Laboratory Flexural-Fatigue Testing of Asphalt-Concrete With Emphasis on Compound-Loading Tests

JOHN A. DEACON and CARL L. MONISMITH
Respectively, U. S. Army Aviation Materiel Laboratories, Fort Eustis, Virginia; and Associate Professor of Civil Engineering, Institute of Transportation and Traffic Engineering, University of California, Berkeley

It has been conclusively established in previous investigations that asphalt-concrete pavement surfaces can exhibit fatigue distress in service. This observation has prompted the instigation of several laboratory investigations concerned with the phenomenological fatigue behavior of asphalt-concrete test specimens.

In this investigation emphasis is placed on the compound-loading fatigue behavior; that is, the fatigue behavior derived from the application of more than one stress level to any particular specimen. Such multilevel loading is meant to stimulate to a limited extent the diverse spectrum of vehicular stresses applied to a point in the pavement surface.

Laboratory equipment was developed through which sequence, repeated-block, and random load histories can be applied to beam specimens. A modification of the linear summation of cycle ratios hypothesis (Miner's hypothesis) was found to be applicable for predicting the arithmetic-mean fracture lives of the test specimens for both repeated-block and random loading.

•THE performance of flexible highway pavements in service is dependent in large measure on the repetitive nature of traffic loads. Grumm (1) and Porter (2) were among the first to recognize this relationship, but both failed to distinguish between the fatigue effects of repetitive loading and other detrimental effects (i.e., rutting) also related to repetitive load application. Repetitive loading causes fatigue distress in flexible pavement surfaces when the strength and stiffness properties of the asphalt surfacing material have been sufficiently altered so that the applied stress level under loading exceeds the flexural strength of the surface. Distortion and cracking of the pavement surface when not accompanied by substantial structural weakening of the surface material (except on a localized scale) are seldom caused by the fatigue effect.

It remained for other investigators (3 through 7) to accumulate the necessary evidence indicating that flexible highway pavement surfaces can exhibit distress due to flexural fatigue cracking as a result of the repetitive application of vehicular loads. Earlier recognition of fatigue failures in flexible pavements may have been partially prevented by the misconception that the pavement surface served only as a load-transmitting medium which was unable to exhibit slab action.

There has been general agreement among investigators that the tendency for fatigue cracking is aggravated by heavy loads, large numbers of load applications, and resilient foundation materials. Additionally, several characteristics of the asphalt surface course are known to influence the development of fatigue cracking. These include

Paper sponsored by Committee on Mechanical Properties of Bituminous Mixtures and presented at the 45th Annual Meeting.

stiffness and thickness, which determine in part the magnitudes of the imposed stresses and strains, and fatigue resistance, which is related to the ability of the surface material to be flexed repeatedly without fracture or undue loss in stiffness and flexural strength.

To examine the nature and cause of the fatigue distress observed in service, the fatigue behavior of asphalt-concrete mixtures was investigated by subjecting them to repetitive loading under carefully controlled laboratory conditions (4, 5, and 7 through 32).

Proper mixture design has been the focal point of the majority of fatigue investigations to date. These particular investigations have sought to determine: (a) what mixture constituents and properties affect fatigue behavior, (b) how fatigue behavior is affected by changes in these constituents and properties, and (c) how maximum potential fatigue resistance for in-service application can be achieved through proper design. A second, but as yet largely unexplored, area for investigation is that of structural pavement design. The basic question here is how best to select the thicknesses of the component pavement layers to minimize potential distress caused by the fatigue phenomenon.

Although much remains to be accomplished in the development of an adequate thickness-design technique, some notable preliminary efforts have been made in this regard. The California Division of Highways, for example, has developed a design procedure based on utilization of the Hveem resiliometer which is intended to reduce fatigue cracking (33). Although admittedly empirical, this procedure was the first available to permit the additional consideration of fatigue in asphalt-concrete pavement design. Recently, another, more theoretical design procedure utilizing elastic stress analyses (34, 35) has been proposed which also attempts in part to control distress caused by fatigue (36, 37). Thicknesses and properties of the various layers are selected so that calculated strains in the asphalt-concrete and at the subgrade surface are restricted to levels below those thought to cause distress. The critical levels of permissible strain in the asphalt-concrete were developed from laboratory fatigue test results modified through in-service correlations.

One of the basic problems inherent in the theoretical design techniques is how best to treat a diverse loading spectrum consisting of a variety of wheel loads repetitively applied and transversely distributed across the pavement. Such loading results in the application of an almost infinite spectrum of stress levels to any particular point in the pavement surface. This study was an attempt to examine some aspects of this problem in the laboratory. Emphasis is placed on the phenomenological fatigue behavior of laboratory-prepared test specimens of an asphalt-concrete mixture subjected to complex forms of loading in which the magnitude of the repetitive, flexural stresses applied to any particular specimen varies according to some predetermined sequence (compound loading). Additionally, some observations are made of the fatigue behavior of similar specimens subjected to simple forms of repetitive loading so designed that any one specimen experiences repetitive loads of a single nature (simple loading). The basic information from which this paper has been derived has been reported by Deacon (23).

