An Experimental Study of Subpressure in a Freezing Soil System

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ABRIDGMENT

•IN MOST regions of the modern world, the frost problem is a factor of major concern to all. The attempts to solve this problem have resulted in many studies to understand the exact nature of frost development in soils. The present paper discusses one aspect of this problem, namely the subpressure developed and its relationship to the causes of frost problems.

The fundamental laws of hydrodynamics indicate that the moisture-migration in the upward direction against gravity in a porous medium is feasible only if there is some form of pressure drop in the direction of flow. In nature, the groundwater table is at atmospheric pressure. In freezing soil, therefore, the absolute pressure in the frozen zone above the groundwater table should be less than atmospheric pressure for upward moisture migration to be feasible. The author is of the opinion that this pressure drop is due to temperature drop. The deviation from this simple cause is due to other side effects such as surface atmospheric pressure.

Factors Affecting Subpressure

The factors influencing pressure conditions in a freezing soil are many, such as porosity of the soil, particle size, shape of particle, intensity of cold spell, deviation of cold spell, and depth of the groundwater table. Any analytical method of evaluation would be quite tedious, if not impossible. The strain and effort involved in such an evaluation would not be commensurate with the problem. The more direct experimental approach is quicker and more dependable in such situations of random variables; therefore, the study undertaken used the experimental method.

In this study the soil freezing system has been considered mainly as a porous medium subject to a freezing thermal gradient. The moisture movement is a secondary effect, although it is the immediate cause of frost heave and breaking up of pavements. The study involved experimental freezing of soil samples in the laboratory under controlled conditions where variation of several influencing factors was possible.

A series of soil freezing experiments was conducted at the Frost Action Research Laboratory of Rutgers University. The material presented in this paper is based on these experiments. The experiments simulated the conditions of natural soil freezing in New Jersey. The particular case selected, since it focused the main points of the study, was one where water exists at a fixed depth from the ground surface at a constant nonfreezing temperature. The study is limited to one type of highly frost-susceptible silty soil, Penn Soil. Figure 1 is a diagram of the instrumented soil sample used in these experiments. Details on the experimental procedure and data are given elsewhere (1).

Relationship Between Influencing Factors and Subpressure

The study correlated the observed phenomenon of induced subpressure in freezing soil to one of the main physical properties of the soil, porosity (which in a way represents other physical properties such as pore size and specific surface) and the freezing

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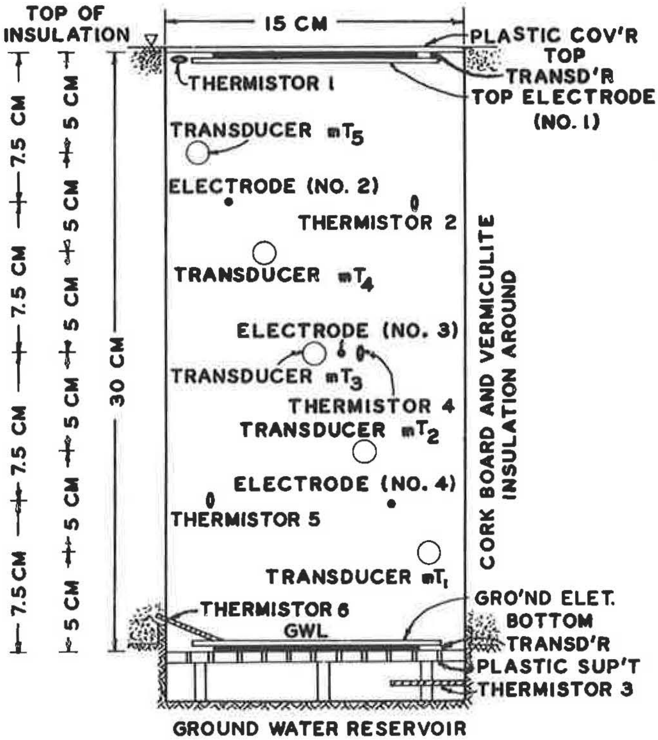

Figure 1. Instrumented soil sample.

