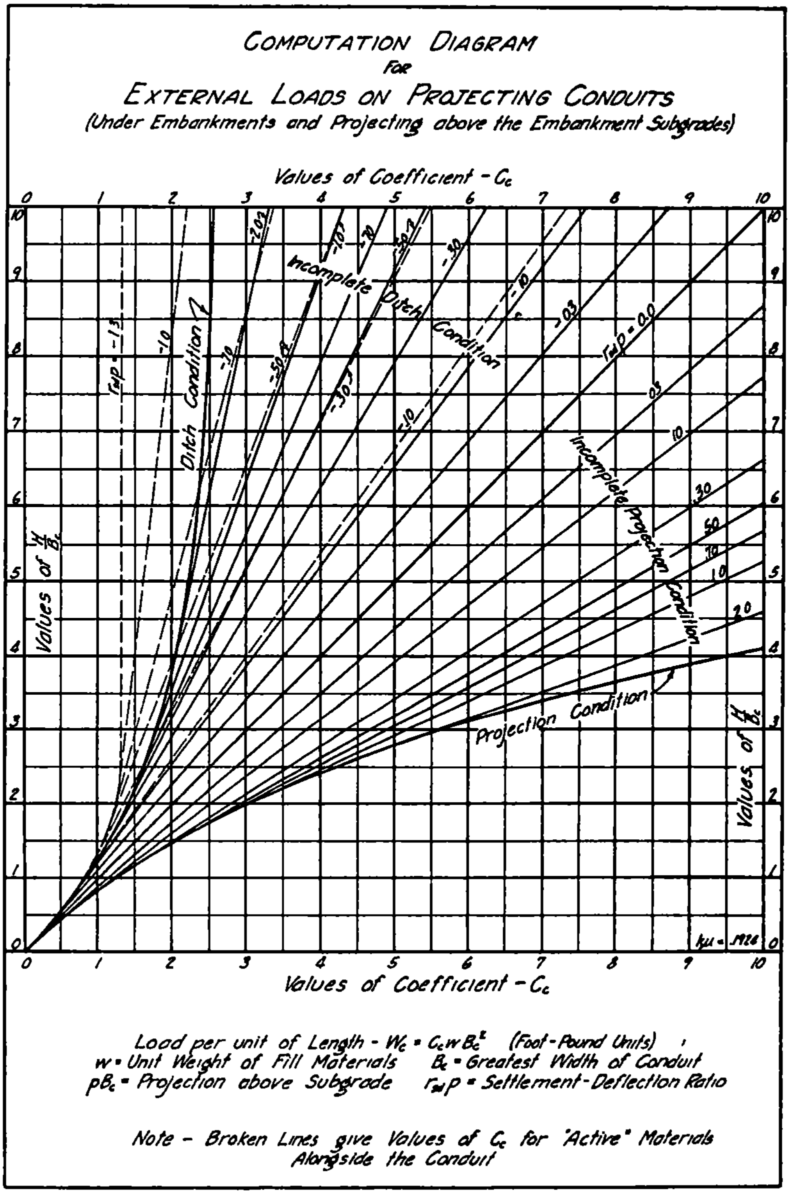
NOTE—The above discussion applies also to μ', the coefficient of sliding friction

2 The Projection, pB_c, of the Conduit Above the Adjacent Embankment Sub-Grade

The projection, pB_c, has an important effect (see Figure 3) on loads on "projecting conduits" (under embankments and projecting above the embankment sub-grade), though it does not materially affect loads on "ditch conduits" (completely buried in ditches).

In many cases, the ratio p should be regarded as a semi-empiric physical factor, representing the best judgment of the engineer as to the characteristic field conditions in actual use. Most conduits under embankments are buried partially, or even entirely in ditches which vary as to depths, widths and side slopes

Whenever practicable, however, p should be determined by actual measurement, or observation, or from correct field construction notes, for each structure.



Load per unit of Length - $W_c = C_c w B_c^2$ (Foot-Pound Units)
w = Unit Weight of Fill Materials B_c = Greatest Width of Conduit
pB_c = Projection above Subgrade $r_{sd}p$ = Settlement-Deflection Ratio

Note - Broken Lines give Values of C_c for "Active" Materials Alongside the Conduit

Figure 3

3 The Settlement-Deflection Ratio, r_{sd}

The settlement-deflection ratio, r_{sd}, is an extremely important factor affecting loads on "projecting conduits" (under embankments and projecting above the embankment sub-grade). It indicates correctly the effects of the conduit deflections, the settlement of the conduit foundations and the embankment sub-grade settlements, relative to the yielding of the embankment fill materials adjacent to the conduits Loads on conduits under embankments of the same height are sometimes three times as great for high values of r_{sd} as for low values. See Figure 3.

The settlement-deflection ratio is a semi-empiric physical factor, the values of which to use in computations must be determined by comparisons of computed loads on conduits with the actual loads, as weighed in experiments and as estimated for actual conduits by their supporting strengths as determined by laboratory tests.

A table of values of r_{sd} for different field and conduit conditions is under preparation by the Iowa Engineering Experiment Station

E. THE HORIZONTAL COMPONENTS OF EXTERNAL LOADS ON CLOSED CONDUITS DUE TO FILL MATERIALS

Study of hundreds of measurements of actual horizontal pressures in granular fill materials, during 21 years of research, has convinced the author of the reliability of Rankine's well-known formulas, which may be stated as follows.

The *Active* horizontal pressure $= K \times$ (vertical pressure).

The *Passive* horizontal pressure $\not> 1/K \times$ (vertical pressure).

$$K = \frac{\sqrt{\mu^2 + 1} - \mu}{\sqrt{\mu^2 + 1} + \mu}$$

1 For "Ditch Conduits" (Completely Buried in Ditches)

The horizontal pressures against "ditch conduits" are wholly dependent on the class of bedding, and are best taken into account by using *conduit supporting strength bedding ratios* (see page 168), of field supporting strength to laboratory test strength

2. For "Projecting Conduits" (Under Embankments and Projecting Above the Embankment Sub-Grades)

(a) For *rigid conduits,* not injuriously cracked:

Active horizontal pressure equal to $K \times$ (vertical pressure) should be assumed against the projecting conduit sides.

(b) For *semi-rigid conduits*·

Somewhat greater active horizontal pressures should be assumed than for rigid conduits.

