Analysis for the Evaluation of the Settlement Ratio for Rigid Positive Projecting Conduits

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THE SETTLEMENT ratio δ is a parameter used in load formulas of positive projecting conduits. Its value is required to determine the height of the plane of equal settlement H_e. The settlement ratio is defined as the ratio of the difference of the additional settlement of the top of the conduit and the additional settlement of the critical plane in the exterior prism to the additional consolidation of the embankment material below the critical plane.

$$\delta = \frac{(s_m + s_g) - (s_f + s_c)}{s_m} \tag{1}$$

where

- s_m = additional consolidation of the embankment material in the exterior prism between the critical plane and the natural ground surface, ft;
- s_g = additional settlement of the natural ground surface below the exterior prism due to the consolidation of the foundation, ft;
- $s_m + s_g$ = additional settlement of the critical plane in the exterior prism, ft;
- s_c = additional deformation of the conduit, ft;
- s_f = additional settlement of the bottom of the conduit (i.e., the surface of the natural ground beneath the conduit) due to the consolidation of the foundation, ft; and
- $s_f + s_c$ = additional settlement of the top of the conduit, ft.

The equation for determining the height of equal settlement H_e is

$$e^{\pm 2K\mu(H_e/b_c)} \mp 2K\mu(H_e/b_c) = \pm 2K\mu\delta\rho + 1 \tag{2}$$

where

- b_c = outside width of conduit, ft;
- e = base of natural logarithms = 2.7183;
- H_e = vertical distance from the plane of equal settlement to top of conduit, ft;
- K = ratio at a point of active lateral pressure to vertical pressure for the backfill or embankment material;
- μ = tan ϕ = tangent of the angle of internal friction of the backfill or embankment material; and
- ρ = projection ratio for positive projecting conduits = ratio of the distance between the natural ground surface and the top of the conduit (when H_c = 0) to the outside width of the conduit.

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The value of H_e can be determined from Eq. 2 if the values for δ and the other variables are known. The upper algebraic operation symbol is applicable for positive projecting conduits, and the lower algebraic operation symbol is to be used for negative projecting conduits.

The present method of estimating the value of δ is by judgment coupled with known boundary values. The value of δ can be determined after a conduit is installed (1), if provisions are made during the installation to obtain certain measurements.

The analytical derivation for the expression of δ presented herein is based on the concept of the existence of a lower plane of equal settlement either in the yielding foundation or at the upper surface of a nonyielding foundation. The existence of a lower plane of equal settlement can be shown by reasoning similar to that used by Marston (2) to prove the existence of an upper plane of equal settlement above the conduit. The parameters used in the derived expression of δ can be readily determined for the particular site conditions for which the conduit is installed.

ASSUMPTIONS

The following assumptions are made in the derivation:

1. Vertical shearing planes exist adjacent to the cradle (or conduit if no cradle). The shearing plane is taken adjacent to the cradle because the total load on the cradle is to be evaluated. The additional load on the cradle is required to evaluate the additional settlements in the interior prism below the conduit.
2. Cohesion is negligible.
3. The magnitude of the shearing stresses is equal to the active lateral pressure at the vertical shearing planes multiplied by the tangent of the angle of internal friction of the embankment material.
4. The weight of the embankment material above the top of the conduit produces a uniform vertical pressure over the entire width of the interior prism.
5. The load on any horizontal differential element in the interior prism below the bottom of the conduit is a uniform vertical pressure over the entire width of the interior prism.
6. The shear at the sides of the interior prism is distributed as uniform vertical pressure (by virtue of internal friction in the embankment or foundation materials) over the entire width of the interior prism.
7. The shear at the sides of the exterior prism is distributed as uniform vertical pressures throughout the embankment and foundation in the infinitely wide exterior prism and its effect on the consolidation in the exterior prism may be neglected.
8. The embankment and foundation materials have constant moduli of consolidation.
9. The weight of the conduit and cradle are neglected.
10. One mathematical approximation is made in the derivation for case c and case d.

