

Effect of Stress on the Dynamic Modulus of Concrete

D. A. LINGER, Associate Professor of Civil Engineering, University of Arizona

This paper presents the results of a preliminary investigation of the effect of stress intensity on the modulus of elasticity of concrete. Stress-strain diagrams are shown which were constructed for static tests and by using the vibratory data for transverse resonant vibrations at various stress intensities. The various tests were performed on the same cylinders, thereby using the same specimen for both the vibratory and static tests. The vibratory test is an adaptation of ASTM C215-60 with the specimens vibrated in their transverse mode.

The results of this investigation indicate that the dynamic modulus of elasticity increases as the intensity of stress on the specimen increases. The increase in the dynamic modulus of elasticity varied from 7 to 18 percent for different strength specimens, whereas the corresponding decrease in the static "tangent" modulus was 54 and 7 percent for the same specimens.

•IN A SENSE, only perfectly elastic materials have a modulus of elasticity. For many structural materials, the stress-strain relations are neither constant nor perfectly elastic. This is particularly true of concrete for which values of this ratio are generally understood to vary with magnitude and period of application of stress, with age and quality of concrete, and with moisture and temperature conditions. Furthermore, concrete is not perfectly elastic even at low stresses because it fails to resume completely its original dimensions on release of stress and continues slowly to deform over a long period of time under the action of sustained stress. Hence, insofar as concrete is concerned, the term modulus of elasticity is not a correct term because the stress-strain ratio includes plastic as well as elastic deformations.

The load deformation of a concrete specimen is dependent on two factors: the amount of stress and the length of time over which the stress is applied. The first type of deformation is due to the elastic properties of the concrete. The second type of deformation is the result of the inelastic or plastic properties of concrete.

In computing the total deformation of a concrete structure, the entire strain—plastic and elastic—must be used. If, however, the dynamic response of a concrete structure is desired, the modulus of elasticity, as found by the sonic or vibratory method, would govern the response of the structure and would be considerably different from that obtained in the static test.

It is difficult to define clearly all the reasons for the difference between the static and the dynamic moduli of elasticity, but the difference is probably in the varying amounts of plastic and elastic deformations that occur. Moreover, because of the stress-strain characteristics of concrete, it is probable that the modulus of elasticity of concrete, regardless of how it is determined, is a function of the stress intensity.

OBJECTIVES

Before undertaking a broad research study of the static, dynamic, and fatigue properties of concrete, an evaluation was made of the various experimental techniques used in concrete research. During this process, the effect of stress intensity on the dynamic modulus of elasticity of concrete was evaluated. The techniques used and results obtained for three representative specimens of different mixes and strengths are de-

scribed. The methods used to study the elastic modulus of concrete used the same specimen for both the vibratory and the static tests.

TEST METHODS

Any method that is based on one specific property will have results that are dependent on the component in the mixture that has the greatest effect on the property measured. To obtain the modulus that governs the response of concrete structures, it is necessary to base the method on the action of the entire mass. Thus, the value determined will be the result of the nonhomogeneous mass acting singularly and not on any one component of the mass.

Static Method

This procedure, which measures the action of the entire mass and includes some plastic as well as the elastic deformation, is similar to the procedure covered in ASTM C469-61T, in which the load is applied slowly while the deformations are measured and the resulting stress-strain diagram obtained. The static method, even when the loads are applied at extremely high rates of speed, includes the plastic flow of the concrete in the measurement of strain.

Dynamic Method

The dynamic method used is similar to ASTM C215-60, in which the resonant frequency of the specimen is determined and by using the size and density of the specimen, the elastic constant is computed. Because of the extremely small deformations that occur as a result of the resonant vibrations, this method incorporates little, if any, plastic flow in the results. Moreover, because this method is based on the vibratory motion of the entire specimen, it should yield a modulus of elasticity that will govern the response of concrete either to vibratory and shock loadings or to static loads after the concrete has been subjected to sustained stresses. Therefore, it should determine what might be called the "purely elastic" modulus of elasticity of a specimen. The disadvantage of this method, however, is that the resonant frequency is determined at a zero stress and, therefore, the dynamic modulus of elasticity is the "purely elastic" property at a zero stress intensity. To overcome this disadvantage, the vibratory method was used with cylindrical specimens that were prestressed by post-tensioning bars along their longitudinal axis.

To reduce the irregularity of results often found in concrete testing, the static and dynamic procedures were performed on the same specimens.

Test Specimens

The specimens were 6 in. in diameter and approximately 30 in. long. The actual lengths were 30.27, 29.00, and 28.48 in. for specimens 1, 2, and 3, respectively. A $1\frac{1}{4}$ -in. diameter hole was cast along the longitudinal axis of the cylinders. The purpose of the hole was to enable a prestressing rod to be inserted in the specimen. Each specimen was tested through the entire series of tests within a reasonable length of time, usually one or two hours. This was done to insure that the results would not be affected by a prolonged stress on the concrete.