(c) For *flexible conduits,* and for all badly cracked conduits:

The development of passive resistance to elongation of the horizontal diameters will often cause horizontal pressures equal to the vertical pressures

NOTE—The Iowa Engineering Experiment Station is endeavoring to establish, experimentally, "moduli of passive resistance," per inch elongation of horizontal diameter, for different materials and classes of conduits

IV. THE MATHEMATICAL THEORY OF LOADS ON CLOSED CONDUITS DUE TO SUPER-LOADS

A. NOTATION

Let A = length of a section of a closed conduit, feet

a = area of each elementary area into which the conduit top is divided for computation purposes, square feet

NOTE—The vertical loads on the tops of cylindrical and arched conduit tops, due to super-loads, are substantially equal to the vertical loads, due to super-loads, on the projection of the conduit section upward on the horizontal plane tangent to the conduit top Hence, the elementary areas, a, may be taken in this "*conduit top horizontal plane projection*"

B_c = the greatest horizontal breadth of the conduit, feet

B_d = the horizontal breadth of a ditch, at top of conduit, feet

C_t = calculation coefficient for vertical loads on conduit tops due to concentrated super-loads, abstract number

C_{us} = calculation coefficient for vertical loads on the top of "ditch conduits" (completely buried in ditches), due to uniformly distributed super-loads, abstract number

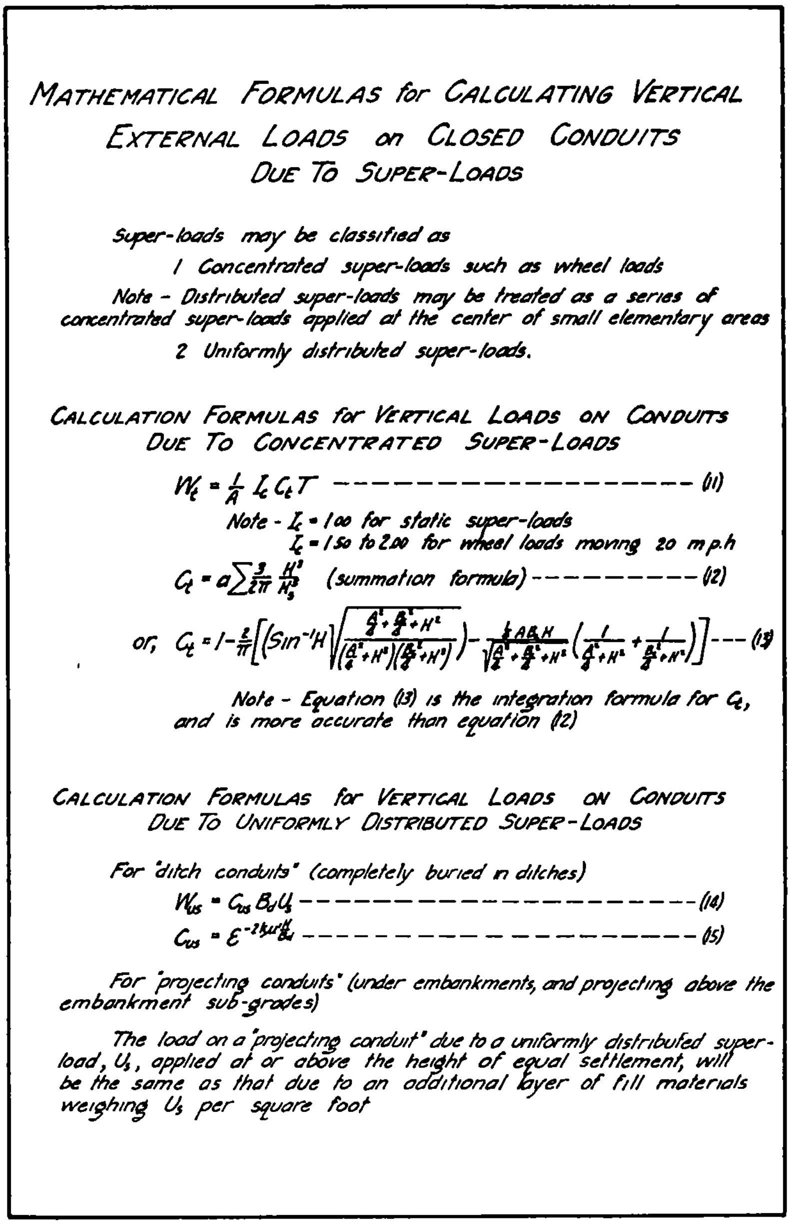
H = the *vertical height* from the level of the conduit top to the level of the point or area of application of a super-load, feet

H_s = the *slant height* from any point x, y in the conduit top horizontal plane projection to the point of application of a concentrated super-load, feet

x and y = the rectangular coordinates of any point x, y in the conduit top horizontal plane projection, with x parallel and y perpendicular to the conduit axis, feet

I_c = impact coefficient for the calculation of impact loads on conduit tops, due to moving concentrated super-loads, abstract number

B. MATHEMATICAL FORMULAS FOR CALCULATING VERTICAL EXTERNAL LOADS ON CLOSED CONDUITS DUE TO SUPER-LOADS



MATHEMATICAL FORMULAS for CALCULATING VERTICAL EXTERNAL LOADS on CLOSED CONDUITS DUE TO SUPER-LOADS

Super-loads may be classified as

1 Concentrated super-loads such as wheel loads

Note - Distributed super-loads may be treated as a series of concentrated super-loads applied at the center of small elementary areas

2 Uniformly distributed super-loads.

CALCULATION FORMULAS for VERTICAL LOADS ON CONDUITS DUE TO CONCENTRATED SUPER-LOADS

$$W_t = \frac{1}{A} I_c C_t T \text{ ------------------- } (11)$$

Note - $I_c = 1.00$ for static super-loads
$I_c = 1.50$ to 2.00 for wheel loads moving 20 m.p.h

$$C_t = a\sum \frac{3}{2\pi}\frac{H^3}{H_s^3} \text{ (summation formula) ---------- } (12)$$

$$\text{or, } C_t = 1 - \frac{2}{\pi}\left[\left(\sin^{-1} H\sqrt{\frac{\frac{A^2}{4}+\frac{B^2}{4}+H^2}{\left(\frac{A^2}{4}+H^2\right)\left(\frac{B^2}{4}+H^2\right)}}\right) - \frac{\frac{1}{4}ABH}{\sqrt{\frac{A^2}{4}+\frac{B^2}{4}+H^2}}\left(\frac{1}{\frac{A^2}{4}+H^2}+\frac{1}{\frac{B^2}{4}+H^2}\right)\right] \text{ --- } (13)$$

Note - Equation (13) is the integration formula for C_t, and is more accurate than equation (12)

CALCULATION FORMULAS for VERTICAL LOADS ON CONDUITS DUE TO UNIFORMLY DISTRIBUTED SUPER-LOADS

For "ditch conduits" (completely buried in ditches)

$$W_{us} = C_{us} B_d U_s \text{ --------------------- } (14)$$

$$C_{us} = \varepsilon^{-2K\mu'\frac{H}{B_d}} \text{ --------------------- } (15)$$

For "projecting conduits" (under embankments, and projecting above the embankment sub-grades)

The load on a "projecting conduit" due to a uniformly distributed super-load, U_s, applied at or above the height of equal settlement, will be the same as that due to an additional layer of fill materials weighing U_s per square foot

Figure 4

T = a concentrated super-load, pounds

U_s = a uniformly distributed super-load, pounds per square foot

W_{us} = the *average* total vertical load on a section of a closed conduit, due to a uniformly distributed super-load U_s, pounds per linear foot

W_t = the *average* total vertical load on a section of a closed conduit, due to a concentrated super-load T, pounds per linear foot

K = ratio of active horizontal pressure at any point in the fill material to the *causal* vertical pressure, abstract number

$$K = \frac{\sqrt{\mu^2 + 1} - \mu}{\sqrt{\mu^2 + 1} + \mu} \text{(Rankine's formula)}$$

μ = *coefficient of internal friction* of the fill material, abstract number

μ' = *coefficient of sliding friction* between the fill material and the sides of a ditch, abstract number

π = 3 14159 = the ratio of a circumference of a circle to its diameter, abstract number

C. COMPUTATION DIAGRAM AND TABLE FOR EXTERNAL VERTICAL LOADS ON CLOSED CONDUITS, DUE TO SUPER-LOADS

The computation of vertical external loads on closed conduits by formulas (11) and (14) (Figure 4) is made very simple and easy by a computation diagram, for formula (11), and a computation table for formula (14), from which the values of the calculation coefficients C_t and C_{us} can be obtained directly instead of by special computations.