SYMBOLS

The following additional symbols are used in the derivation:

$a = \dfrac{2K\mu}{b}$

$a_f = \dfrac{2K_f\mu_f}{b}$

b = bottom width of cradle, ft. When no cradle is used, $b = b_c$ = outside width of conduit, ft;

E = modulus of consolidation of the embankment material, tons/ft^2;

E_f = modulus of consolidation of the foundation material, tons/ft^2;

H = for the complete condition—vertical distance from top of backfill to a horizontal element of fill material having a height dH, ft; for the incomplete condition—vertical distance from plane of equal settlement to a horizontal element of fill material having a height dH, ft;

H_c = vertical distance from top of backfill or embankment to top of conduit, ft;

H'_e = distance between the top of the conduit and the upper plane of equal settlement when the interior prism has a width b;

H_f = distance between the bottom of the cradle and the nonyielding foundation material, ft. When no cradle is used, it is the distance between the bottom of the conduit and the nonyielding foundation material.

H_l = distance between the bottom of the cradle and the lower plane of equal settlement, ft. When no cradle is used, it is the distance between the bottom of the conduit and the lower plane of equal settlement;

K_f = ratio at a point of active lateral pressure to vertical pressure on the foundation material;

P' = vertical pressure on a horizontal plane within the interior prism when the embankment height is equal to or less than the height of equal settlement, lbs/ft length of conduit;

P'' = additional vertical pressure on a horizontal plane within the interior prism due to the weight of the material above the plane of equal settlement, lbs/ft length of conduit;

λ_e = additional consolidation of the embankment material in the exterior prism between the critical plane and the plane of equal settlement, ft (positive projecting conduits);

λ_i = additional consolidation of the embankment material in the interior prism between the top of the conduit and the plane of equal settlement, ft (positive projecting conduits);

μ_f = tangent of the angle of internal friction ϕ_f for the foundation material;

ϕ_f = angle of internal friction of the foundation material; and

Ψb_c = vertical distance between the natural ground line in the exterior prism and the bottom of the cradle (or the bottom of the conduit if no cradle is used), ft.

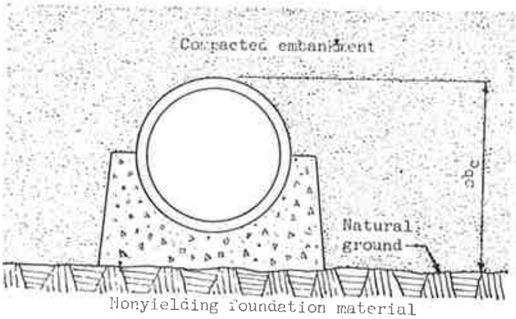
CASES

The four cases represented by the drawings shown in Figures 1, 2, 3, and 5 will be considered separately.

Case a.

Value of δ for conduits resting on rock foundation. When the conduit and embankment are on nonyielding foundation (Fig. 1), the values of s_g, s_c and s_f are zero. Thus by Eq. 1

$$\delta = \frac{s_m}{s_m} = 1 \tag{3}$$



The surface of the nonyielding foundation is considered as the natural ground line. The distance between the top of the conduit and the natural ground line is ρb_c.

Since the foundation is nonyielding, the additional settlements s_f and s_g are both zero, and

$$\delta = 1$$

Figure 1. Conduit resting in or on nonyielding foundations with embankment extending to the nonyielding foundation.

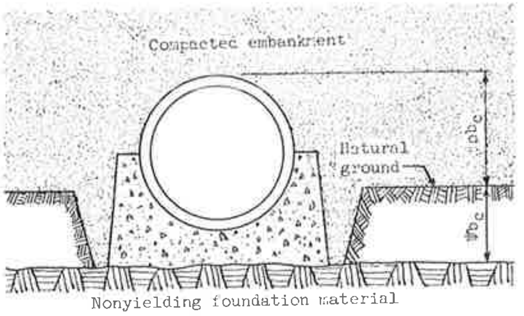


Figure 2. Conduit resting on nonyielding support with compressible foundation materials adjacent to the conduit.