TERMINOLOGY

Fatigue failure of laboratory test specimens is often a rather arbitrarily defined point related to the ability of the specimens to continue to perform satisfactorily as load-carrying entities under repetitive loading. Investigators have chosen to identify the failure condition in a number of different ways. The most common basis for selection of an appropriate definition is that of ease in identification for the particular testing techniques and equipment employed. The service life (N_s) is the accumulated number of load applications necessary to cause failure in the test specimen. Service life, as defined here, is closely analogous to that which has been termed the fatigue life elsewhere (38). The accumulated number of load applications necessary to fracture a specimen completely is termed its fracture life (N_f). The service and fracture lives are identical only when failure is designated as meaning complete rupture of the specimens under continued, repetitive load applications.

TABLE 1

LABORATORY TEST VARIABLES AFFECTING FATIGUE BEHAVIOR

I. Load variables
   A. Pattern of stressing
      1. Types of stresses
      2. Geometrical stress distribution
   B. Time distribution of loading
      1. Distribution of time between successive load applications
      2. Mean rate of loading
      3. Shape of load curve
      4. Duration of loading
   C. Testing method
      1. Mode of loading
      2. Simple loading
      3. Compound loading
         a. Sequence tests
         b. Repeated-block tests
         c. Random tests
         d. Simulation tests

II. Environmental variables
   A. Temperature
   B. Moisture
   C. Alteration of material properties during service life

III. Mixture and specimen variables
   A. Aggregate
      1. Type
      2. Gradation
   B. Binder
      1. Type
      2. Hardness
   C. Specimens
      1. Bitumen content
      2. Surface texture
      3. Air void content
      4. Anisotropy
      5. Shape
      6. Size
      7. Stiffness

Many of the test variables of potential influence in determining the laboratory fatigue behavior of asphalt mixtures are given in Table 1. The load condition refers to a particular set of values which the appropriate load and environmental variables in Table 1 assume for a particular load application. If the load condition remains unchanged throughout the service life of any single specimen, that specimen is said to be subjected to simple loading. Compound loading results from the repeated application of loads in which the load condition changes during the service life of any particular specimen in some prescribed manner, this being called the load history. Compound-loading tests in which all variables except stress level are held constant are sometimes called variable-stress level tests (39) or multilevel tests (23).

Typical types of compound-loading tests are sequence, repeated-block, and random tests. When a specimen is subjected first to a fixed number of applications of a given load condition followed by a fixed number of applications of a second, different load condition, etc., until failure occurs, the specimen is said to have been subjected to a

sequence test. In a repeated-block test, a block of load applications is applied repetitively until failure occurs, each subsequent block being identical to that which precedes it. Two or more load conditions are applied within each block in any prescribed manner, with the total, preset number of load applications within each block termed the block size. The random test is a compound-loading test in which the probability that any one of the several load conditions will be selected for any particular load application is constant regardless of the preceding order of applied load conditions. The accumulated number of applications of the various load conditions is jointly distributed, random variables having a multinomial distribution.

A compound-loading hypothesis attempts to predict or describe the fatigue behavior under various forms of compound loading. The basis for such hypotheses may be either theoretical or empirical in origin, and their application may be limited to specific load histories.

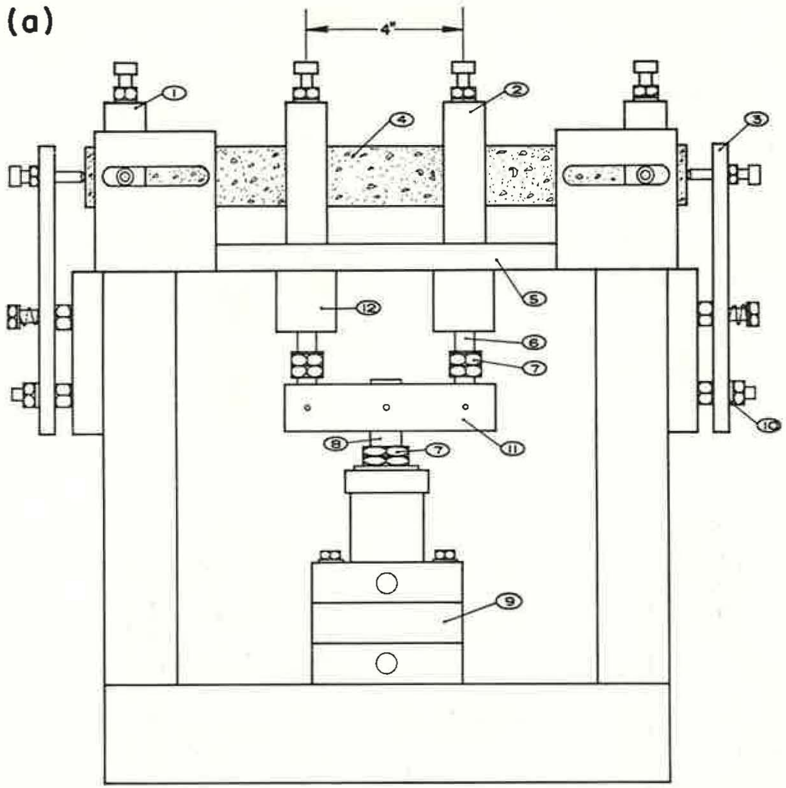
Mode of loading is a term used to describe how stress and strain levels are permitted to vary during repetitive fatigue loading. If the nominal stress level (or levels) is (or are) maintained constant throughout the service life, the testing is of the controlled-stress mode. Controlled-load testing is analogous in most respects to controlled-stress testing. If, however, the nominal strain level (or levels) is (or are) maintained constant throughout the service life, the testing is of the controlled-strain mode. Controlled-deflection testing is analogous in most respects to controlled-strain testing. Controlled-stress and controlled-strain modes represent the limits of an infinite spectrum of possible modes of loading. Intermediate points remain to be precisely defined. Only when the dynamic stress-strain relationship is invariant throughout the service life does the significance of mode of loading disappear. Such is thought to be rarely the case with asphalt specimens.