The specimens were all cast of the same aggregate. The coarse aggregate was a subrounded material with a maximum size of $\frac{3}{4}$ in. Specimen 1 had a water-cement ratio of 0.684, and the mix by weight was 1, 2.92, 2.10. The average ultimate 28-day strength of this mix was 3,200 psi. Specimen 2 had a water-cement ratio of 0.60 and a mix by weight of 1, 2.18, 1.79. The average 28-day strength of this mix was 3,800 psi. Specimen 3 had a water-cement ratio of 0.546, and a mix by weight of 1, 2.05, 1.63. The average 28-day strength of this mix was 4,500 psi.

Static Equipment

The compressometer used to measure the longitudinal deformation of the specimen and thereby to determine the axial stress-strain curve was made of two 8-in. diameter

cast steel yokes (Fig. 1). Each yoke was mounted on the specimen with three set screws, thus enabling each yoke to be entirely isolated and to form a plane perpendicular to the axis of the cylinder. The gage length of the compressometer was 8 in. This was determined by the two spacers that allowed the yokes to be 8 in. center to center. The unit deformation was found by averaging the total deformation as indicated on the two dial gages and then dividing by the gage length. The smallest unit deformation that could be read was approximately 3 micro-in. per in.

Static Procedure

To determine the static stress-strain curve of the test specimens, a 200,000-lb Olsen testing machine was used. The load was applied at a rate of 0.05 in. per min. and readings were taken at approximately 5,000-lb increments.

Prestressing Procedure

The flexural resonance was determined for the prestressed specimens in order to compute the dynamic modulus of elasticity at elevated stresses. In applying the prestress, the specimen was first subjected to a compressive stress, then the $1\frac{1}{16}$ -in. diameter prestressing rod was tightened. As the load was removed from the specimen, the rod absorbed the prestress. To determine the amount of prestress accurately, the prestress rod was instrumented with an SR-4 strain gage. This strain gage was calibrated after being mounted on the prestress rod to insure accuracy in the computation of the amount of the prestress. A specimen being prestressed in the Olsen testing machine is shown in Figure 2. The SR-4 strain indicator and switching box are shown with the dummy gage and the wires leading to the capping plate and thence to the strain gage mounted on the prestress rod. Concurrently, with the first prestressing of the specimen, a static stress-strain curve was determined.

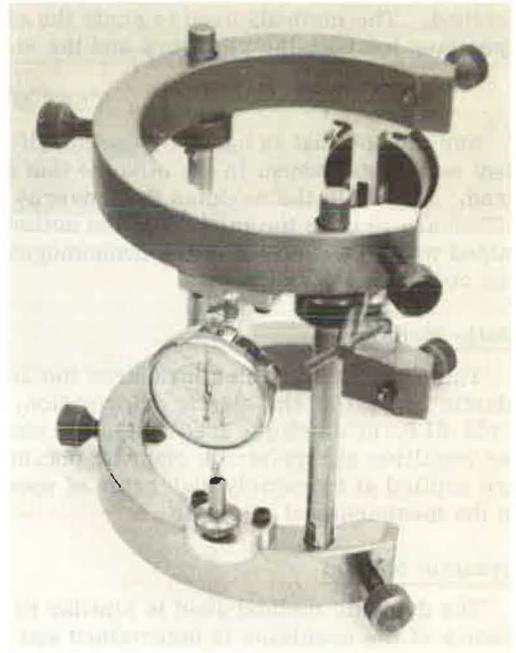


Figure 1. Compressometer.



Figure 2. Prestressing specimen.

Dynamic Equipment

The equipment used to determine the resonant frequency of the specimens consisted of an audio-oscillator, two amplifiers, an oscilloscope, a vibration pickup, and a driver. The equipment is shown in Figure 3 and corresponds to the equipment specified in ASTM C215-60.

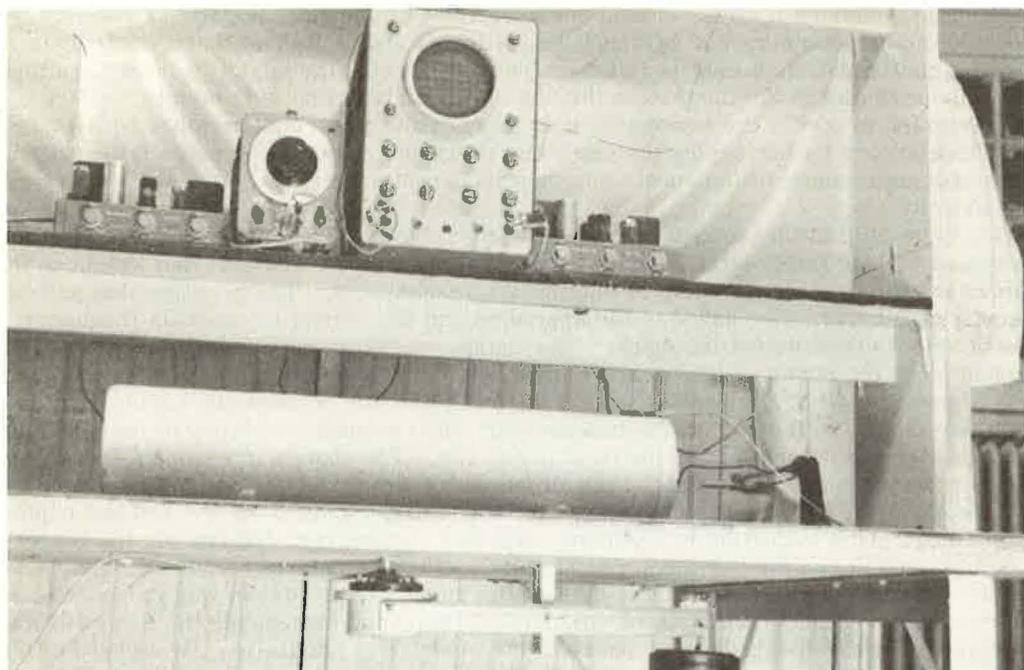


Figure 3. Vibratory modulus equipment.