The value of δ is defined by the following relation

$$\delta = 1 + \left[\frac{\gamma_f}{\gamma}\frac{E}{E_f}\right]\frac{\Psi}{\rho}$$

Case b.

Value of δ for conduits resting on rigid support with compressible adjacent foundation and embankment materials. When the conduit is on nonyielding foundation (Fig. 2), the values of s_f and s_c are zero.

$$\delta = \frac{(s_m + s_g)}{s_m} = 1 + \frac{s_g}{s_m}$$

The additional consolidation s_m of the adjacent material between the top of the conduit and the natural ground is that consolidation due to the additional load. The additional load is the weight of the embankment between $H = H_c$ and $H = H'_e$ (Assumption 7)

$$s_m = \frac{\gamma(H_c - H'_e)}{E}\rho b_c \tag{4}$$

Similarly, the additional consolidation s_g of the material between the natural ground and the bottom of the cradle is (Assumption 7)

$$s_g = \frac{\gamma(H_c - H'_e)}{E_f}\Psi b_c \tag{5}$$

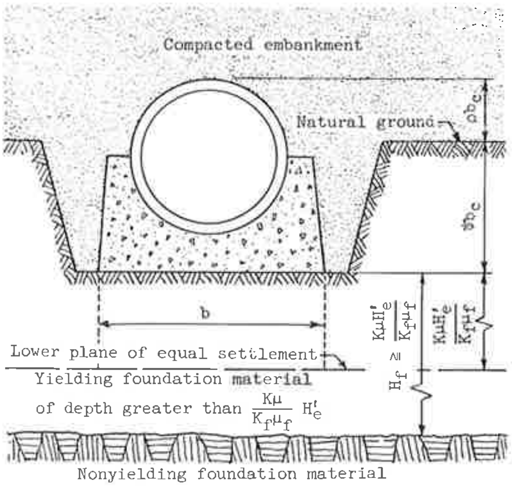
On substituting these values of s_m and s_g, obtain

$$\delta = 1 + \left[\frac{E}{E_f}\right]\frac{\Psi}{\rho} \tag{6}$$

Case c.

Determination of the settlement ratio δ when the foundation material below the top of the conduit is homogeneous material of sufficient depth (Fig. 3). By Eq. 15 when $s_c = 0$, the value of δ for rigid conduits is

$$\delta = \frac{s_m + s_g - s_f}{s_m} \tag{7}$$



The values of δ and H'_e are determined from the following two derived relations:

$$\delta = \frac{1 + \left[\frac{\gamma_f}{\gamma}\frac{E}{E_f}\right]\frac{\psi}{\rho}}{1 + \left[\frac{\gamma_f}{\gamma}\frac{E}{E_f}\right]\frac{K\mu}{K_f\mu_f}}$$

and

$$e^{2K\mu(H'_e/b)} - 2K\mu(H'_e/b) = 2K\mu\delta\rho + 1$$

When the value of δ has been determined from the first of these two relations, the value of H'_e may be obtained from the second relation. The values of δ and H'_e determined in this manner are the correct values of δ and H'_e if $H_f \geq \frac{K\mu}{K_f\mu_f} H'_e$.

Figure 3. Conduit resting on yielding foundation material of sufficiently great depth.

But by definition the upper plane of equal settlement is the lowest horizontal plane at which the additional settlement at the plane for the top of the interior prism is equal to the settlement at the plane for the exterior prism, i.e.,

$$s_f + \lambda_i = s_g + \lambda_e + s_m \tag{8}$$

The evaluation of s_f, s_g, s_m, λ_i, and λ_e is made later.