For Concentrated Super-Loads

$$W_t = 1/AI_cC_tT \qquad (11)$$

The values of C_t may read directly from computation diagrams, of which Figure 5 is a sample

For Loads on Conduits in Ditches, Due to Uniform Super-Loads U_s

$$W_{us} = C_{us}B_dU_s \qquad (14)$$

The values of C_{us} may be taken directly from Table 1, herewith, reproduced from Bulletin 31, Iowa Engineering Experiment Station, 1913.

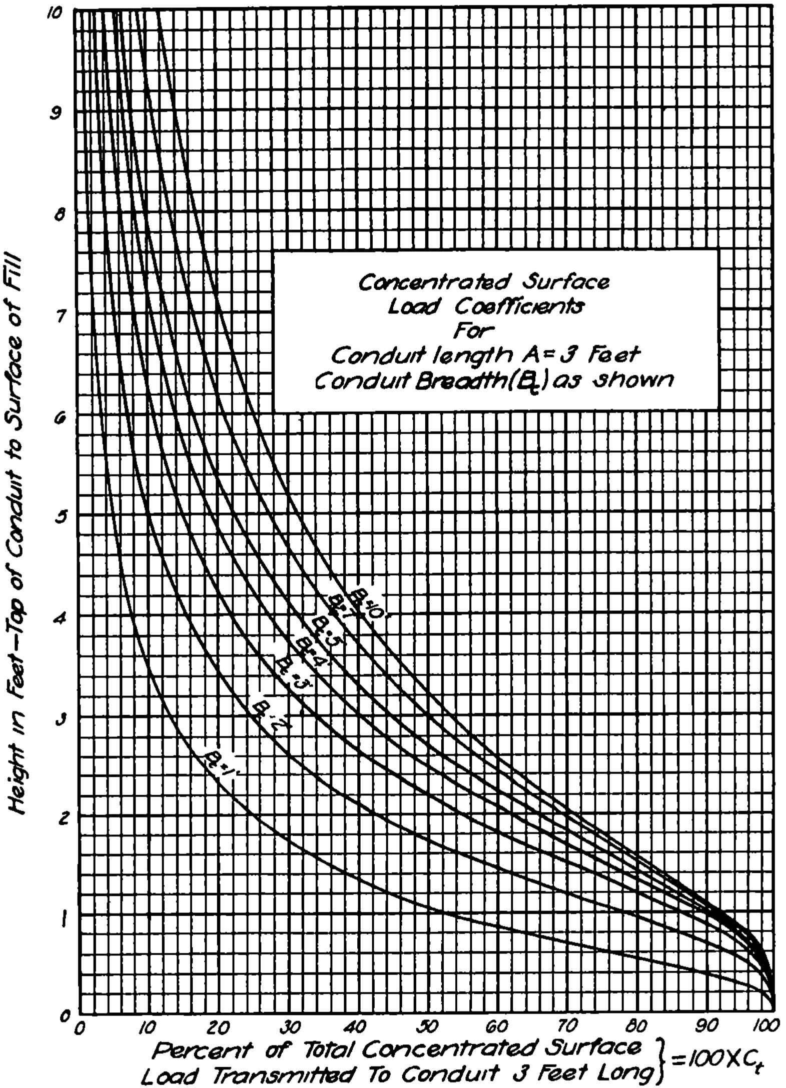

Concentrated Surface
Load Coefficients
For
Conduit length A = 3 Feet
Conduit Breadth (B_c) as shown
B_c=10'
B_c=7'
B_c=5'
B_c=4'
B_c=3'
B_c=2'
B_c=1'
Height in Feet—Top of Conduit to Surface of Fill
0
1
2
3
4
5
6
7
8
9
10
0
10
20
30
40
50
60
70
80
90
100
Percent of Total Concentrated Surface Load Transmitted To Conduit 3 Feet Long $\} = 100 \times C_t$

Figure 5

TABLE I

SAFE VALUES OF C_{us} TO USE IN FORMULA (14)

H/B_d	Sand and damp top soil	Saturated top soil	Damp yellow clay	Saturated yellow clay	H/B_d
0 0	1 00	1 00	1 00	1 00	0 0
0.5	0 85	0 86	0.88	0.89	0.5
1 0	0 72	0 75	0 77	0.80	1 0
1 5	0 61	0 64	0 67	0 72	1 5
2 0	0 52	0.55	0.59	0 64	2 0
2 5	0 44	0 48	0.52	0.57	2.5
3 0	0 37	0 41	0 45	0.51	3 0
4 0	0.27	0 31	0.35	0 41	4 0
5 0	0 19	0 23	0.27	0 33	5 0
6 0	0 14	0 17	0.20	0.26	6 0
8 0	0 07	0 09	0 12	0 17	8 0
10 0	0 04	0 05	0 07	0 11	10 0

V. THE CHARACTERISTICS OF LOADS DUE TO OR TRANSMITTED BY GRANULAR FILL MATERIALS

The mathematical theory of external loads on closed conduits, as presented on pages 142 to 154, applies to loads due to or transmitted by granular fill materials. Such loads have certain characteristics which must be clearly understood in applying the mathematical theory correctly.

1. Loads due to or transmitted by granular fill materials vary greatly with variations in the properties of the materials; as to weight, settlement, moisture, temperature, internal friction and cohesion.
2. Formulas (1) to (15), of the mathematical theory of external loads on closed conduits, with proper selection of the physical factors μ, p and r_{sd}, give the *ultimate limiting loads,* to which *any* particular conduit *may* be subjected eventually; and which, on the other hand, any particular conduit *may* escape for a long time, sometimes until its removal for other causes than load failure. See Figure 6, reproduced from Bulletin 79, Iowa Engineering Experiment Station, 1926. (Cooperative with the U. S Bureau of Public Roads).
3. The external loads on closed conduits vary greatly with time. In 7 experiments at Ames, Iowa, daily observations over periods of 3, 7½, 8, 24, 24, 24, and 28 months, respectively, have proven that loads on conduits measured soon after the completion of the fill over them should be increased 20 to 25 per cent to allow safely for the *ultimate* loads which would be

