Static and Dynamic Visual Fields in Vehicular Guidance

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•TO UNDERSTAND driving, the visual input must be analyzed. Until the effective stimuli are known, driving cannot be thoroughly understood; when these inputs are identified, driving itself will be described to a considerable extent. It is appropriate that the investigation of visual inputs begin with the study of the positional, velocity and acceleration fields around the moving vehicle. These fields are general and persistent aspects of the visual environment. The velocity and acceleration fields, which present time-varying aspects of the environment, are of particular interest, since they provide information not available in static viewing.

The numerous "cues" available in spatial perception have been discussed previously (4, 20, 22) and the terrain characteristics which may orient the human in his spatial environment have also been described (10). These studies indicate methods which might be employed in vehicular guidance, but we are interested in identifying those which the driver actually uses. Other studies have concerned themselves with human errors in space perception, such as the systematic overestimation of size in distant vision (15) and the hyperbolic metric shown in the judgment of space in certain reduction situations (1). In the present context, the characteristic of the driver's judgment of space of concern is its accuracy rather than incompatibility with physical space.

Two main problems are considered in this paper. The first is concerned with the mathematical description of the moving ground plane from the driver's point of view. The environment seen by the driver involves a perspective transformation of ground position, velocity, and acceleration. The formulas governing these transformations are developed, and the fields themselves are plotted. The positional field, which includes the angular coordinates from eye position of points in the driver's environment, is related to linear perspective. The velocity field includes the vectors of angular motion around the driver's eye as he moves along the road. The acceleration field presents vectors of angular acceleration rather than velocity.

The second problem discussed is the use made by the driver of the positional, velocity, and acceleration fields. To affect driving, these characteristics have to be registered and the driver's sensitivity to them influences their utility. This analysis covers the condition of steady-state driving, where the vehicle moves rectilinearly or curvilinearly with constant velocity. Departures from steady-state driving, in turning, braking, avoidance, and other maneuvers, will be considered in a subsequent paper.

DERIVATION OF EQUATIONS OF POSITION AND MOTION

The coordinate system used is shown in Figure 1. The driver's eye is at the origin and the road is considered to be on an infinite plane at some distance, z, below his eye. Distance ahead of the eye is represented by x and to the side by y. Distance from the eye to any point of the field is represented by ρ, whose projection on the xy plane is r.

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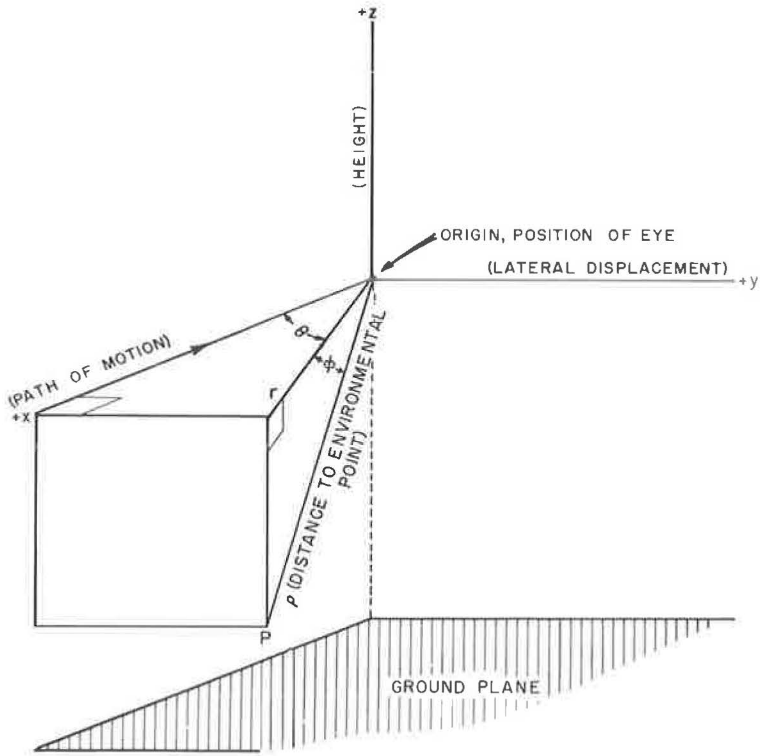

Figure 1. Basic coordinate relationships.

Angular Position

$$\theta = \arctan \frac{y}{x} \tag{1}$$

$$\phi = \arcsin \frac{z}{\rho} \tag{2}$$

$$r = \left(x^2 + y^2\right)^{1/2} \tag{3}$$

$$\rho = \left(x^2 + y^2 + z^2\right)^{1/2} \tag{4}$$

Angular Velocity

$$\frac{d\theta}{dt} = \frac{d}{dt}\left(\arctan \frac{y}{x}\right) = \frac{\partial}{\partial x} \arctan \frac{y}{x} + \frac{\partial}{\partial y} \arctan \frac{y}{x} = -\frac{y}{r^2}\frac{dx}{dt} + \frac{x}{r^2}\frac{dy}{dt} =$$

$$-\frac{y}{r^2}\frac{dx}{dt} + \frac{x}{r^2}\frac{dy}{dt} = \frac{1}{r^2}\left(-y\frac{dx}{dt} + x\frac{dy}{dt}\right) \text{ (rad/sec)} \tag{5}$$

$$\frac{d\phi}{dt} = \frac{d}{dt}\left(\arcsin\frac{z}{\rho}\right) = \frac{\partial}{\partial x}\arcsin\frac{z}{\rho}\frac{dx}{dt} + \frac{\partial}{\partial y}\arcsin\frac{z}{\rho}\frac{dy}{dt} + \frac{\partial}{\partial z}\arcsin$$

$$\frac{z}{\rho}\frac{dz}{dt} = \frac{1}{r\rho^2}\left(-zx\frac{dx}{dt} - zy\frac{dy}{dt} + r^2\frac{dz}{dt}\right) \text{ (rad/sec)} \tag{6}$$

The analysis of ground motion into separate azimuth ($d\theta/dt$) and declination ($d\theta/dt$) components seemed more appropriate than the development of a formula for the total angular velocity (14, 21). In some situations, the driver reacts differently to $d\theta/dt$ and $d\phi/dt$ components of motion (28).