Dynamic Procedure

The flexural or transverse mode of vibration was used in this investigation. The flexural mode of vibration was used to reduce the effect of the steel prestressing rod and capping plate. The steel prestressing rod and capping plates were needed to place the specimens in a stressed condition. The prestressing rod was mounted axially in the specimen so that there were no points of contact between the concrete specimen and the steel prestress rod. The weight of the rod, therefore, was carried by the capping plates which were in contact with the end of the concrete specimen. This was very carefully done to insure that the effect of the addition of the steel rod and capping plates could be considered the same as the addition of a mass at each end of a transversely vibrating beam.

Resonance in the specimen was indicated in one of two ways. The first method was that of noting the maximum vertical amplitude of the signal from the crystal pickup as applied to the vertical axis of the oscilloscope. This method makes use of the maximum amplitude of the specimen when vibrating at its resonance. The second method makes use of the frequency relationship between the driver and the specimen. When the specimen is in resonance, the driver frequency is exactly the same as the frequency of the specimen. By using a Lissajous figure, in which the driving frequency is entered on one axis and the specimen frequency is entered on the other axis, the frequency relationship can be easily determined. An oscilloscope was used in this capacity.

The specimen to be tested was mounted at its nodal points which are $0.22L$ from each end where L is the length of the specimen. In the case of the specimens that had been prestressed, the nodal points moved out toward the ends because of the added weight of the prestressing rod and the capping plates. Each time the specimen was thought to be in resonance, the pickup was moved along the specimen in such a manner as to show the mode of vibration at which the specimen was vibrating.

The method used in this study is based on the vibratory method of determining modulus of elasticity at zero stress. Using this method at various stress intensities, it is

possible to construct a stress-strain curve entirely of dynamic moduli. To do this, the moduli are determined at different stress levels and plotted so that a continuous curve joins the points of tangency at the stresses at which the moduli were determined. The only difficulty in this method is the use of an additional outside member which must be used to stress the concrete. It is this outside or foreign member which may introduce errors by increasing the mass end fixity and possibly the potential energy in the restoring moment of the specimen, thus incorrectly changing the dynamic modulus of elasticity.

The first step in this testing procedure was to obtain the resonant frequency of the specimen without the prestress rod. By using this value and the size and weight of the specimen, the dynamic modulus of elasticity was computed. The prestressing rod and capping plates were then added to the specimen and the change in resonant frequency was noted. In this step of the testing, the change was often very irregular. This was thought to be the result of the prestress rod being only hand-tight on the specimen, and thus allowing additional vibrations to occur in the junction between the capping plates and the washers on the rod. To eliminate this, the resonant frequency of the specimen with the rod was determined at the first prestress. The change in resonant frequency between the plain specimen and the specimen with the rod and capping plates prestressed to approximately 400 psi was considered to be due to the addition of the rod and capping plates only. This reduction in resonant frequency varied from 228 cps for the longer specimen to 350 cps for the shorter specimen. After the natural frequency was obtained for the specimens with the first prestress, the amount of prestress was increased and the resonance again determined. If the natural frequency increased, the increase was assumed to be the result of an increase in the modulus of elasticity. By using the first prestress resonance as the base resonance for the specimen with the rod and plates, it was assumed that the modulus of elasticity for the base resonance or first prestress was equal to the dynamic modulus of elasticity of the specimen at zero stress. The additional moduli were computed by noting that the modulus of elasticity of a particular