Lower plane of equal settlement. In the derivation of δ for this case, a lower plane of equal settlement is recognized. At this plane the intensities of pressure of the interior prism are equal to those of the exterior prism. Furthermore, the additional consolidation of every portion of each horizontal plane below this plane of equal settlement is equal. When loads are transferred into the upper interior prism, loads are transferred out of the lower interior prism. Thus, shearing forces of the upper interior prism are oppositely directed from those of the lower interior prism. The additional consolidation in the interior prism is equal to the additional consolidation in the exterior prism between the upper and lower planes of equal settlement when a rigid conduit is installed. Hence,

$$\lambda_i + \lambda'_i = \lambda_e + s_m + \lambda'_e \tag{9}$$

The evaluation of λ involves the summation of the additional consolidations resulting from the variable additional pressures of each horizontal differential element. These pressures are evaluated next. The top sign in all of the following expressions pertains to the projection condition and the bottom sign pertains to the ditch condition.

Expressions for P''_c and P''_l. Equating the vertical forces on the differential element ΔH (Fig. 4) for the interval $(H_c - H'_e) < H < H_c$

$$\frac{dP}{dH} \mp aP = \gamma b \tag{10}$$

where $P = P' + P''$.

For the interval $(H_c + \rho b_c + \Psi b_c) < H < (H_c + \rho b_c + \Psi b_c + H_l)$