Angular Acceleration

$$\frac{d^2\theta}{dt^2} = \frac{d}{dt}\left(-\frac{y}{r^2}\frac{dx}{dt} + \frac{x}{r^2}\frac{dy}{dt}\right) = \left[\frac{\partial}{\partial x}\left(-\frac{y}{r^2}\right)\left(\frac{dx}{dt}\right) + \frac{\partial}{\partial y}\left(-\frac{y}{r^2}\right)\left(\frac{dy}{dt}\right)\right]$$

$$\frac{dx}{dt} - \frac{y}{r^2}\frac{d^2x}{dt^2} + \left[\frac{\partial}{\partial x}\frac{x}{r^2}\left(\frac{dx}{dt}\right) + \frac{\partial}{\partial y}\frac{x}{r^2}\left(\frac{dy}{dt}\right)\right]\frac{dy}{dt} + \frac{x}{r^2}\frac{d^2y}{dt^2}$$

$$= \frac{2xy}{r^4}\left(\frac{dx}{dt}\right)^2 + \frac{y^2 - x^2}{r^4}\left(\frac{dx}{dt}\frac{dy}{dt}\right) - \frac{y}{r^2}\frac{d^2x}{dt^2} + \frac{y^2 - x^2}{r^4}\left(\frac{dx}{dt}\frac{dy}{dt}\right)$$

$$-\frac{2xy}{r^4}\left(\frac{dy}{dt}\right)^2 + \frac{x}{r^2}\frac{d^2y}{dt^2} = \frac{2xy}{r^4}\left[\left(\frac{dx}{dt}\right)^2 - \left(\frac{dy}{dt}\right)^2\right] -$$

$$\frac{1}{r^2}\left(y\frac{d^2x}{dt^2} - x\frac{d^2y}{dt^2}\right) + 2\left[\frac{y^2 - x^2}{r^4}\left(\frac{dx}{dt}\frac{dy}{dt}\right)\right] \text{ (rad/sec/sec)} \tag{7}$$

$$\frac{d^2\phi}{dt^2} = \frac{d}{dt}\left(-\frac{zx}{r\rho^2}\frac{dx}{dt} - \frac{zy}{r\rho^2}\frac{dy}{dt} + \frac{r}{\rho^2}\frac{dz}{dt}\right) = \left[\frac{\partial}{\partial x}\left(-\frac{zx}{r\rho^2}\right)\left(\frac{dx}{dt}\right) +\right.$$

$$\frac{\partial}{\partial y}\left(-\frac{zx}{r\rho^2}\right)\left(\frac{dy}{dt}\right) + \left.\frac{\partial}{\partial z}\left(-\frac{zx}{r\rho^2}\right)\left(\frac{dz}{dt}\right)\right]\frac{dx}{dt} - \frac{zx}{r\rho^2}\frac{d^2x}{dt^2} +$$

$$\left[\frac{\partial}{\partial x}\left(\frac{zy}{r\rho^2}\right)\left(\frac{dx}{dt}\right) + \frac{\partial}{\partial y}\left(-\frac{zy}{r\rho^2}\right)\left(\frac{dy}{dt}\right) + \frac{\partial}{\partial z}\left(-\frac{zy}{r\rho^2}\right)\right.$$

$$\left.\left(\frac{dz}{dt}\right)\right]\frac{dy}{dt} - \frac{zy}{r\rho^2}\frac{d^2y}{dt^2} + \left[\frac{\partial}{\partial x}\frac{r}{\rho^2}\left(\frac{dx}{dt}\right) + \frac{\partial}{\partial y}\frac{r}{\rho^2}\left(\frac{dy}{dt}\right) +\right.$$

$$\left.\frac{\partial}{\partial z}\frac{r}{\rho^2}\left(\frac{dz}{dt}\right)\right]\frac{dz}{dt} + \frac{r}{\rho^2}\frac{d^2z}{dt^2} = \frac{-z}{\rho^4 r^3}\left[\left(\rho^2y^2 - 2r^2x^2\right)\right.$$

$$\left.\left(\frac{dx}{dt}\right)^2 + \left(\rho^2x^2 - 2r^2y^2\right)\left(\frac{dy}{dt}\right)^2\right] - \frac{2rz}{\rho^4}\left(\frac{dz}{dt}\right)^2 +$$

$$\left(\frac{z^2 - r^2}{\rho^4 r}\right)\left(2x\frac{dx}{dt}\frac{dz}{dt} + 2y\frac{dy}{dt}\frac{dz}{dt}\right) +$$

$$\frac{2xyz}{\rho^4 r^3}\left(2r^2 + \rho^2\right)\frac{dx}{dt}\frac{dy}{dt} -$$

$$\frac{zx}{r\rho^2}\frac{d^2x}{dt^2} - \frac{zy}{r\rho^2}\frac{d^2y}{dt^2} + \frac{r}{\rho^2}\frac{d^2z}{dt^2} \text{ (rad/sec/sec)} \tag{8}$$

Rectilinear Motion

In rectilinear motion, where the eye moves with constant velocity in a straight line, the environment translates in x at a rate of $-dx/dt$ (negative of the speed of forward motion). The terms dy/dt and dz/dt equal zero. The equations in spherical coordinates are again Eqs. 1 and 2, as well as:

$$\frac{d\theta}{dt} = -\frac{y}{r^2}\frac{dx}{dt} \text{ (rad/sec)} \tag{9}$$

$$\frac{d\phi}{dt} = -\frac{zx}{\rho^2 r}\frac{dx}{dt} \text{ (rad/sec)} \tag{10}$$

$$\frac{d^2\theta}{dt^2} = \frac{2xy}{r^4}\left(\frac{dx}{dt}\right)^2 \text{ (rad/sec/sec)} \tag{11}$$

$$\frac{d^2\phi}{dt^2} = -z\left[\frac{\rho^2 y^2 - 2r^2x^2}{\rho^4 r^3}\right]\left(\frac{dx}{dt}\right)^2 \text{ (rad/sec/sec)} \tag{12}$$

These equations are applied in Figures 2, 4, 6 and 9. Modifications of Eqs. 5 and 6 are applied in the illustrations of horizontally curved motion shown in Figures 5 and 7. The equations apply to the induced motion of a flat ground plane, to objects above the ground, other cars, and all other environmental points viewed from a moving vehicle.