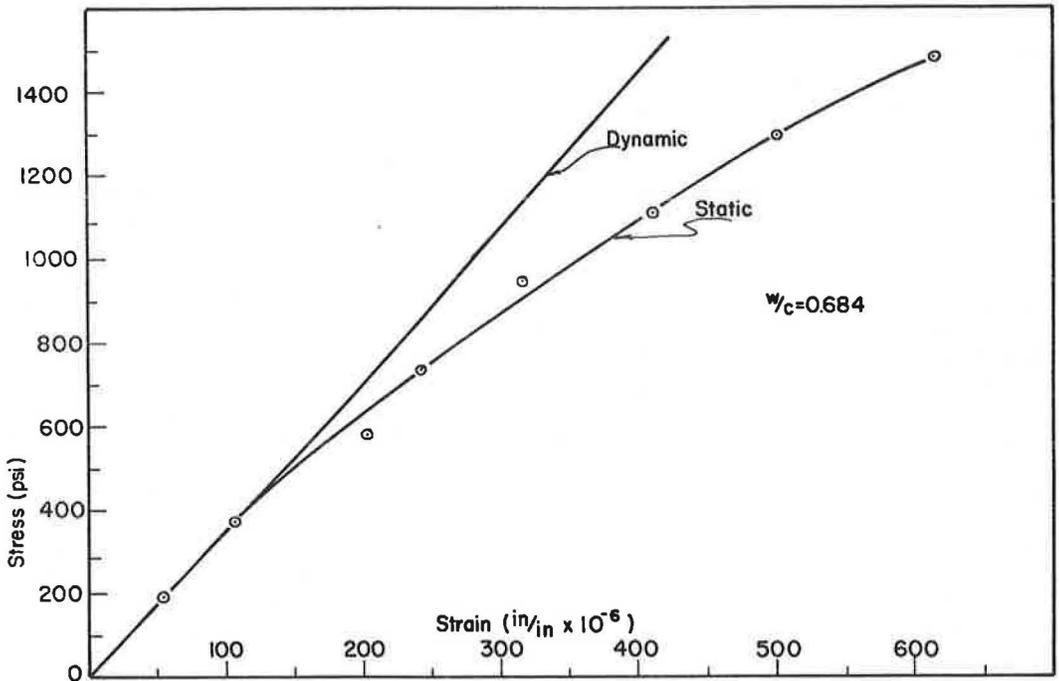


Figure 4. Stress-strain curve for specimen 1.

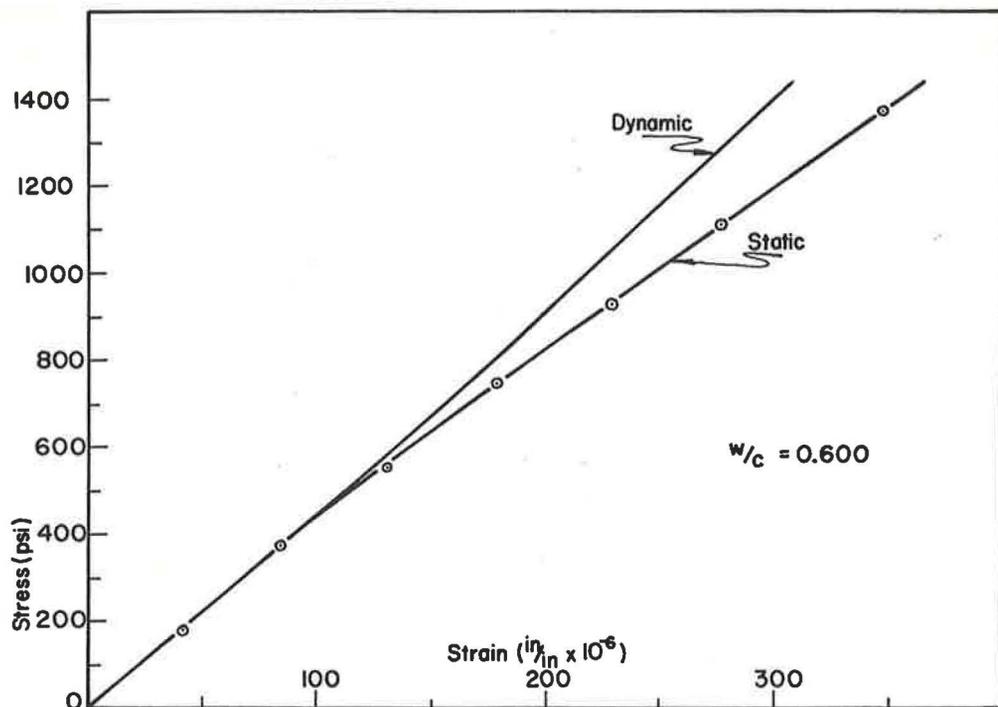


Figure 5. Stress-strain curve for specimen 2.