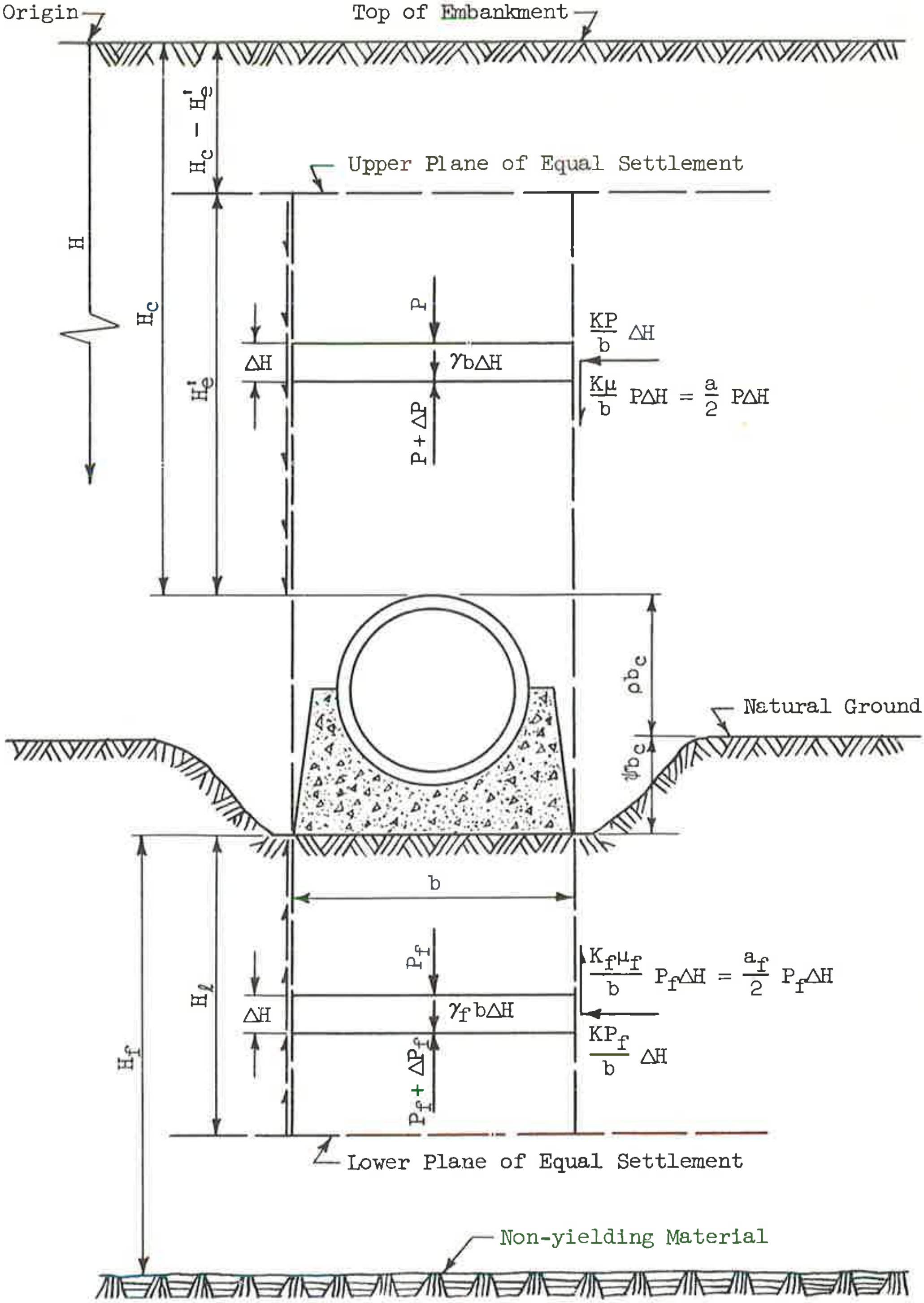


Figure 4. Analysis of settlement ratio δ.

$$\frac{dP}{dH} \pm a_f P = \gamma b \tag{11}$$

On observing the existence of the plane of equal settlement and since Eq. 10 is a linear homogenous differential equation, it may be written in two components

$$\frac{dP'}{dH} \mp aP' = \gamma b \tag{10a}$$

$$\frac{dP''}{dH} \mp aP'' = 0 \tag{10b}$$

and similarly Eq. 11 is written

$$\frac{dP'_1}{dH} \mp a_f P'_1 = \gamma b \tag{11a}$$

$$\frac{dP''_1}{dH} \mp a_f P''_1 = 0 \tag{11b}$$

The general solution of Eq. 10b is

$$P'' = c\, e^{\pm aH} \tag{12a}$$

where c is an arbitrary constant.
When $H = H_c - H'_e$, $P'' = \gamma b(H_c - H'_e)$, the value of c is

$$c = \gamma b(H_c - H'_e)\, e^{\mp a(H_c - H'_e)} \tag{12b}$$

At the top of the conduit $H = H_c$ and $P'' = P''_c$.

$$P'' = \gamma b(H_c - H'_e)\, e^{\pm aH'_e} \tag{13}$$

where P''_c = additional vertical pressure on a horizontal plane at the top of the conduit.
The general solution of Eq. 11b is

$$P''_1 = c\, e^{\mp a_f H} \tag{14a}$$

where c is an arbitrary constant.
When $H = H_c + \rho b_c + \Psi b_c$, then $P''_1 = P''_c$, the value of c is

$$c = \gamma b(H_c - H'_e)\, e^{\pm\left[aH'_e + a_f(H_c + \rho b_c + \Psi b_c)\right]} \tag{14b}$$

At the lower plane of equal settlement $H = H_c + \rho b_c + \Psi b_c + H_l$ and $P'' = P''_l$

$$P''_l = \gamma b(H_c - H'_e)\, e^{\pm(aH'_e - a_f H_l)} \tag{15}$$

where P''_1 = additional vertical pressure on a horizontal plane at the lower plane of equal settlement.

Expressions for λ_e, λ_i, λ'_e and λ'_i. The differential equation expressing the additional consolidation in the interior prism λ_i is

$$d\lambda_i = \frac{P''}{bE} dH$$

Substituting the value of P'' previously determined (Eqs. 12a and 12b)

$$d\lambda_i = \frac{\gamma}{E}(H_c - H'_e)\, e^{\pm a\left[H - (H_c - H'_e)\right]} dH$$

The general solution is

$$\lambda_i = \frac{\gamma}{E}(H_c - H'_e)\, e^{\mp a(H_c - H'_e)} \left[\frac{1}{\pm a}(e^{\pm aH} + c)\right] \tag{16a}$$

where c is an arbitrary constant.