POSITIONAL FIELD AND VEHICULAR GUIDANCE

The angular coordinates θ and ϕ of the ground plane from 0 to 50 ft to the left and from 0 to 100 ft in front of the driver are given in Figure 2. The coordinate system

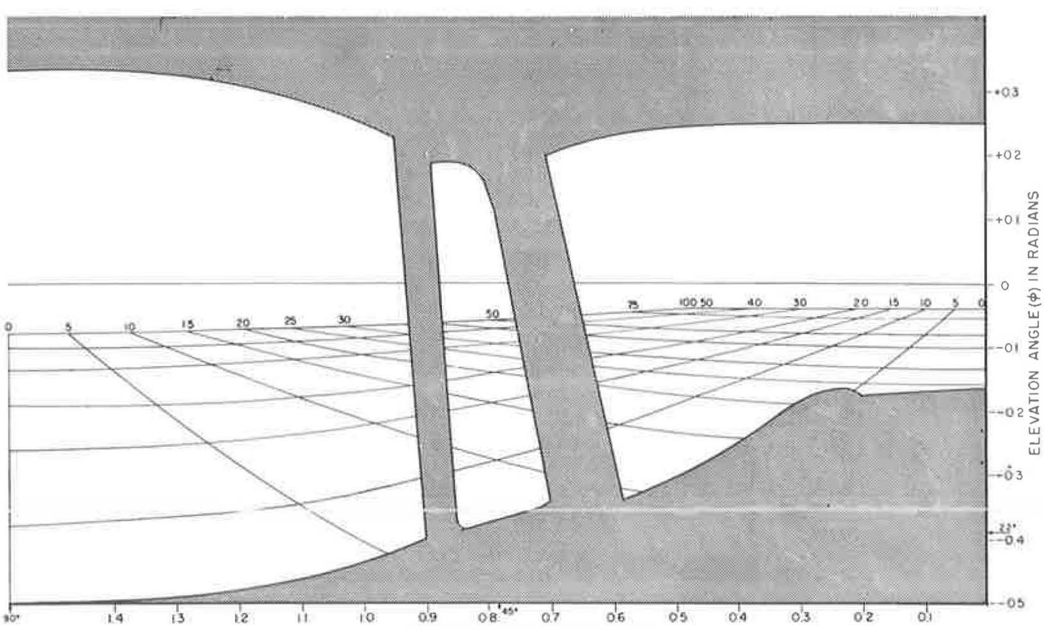

Figure 2. Perspective through windshield and side window.

has been described in Figure 1. Since the driver sits on the left of the vehicle, the window areas and road views are asymmetrical. The left view is described in Figure 2. The flow lines also apply to the right side of the vehicle, if the appropriate window area is superimposed, and to the rear areas if the grid is reflected about the $x = 0$ line. The driver's eye is placed at a representative height of 4 ft above the ground. The empty area at the right of the figure is the automobile cab and hood which partially cut off the view of the road. The blind areas at 0.65 and 0.9 rad are the roof support and window edging.

The equirectangular projection shown in Figure 2 is one of many possible ways of representing a three-dimensional environment on a flat page surface. The figure is distorted because at the zenith and nadir of actual space, 360° of azimuth are reduced to a point; in the figure they would occupy the same extent as on the horizontal meridian. A rectification in the θ dimension may be achieved by curving the page through 90° and viewing from a point close to the center of curvature. The ϕ dimension is not rectified, but in practice need not be, since it covers a limited ½-rad range.

<u>Linear Perspective and Interpretive Scale</u>

Linear perspective, the diminution of angular size with distance, is related to the positional field. The angular scale of the positional field may be expressed in terms such as, $\Delta\theta/\Delta x$, in this case indicating the change in θ angle associated with a small change in x. If Δl is a small change in x, y, and z (i.e., $\sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$) and $\Delta\alpha$ is the change in angle associated with Δl, then angular scale is $\Delta\alpha/\Delta l$ in radians per foot or equivalent units. It is seen that angular scale expresses the angular effects underlying perspective and establishes a relationship between linear perspective and the positional field.

Interpretive scale may be defined as the inverse of the angular scaling effects underlying perspective. If angular scale is $\Delta\alpha/\Delta l$, then the corresponding expression of interpretive scale is $\Delta l/\Delta\alpha$ in feet per radian or equivalent units. As applied to a common road map, interpretive scale would represent the miles per inch required to interpret map lengths rather than the inches per mile used to draft the map.

The perception of interpretive scale enables the driver to calibrate visual angle, in terms of length, on the road. Scaling may be explicitly in terms of feet and inches, but more commonly it involves distance implicitly, as exemplifed by the driver's estimation of the space to a road point. The driver's conception of scale may be designated by primes ($\Delta l'/\Delta\alpha'$) to connote that a subjective estimate is implied.

Since scaling involves the inverse of the angular effects underlying linear perspective, it would be expected to play some part in general space perception. It appears as a key factor when the observer takes the attitude of making quantitative judgments of size, distance or motion.

In size perception, the observer estimates scale and then evaluates the visual extent of interest, in this metric. Gibson describes the process as follows:

> ...with fixed monocular stimulation the size of an object is given by the size of the elements of texture or structure in the adjacent optical array....Size is perceived relative to the size scale of the place where the object is seen. (<u>11</u>)

Scaling also applies to distance judgments. Gogel finds that judgments of the relative distance of objects can be explained in terms of the ratio of familiar size to retinal size (<u>17</u>). This ratio is equivalent to what we have called interpretive scale. Gogel states that it underlies the judged distance between objects of similar shape but different size (<u>4</u>, <u>8</u>, <u>16</u>), of different shapes (<u>23</u>, <u>29</u>), and familiar objects of different size and shape (<u>17</u>, <u>23</u>). If the scale of one object appears smaller than that of another, it will be judged as closer; if it is seen as equal, it will be judged equidistant; and if in-

terpretive scale is seen as larger, it will be judged as farther away. In these experiments, where background cues are minimized, judgments of relative distance are based on relative scale.

Often distance judgments are made between widely separated objects. If distance is large, scale changes along the path and the judgment would be expected to take the following form:

$$\text{Judged distance} = \sum_{l'=1}^{l_1=n} \left(\text{scale of } \Delta_{l'}\right) \Delta_{l'} \tag{13}$$

where the $\Delta_{l'}$'s are convenient seen lengths between the objects. It may be predicted that the observer would sum lengths in situations where the scale changes. A similar approach holds for judging distances from the eye, except that scale is learned through experience and does not have to be repeatedly resolved.