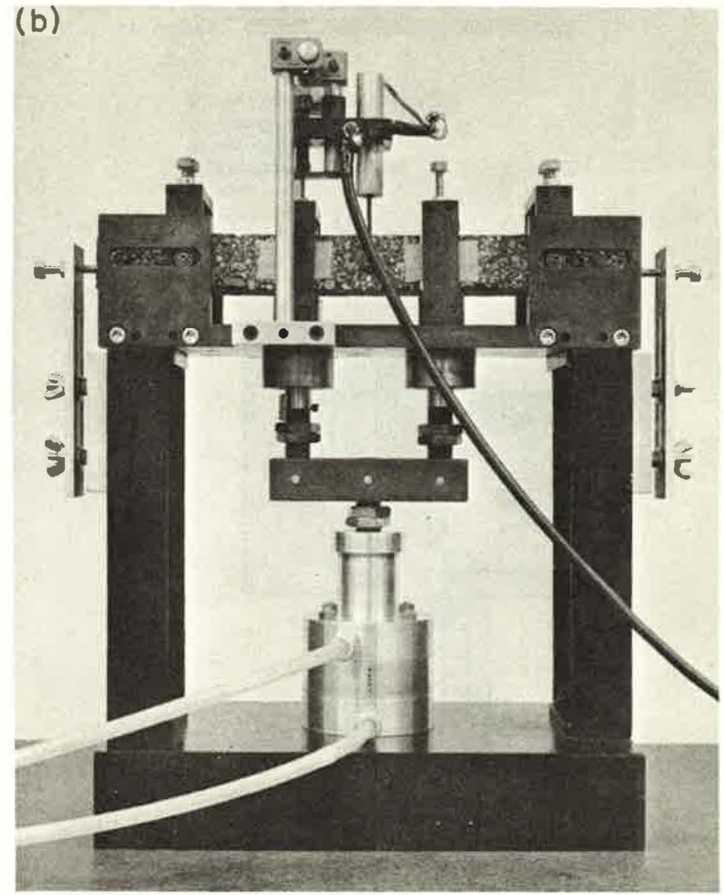
TEST EQUIPMENT

A primary objective of this research was the development of suitable equipment capable of applying repetitive loads to asphalt-concrete test specimens. The primary characteristic distinguishing this equipment from most other previously developed equipment of a similar nature rests in its ability to apply multilevel loading of both a repeated-block and random nature. The load applied to a specimen results from the application of controlled pneumatic pressures to a frictionless pressure cylinder. By varying the magnitude of the pneumatic pressure, the applied load can be suitably controlled.

The three major components of the loading system are the repeated-flexure apparatus, the pneumatic-pressure system, and the control and counting system. The repeated-flexure apparatus with a specimen positioned for loading is shown in Figure 1. Major components of this apparatus are detailed in Figure 2. The function of the pneumatic-pressure system is to supply controlled pneumatic pressures to the double-acting Bellofram load cylinder for conversion to dynamically applied loads. This system consists of a bank of individually regulated pressure cylinders connected to the appropriate chambers of the Bellofram cylinder through mechanical air control (MAC) valves which control the flow of air. The MAC valves are three-way solenoid-operated pressure valves. The control and counting system is used to actuate the appropriate MAC valves of the pneumatic-pressure system and to count the total number of loads of each magnitude applied to the specimen. Multilevel loads are applied by pressurizing the pressure cylinders to different levels and selecting the appropriate cylinder corresponding to the desired load.

The symmetrical, two-point load system of the repeated-flexure apparatus applies unidirectional bending stresses to the simply supported asphalt-concrete beam specimens. The restrainers (Fig. 1a) maintain the symmetrical positioning of the specimen without the imposition of significant axial forces but yet permit outward longitudinal movement at fracture. The lubricated rockers of the load and reaction clamps (Fig. 2) assist in eliminating torsional stresses resulting from distorted specimen surfaces. The specimen is firmly clamped in the reaction clamps so that there is no relative movement between these clamps and the specimen. Ball bearings on these clamps,





Key:

1.	Reaction clamp	5.	Base plate	9.	Double-acting, Bellofram cylinder
2.	Load clamp	6.	Loading rod	10.	Rubber washer
3.	Restrainer	7.	Stop nut	11.	Load bar
4.	Specimen	8.	Piston rod	12.	Thomson ball bushing

Figure 1. Repeated-flexure apparatus.