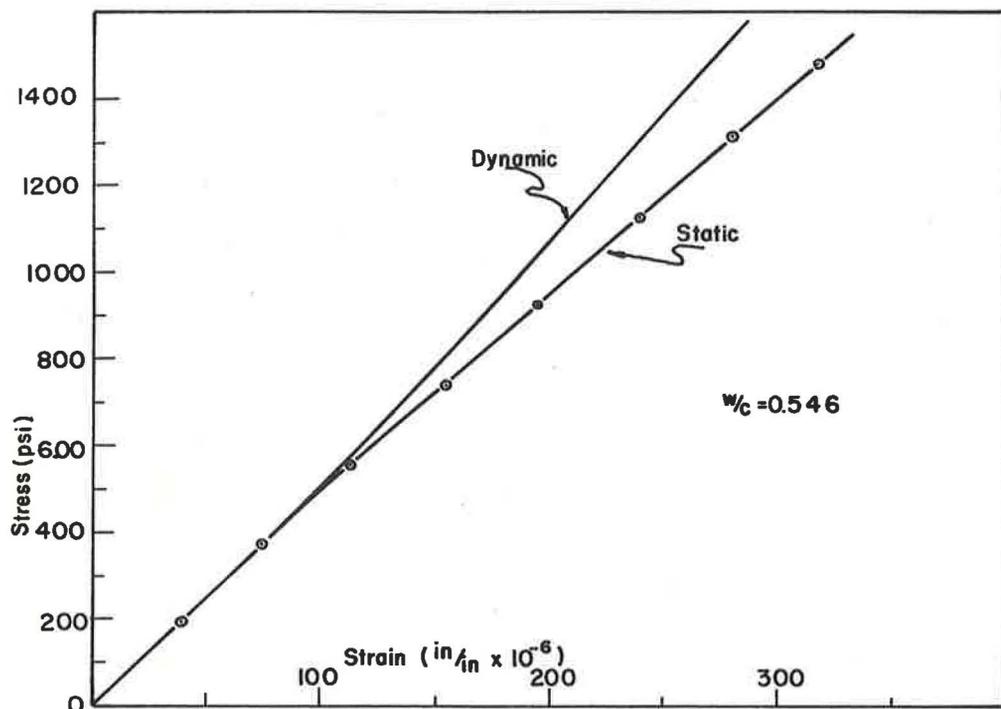


Figure 6. Stress-strain curve for specimen 3.

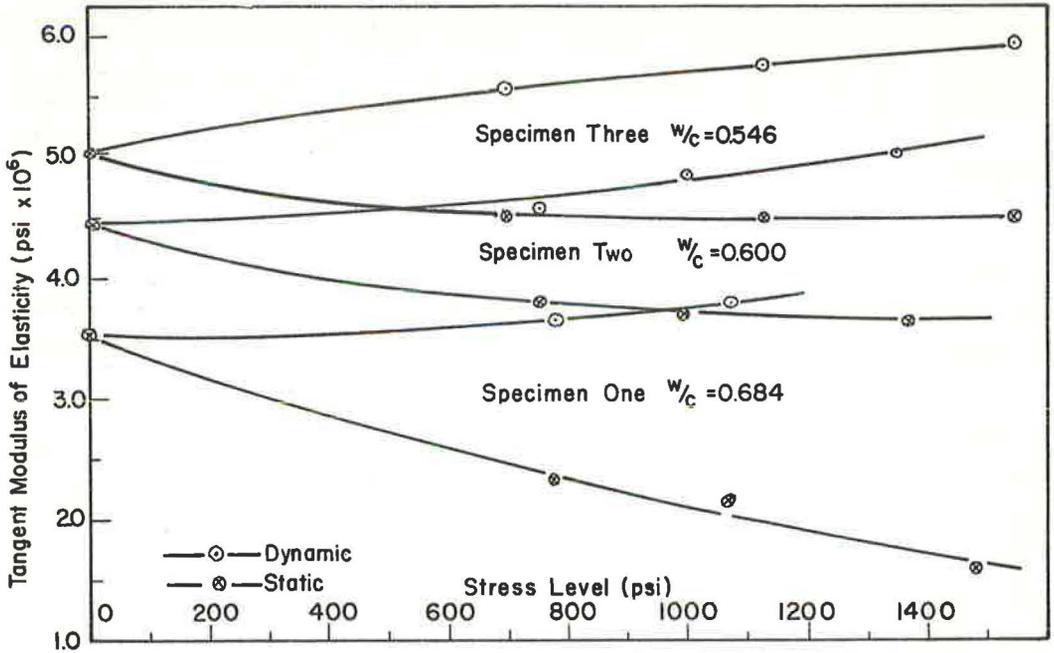


Figure 7. Variation in tangent modulus of elasticity with stress level.

specimen is directly proportional to the square of the resonant frequency. The dynamic modulus of elasticity was then determined for each specimen at each prestress by multiplying the square of the ratio of the stressed natural frequency to the base natural frequency by the unstressed dynamic modulus of elasticity. Because the maximum change in stress during the vibration is very small, the values of the moduli from which the results are based are values obtained at very small increments of stress. Therefore, the results should not include any appreciable amount of plastic flow. The stress-strain curves as obtained by the procedures previously discussed for the static and dynamic tests are shown in Figures 4, 5, and 6 for specimens 1, 2, and 3, respectively.

RESULTS

It is evident from Figures 4, 5, and 6, that the dynamic modulus at zero stress agrees closely with the initial tangent modulus of elasticity for the static results. Moreover, both the static initial tangent modulus and the dynamic modulus at zero stress increase for the higher strength specimens. The static compression test results show the typical stress-strain curve which is concave downward. The modulus of elasticity for this curve decreases with increasing stress. However, the dynamic moduli increase for an increasing stress and form a stress-strain curve that is uniquely concave upward. This is not unreasonable, however, if the Watertown Arsenal Report of 1899 and other studies (1) of natural building stones are consulted. These studies indicate that natural stone has a concave upward stress-strain curve. Inasmuch as concrete might be considered a "green" form of natural building stone, it is evident that by eliminating creep it might be expected to act as a natural stone. Moreover, the results presented herein agree with and substantiate the results obtained by Obert in 1941 with a different method (2).

The area between the dynamic curve and the static curve represents what might be considered "creep strain," which decreased for the higher strength specimens.

The best indication of the difference between the static and dynamic results can be

represented by plotting the dynamic moduli at elevated stresses and the static tangent moduli at each similar stress against the stress magnitude, as shown in Figure 7. The increase in the dynamic modulus of elasticity was 7.4, 13.2, and 18.3 percent, and the decrease in the static tangent modulus was 54.8, 16.3, and 6.96 percent for specimens 1, 2, and 3, respectively. Obert found an increase in the dynamic modulus of 8.2 to 13.0 percent of the initial tangent modulus of elasticity (2), which is in the range obtained in this research.