It seems reasonable to believe that interpretive scale, once resolved, would be generally applied to size, distance and motion judgments in a situation. Thus, the scale of a familiar-sized object may be applied to distance and speed judgments in the same setting. Speed judgments may depend on interpretive scale; the angular movement of objects seen by a stationary eye is converted to speed by the same rule that governs angle and length. Eqs. 5 and 6 may be expressed in terms of differentials instead of time derivatives; for example, Eq. 9 may be expressed as:

$$\Delta\theta = -\frac{y}{r^2}\,\Delta x \tag{14}$$

It is seen that Δx is converted into $\Delta\theta$ by the same function of y and r^2 as converts dx/dt into $d\theta/dt$. The same approach holds for $\Delta\phi$. Since angular stimulus is the vectorial sum of $\Delta\theta$ and $\Delta\phi$ components, and angular movement stimulus is the sum of $d\theta/dt$ and $d\phi/dt$ components of motion, the communability of angle and angular velocity effects is demonstrated, as is the similarity of interpretive scales required for the correct perception of length and speed.

The relationship between length and distance judgments is discussed in the literature under the "size-distance invariance hypothesis" (2, 9, 30). The hypothesis states: "A visual angle of given size determines a unique ratio of apparent size to apparent distance" (25). Since the hypothesis describes a perceptual relationship, it must be shown valid on the basis of perceptual rather than geometrical evidence. Thus, if size and distance judgments are shown to be based on the same interpretive scale, a rational basis would be provided for an invariance hypothesis. In some circumstances, the scaling underlying size and distance judgments differs. The scale in a film protection may be established by familiar objects included in the scene which would not directly indicate the scale of eye distance. The same principle is true in size judgments of geological specimens. A hammer included in a page illustration may give the size scale, but distance from the eye would not be thereby revealed. The moon illusion is a puzzling example where seen size and distance are paradoxically unrelated (24). The relation of linear and velocity scales has received less research attention. Experiments by J. F. Brown may be interpreted as indicating that linear scale is actually applied in speed judgments (3).

It is tempting to think that slant and shape perception could also be subsumed under the attainment of interpretive scale. Slant could be dealt with as the seen ratio of front-parallel scale to surface scale perpendicular to this line. If the ratio of these scales is ½, the surface lies 60° to the line of sight; i.e., cosine 60° = ½. This explanation of slant perception is objectionable because it seems more complex than the judgment explained. We tend to believe that slant and the perception of shape are basic perceptions which precede, rather than follow, scaling judgments. However, if the observer in a shape constancy experiment makes estimates of the dimensions of

simple rectangles or ellipses projected at a slant to the eye, the responses may be considered to involve relative scaling.

The question remains of how scale is obtained. In theory, a decoding could be simply achieved because every x, y point on the ground plane has a unique θ, ϕ representation. A mechanical robot could steer down a level road and avoid obstacles if it could sense θ and ϕ and if obstacles and the road were correctly coded for it. This two-dimensional approach has an appealing simplicity, but introspection compellingly reveals the three-dimensional nature of the visual world.

Scale is most obviously revealed by familiar-sized objects, particularly those with parallel sides. The quantity $\Delta l'$ is then the known length of the familiar object; $\Delta \alpha'$ is its seen angular extent. An example is provided by the space perceptions of a motorist. An obvious key to the terrain configuration is the shape of the road ahead. The roadway is of almost constant width, converges rapidly in perspective, and gives a ready scale for converting visual angle to linear extent which may be applied to other objects (Fig. 3). If the borders of the road are straight, the road itself has no vertical curvature. If the borders are concave, the road surface is concave. If the borders are convex, the road is also. On an uphill, the road may rise above eye height; downhill it will be below eye level and may, in fact, be overlapped and hidden. Light and shadow, texture, ocular adjustments, intersections of surfaces, and overlapping of contours may enhance these perceptions. The roadway is a particularly convenient rule since it is always available, has known characteristics, and provides continual feedback to the driver on the accuracy of his perceptions. However, other familiar objects—vehicles, pedestrians, houses, fence posts, sidewalks and crosswalks--are usually in the field of view and may also serve to calibrate visual angle.

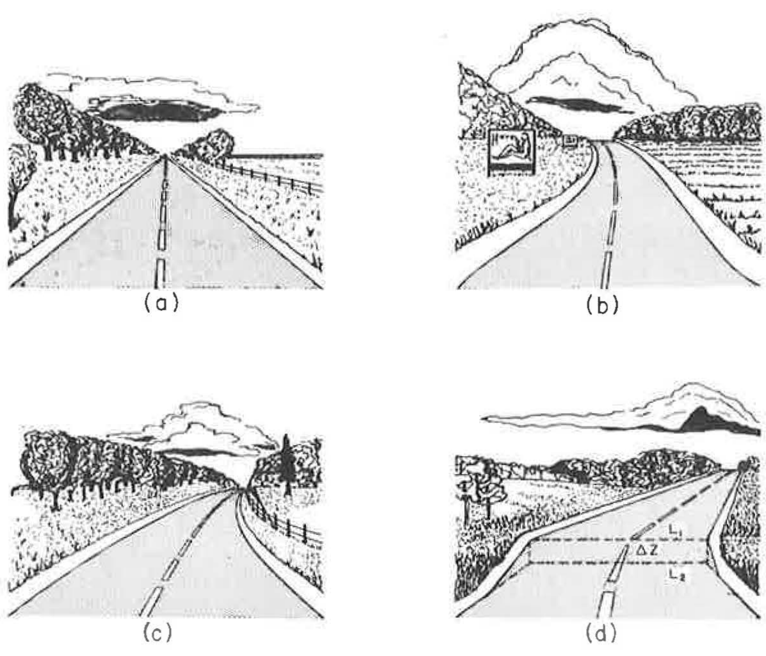

Figure 3. Road configurations: (a) width of road gives the linear scale; (b) and (c) convergence pattern reveals vertical curvature; and (d) pattern of road boundaries shows the vertical rise, Δ Z.