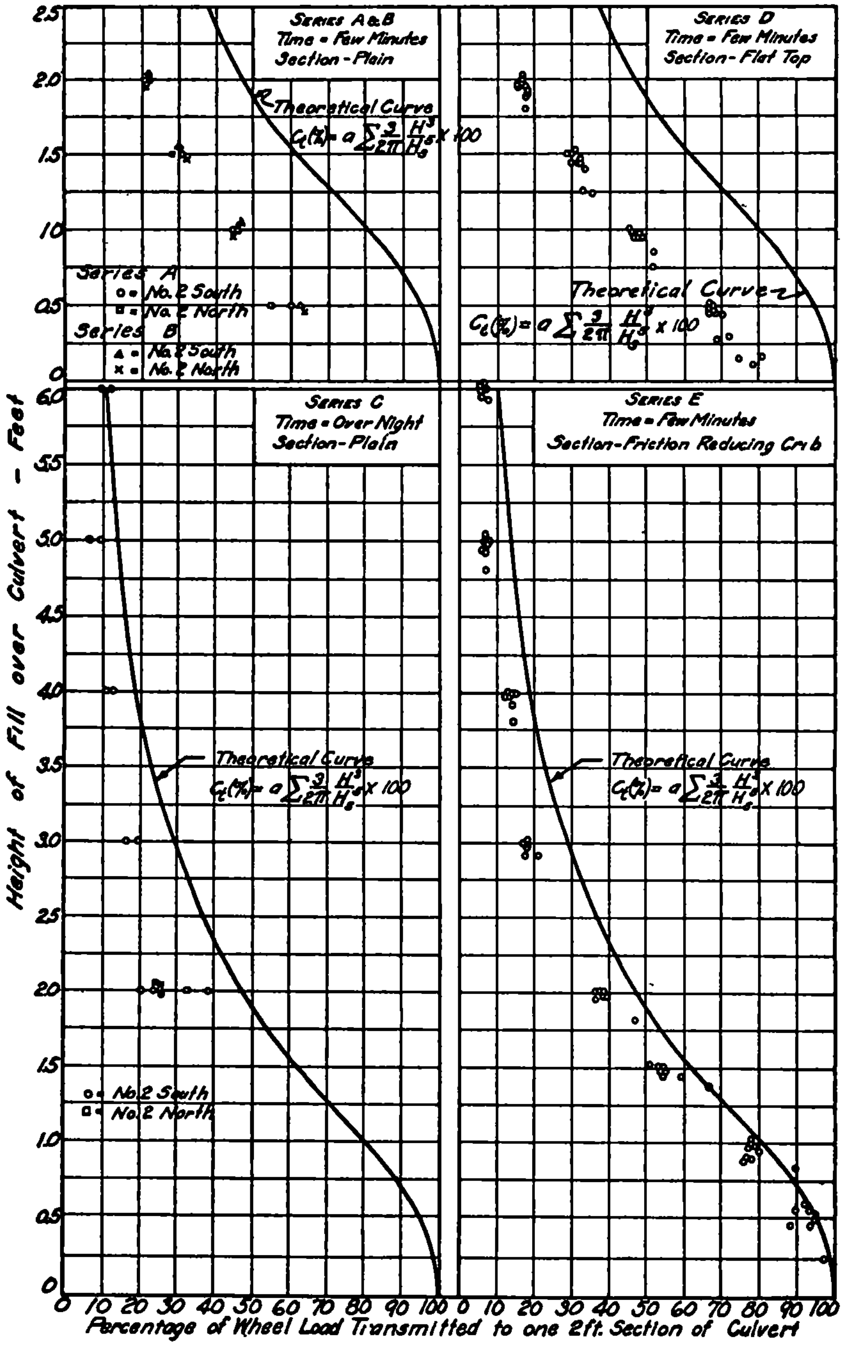

Series A & B
Time = Few Minutes
Section - Plain
Theoretical Curve
$C_t(\%) = a\sum\frac{3}{2\pi}\frac{H^3}{H_s^5} \times 100$
Series A
o = No. 2 South
□ = No. 2 North
Series B
▲ = No. 2 South
× = No. 2 North
Series D
Time = Few Minutes
Section - Flat Top
Theoretical Curve
$C_t(\%) = a\sum\frac{3}{2\pi}\frac{H^3}{H_s^5} \times 100$
Series C
Time = Over Night
Section - Plain
Theoretical Curve
$C_t(\%) = a\sum\frac{3}{2\pi}\frac{H^3}{H_s^5} \times 100$
o = No. 2 South
□ = No. 2 North
Series E
Time = Few Minutes
Section - Friction Reducing Crib
Theoretical Curve
$C_t(\%) = a\sum\frac{3}{2\pi}\frac{H^3}{H_s^5} \times 100$
2.5
2.0
1.5
1.0
0.5
0
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
Height of Fill over Culvert - Feet
0 10 20 30 40 50 60 70 80 90 100 0 10 20 30 40 50 60 70 80 90 100
Percentage of Wheel Load Transmitted to one 2 ft. Section of Culvert

Figure 6

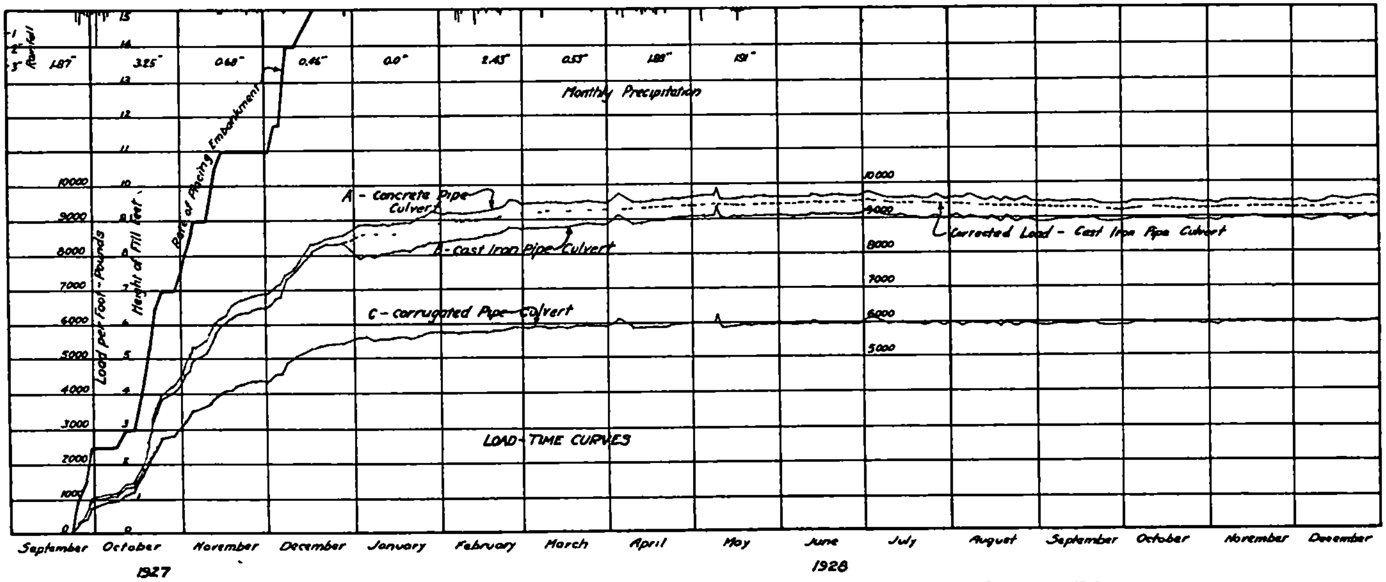


Figure 7. Experiments at Ames, Iowa, with Three Culverts of 49" Outside Diameter

found if the observations were extended to cover a period of several years See Figure 7, reproduced from a forthcoming bulletin of the Iowa Engineering Experiment Station (Cooperative with the U. S. Bureau of Public Roads).

Field Observations of cracked conduits show that such cracks often develop months or even years after construction

A THE HEIGHT OF EQUAL SETTLEMENT IN EMBANKMENTS OVER CLOSED CONDUITS

The key to the mathematical solution of the theory of loads on "projecting conduits" (under embankments and projecting above the adjacent embankment sub-grade), is the discovery and publication by the author, in 1922, of the existence and the method of computation of the "height of equal settlement," at and above which, in the "incomplete" ditch and projection conditions, all additional uniform horizontal layers of embankment materials settle equally in and on each side of the vertical prism of fill materials directly over the conduit

Below the height of equal settlement, the horizontal layers of fill material each side of the vertical prism over the conduit, settle, less in the "incomplete ditch condition" and more in the "incomplete projection condition," than the fill materials originally at the same level in the prism over the conduit.