CONCLUSIONS

The results of the tests performed in this investigation show that the modulus of elasticity of concrete has a tendency to increase as the stress increases, provided the inelastic creep can be eliminated. In this study, the creep was eliminated or preactivated by prestressing the specimens and by the use of a very small superimposed stress during the determination of the elastic modulus by the vibratory method.

The difference between the static modulus and the dynamic modulus was less for the higher strength specimens, which indicates the possible application of this method in studying the elastic properties used for deflection calculations in high strength concrete. The results also suggest the use of this method in studying the repeated load response of concrete.

REFERENCES

1. Griffith, J. H., "Physical Properties of Typical American Rocks." Iowa Engineering Experimentation Station Bull. 131 (1937).
2. Obert, L., "Sonic Method of Determining the Modulus of Elasticity of Building Materials Under Pressure." Proc., ASTM, 39:987 (1939).
3. Philleo, R. E., "Comparison of Results of Three Methods for Determining Young's Modulus of Elasticity of Concrete." ACI, Proc., 51:461 (1955).
4. Stanton, T. E., "Tests Comparing the Modulus of Elasticity of Portland Cement Concrete as Determined by the Dynamic (Sonic) and Compression (Secant at 1000 Psi) Methods." ASTM Bull. 131, 17 (1944).

Discussion

ERNEST E. MC COY, Chief, Thermal Research and Field Durability Section, Concrete Division, U. S. Army Engineer Waterways Experiment Station, Jackson, Mississippi—

The author is to be congratulated for his excellent paper on a subject that is currently of increasing interest and importance. In 1939 Obert (5) described results of vibration tests of concrete beam specimens under stress; he used the simply supported (hinged-ends) mode of vibration. After the author's paper was presented, a single concrete specimen was tested at the writer's laboratory to investigate whether it is feasible to vibrate a specimen under longitudinal stress (compression), in the fixed-ends mode of vibration.

The specimen was a 6- by 12-in. cylinder. Its age was 8 months, weight 28.24 lb, air content 1.8 percent, and dynamic modulus 3.30×10^6 psi. The specimen was from a set having approximately 2,400-psi compressive strength at 28 days age, at which age it had been subjected to a hydraulic pressure of 4,000 psi in a test for air content. It had then dried in laboratory air for several months and its surface presumably was covered with microscopic shrinkage cracks at the time of the tests described below.

The specimen was subjected to two vibration tests with incremental compressive stress. For the first test, load was applied in five 100-psi increments and a final 500-

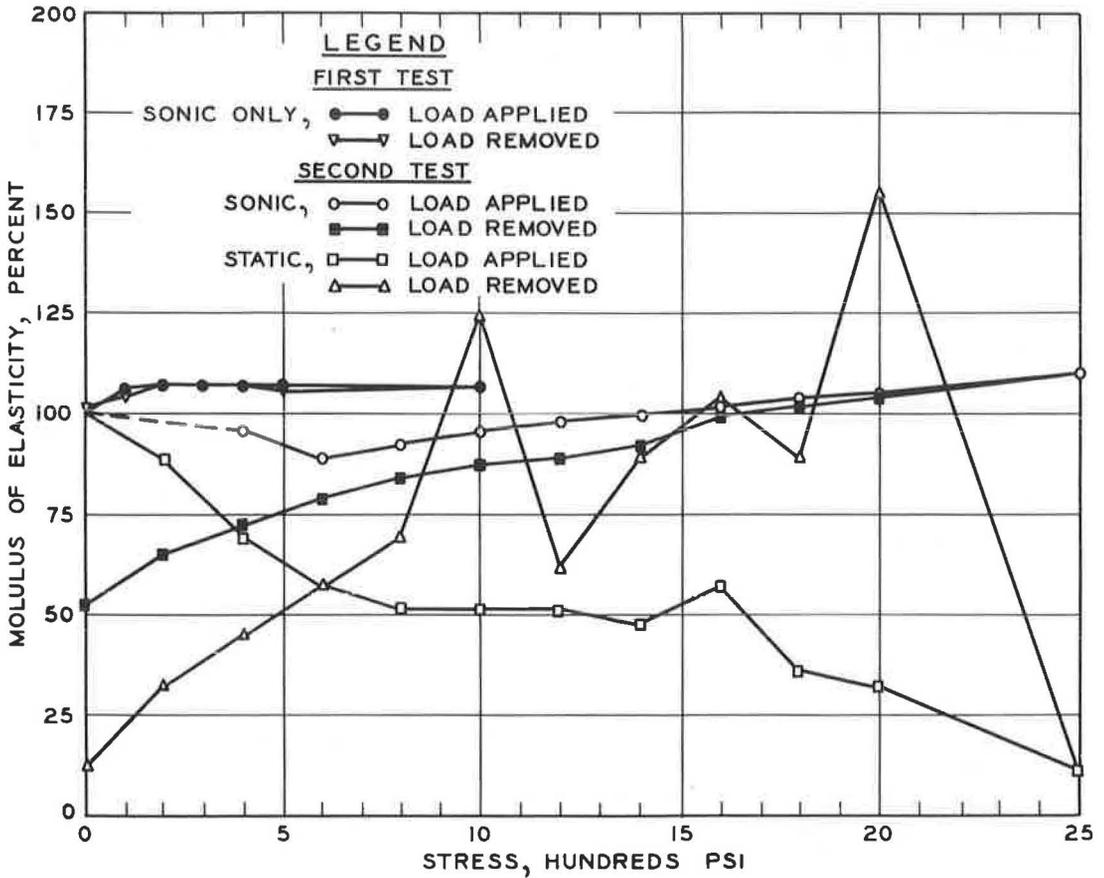


Figure 8. Variations of dynamic and static modulus with stress.