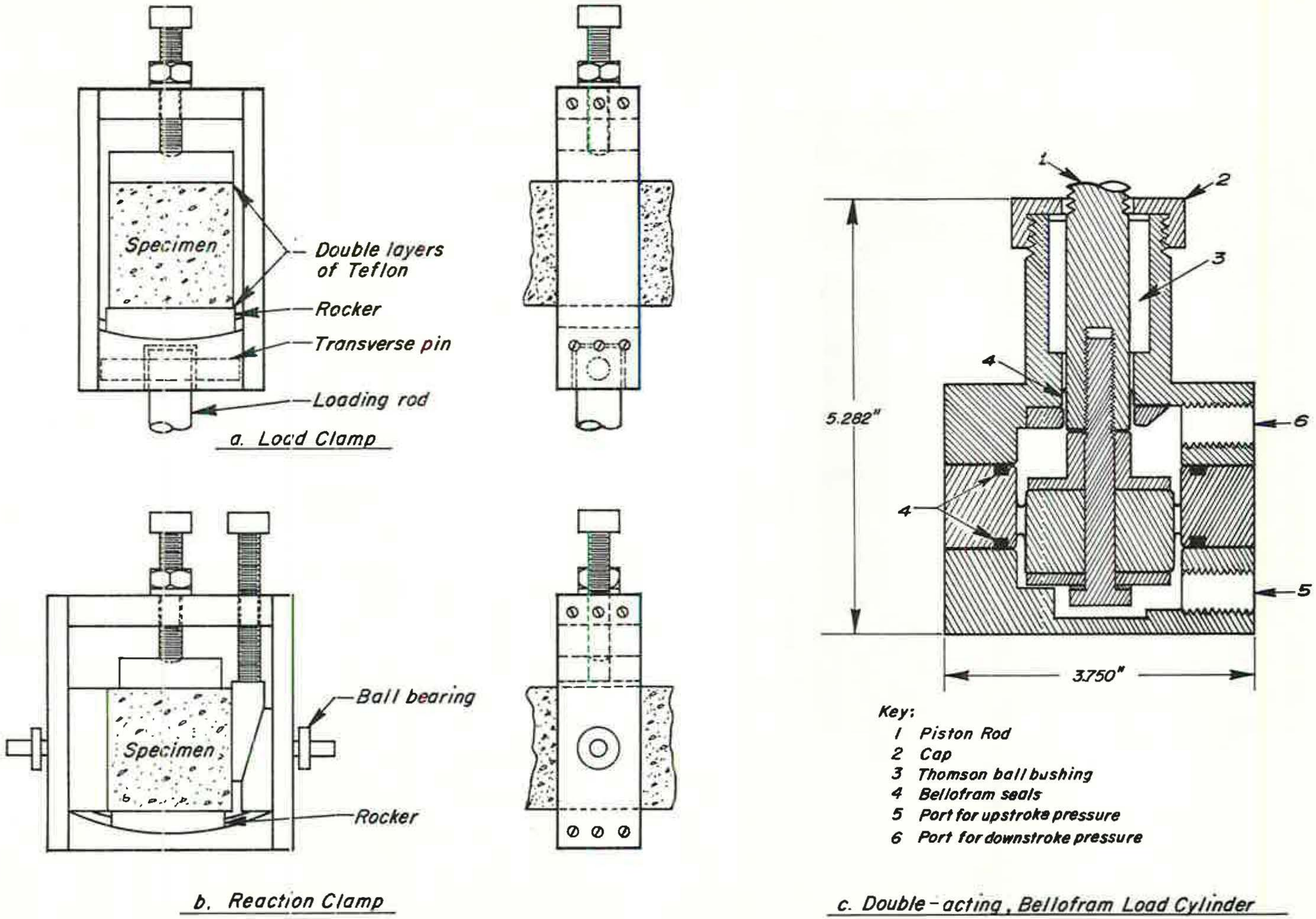


Figure 2. Components of repeated-flexure apparatus.

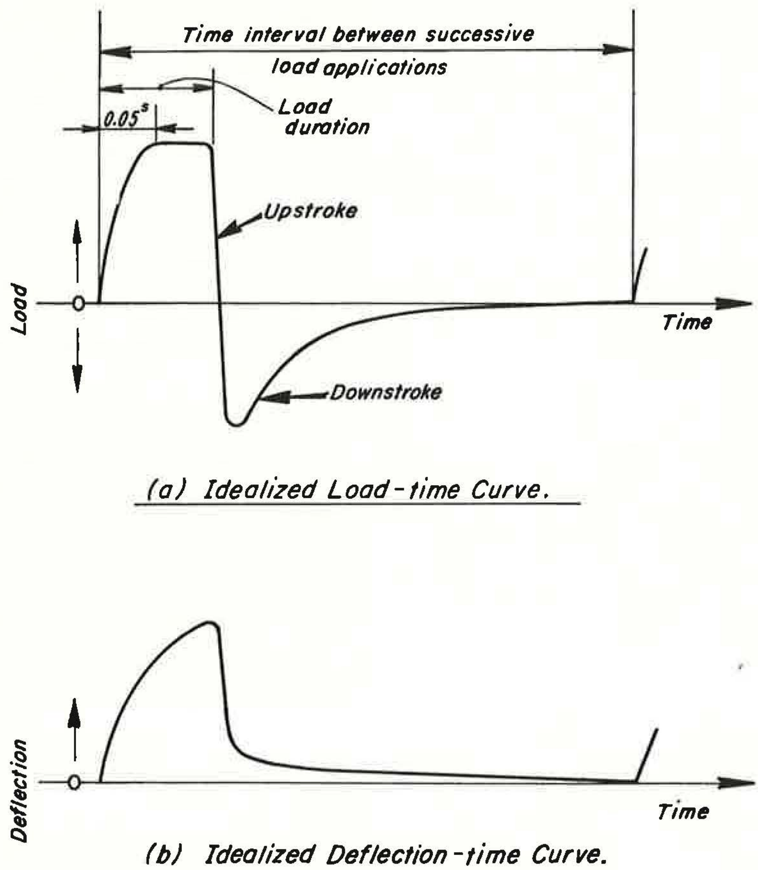


Figure 3. Load vs time and deflection vs time relationships for controlled-stress test equipment.