ANGULAR VELOCITY FIELD AND VEHICULAR GUIDANCE

Velocity vectors along the flat ground plane are plotted in Figure 4. The ground area covered is superimposed on Figure 2; the magnitude and direction of the ground flow is shown by the length and direction of the vectors in the figure. The angular velocity field of Figure 4 appears to fit almost exactly over the positional field of Figure 2. The resemblance is due to the approximate equivalence of $d\theta/dx$ and $d\phi/dx$ vectors to $\Delta\theta/\Delta x$ and $\Delta\phi/\Delta x$ angular extents. As the eye moves in x, velocity vectors give the effect of equally sized rods parallel to the x axis. On the ground plane, these vectors fall on perspective lines.

Figure 4 indicates that $d\theta/dx$ is zero along the median plane, approaches zero at $\theta = \pi/2$, and both $d\theta/dx$ and $d\phi/dx$ are zero at the vanishing point at eye level directly ahead. Objects off this path change their $d\phi/dx$ velocity component to predominating $d\theta/dx$ as they are approached.

Applications of Velocity Field

Perception of Motion.—The psychological basis of motion perception is discussed in the literature (20, 22). At slow rates, motion is inferred from a change in position, as in the minute hand of a watch. At more rapid rates, motion is directly perceived, as in the motion of the second hand of a watch, which is actually seen. At still more rapid rates, motion appears as a blur. Similar judgments and perceptions enable the driver to register impressions of motion (not arrows) related to the velocity field around him.

Velocity Field as Basic Reference for Object Motion.—Just as the positional field gives the scale of object size, the velocity field might be expected to provide the background for the seen movement of objects. A difficulty in applying this relationship is that the driver may see the ground as a still reference or, alternately, he may conceive of his vehicle as still and other cars as moving relative to it. For example, the driver reacts promptly when a vehicle approaching his moving car has no apparent sidewise velocity vector. This is visual warning of an impending collision. (This relationship is used by Michaels and Cozan to explain vehicular avoidance reactions (28). The lateral displacement of a moving vehicle to a road obstacle was found to be inversely related to the seen angular velocity of the object.) An analogous situation exists where

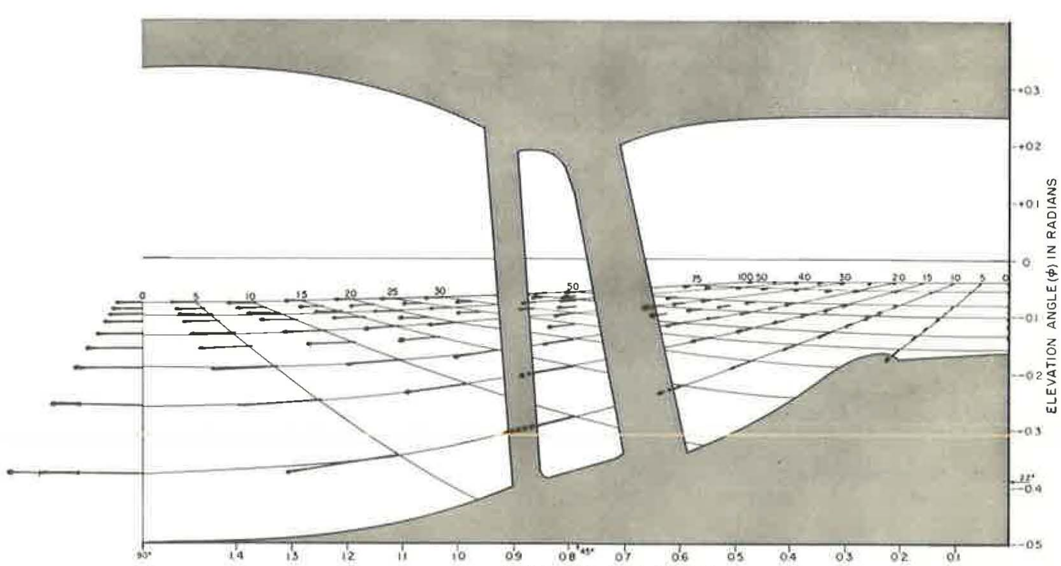

Figure 4. Velocity vectors on basic ground plane.

the resultant vector is directed towards the median plane. The intruding vehicle will cross one's path, and if its course is changed it may collide. The driver takes his own vehicle as reference when he reacts to these situations.

The velocity field also serves as background for the driver's sensitivity to seen motion. As an example, the relatively small angular motions ahead of the vehicle favor the perception of object motion, particularly in the θ dimension. In contrast, the large velocities at $\theta = \pi/2$ inhibit the perception of object motion. If the vehicle is moving very rapidly, the resolution of independent motion is further impaired by blurring of the image. Angular velocities of several hundred degrees per second are generated at the side of a rapidly moving vehicle, and the driver's vision will be severely reduced even if he follows object movement with his head and eyes (5, 26, 27).

Perception of Speed and Direction of Vehicular Motion.--The velocity field indicates the speed and direction of the vehicle's forward motion. Although observed motion is the most direct indicator of vehicular motion, it is only one of its accompaniments. Vehicular speed is also indicated by direct speedometer readings, the pull of the steering wheel, the gear in use, the response to the accelerator, the pitch of the engine and tire squeal, the roughness of the ride, and centrifugal force when a curve is rounded. Apparently the visual appearance of motion cannot be claimed to be the sole, or even the most useful, input for the estimation of speed.

The direction of vehicular motion is indicated by the flow characteristics of the velocity field. There is no $d\theta/dt$ component in the median plane ahead of the driver. This lack of motion may be used by the driver, along with information of posture and the position of objects in the windshield to indicate the direction of the vehicle's motion.

The importance of expansion patterns as an indication of the vehicle's direction of movement has been pointed out by James Gibson (10):

> When an observer approaches a surface instead of moving parallel to it, a modification of its deformation is introduced in that the focus of expansion is no longer on the horizon of that surface but at a particular spot on it—the point of collision with the surface. The rule is that all deformation in a forward visual field radiates from this point. Crudely speaking, the environmental scene expands as we move into it, and the focus of expansion provides us with a point of aim for our locomotion. An object in our line of travel, regarded as a patch of color, enlarges as we approach. It is not difficult to understand, therefore, why this expansion should be a stimulus for sensed locomotion as well as a stimulus for sensing the lay of the land. The behavior involved in steering an automobile, for instance, has usually been misunderstood. It is less a matter of aligning the car with the road than it is a matter of keeping the focus of expansion in the direction one must go.