psi increment to 1,000 psi in a universal testing machine, and load was removed by the same increments. The resonant frequency was recorded for each increment. After six days the specimen was capped with a very thin layer of high-strength plaster, and four strain gages of $2\frac{1}{2}$ -in. gage length (wired to read as one gage) were mounted at 90° intervals. The next day the specimen was tested to 2,500 psi by increments of 200 to 2,000 psi and a final increment of 500 psi. Load was removed by the same increments. The rate of loading was 175 psi per min for each test. The initial frequency for the second test was obtained with the specimen properly supported for the free-free mode, and after the loading test was completed, the frequency was checked in the free-free mode, agreeing to 1 percent with the final frequency obtained for the loading test.

Figures 8 and 9 show the results. All frequencies on which (when squared) changes of modulus were based were assumed to be for vibration in the fixed-ends mode. According to simple formulas (2), there should be no difference between free-free and fixed-ends resonant frequencies.

The frequency at each point for the unloading cycle of the first test agreed to 1 percent with the loading cycle frequency. All the frequencies for this test, however, were about 5 percent lower than the predetermined free-free frequency. The pickup was taped to the specimen for the free-free test and was not removed for the incremental test; however, for the latter the pressure of the driver against the specimen may have been slightly higher.

It was thought that capping the specimen would improve free-free and fixed-ends frequency agreement for the second test. As mentioned, the initial frequency for the

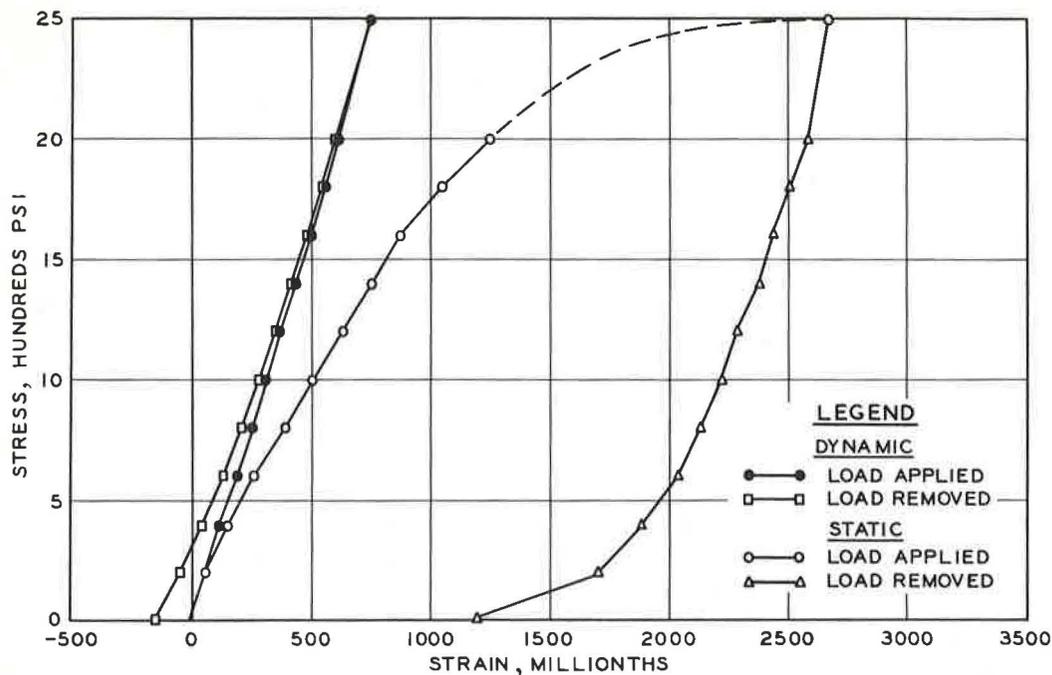


Figure 9. Stress-strain diagram, second test.

second test was obtained with certainty for the free-free mode. At zero load and at 200 psi, however, it appeared that the only frequencies obtainable were not relevant and they were not used. On removal of load, the squared frequencies, including the zero-load frequency, followed the smooth curve shown. The dip in the loading cycle curve can hardly be understood without further testing and study. A very subdued tendency for the vibration to be influenced by the simply supported mode, which has a lower frequency, may have affected the frequency for this test, and also for the first test. Beginning at 600 psi, the dynamic modulus curve closely resembles that for specimen 3 of the author's investigation, increasing 21 percent as compared with 18.3 percent. The increase of dynamic modulus beyond about 2,000 psi is noteworthy because the specimen had evidently been loaded critically beyond the yield point, as shown by strain measurements.