however, permit differential longitudinal movement and rotation under load. During loading, the load clamps are free to rotate about the lower transverse pins. Double layers of Teflon tape, lubricated with Molykote powder, reduce the friction between the load clamps and the specimen and, hence, the longitudinal restraint that might be imposed on the specimens by these clamps.

During each load application, the specimen is flexed for a given duration after which it is forced to return to its original undeflected position. The load duration and constant time interval between successive load applications are controlled by a mechanically powered cam and microswitch arrangement in the control and counting system. This form of load pulsation produces stress reversal without corresponding strain reversal. Figure 3 shows typical load-time and deflection-time curves. The rate of loading can be made to vary up to approximately 120 applications per minute. The load duration can be increased above a minimum of about 0.05 sec to any desired value compatible with the period determined by the chosen rate of loading.

A controlled-stress mode of loading has been used in this investigation. For such loading, failure is defined as the fracture condition in this study. Multilevel, controlled-strain loading is possible only by continuous monitoring of the applied strain levels and suitable adjustment of the pneumatic pressures.

Both simple and compound loading can be applied with the repeated-flexure system. Types of compound loading permissible include sequence, repeated-block, and random loading. Any number of stress levels may be applied for sequence histories, but a manual change in stress level is required. For repeated-block loading, the number of

TABLE 2

LABORATORY TEST RESULTS—ASPHALT CEMENT

Test	Sp. Gr.	Penetration 77 F, 100 g, 5 sec (0.1 mm)	Penetration 39.2 F, 200 g, 60 sec (0.1 mm)	Penetration Ratio	Flash Point P.M.C.T. (F)	Viscosity Saybolt Furol, 275 F (sec)	Solubility in CCl_4 (%)	Softening Point Ring and Ball (F)
Original Asphalt								
AASHO method[a]	T43-54	T49-53			T73-60	T72-57	T45-56	T53-42
Calif. spec.[b]	—	85-100		25 (min)	440 (min)	85-260	99 (min)	—
Test result	1.020	92	28	30.4	465	164.5	99.85	117
Recovered Asphalt								
AASHO method[a]								
Calif. spec.[b]		—						—
Test result		40						121

[a]Ref. 40.
[b]Ref. 41.

stress levels may vary from two to five, the maximum number being determined by the number of pressure cylinders in the pneumatic-pressure system. A 20-pole stepping relay in the control and counting system provides a restriction on the maximum block size of 20 load applications. Likewise, any number of stress levels from two to five may be applied for the random loading. The probability of application of any particular stress level may be set in increments of 0.05. The random mode of operation results from the random advancement of the 20-pole stepping relay by a suitably amplified pulse from a Geiger tube.

For most of the specimens tested, continuous, dynamic-deflection measurements were taken under the application of the controlled-stress loading. These deflections were measured and recorded through the use of a linear variable differential transformer and a Sanborn strip chart recorder. From the measured deflection, a dynamic deflection-based stiffness modulus is calculated by means of the following relation:

$$E = \frac{KP}{I\Delta} \qquad (1)$$

where

- E = deflection-based, stiffness modulus,
- K = constant dependent on system geometry,
- P = total dynamic load applied upward to specimen,
- I = specimen moment of inertia, and
- Δ = dynamic, center deflection.

TEST SPECIMENS

The effects of mixture and specimen variables (Table 1) were excluded in large measure from the scope of this investigation, and a single asphalt-concrete mixture was employed throughout the testing program. This mixture, essentially a dense-graded asphalt concrete, is similar to surface course mixtures used on heavy-duty highways in California.

Composition

The asphalt cement was an 85-100 penetration material. Pertinent characteristics of this material are indicated in Table 2, together with the appropriate standard specifications of the California Division of Highways. Penetration and

TABLE 3

LABORATORY TEST RESULTS—AGGREGATE

Test	California Test Method No.[a]	Test Result
Wet shot rattler test (% abrasion loss)	210-C	27
Los Angeles rattler test (% wear)	211-C	
100 revolutions		6
500 revolutions		27
Sand equivalent test (sand equivalent)	217-E	71
Film stripping test	302-C	Slight stripping

[a]Ref. 42.

softening points of the asphalt recovered from a typical test specimen are also included in Table 2. The design asphalt content, based on current California mix design procedure, was 6 percent (by weight of dry aggregate).

The aggregate was a crushed granite from Watsonville, Calif., having a uniform apparent specific gravity of 2.92. Results of laboratory tests performed on this aggregate are given in Table 3. The gradation conforms with the aggregate gradation requirements of the division of highways for surface courses having a ³/₈-in. maximum aggregate size. Figure 4 shows this gradation together with the specification limits.