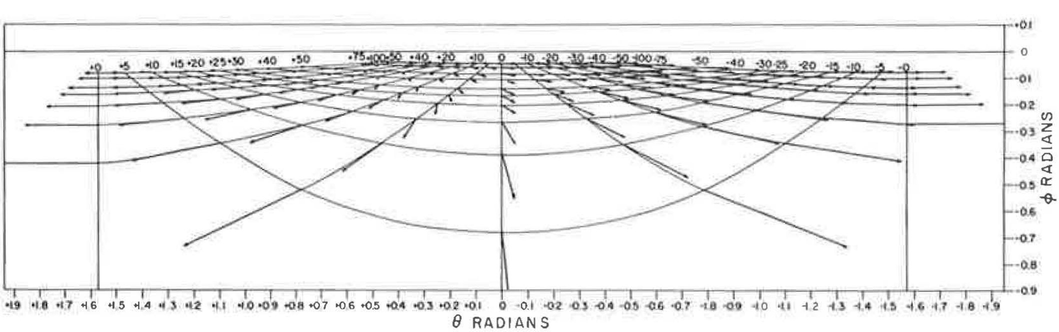

Figure 5. Vector velocity field of horizontal curved motion—Radius of curvature is 100 ft to left of eye (origin). The field is asymmetrical and has null point at center of curvature. It approaches I/R · dx/dt at infinite distance.

Although the direction of vehicular motion is related to the focus of expansion, the focus itself is not the effective cue. The focus of expansion of a flat horizontal plane lies at the vanishing point in the sky, or it will occupy points on trees or buildings if the road is curved. It is generally difficult, if not impossible, for the driver to locate the focus of expansion (Figs. 4 and 5), and contrary to Gibson, the borders and lane markings are used in vehicular guidance. (These results are derived from studies conducted at the U. S. Bureau of Public Roads.) When the vehicle is off course, these lines are at an angle with the y axis and will have a lateral component of movement as the field translates towards the eye.

Motion Parallax, Parallax Curl.—As the eye moves, objects pass on either side; distant things seem to move more slowly than those close by. The relative speed of angular motion (motion parallax) might be expected to provide an indication of distance. Discussions of human depth perception in the psychological literature mention motion parallax as a classical cue to distance, along with perspective, interposition, aerial perspective and shadows (10, 20, 22).

The first thorough discussion of motion parallax as an indicator of distance is given by H. von Helmholtz (22):

> In walking along, the objects that are at rest by the wayside stay behind us; that is, they appear to glide past us in our field of view in the opposite direction to that in which we are advancing. More distant objects do the same way, only more slowly, while very remote bodies like the stars maintain their permanent positions in the field of view, provided the direction of the head and body keep in the same directions. Evidently, under these circumstances, the apparent angular velocities of objects in the field of view will be inversely proportional to their real distances away; and consequently, safe conclusions can be drawn as to the real distance of the body from its apparent angular velocity.

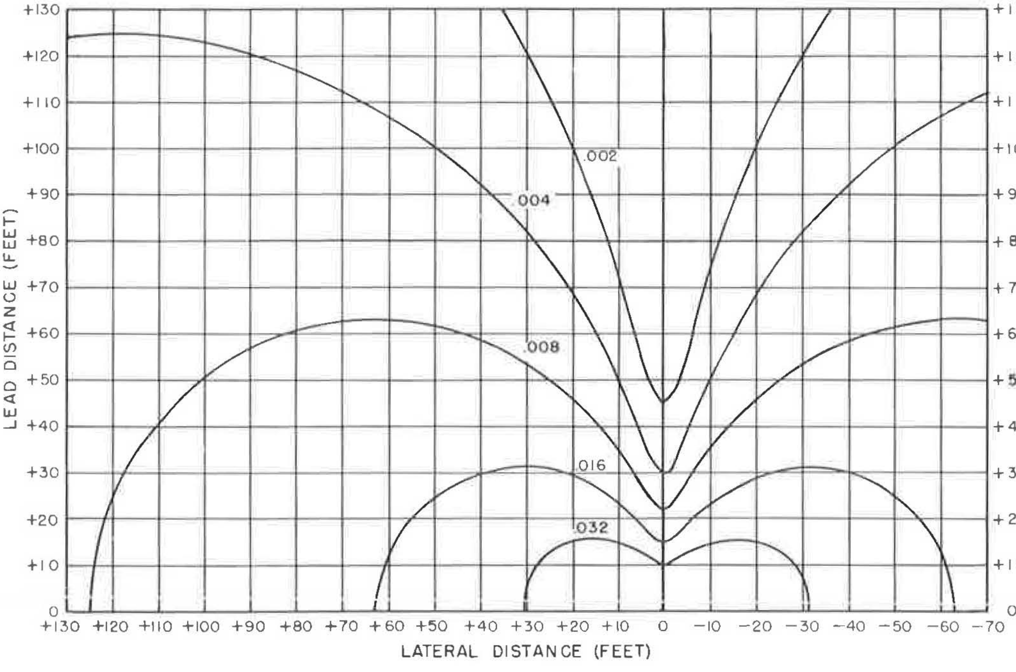

Figure 6. Isoangular-velocity curves under linear motion—Angular velocity in radians per second of each contour can be obtained by multiplying value shown by vehicular speed in feet per second. If speed is increased, angular velocity is increased but pattern remains same.

The position that "... safe conclusions can be drawn as to the real distance of the body from its angular velocity" is subject to modification. When the observer follows a curved path, angular motion of ground points becomes an unreliable indicator of distance. The basic geometry of the situation is altered so that motion of the ground (or other points) does not decrease regularly along a sight line. This fact may be seen from the velocity field of curvilinear motion (Fig. 5) and by a comparison of isoangular-velocity curves of linear and curvilinear observer motions (Figs. 6 and 7).

The vector field of horizontally curved observer motion (Fig. 5) shows a number of features which complicate a motion parallax interpretation. As distance from the eye increases along an azimuth line, the direction and magnitude of ground motion change. Motion on the 0.3-rad azimuth line is to the right at far distances and to the left at close points. The interpretation of distance from these motions alone would be very difficult.