The zero static modulus (3.2×10^6 psi) for the strain gage measurement was obtained by extrapolation of the first three points. The percentage changes (Fig. 8) are for the chord modulus based on the measured stress and strain increments between consecutive points. The plateau between about 600 and 1,600 psi is considered to be evidence of the closing of microscopic cracks before critical yielding. A very considerable amount of creep or yield was observed to be in progress above 2,000 psi.

The results of the second test show that there was a very considerable divergence between static and dynamic modulus for the loading cycle, as did the results of the tests reported by the author and by Obert. The method employed by the author does not permit rapid simultaneous testing for both the static and dynamic modulus; that employed by Obert would do so with the addition of strain gages. Both methods require the use of end plates for which satisfactory corrections can be applied.

For the stress-strain diagram (Fig. 9), the apparent strains for the second dynamic test were computed by the method used by the author. The negative strains for a portion of the unloading curve are without special significance, merely reflecting the change in sonic modulus to a considerably reduced value as a result of overloading. The loading parts of the curves in Figure 9 are quite similar to the curves shown in Figures 4, 5, and 6.

The results of these tests provide evidence of the possible usefulness of the fixed-ends mode of vibration. The advantage of the method, if it can be perfected, would be continuous rapid testing of a relatively unencumbered specimen. It appears, however, that for further investigation of vibration in this mode the specimen should be firmly fixed to the testing machine platens or to relatively large and heavy auxiliary plates. It is contemplated that high-strength plaster or epoxy will be used for this purpose in an additional test. Apparently the amplitude of the vibrations at the ends is so small (only a few microinches) that imperfect contact, especially at low load, affects the test.

REFERENCES

5. Obert, L., "Sonic Method of Determining the Modulus of Elasticity of Building Materials Under Pressure." Proc., ASTM, Vol. 39 (1939).
6. Van Santen, G.W., "Mechanical Vibration." MacMillan, 2nd ed. (1958).

D. A. LINGER, Closure--Mr. McCoy's discussion presents several important aspects and adds to the understanding of the dynamic modulus characteristic of concrete. The research of Obert previously mentioned is significant because in it are the basic ideas enlarged on by Mr. McCoy and the author. However, the limitations of the various methods used in determining the dynamic modulus at elevated stresses are noteworthy. Obert's method necessitated corrections amounting to 25 percent of the total change in frequency. These corrections were needed to account for the mass of the capping plates and the change in the vibrating condition. Hence, the correction applied by Obert was larger than the increase in the dynamic modulus obtained. Other problems exist inasmuch as it has been shown by Timoshenko (7) that an axial load changes the vibrating system and hence changes the natural frequency of vibration. The effect of an axial compressive load on the vibration of a hinged-end beam or a free-free beam results in a decrease in the frequency of the system by an amount equal to $\sqrt{1 - P/P_C}$ in which P is the axial load, and P_C is the Euler buckling load. If this is the only effect of the axial load, it is found that the reduction in frequency for a 6- by 12-in. cylinder is less than 0.1 percent, and for a 6- by 30-in. specimen is only approximately 0.1 percent. Therefore, the effect of the axial load on the hinged-end or a free-free vibrating system is negligibly small, if the end conditions and the fundamental mode of vibration are maintained.

The effect of changes in mode or type of vibration can, however, be quite large. The fixed end condition necessary for the fixed-fixed mode of vibration is extremely difficult to obtain, and Mr. McCoy is to be congratulated on the close agreement of the fixed-fixed vibration frequency of the second loading with the anticipated value. Vibration tests similar to Mr. McCoy's have been conducted by the author and it was found that the capping plates, type of testing machine, and the amount of axial load affected the end fixity of the cylinder and hence, the mode of vibration. This can be exemplified theoretically by noting that the percentage reduction in natural frequency of a beam from the fixed-fixed condition to the hinged-end condition is 57 percent. During a study of the elastic properties of the beam specimen, this change (or any small part of it) would be included in the calculations for the change in the dynamic modulus. Therefore, it is apparent that the type of test used to evaluate the dynamic modulus of materials needs close and careful study.

This brief discussion of the methods used should not detract from the results obtained. It is important to point out that the "unknown" factors just mentioned have a tendency to reduce the natural frequency or the resulting modulus of elasticity. From this it would seem reasonable that the increase in the dynamic modulus might have been even larger if the other "unknown" factors could have been held constant. Moreover, the increase that does occur in the dynamic modulus is extremely important in the dynamic analysis of concrete structures. During a recent comprehensive evaluation of the dynamics of highway bridges, it was found that the modulus of elasticity of

the structural concrete needed to be considerably larger in order to obtain any agreement between the theoretical natural frequency of the bridges and the experimentally obtained actual natural frequency. In this case any theoretical dynamic analysis of the structures would have included considerable error if the static value of the modulus of elasticity was used in the computations.

The author is indebted to Mr. McCoy for his discussion, and would like to again point out the need for additional research on this subject in an era in which dynamic or blast loadings are becoming important concepts in design.

REFERENCES

7. Timoshenko, S. P., "Vibration Problems in Engineering." New York, p. 374 (1955).
8. Linger, D., and Hulsbos, C. L., "Dynamics of Highway Bridges." Iowa Engineering Experiment Station Bull. 188 (1960).