The existence of the "height of equal settlement" was discovered and first announced by the author on purely mathematical grounds. There are many difficulties in the way of measuring the actual settlements at different points in the interiors of embankment masses, which, however, have been overcome by very ingenious apparatus devised by the staff of the Iowa Engineering Experiment Station. The actual existence of the height of equal settlement is now amply verified by many actual measurements, made during a period of years, of the actual settlements at different points in embankment interiors.

Such measurements are illustrated by Figure 8, reproduced from a forthcoming bulletin of the Iowa Engineering Experiment Station (Cooperative with the U. S Bureau of Public Roads).

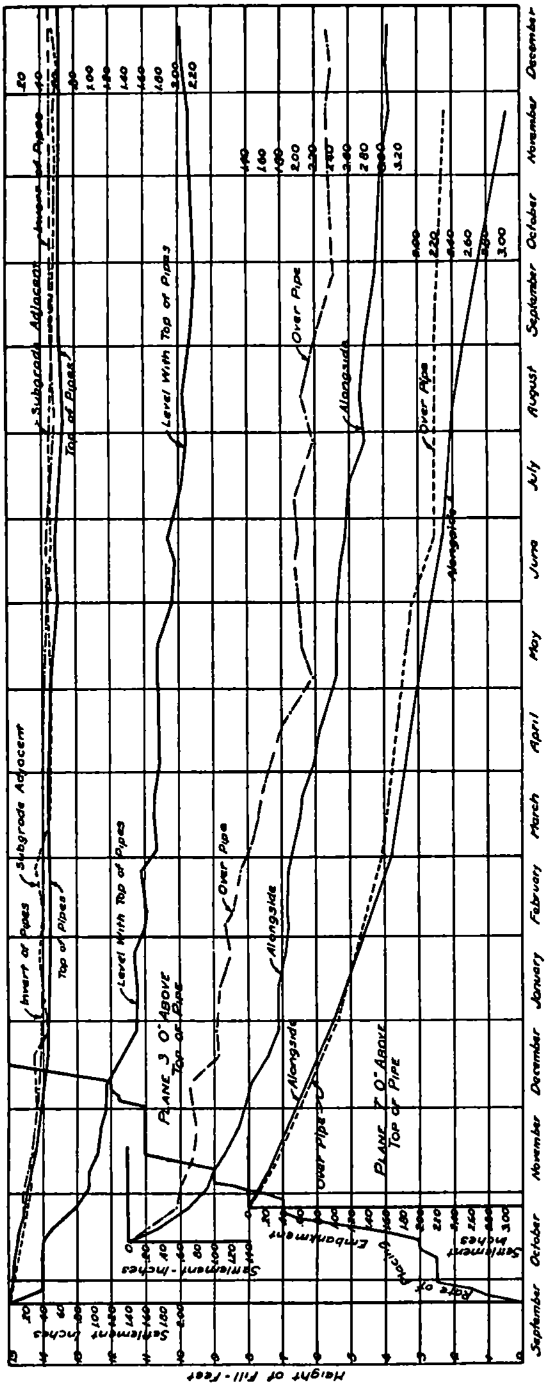


Figure 8 Time-Settlement Curves. Concrete Pipe Culverts

VI VERIFICATION OF THE MATHEMATICAL THEORY OF EXTFRNAL LOADS ON CLOSED CONDUITS, BY EXPERIMENTS AND BY FIELD OBSERVATIONS

The mathematical theory of external loads on closed conduits, as presented in equations (1) to (15) (Figures 1, 4), has been verified in the amplest manner:

First, by actually weighing the loads on full size conduit sections, in experiments which doubtless have cost more than $150,000, at Ames, Iowa, and elsewhere, and which at Ames, Iowa, have extended continuously for 18 years and are still in progress.

Second, by extensive and expensive failures and partial failures, of conduits in service, by injurious cracking and sometimes by collapse. Many such failures have been investigated, and they have been found to verify the mathematical theory Important changes in drainage, sewerage and culvert engineering practice are now developing to make safe practice conform to this theory.

Third, by extensive field examinations of actual conduits in service, some sound and others cracked Most cracked conduits do not collapse, but many cases have been found of objectionable deformation, and a number of cases of collapse

A EXPERIMENTAL AND FIELD VERIFICATION OF THE THEORY OF EXTERNAL LOADS ON CLOSED CONDUITS IN DITCHES (SEE EQUATIONS (2) AND (3) (FIGURE 1)

The theory of external loads on "ditch conduits" (completely buried in ditches) has been verified in the amplest manner by the experiments and other researches of the Iowa Engineering Experiment Station at Ames, Iowa, from 1908 to 1929, still continuing The theory is also verified by Barbour's experiments at Boston, Mass, in 1897 *

The theory of external loads on "ditch conduits" has also been verified in the amplest manner by actual experience with sewers, drains and culverts in use The best sanitary engineers are now more and more coming to require concrete cradling of pipe sewers and drains wherever the author's theory indicates danger of injurious cracking by the computed loads.

The author's theory has been republished in highly condensed form in several standard textbooks on sewerage and drainage.

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* Journal of Association of Engineering Societies, Vol 19, Dec, 1897

B. EXPERIMENTAL AND FIELD VERIFICATION OF THE THEORY OF EXTERNAL LOADS ON "PROJECTING CONDUITS" (UNDER EMBANKMENTS AND PROJECTING ABOVE THE ADJACENT EMBANKMENT SUB-GRADE) (SEE EQUATIONS (4) TO (15) (FIGURES 1, 4)

1 *Experimental Verification*

The mathematical theory of external loads on closed conduits under embankments, and projecting above the adjacent embankment subgrade, has been verified in the amplest manner by the following experiments:

> *Iowa Engineering Experiment Station, Ames, Iowa, 1917 to 1929, still continuing* Actual weighings were made of the total loads on pipe culverts, of 9, 24, 36 and 42 inches inside diameters, under embankment fills of heights above the tops of the pipes up to 4 feet for the 9 inch, 10 feet for the 24 inch, and 15 feet to 20 feet for the 36 and 42 inch culverts The fill materials have included top-soil, sand, and gravel, weighing 96 to over 130 pounds per cubic foot. The culverts have included thin smooth steel, corrugated metal, cast iron, clay, plain concrete, reinforced concrete, and wood staves on steel forms. Since 1925, the experiments have included measurements of the settlements at various points on the embankment sub-grades and in the interiors of the embankments, as well as of the deflections of the culvert tops and the settlements of the culvert foundations
>
> The experiments have covered ranges of the settlement-deflection ratio, r_{sd}, from -3.0 to $+1.0$
>
> In all the experiments, the *total* load was weighed In all cases except the 9 inch and the 24 inch culverts, the *total* load on *each* of several sections of each culvert was weighed. Many measurements of horizontal pressures have been made.
>
> Regular weighings of the loads and other observations have been continued 3 months to 24 months after the completion of the embankment fills in the different experiments, and are still continuing for 3 culverts which have already been under observation 24 months.
>
> All the Ames culvert tests have been for approximately 90 per cent projection ($p = 0.90$).