Preparation

Proportioned aggregate and asphalt cement were mixed by mechanical means at 250 F. After mixing, each batch was allowed to cure for approximately 20 to 24 hr at 140 F. The mix was then compacted at 250 F, using a Triaxial Institute kneading compactor, into bars having lengths of 15 in. and rectangular cross-sections of approximately 3.25 by 3.5 in. From each of these bars four test specimens having 1.5-in. square cross-sections and lengths of 15 in. were sawed with a diamond-tipped arbor saw. All specimens were tested within a period of 1 to 4 weeks following compaction.

Properties

Resonant-frequency measurements were made on four specimens at several temperature levels to establish the approximate magnitude of the dynamic modulus of elasticity

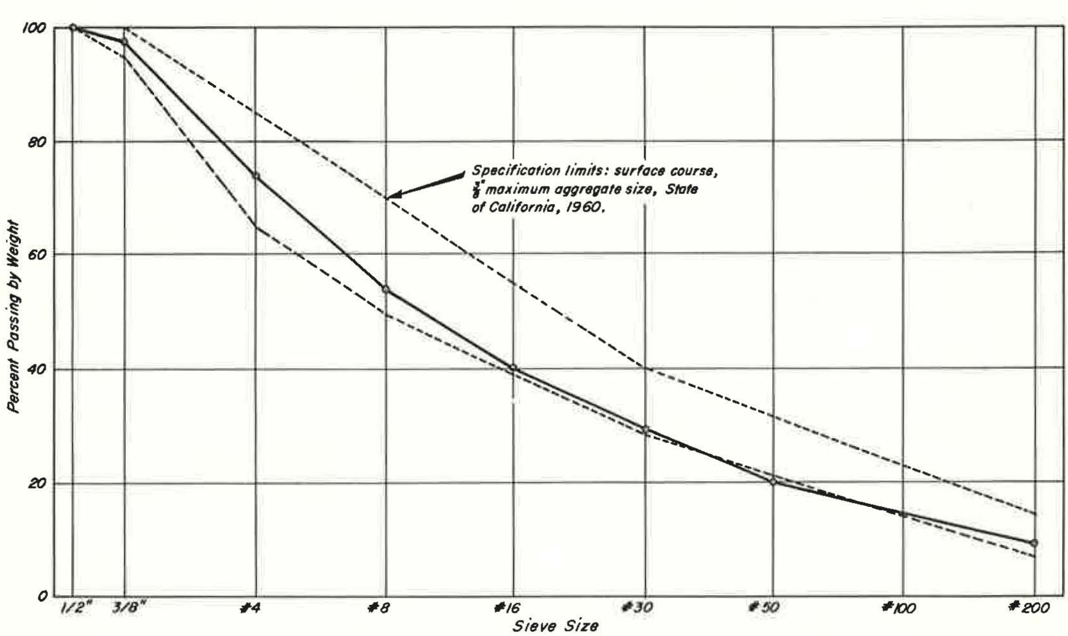


Figure 4. Aggregate-gradation curve.

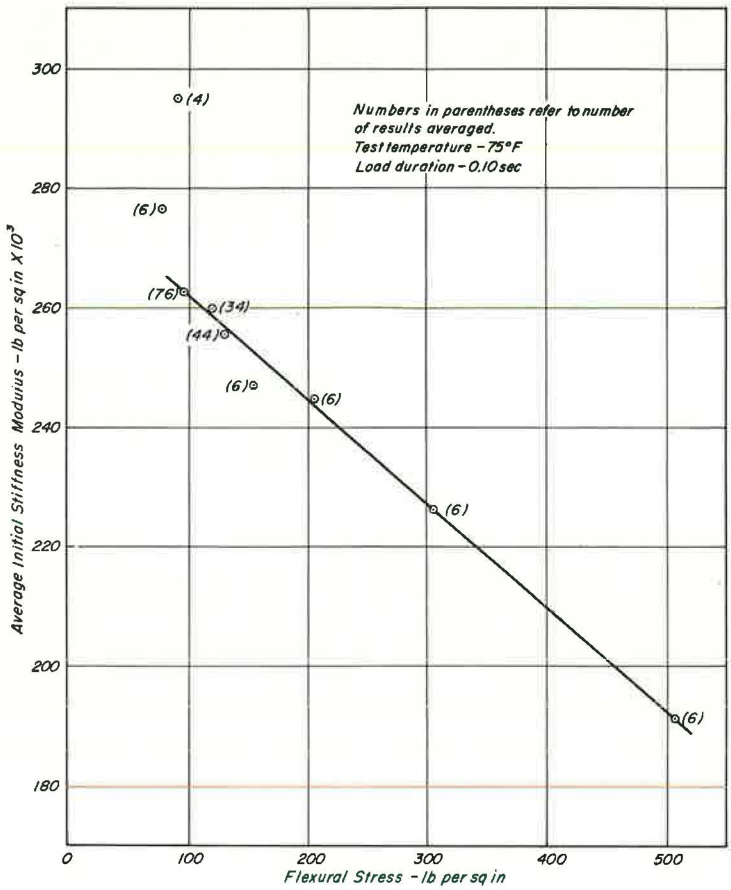


Figure 5. Average initial stiffness modulus vs flexural stress.