When $H = H_c - H'_e$, then $\lambda_i = 0$, and the value of c is

$$c = -e^{\pm a(H_c - H'_e)} \tag{16b}$$

The total additional consolidation in the interior prism λ_i between $H = H'_c - H'_e$ and $H = H_c$ is

$$\lambda_i = \frac{\gamma(H_c - H'_e)}{\pm aE}(e^{\pm aH'_e} - 1) \tag{17}$$

The additional consolidation in the exterior prism λ_e for the interval $(H_c - H'_e) < H < H_c$ is

$$\lambda_e = \frac{\gamma(H_c - H'_e)}{E} H'_e \tag{18}$$

The differential equation expressing the additional consolidation in the lower interior prism λ'_i is

$$d\lambda'_i = \frac{P''_1}{bE_f} dH$$

Substituting the value of P''_1 previously determined for Eqs. 14a and 14b, the general solution is

$$\lambda'_i = \frac{\gamma(H_c - H'_e)}{E_f}\, e^{\pm\left[aH'_e + a_f(H_c + \rho b_c + \Psi b_c)\right]} \left[\frac{1}{\mp a_f}(e^{\mp a_f H} + c)\right] \tag{19a}$$

where c is an arbitrary constant.

When $H = H_c + \rho b_c + \Psi b_c$, then $\lambda'_i = 0$, and the value of c is

$$c = -e^{\mp a_f(H_c + \rho b_c + \Psi b_c)} \tag{19b}$$

The total additional consolidation in the lower interior prism λ'_i between $H = H_c + \rho b_c + \Psi b_c$ and $H = H_c + \rho b_c + \Psi b_c + H_l$ is

$$\lambda'_i = \frac{\gamma(H_c - H'_e)}{\pm a_f E_f} e^{\pm aH'_e}(1 - e^{\mp a_f H_l}) \tag{20}$$

The additional consolidation λ' in the lower exterior prism is

$$\lambda'_e = \frac{\gamma(H_c - H'_e)}{E_f}(H_l + \Psi b_c) \tag{21}$$

The additional settlement s_m of the material adjacent to the conduit is

$$s_m = \frac{\gamma(H_c - H'_e)}{E} \rho b_c \tag{4}$$

The expression for H'_e is obtained by substituting the evaluations of λ_i and λ_e previously determined by Eqs. 17 and 18 into Eq. 8.

$$s_f + \frac{\gamma(H_c - H'_e)}{\pm aE}(e^{\pm aH'_e} - 1) = s_g + s_m + \frac{\gamma(H_c - H'_e)}{E} H'_e$$

Rearranging and using Eq. 1, $\delta s_m = s_m + s_g - s_f$

$$\delta \frac{\gamma(H_c - H'_e)}{E} \rho b_c = \frac{\gamma(H_c - H'_e)}{\pm aE}(e^{\pm aH'_e} - 1) - \frac{\gamma(H_c - H'_e)}{E} H'_e$$

which reduces to

$$e^{\pm aH'_e} - 1 = \pm a\delta\rho b_c \pm aH'_e$$

or

$$e^{\pm aH'_e} \mp aH'_e = \pm a\delta\rho b_c + 1 \tag{22}$$

This relation evaluates the position of the plane of equal settlement for the conduit and cradle. This relation differs from Eq. 2, which evaluates the plane of equal settlement for the conduit.

Expression for H_l. The location of the lower plane of equal settlement is determined by observing that the additional vertical pressure at the lower plane of equal settlement is equal to the additional vertical pressure at the upper plane of equal settlement.