The Iowa Engineering Experiment Station experiments verify the mathematical theory stated in this paper in the most complete manner.

> *American Railway Engineering Association Roadway Committee Culvert Load Tests at Farina, Ill,* 1925 to 1927.* Measurements by Goldbeck-Smith pressure cells were made at different points on the circumferences of 6 corrugated pipe culverts 1-24, 4-42, and 1-48 inches diameter, under 6 0 to 34 9 feet height of embankment, with similar measurements for a 24 inch by 27 inch concrete culvert under 32 2 feet height of embankment and a 42 inch cast iron culvert under 34 2 feet height of embankment The projection of the culverts was approximately 90 per cent ($p = 0\ 90$) The total loads were not weighed
>
> In 1928 the committee published the " theory of culvert loads " which it considered would explain its observed pressure data The committee stated in this publication that, for " Type I " culverts (rigid and semi-rigid), " the above theory, with a few slight changes in nomenclature, has been taken from various bulletins and papers by Dean Anson Marston " For " Type II " culverts (flexible), the committee repeated, with due credit to Dean Anson Marston, his solution for the case of a culvert actually buried in a ditch In consultation with Dean Marston, and acting on a suggestion made by him, the committee worked out and published a modification of Dean Marston's solution for the " incomplete ditch condition," to apply to the flexible culverts in their experiments

Thus the Farina experiments verify, in a general way, the author's theory presented in this paper.

> *University of North Carolina Culvert Load Tests, 1924-1927* The University of North Carolina, Chapel Hill, North Carolina, has conducted valuable culvert load tests in two series
>
> *First Series,*† 1924-1926 Three tests were made with 30 inch cast iron culvert pipe sections (32 inches outside diameter), projecting 50 per cent above the embankment sub-grade In the first test, the embankment was of sand, 20 feet high; in the second, of clay 14 feet high, and in the third of clay, 11 feet high. In the third test, at the height of 4 feet a ditch was dug over the pipe and refilled with loose material, following a plan tried at Ames, in 1920, to establish an artificial ditch condition. All embankments were protected during construction by tar-

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* See page 794, Vol 27, 1926, Proc A R E A, and page 527, Vol 29, 1928, Proc A R E A

† See Public Roads, Vol 7, No 11, January, 1927, for full report

> paulins from rain The first and second embankments were removed promptly after completion The third was continued in place from February, 1925, to May, 1926, but doubtless without real penetration of moisture through the clay cover to the ditch immediately over the pipe, for the load decreased (as transfer of load to the sides of the ditch might occur through the development of cohesion in the clay) to August, 1925, and the later increase had not yet exceeded or even reached the original load, up to the date of removal in May, 1926
>
> *Second Series,* 1926-1927. In the second series of tests, six 30-inch culverts, with 100 per cent projection, and respectively of smooth ingot iron, corrugated pipe, steel tube, cast iron pipe, reinforced concrete pipe, solid concrete plug, were tested successively on the same load weighing apparatus, under sand embankments 12 feet high above the culvert tops, protected during construction from rainfall and removed promptly after completion. No full report of these tests has been published, but a preliminary partial report has been published in the Proceedings of the Sixth Annual Meeting of the Highway Research Board, December 2-3, 1927, and the author is informed that the complete final report is to appear immediately
>
> In all the North Carolina culvert load tests, the total loads were weighed by reliable apparatus, and the results are of great value.
>
> However, as already explained (see p. 154), the ultimate loads on conduits under embankments are practically always at least 20 per cent to 25 per cent greater than the loads observed immediately on completion of the embankments over them, and additions of 25 per cent should be made to the North Carolina culvert load test results on this account

The data already published are not sufficient to enable a comparison of the loads measured with those computed by the author's theory, except in the case of the " ditch condition " culvert in the first series, and the concrete plug in the second series In both these cases the observed loads closely support the formulas of the author's theory Moreover, the load curves of the other tests already published are of such character that it is plainly evident that they conform closely to the curves of the author's theory, with suitable values of the settlement deflection ratio, r_{sd}, treated as a semi-empiric physical factor.

2 Verification by Field Experience

The mathematical theory of external loads on closed conduits under embankments, and projecting above the adjacent-embankment subgrades is also verified amply by field experience with actual culverts

In actual culverts many cases of injurious cracking or deformation have been encountered: a number of extensive field examinations of culverts have accumulated a mass of data on such cases

Upon comparison of the loads computed by the author's formulas with the known or probable supporting strengths of the culvert pipes used, it is found that the observed facts can readily be harmonized with the theory.

VII THE LATEST CLOSED CONDUIT LOAD EXPERIMENTS

Mr. M. G Spangler, of the staff of the Iowa Engineering Experiment Station, is conducting tests of three culverts, of 44 inches outside diameter, 25 feet apart under the same sandy loam top soil embankment, 15 feet high above the tops of the culverts, which are respectively of reinforced concrete, cast iron and corrugated metal. Extensive observations are being made of the settlements at a number of points on the embankment sub grades and in the interior of the embankment at different levels These culverts have already been under observation for two complete years, and the work is continuing Most valuable results have already been secured Figures 7, 8, 9, 10, and 11 are all from Mr Spangler's last progress report, which is to be published soon as a cooperative bulletin of the U S Bureau of Public Roads and the Iowa Engineering Experiment Station

Mr Spangler is also continuing the cooperative investigation of the ratio of the actual supporting strength of rigid pipe culverts to their laboratory test strengths with standard 3-edge bearings See cooperative bulletin 76 (1926) Iowa Engineering Experiment Station and U. S Bureau of Public Roads Seven more embankments have been erected and six removed since 1926 The latest results support a ratio of about 2.25

Mr. W. J. Schlick, also of the Iowa Engineering Experiment Station Staff, is conducting a series of cooperative experiments, partially financed by the Clay Products Association, Chicago, Illinois, upon the loads on sewer pipes in wide ditches, with vertical and with sloping sides.

His results verify the principle announced in 1913, in Bulletin 31, Iowa Engineering Experiment Station, that the load on a sewer pipe in a ditch with sloping sides is the same as in a vertical sided ditch of width equal to the width in the sloping sided ditch at the level of the top of the pipe

Mr Schlick finds that:

(1) The "ditch formula" (2) (Figure 1) gives the correct loads for widths of ditch not greater than $B_d = 1\frac{1}{2}B_c$.

(2) The "projection" formula (4) (Figure 1) gives the correct loads for widths of ditch greater than $B_d = 3B_c$.

(3) For widths of ditch between $1\frac{1}{2}B_c$ and $3B_c$, the values of H and of r_{sd} affect the decision between formula (2) and formula (4), in accordance with a table which Mr Schlick is preparing.