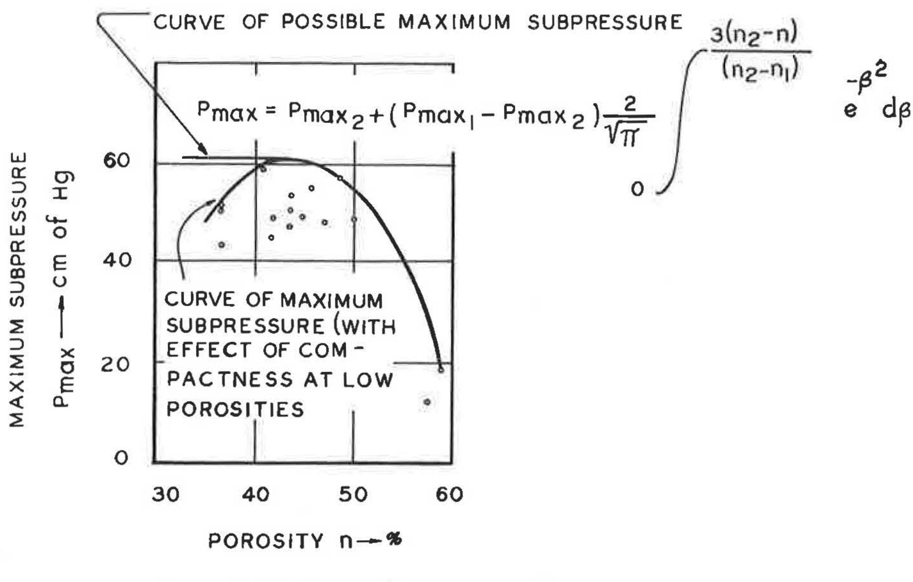

Figure 2. Maximum subpressure as a function of porosity.

thermal gradient, the chief cause which triggers the whole freezing moisture-migration process. The effect of the time element involving the development of the subpressure in an unsteady state of thermal condition was recognized. The findings show the relationships between the following factors: (a) the maximum induced subpressure and porosity of the soil; (b) the maximum subpressure and frost depth; (c) the maximum subpressure and the surface subfreezing temperature; (d) the maximum subpressure and the freezing thermal gradient; and (e) the subpressure at a point and depth (subpressure profile). A combined empirical equation for the maximum induced subpressure as a function of all the above influencing factors is also evolved which is a combination of the individual relationships developed.

Notation

p_{max_1}, p_{max_2}, p_{max} = maximum induced subpressure in freezing soil of porosities n_1, n_2, and n;
β = arbitrary variable of integration;
p_m = subpressure induced at the frost boundary;
h = depth of groundwater table from the ground surface;
ξ, ξ_0 = frost depth and optimum frost depth;
m_1, m_2, a_2, b_2, c_2, k_1, k_2 = constants;
T_s = ground surface subfreezing temperature; and
p = induced subpressure in the freezing soil layer at a depth z from the ground surface and above groundwater table.

Subpressure and Porosity of the Soil

In a given type of soil, the maximum induced subpressure under any subfreezing thermal gradient depends on the porosity of the soil. In a practical experimental range of porosity, the experimental results indicate that the maximum induced subpressure decreases with increasing porosity. The pattern of the decaying curve conforms to the curve represented by Gauss' error function, which is given by

$$p_{max} = p_{max_2} + \left(p_{max_1} - p_{max_2}\right) \frac{z}{\sqrt{\pi}} \int_0^{\frac{3(n_2 - n)}{(n_2 - n_1)}} e^{-\beta^2} d\beta \qquad (1)$$

This relationship is illustrated in Figure 2. The curve gets modified according to the range of limits of error function chosen in any particular case.