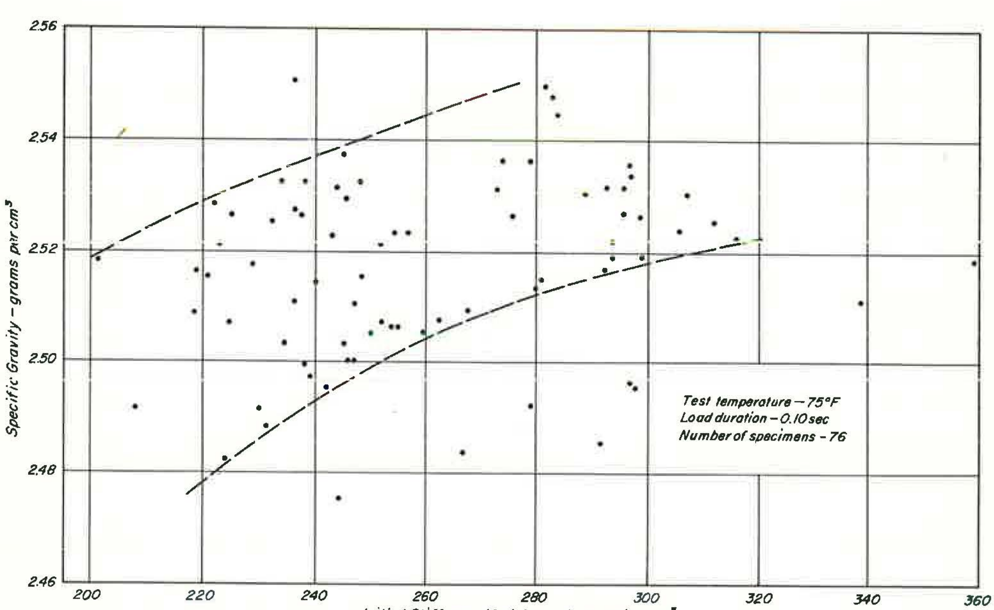


Figure 6. Relation between specific gravity and initial stiffness modulus (98.5 psi stress).

TABLE 4

NUMBER OF TEST SPECIMENS ASSIGNED TO SIMPLE-LOADING TEST SERIES

Test Series	No. of Specimens[a]	Stress Levels (psi)	Load Duration (sec)	Rate of Loading (applic./min)	Temperature (F)
A	58	78.5 to 507.2	0.1	100	75
B	3	90 to 125	0.1	100	75
	4	360 to 490	0.1	60 to 100	40
C	6	105	0.1	30 to 100	75
	24	400	0.1	30 to 100	40
D	12[b]	88.5	0.1 to 0.18	100	75
E	10	93.5 to 128.5	0.1	100	75

[a]Total number of specimens included in this table is 117.
[b]Twelve specimens of Test Series D were supplemented with six of Test Series A.

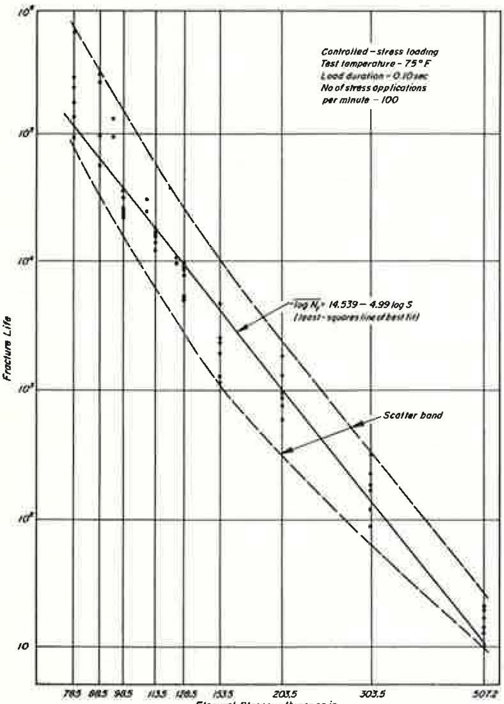


Figure 7. Nf-S diagram (logarithmic plot).

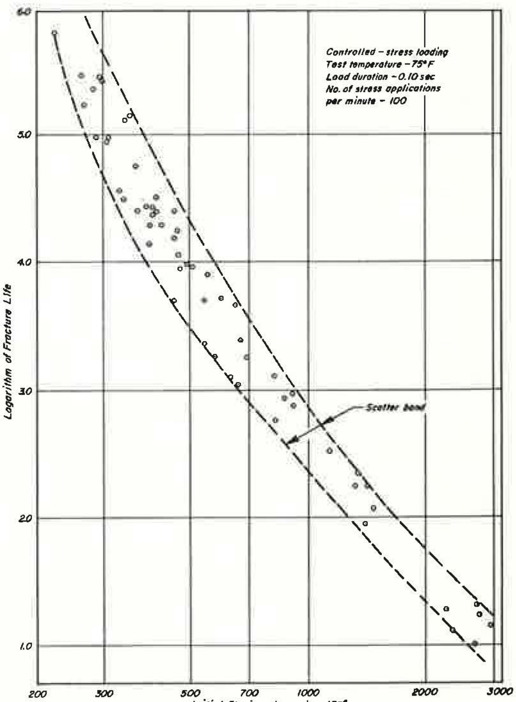


Figure 8. Nf-e diagram (logarithmic plot).

and to ascertain the effect of temperature on this modulus. The average dynamic modulus of elasticity was 6.28×10^6 psi at - 20 F and 3.09×10^6 psi at 75 F, the temperature most commonly used for the fatigue testing.