VIII. The Theory of the Supporting Strength of Pipe Conduits

In designing conduits to carry safely the loads to which they will be subjected in use, it is necessary to know, in advance, both the load and the supporting strength of the conduit

The supporting strengths of conduit pipes can be determined in advance only by laboratory tests, but the laboratory test strengths in general are not the same as the supporting strengths of the pipe in actual structures, because of the differences in the distribution of the pressures on the pipe

The actual distributions of the *normal* pressures around the three experimental culverts in the experiments at Ames, Iowa, are shown in Figures 9, 10, and 11, herewith

Laboratory tests should be made with,

(1) A S T. M. Standard "Sand Bearings"
(2) A S T M Standard "3-edge Bearings"

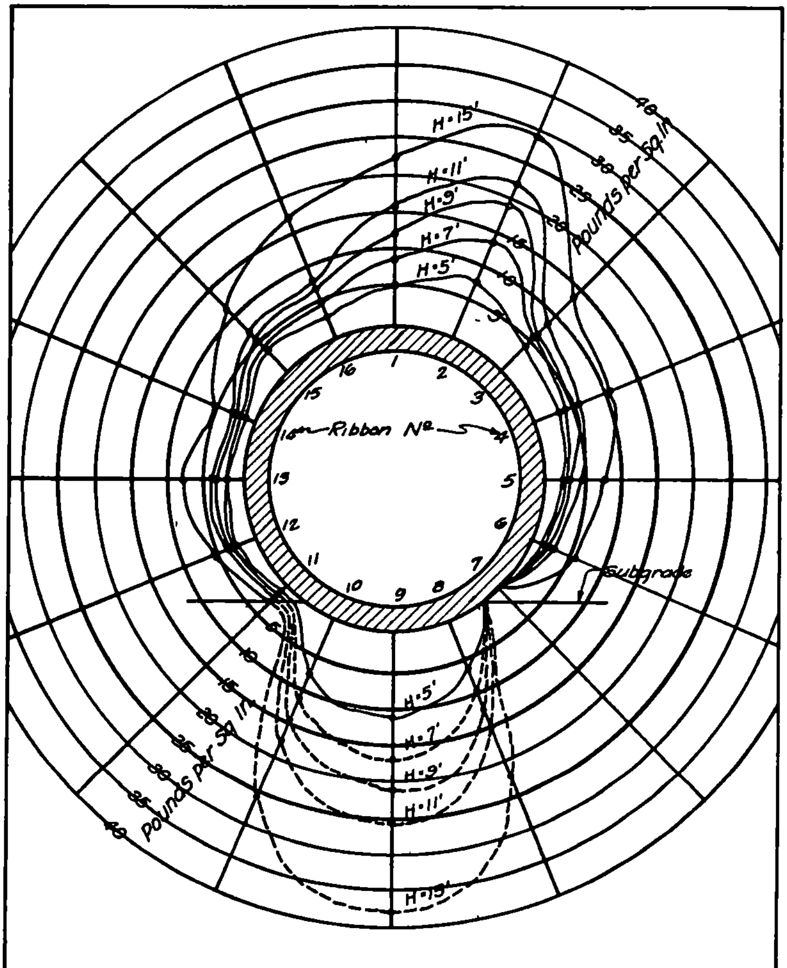
Let S_{sb} = "sand bearing" laboratory strength, pounds per foot length

S_{eb} = "3-edge bearing" laboratory strength, pounds per foot length

Notation for Laboratory and Supporting Strengths

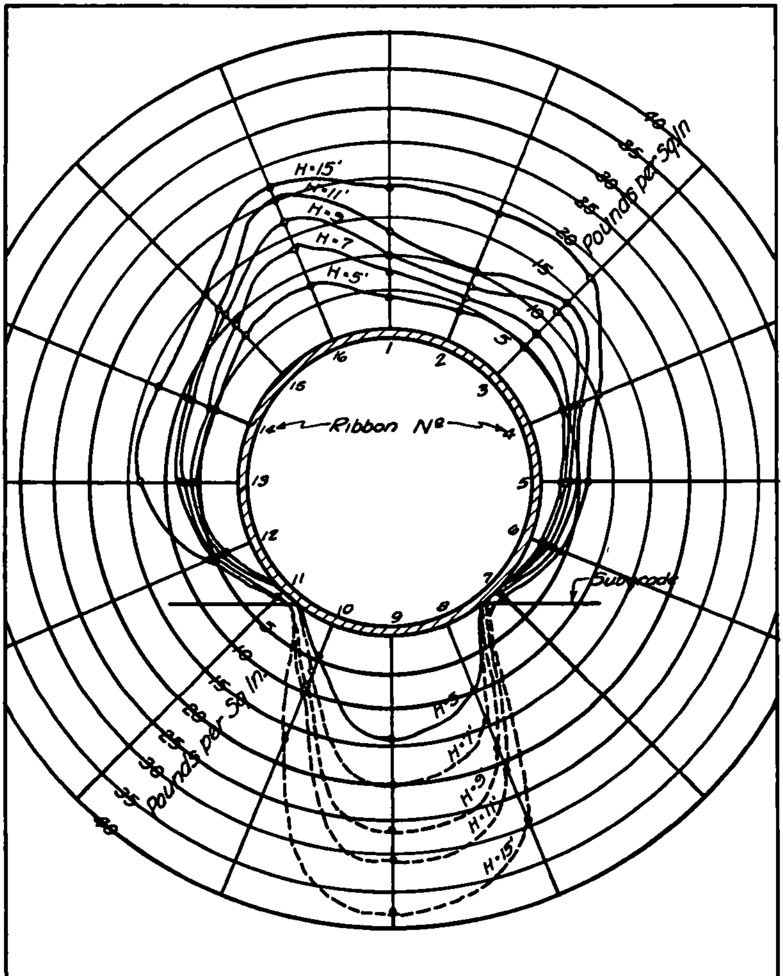
For "Ditch Conduits" (Completely Buried in Ditches)

S_{mb} = Supporting strength, "impermissible bedding," pounds per foot length



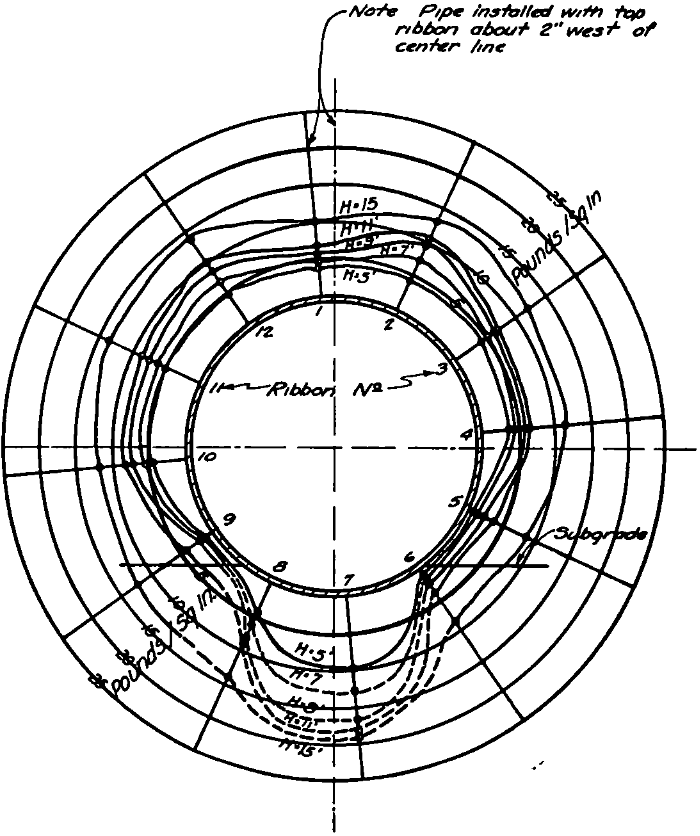
RADIAL PRESSURE ON CONCRETE PIPE CULVERT
SECTION A-3
MEASURED BY PRESSURE RIBBONS

Figure 9



RADIAL PRESSURE ON CAST IRON PIPE CULVERT
SECTION B-3
MEASURED BY PRESSURE RIBBONS

Figure 10



RADIAL PRESSURE ON CORRUGATED IRON PIPE CULVERT
SECTION C-3
MEASURED BY PRESSURE RIBBONS

Figure 11

S_{ob} = Supporting strength, "ordinary bedding," pounds per foot length

S_{cb} = Supporting strength, "1st class bedding," pounds per foot length

S_{cc} = Supporting strength, "concrete-cradle bedding," pounds per foot length.