Subpressure and Frost Depth

The next area studied relates to the relationship between the maximum induced subpressure and the frost depth. The maximum induced subpressure in a freezing soil depends on the frost depth. The experimental results indicate that the maximum subpressure increases rapidly with the increase in frost depth, starting from zero at the ground surface. It reaches a maximum value at the optimum frost depth, which is usually a little closer to the ground surface than to the groundwater table. The optimum frost depth is defined as the frost depth at which the induced subpressure profile is maximum for a given soil of a given porosity. Thereafter it will begin to decrease with further increase in the frost depth, reaching zero value when the frost depth reaches the groundwater table. Both the increasing subpressure from the ground surface to the optimum frost depth and the decreasing subpressure from optimum frost depth to the groundwater table vary parabolically with depth but at different rates. The maximum subpressure at any porosity as indicated by Eq. 1 corresponds to the peak value of the subpressure profile at optimum frost depth. The peak value of the subpressure at any frost depth is given by

$$p_m = p_{max} \left[1 - \left(1 - \frac{\xi}{\xi_0}\right)^{m_1}\right] \qquad (2)$$

for frost depth ξ less than optimum frost depth ξ_0, and for frost depth ξ greater than the optimum frost depth ξ_0,

$$p_m = p_{max} \left[1 - \left(\frac{\xi - \xi_0}{h - \xi_0}\right)^{m_2}\right] \qquad (3)$$

These relationships are illustrated in Figure 3. The dotted line in the figure indicates the theoretical case of frost depth approaching the groundwater table of constant nonfreezing temperature.

Subpressure, Freezing Thermal Gradient, and Frost Depth

The study also included the correlation of the maximum induced subpressure to the freezing ground surface temperature and freezing thermal gradient. The experimental evidence indicated the following relationship between the surface subfreezing temperature T_s and the maximum frost depth ξ that it can induce in the given soil

$$\xi = a_2 T_s^2 + b_2 T_s + c_2 \qquad (4)$$

The maximum induced subpressure p_m can now be expressed as a function of porosity of the soil (n) and the surface subfreezing temperature T_s by combining Eqs. 1, 2 and 4 or 1, 3 and 4 as the case may be, depending on whether the frost depth ξ is less than or greater than the optimum frost depth ξ_0. The expression will be