$$\gamma(H_c - H'_e)b = \gamma b(H_c - H'_e)\, e^{\pm(aH'_e - a_f H_l)}$$

Rewriting

$$1 = e^{\pm(aH'_e - a_f H_l)} \tag{23}$$

or

$$aH'_e - a_f H_l = 0$$

and

$$H_l = \frac{a}{a_f} H'_e \tag{24}$$

Expression for evaluation of δ. Substituting the evaluations of s_m, λ_i, λ_e, λ'_i and λ'_e as given by Eqs. 4, 17, 18, 20 and 21 into Eq. 9, obtain

$$\frac{\gamma(H_c - H'_e)}{\pm aE}(e^{\pm aH'_e} - 1) + \frac{\gamma(H_c - H'_e)}{\pm a_f E_f}\left[e^{\pm aH'_e} - e^{\pm aH'_e \mp a_f H_l}\right] =$$

$$\frac{\gamma(H_c - H'_e)}{E}(H'_e + \rho b_c) + \frac{\gamma(H_c - H'_e)}{E_f}(H_l + \Psi b_c) \quad \frac{\gamma(H_c - H'_e)}{\pm aE}$$

$$(e^{\pm aH'_e} - 1) + \frac{\gamma(H_c - H'_e)}{\pm a_f E_f} e^{\pm aH'_e}(1 - e^{\mp a_f H_l}) =$$

$$\frac{\gamma(H_c - H'_e)}{E}(\rho b_c + H'_e) + \frac{\gamma(H_c - H'_e)}{E_f}(H_l + \Psi b_c) \tag{25}$$

Multiplying by $\frac{\pm a}{H_c - H'_e}$ and substituting Eqs. 22 and 24, obtain on rearranging

$$\frac{\pm a\gamma}{E}(\delta\rho b_c + H'_e) + \frac{\gamma}{E_f}\frac{a}{a_f}\left[\pm a\delta\rho b_c \pm aH'_e\right] =$$

$$\frac{\pm a\gamma}{E}(H'_e + \rho b_c) \frac{\pm a\gamma}{E_f}\left[\frac{a}{a_f} H'_e + \Psi b_c\right]$$

On rearranging

$$\delta = \frac{1 + \left[\frac{E}{E_f}\right]\frac{\Psi}{\rho}}{1 + \left[\frac{E}{E_f}\right]\frac{a}{a_f}} \tag{26}$$

obtain

$$\delta = \frac{1 + \left[\frac{E}{E_f}\right]\frac{\Psi}{\rho}}{1 + \left[\frac{E}{E_f}\right]\frac{K\mu}{K_f\mu_f}} \qquad (27)$$

Equation 27 gives the expression for δ when the foundation material is homogeneous for a sufficiently great depth. The depth H_f is sufficiently great if $H_f \geq H_l$, or from Eq. 24, $H \geq \frac{K\mu}{K_f\mu_f} H'_e$.

Case d.

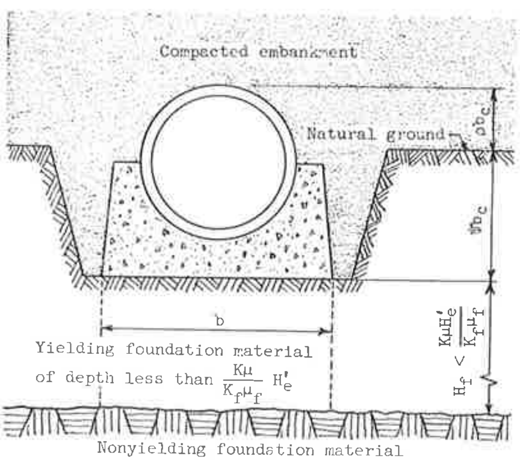
Determination of the settlement ratio δ when the foundation material is shallow ($H_f < H_l$). The additional settlements of the interior prism and exterior prisms for Case d at $H = H + (\rho + \Psi)\, b_c + H_f$ are equal. By the definition of the plane of equal settlement, the additional consolidations in the interior prism and exterior prisms between the plane of equal settlement and the nonyielding foundation are equal (Fig. 5). This results in the relation

$$\lambda'_i + \lambda_i = \lambda'_e + \lambda_e + s_m$$

The evaluations of the terms s_m, λ_i, and λ_e are given by Eqs. 4, 17 and 18. The evaluations of λ'_i and λ'_e are obtained by substituting H_f for H_l in Eqs. 20 and 21. Making these substitutions, obtain