For "Projecting Conduits" (Under Embankments and Projecting Above the Embankment Sub-Grades)

S_{opb} = Supporting strength, "ordinary projection bedding," pounds per foot length

A THE RELATION OF ACTUAL SUPPORTING STRENGTHS TO LABORATORY TEST STRENGTHS FOR "DITCH CONDUITS" (COMPLETELY BURIED IN DITCHES)

By several years of experiments, partly cooperative with the U S. Bureau of Public Roads, The Iowa Engineering Experiment Station has established the following relations between laboratory test strengths of conduit pipes and the actual supporting strengths of equivalent pipe used in conduits in ditches.

For "impermissible bedding," $S_{mb} = 0.75\ S_{sb} = 1.12\ S_{eb}$

For "ordinary bedding," $S_{ob} = 1\ 00\ S_{sb} = 1.50\ S_{eb}$.

For "1st class bedding," $S_{cb} = 1.25\ S_{sb} = 1\ 87\ S_{ob}$

For "concrete-cradle bedding," $S_{cc} = \begin{matrix}(1.50)\\(\text{to})\\(2\ 25)\end{matrix}\ S_{sb} = \begin{matrix}(2.25)\\(\text{to})\\(3\ 37)\end{matrix}\ S_{eb}$.

B THE RELATION OF ACTUAL SUPPORTING STRENGTHS TO LABORATORY TEST STRENGTHS FOR "PROJECTING CONDUITS" (UNDER EMBANKMENTS AND PROJECTING ABOVE THE EMBANKMENT SUB-GRADE)

In several years experimentation, cooperative with the U S Bureau of Public Roads, the Iowa Engineering Experiment Station has determined the ratio between laboratory test strengths and actual supporting strengths, of 90 per cent projection culverts with "ordinary projection beddings," in 9 cases of 36 inch clay and concrete culverts These seem to justify the relation (still somewhat tentative).

For "ordinary projection bedding," $S_{opb} = 1.50\ S_{sb} = 2\ 25\ S_{eb}$.

APPENDIX A

ACKNOWLEDGMENTS

During the past 21 years, the author has been assisted by a large number of different persons, committees, and organizations in the research work connected with the development of the theory of loads on conduits presented in this paper A detailed statement will be made in the near future, in a forthcoming bulletin of the Iowa Engineering Experiment Station, of the assistance rendered in each case In this paper there is room only to give, in approximately chronological order, the following list of names.

1 A. O Anderson
2 F. M. Okey.
3 H. W. Wagner.
4. A. N. Talbot
5 R W. Crum.
6 H F Clemmer.
7. W J Schlick.
8 Clyde Mason.
9 J H Griffith
10. J H. Ames
11 Bert Myers
12 Mark Morris.
13. Robley Winfrey.
14 M G. Spangler
15. J. C. Everds
16 J W. Johnson.
17 D G. Miller.
18 E B. Smith
19 Geo E Shafer, A R. E A Comm
20 D. L. Holl
21 Comm C-4, Am Soc Test Mat
22 Comm C-6, Am Soc Test Mat.
23 Joint Conc Cul Pipe Comm
24 U. S Bureau of Public Roads
25 Iowa State Highway Commission
26 Clay Products Association

At this point the writer wishes to make special mention of the assistance rendered by Mr George E Shafer in developing the theory for the incomplete ditch condition, Mr Robley Winfrey and Mr M G Spangler in research work connected with the theory of loads due to concentrated super-loads, including Equation 13, to Mr W J Schlick in connection with the theory of supporting strength of pipe culverts, and to the U S Bureau of Public Roads

APPENDIX B

PUBLICATION OF THE THEORY OF LOADS ON CONDUITS

The publication of the theory of loads on conduits has been made in successive stages in several papers, progress reports and bulletins, including the following:

1 Marston, Anson Standard Tests for Drain Tile and Sewer Pipe. A S. T M. Proc 11: 833-844, 1911.

2 Marston, Anson Report on Standard Tests and Specifications for Drain Tile (Committee C-6 Report) A. S T M. Proc 13: 303-312, 1913

3 Marston, Anson, and Anderson, A. O The Theory of Loads on Pipes in Ditches and Tests of Cement and Clay Drain Tile and Sewer Pipe. Ia Eng. Exp. Sta Bul 31, 1913.

4. Marston, Anson, Chr., and Stewart, J. T , Sec. Report of the Investigations on Drain Tile of Committee C-6 on Standard Tests and Specifications for Drain Tile American Society for Testing Materials. Bul. 36, Ia Eng Exp Sta , 1914.

5. Marston, Anson Report on Standard Tests and Specifications for Drain Tile. (Committee C-6 Report) A S. T. M Proc 14: pt. 1: 208-212, 1914.

6 Crum, R W. Failure of a 30-inch Tile Drain at Albert Lea, Minn. A. S. T. M. Proc 17: 453-463, 1917.

7. Marston, Anson, Schlick, W. J., and Clemmer, H F The Supporting Strength of Sewer Pipe in Ditches and Methods of Testing Sewer Pipe in Laboratories to Determine Their Ordinary Supporting Strength Iowa Eng Exp Sta Bul 47, 1917.

8. Schlick, W. J. An Investigation of Tests of Iowa Shale Drain Tile. Ia. Eng. Exp Sta Bul 49, 1918.

9. Schlick, W. J. Supporting Strength of Drain Tile and Sewer Pipe Under Different Pipe Laying Conditions Ia. Eng Exp. Sta Bul 57, 1920.

10. Marston, Anson. Second Progress Report to the Joint Concrete Culvert Pipe Committee Ia Eng Exp Sta Mimeograph Report, 1922.

11. Marston, Anson Culvert Research Report to the Advisory Board on Highway Research. Highway Research Bd. Proc. 5 . pt 1: 284-291, 1925.

12. Spangler, M. G. A Preliminary Experiment on the Supporting Strength of Culvert Pipes in an Actual Embankment Iowa Eng. Exp Sta. Bul. 76, 1926.

13. Spangler, M G., Mason, Clyde, and Winfrey, Robley Experimental Determinations of Static and Impact Loads Transmitted to Culverts Ia Eng. Exp Station Bul. 79, 1926.

14. Schlick, W. J , and Johnson, J W Concrete Cradles for Large Pipe Conduits Ia Eng Exp. Sta. Bul. 80, 1926.

15. Marston, Anson. Recent Research Relative to Culvert Pipe. (12th Annual Meeting of the American Association of State Highway Officials. Pinehurst, N. C , 1926.) Public Roads, 7: 226-229, 1927.

16. Schlick, W. J. Tests of Clay and Concrete Load Bearing Pipe A. S. T. M Proc. 28, pt 2 · 635, 1928.

17 Schlick, W. J. Supporting Strength of Concrete-Incased Clay Pipe. Bul. 93, Ia. Eng. Exp. Station, 1929.