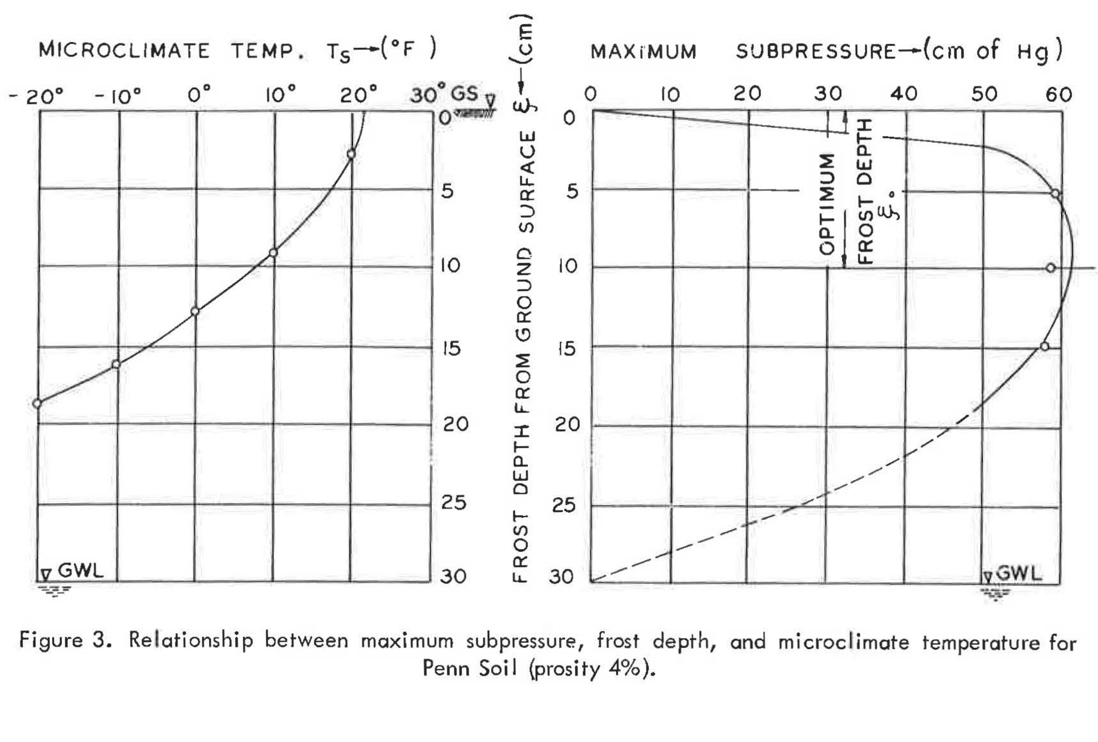

Figure 3. Relationship between maximum subpressure, frost depth, and microclimate temperature for Penn Soil (prosity 4%).

$$p_m = \left[p_{max_2} + \left(p_{max_1} - p_{max_2} \right) \frac{2}{\sqrt{\pi}} \int_0^{\frac{3(n_2 - n)}{(n_2 - n_1)}} e^{-\beta^2} d\beta \right] \left[1 - \left(1 - \frac{a_2 T_S^2 + b_2 T_S + c_2}{\xi_0} \right)^{m_1} \right] \quad (5)$$

$$p_m = \left[p_{max_2} + \left(p_{max_1} - p_{max_2} \right) \frac{2}{\sqrt{\pi}} \int_0^{\frac{3(n_2 - n)}{(n_2 - n_1)}} e^{-\beta^2} d\beta \right] \left[1 - \left(\frac{a_2 T_S^2 + b_2 T_S + c_2 - \xi_0}{h - \xi_0} \right)^{m_2} \right] \quad (6)$$

Subpressure Profile

The foregoing discussion relates to the maximum induced subpressure (p_m) in a given soil of known porosity (n) under a given freezing surface temperature T_S. However, at any instant the subpressure in the soil layer varies between ground surface and the groundwater table (both are at atmospheric pressure). It is observed that the maximum subpressure (p_m) always occurs at the frost boundary. This is consistent with the theoretical reasoning based on the concept of the varying internal energy of

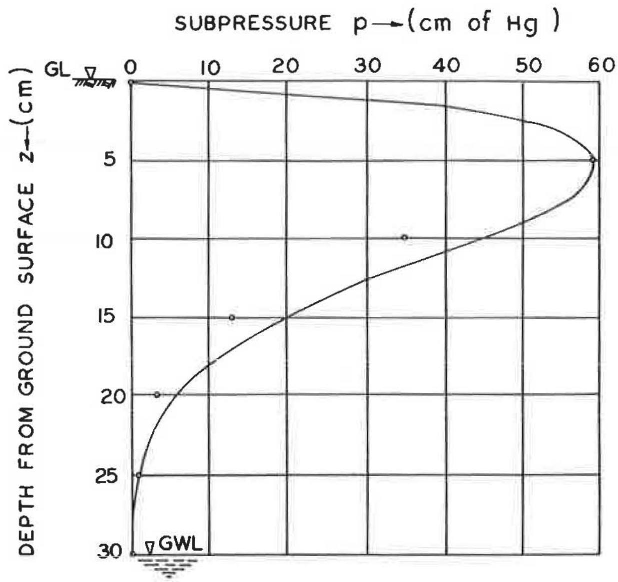

Figure 4. Subpressure profile in freezing Penn Soil (porosity 42%, frost depth 5 cm).

soil moisture at varying temperature from the ground surface to the groundwater table. The subpressure profile resembles a skew frequency distribution curve. The subpressure at any depth as a function of depth of soil layer z can be expressed as

$$p = p_m \left(\frac{z}{\xi}\right)^{k_1} \left(1 - \frac{z - \xi}{h - \xi}\right)^{k_2} \tag{7}$$