The same conclusion is brought out by the isoangular-velocity plots. These curves show the locus of terrain points of equal angular velocity, regardless of direction of motion. It may be seen that linear isoangular-velocity curves decrease fairly systematically with the reciprocal of distance on each azimuth. The curvilinear isoangular-velocity pattern does not follow the same rule. The functions are asymmetrical and approach a limiting value of $1/R\ dx/dt$, where R is the radius of curvature. Angular movement in Figure 7 is seen to reverse itself on the sight line through the center of rotation at $x = 0$ and $y = 100$ ft. An interpretation in line with the motion parallax approach would place the stationary center of rotation at an infinite distance from the eye. Positions beyond the center of rotation on the same azimuth line increase in angular velocity and would be interpreted as decreasing in distance. Evidently, distance is not related to ground motion in the motion parallax manner when the eye is following a curved path.

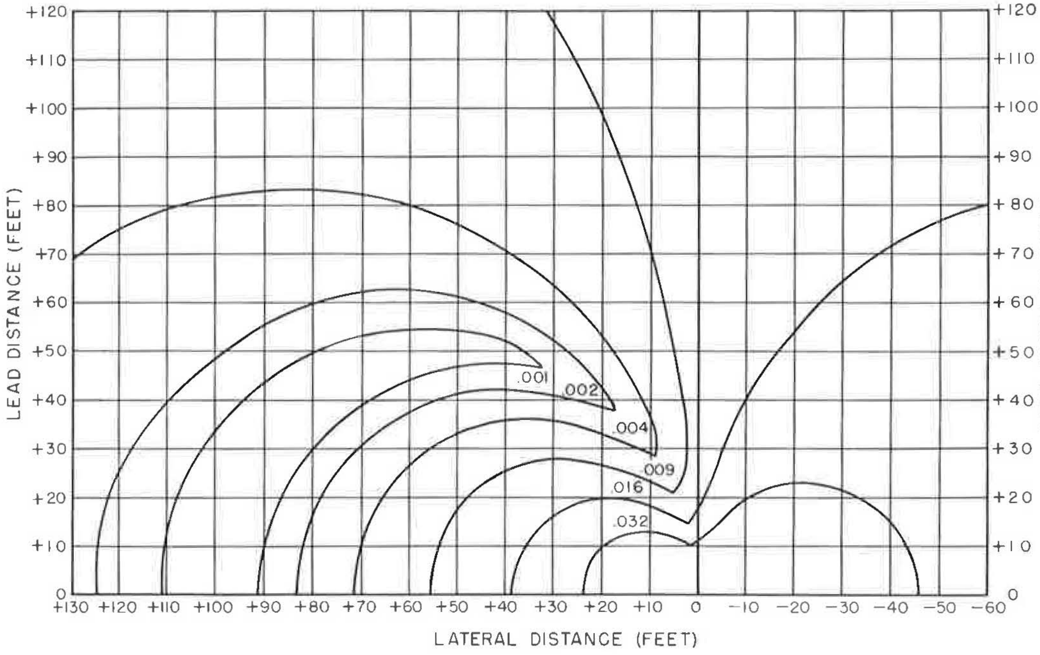

Figure 7. Isoangular-velocity curves under curvilinear motion—Center of curvature is 100 ft to left of origin. Angular velocity in radians per second of each contour can be obtained by multiplying value shown by vehicular speed in feet per second. If speed is increased, angular velocity is increased but pattern remains same.

On a straight trajectory, distances at right angles to the line of movement where angular velocity is high are not noticeably easier to estimate than those ahead where angular velocity is minimal. On the contrary, illusory movements of the terrain, which depend on the observer's visual fixation position, are seen to the side. If the foreground is viewed from an automobile, the background seems to move and rotate forward around it. If the background is fixated, the foreground turns. This illusion of rotation, which may be called motion parallax curl, is based on differences in angular velocity between the foreground and background (Fig. 8). The point of reference of seen movement is ambiguous and is not necessarily the position of the moving eye itself (as a motion parallax formulation would require).

If the velocity field is considered a positional field in motion, the relation of this field to the perception of distance is clarified. Distance enters into the perception of the velocity field, rather than being revealed by it. This approach is supported by the experimental studies of motion parallax, which show that the context of motion must be revealed to an observer if he is to make an accurate estimate of distance. Differential motion simulating the ground projection of the velocity field is not sufficient to permit an accurate judgment of depth to be made (12, 13).

ACCELERATION FIELD

The projection of the acceleration field on the ground plane is shown in Figure 9. The vectors on the field represent the differences in successive velocity vectors, divided by time, as time approaches zero. The cab area of Figure 9 is the same as in Figures 2 and 4, but the vector scale is 10 times as large. As may be seen in Figure

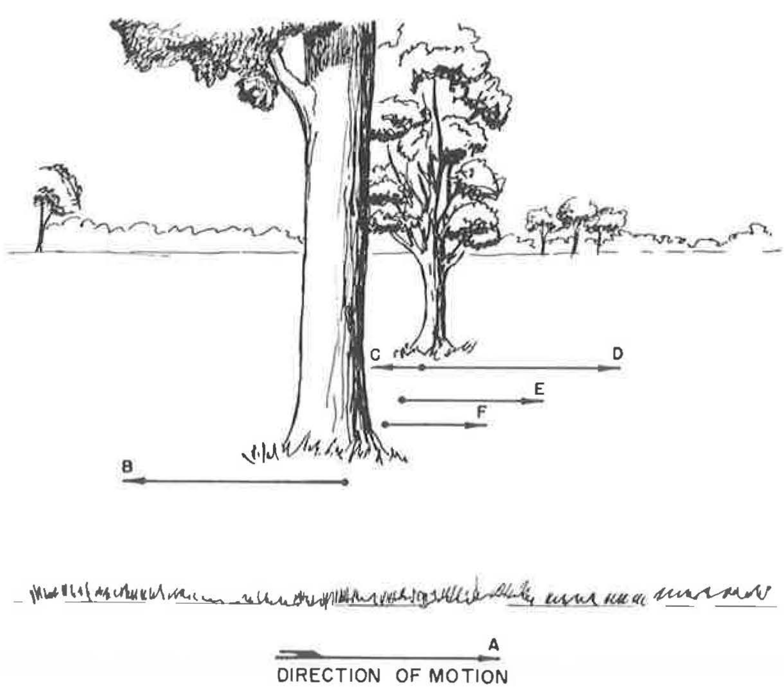

Figure 8. Motion parallax curl—Illusion is shown at right angles to vehicle's line of movement. Vehicle is moving in direction A, inducing vectors B in foreground tree and C in background tree. If foreground tree is fixated, the background moves to right with velocity D (difference between B and C). Ground positions between the trees have linearly decreasing velocity vectors which produce appearance of rotation.