$$\frac{\gamma(H_c - H'_e)}{\pm a_f E_f}\, e^{\pm aH'_e}\left[1 - e^{\mp a_f H_f}\right] + \frac{\gamma(H_c - H'_e)}{\pm aE}\left[e^{\pm aH'_e} - 1\right] =$$

$$\frac{\gamma(H_c - H'_e)}{E_f}(H_f + \Psi b_c) + \frac{\gamma(H_c - H'_e)}{E}(H'_e + \rho b_c)$$



The values of δ and H'_e depend, among other parameters, on the value of H_f. They are determined by the simultaneous solution of the following two derived relations:

$$\delta = \frac{1 + \left[\frac{\gamma_f}{\gamma}\frac{E}{E_f}\right]\frac{\psi}{\rho}}{1 + \left[\frac{\gamma_f}{\gamma}\frac{E}{E_f}\right]\frac{H_f}{H'_e}}$$

and

$$e^{2K\mu(H'_e/b)} - 2K\mu(H'_e/b) = 2K\mu\delta\rho + 1$$

Figure 5. Conduit resting on yielding foundation material of shallow depth.

Multiplying by $\dfrac{\pm a_f E_f}{\gamma(H_c - H'_e)}$ and substituting xaH'_e for $a_f H_f$

$$e^{\pm aH'_e}\left[1 - e^{\mp axH'_e}\right] + \frac{a_f}{a}\frac{E_f}{E}\left[e^{\pm aH'_e} - 1\right] =$$

$$\pm(axH'_e + a_f\Psi b_c) \pm a_f\frac{E_f}{E}(H'_e + \rho b_c)$$

Recognizing that $e^{\pm aH'_e} - 1 = \pm a\delta\rho b_c \pm aH'_e$

$$\left[1 + \frac{a_f}{a}\frac{E_f}{E}\right]\left[\pm a\delta\rho b_c \pm aH'_e\right] + \left[1 - e^{\pm a(1-x)H'_e}\right] =$$

$$\pm(axH'_e + a_f\Psi b_c) \pm a_f\frac{E_f}{E}(H'_e + \rho b_c)$$

or

$$\left[1 + \frac{a_f}{a}\frac{E_f}{E}\right](\delta\rho b_c) + \left\{\frac{1}{\pm a}\left[1 - e^{\pm a(1-x)H'_e}\right] + (1-x)H'_e\right\} =$$

$$\frac{a_f}{a}\Psi b_c + \frac{a_f}{a}\frac{E_f}{E}\rho b_c$$

Make the approximation

$$\frac{1}{\pm a}\left[1 - e^{\pm a(1-x)H'_e}\right] + (1-x)H'_e = (x-1)\delta\rho b_c$$

Obtain

$$\left[x + \frac{a_f}{a}\frac{E_f}{E}\right](\delta\rho b_c) = \frac{a_f}{a}\Psi b_c + \frac{a_f}{a}\frac{E_f}{E}\rho b_c$$

or

$$\delta = \frac{1 + \left[\dfrac{E}{E_f}\right]\dfrac{\Psi}{\rho}}{1 + \left[\dfrac{E}{E_f}\right]\dfrac{H_f}{H'_e}} \qquad (28)$$

The simultaneous solution of Eqs. 2 and 28 can be given graphically. An illustration of such a graphical solution can be found in the technical publication of the Engineering Division of the Soil Conservation Service, Technical Release No. 5: The Structural Design of Underground Conduits.

REFERENCES

1. Spangler, M. G. Field Measurements of the Settlement Ratios of Various Highway Culverts. Bull. 170, Iowa Engineering Experiment Station, Ames, 1950.
2. Marston, A. The Theory of External Loads on Close Conduits in the Light of the Latest Experiments. Bull. 96, Iowa Engineering Experiment Station, Ames, 1930.