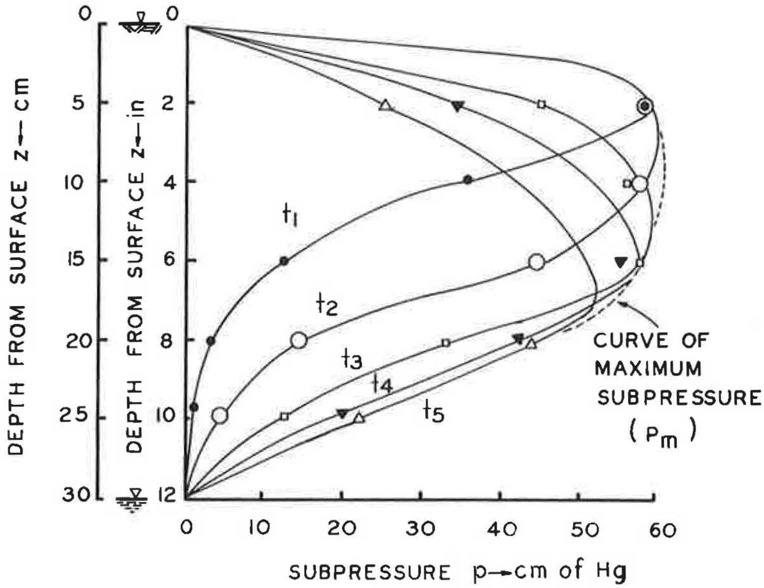

Figure 5. Subpressure profiles in the freezing soil system at different intervals of time after start of cold spell (Penn Soil, porosity 42%).

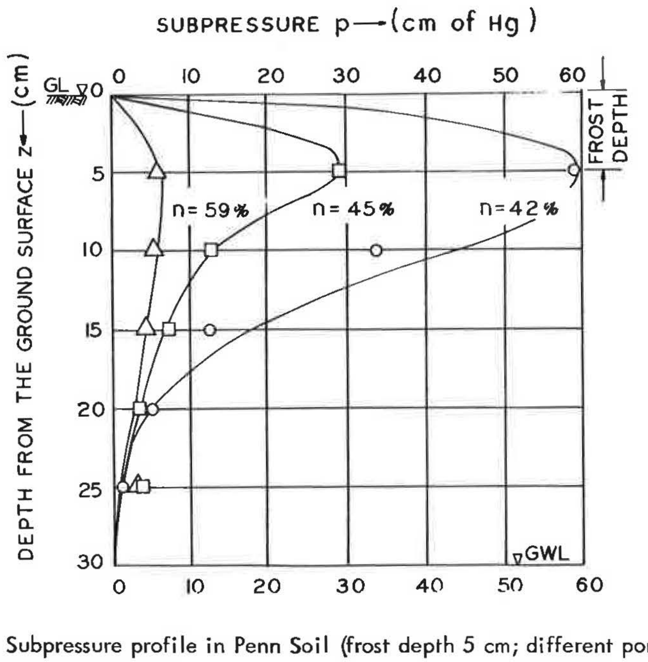

Figure 6. Subpressure profile in Penn Soil (frost depth 5 cm; different porosities).

This is illustrated in Figure 4. Figure 5 shows the relationship of subpressure profile and p_m curve. The variation of subpressure profile for the same frost depth and for the same soil but under different porosities is illustrated in Figure 6. The subpressure p can be expressed as a function of T_s and n only by substituting the values of p_m and ξ in Eq. 7 from Eqs. 4, 5, and 6.

Application and Limitations of the Study

The study presented in this paper has its limitations. It has only indicated the pattern of relationship. Extensive experimental data are necessary to get a practical set of quantitative charts and equations, since many of the constants in the equations presented in this paper have to be obtained from such experimentation.

There is also room for broadening the scope of the study by extending the problem into the quality of water and the pattern of the cold spell. The charts and graphs thus developed could readily be used by practicing engineers.

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