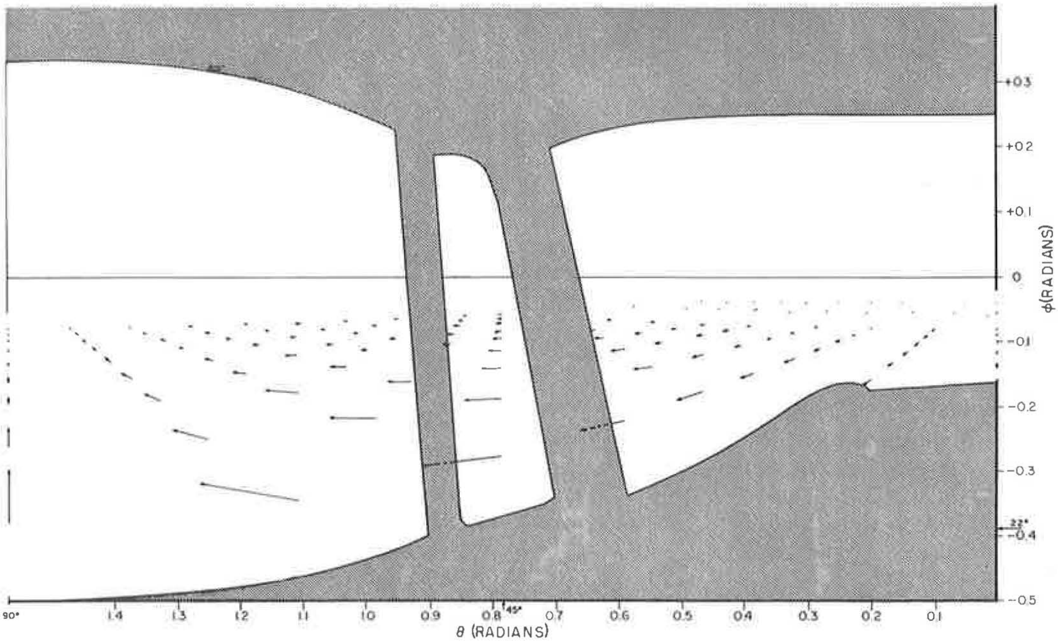

Figure 9. Acceleration vectors on basic ground plane.

9, there is no azimuthal component ahead of the eye under rectilinear motion, and $d^2\phi/dx^2$ is directed towards the eye in those positions. Vectors are directed away from the eye at $\theta = \pi/2$ where $d\theta/dx$ goes through a maximum. At angles between $\theta = 0$ and $\theta = \pi/2$, the vectors shift from an approaching to a receding direction and are generally largest close to the eye.

Perceptual Problems of Acceleration Field

The major perceptual problem of the acceleration input is whether or not it is directly sensed. The human can distinguish accelerations, but it is not certain that they are detected as such. They may possibly be inferred from successive impressions of changing rates. Gottsdanker et al. showed that group performance is more clearly ordered in terms of a threshold based on total change in velocity than in terms of direct sensory impression of acceleration (18, 19).

The vector field shown in Figure 9 furnishes evidence on the sensing of acceleration. The field appears unnatural and no characteristic of experience can be associated with the pattern of vectors shown. This case differs from the psychophysical correspondence between light wavelength and hue, physical energy and brightness, etc. Since the observer does not directly or precisely register accelerations, the acceleration field and the gradients within it cannot be considered as a primary visual input. The same conclusion probably holds, by extension, to higher velocity derivatives. This does not suggest that accelerations may not be perceived as changes in velocity.

Acceleration varies as the square of speed, as may be seen from Eqs. 11 and 12. If speed of the moving eye is doubled, angular acceleration is quadrupled. The same condition holds for the eye on a curved trajectory. This relationship leads to the paradoxical situation that the angular acceleration stimulus is more sensitive to speed than is angular velocity. It would be expected that the appearance of the environment would change markedly as linear speed is increased, and that there would be acceleratory indications of velocity. The visual appearance of increased velocity on a roadway may be a sharp swoop of objects and road features as they change from a ϕ to a θ direction. A jitter due to acceleratory movements may also be seen in the imperfections of lane markers and road edges. However, these acceleratory effects have not been systematically verified.

SUMMARY

Perceptual problems in vehicular guidance are considered here in the context of the positional, velocity and acceleration fields around the moving vehicle. These are very general and persistent aspects of the driver's visual environment. The approach is to examine the equations governing these fields, and the fields themselves, for features and regularities which might serve to explain human spatial perception.

The following findings emerge from the analysis:

1. The interpretive scaling of visual angle, which is the inverse of perspective effects in the positional field, is shown to be a key factor in size, distance and motion perception.

2. Simple and obvious features of the visual environment, often ignored in explanations of space perception, are believed to provide the most important aids for vehicular guidance. The roadway ahead of the vehicle, for example, may be used to obtain the scale of the terrain and objects in it.

3. The velocity field furnishes a reference for the seen movement of objects. However, the driver may see the field, his own vehicle, or part of the field as reference. If the foreground is taken as reference, a curious illusion of motion is seen. The background seems to rotate forward and around the foreground. This velocity parallax curl is based on the difference in velocity vectors in the foreground and background.

4. Some difficulties are pointed out in the motion parallax indication of distance.

5. Roadway boundaries and lane markings are used in aligning the moving vehicle with the road. This conclusion challenges the widely quoted view that the focus of expansion is the cue for the direction of sensed locomotion.

6. The formulas derived indicate that angular acceleration increases as the square of vehicular speed. The consequences of this interesting relationship for the perception of vehicular speed are indicated.

7. Since the pattern of the angular acceleration field does not resemble any familiar pattern of visual experience, evidence is provided that angular acceleration is not directly sensed. By extension, it is doubtful that higher derivatives of motion are seen as such.

The analysis pursued in this paper is concerned with basic aspects of the perception of static and moving visual fields. The great need in future work is to show how the driver responds to these fields in obtaining a correct perception.

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