Using the data in Table 2 for recovered asphalt, the nomographs of Van der Poel (43) were employed to estimate the stiffness modulus of the test specimens. For a temperature of 75 F and a time of loading of 0.1 sec, the stiffness modulus obtained was approximately 2 to 2.5×10^5 psi. This modulus can be compared with the experimental deflection-based stiffness moduli calculated on the basis of Eq. 1. Figure 5 shows the experimentally determined relationship between the average stiffness modulus calculated using the center deflection of the specimen at the first load application and the extreme-fiber bending stress. The approximate stiffness modulus estimated using these nomographs conforms rather well with the experimental moduli obtained in this study.

Figure 5 also shows that the stiffness modulus is a function of stress level for the range of stresses employed; thus, the specimens do not exhibit true linear viscoelastic behavior in flexure under these particular test conditions.

The average specific gravity for the 314 specimens prepared for this study was 2.523 g/cc with a corresponding standard deviation of 0.016 g/cc. The average percent voids in the total mix was 4.53 percent, and the average percent voids in the aggregate filled with asphalt was 75.8 percent. Despite the fact that specimens exhibiting wide ranges in specific gravities were not intentionally prepared, it became evident during the testing program that a relationship appeared to exist between specimen specific gravity and initial stiffness modulus. Figure 6 shows the relationship for one level of flexural stress. Similar relationships were observed at other stress levels. It was concluded, despite the variability in the observed data, that more dense specimens tend to exhibit larger initial, deflection-based stiffness moduli.

SIMPLE-LOADING CONSIDERATIONS

A considerable number of the simple-loading tests were performed primarily to provide necessary data for the compound-loading phases of the effort. Additionally, however, a few simple-loading tests were designed to enable other independent investigations. A total of five test series was programmed for the simple-loading studies. Table 4 gives the number of specimens assigned to each of these test series and the range of values assigned to the major test variables. Test Series A provided the basic simple-loading data for a wide range of stress levels. Test Series B was used to investigate possible internal temperature changes associated with repetitive fatigue loading. The effects of rate of loading and load duration were studied using Test Series C and D, respectively. Finally, Test Series E employed strain measurements in an attempt to correlate measured strains and deflections. Continuous measurements of dynamic deflection under load were made for all of the specimens except those of Test Series B and some of Test Series C.

The following represent some of the most significant conclusions drawn from the simple-loading investigations.

1. The relationship between the mean logarithm of the fracture life and the logarithm of the applied flexural stress for these controlled-stress tests can be approximated by a linear function. Figure 7 shows the basic simple-loading test data and the corresponding least-squares line of best fit which were used, in part, to support this conclusion.

2. On the basis of the data of Figure 8 and other supporting considerations, the derived relationship between the mean logarithm of the fracture life and the logarithm of the initial strain level was found to be curvilinear for the controlled-stress mode of loading.

3. Because of the important influence of initial stiffness modulus on fatigue life, a plot of the fracture life against the initial strain level (Fig. 8) exhibits noticeably less variability than a corresponding plot of the fracture life against stress level (Fig. 7) for controlled-stress tests.

4. Results of previous investigations and the test data of this study show that one of the most significant variables in laboratory fatigue testing of asphalt-concrete is the mode of loading. Significant interrelationships exist among specimen stiffness, mode of loading, and observed fatigue behavior (23). In general it appears that load, environmental, and mixture and specimen variables (Table 1) tending to increase the stiffness moduli of asphalt specimens (at least for nonbrittle mixtures having a reasonable balance among the proportions of their constituent materials) tend also: (a) to increase the observed service lives for controlled-stress loading at any given stress level, but (b) to decrease the observed service lives for controlled-strain testing at any given strain level.

5. As anticipated, the standard deviation of fracture life decreased substantially as the stress level increased. At the same time, the coefficient of variation of fracture life was relatively unaffected by the stress level.

6. The stiffness modulus of asphalt-concrete specimens is considerably altered by damaging fatigue loading. The modulus of a specimen decreases rather drastically during the initial and final phases of a test and more gradually throughout the remaining portions. The change in modulus is a function of the severity of the damaging load condition.

7. As expected, the compressive stiffness modulus exceeded the tensile stiffness modulus of asphalt specimens subjected to dynamic flexural stresses, but by a small magnitude (less than 30%). The deflection-based stiffness modulus seems to be more satisfactory than the strain-based stiffness modulus as a working modulus for potential use in elastic analyses.

8. For moderate stress levels and rates of loading less than about 100 applications per minute, the internal temperature of the asphalt-concrete specimens did not increase measurably.

9. Increases in the rate of loading significantly decrease the fracture lives for the type of test employed and rates between 30 and 100 applications per minute. For two different sets of test conditions, the fracture lives at 100 applications per minute were approximately 22 percent of those at 30 applications per minute.

10. Load duration also considerably affected fatigue behavior, with a larger load duration resulting in a decreased fracture life for a constant rate of loading. In one instance, a change in duration from 0.1 to 0.18 sec resulted in a reduction of the mean fracture life from approximately 100,000 to 7,000 load applications.

11. Specimens having larger specific gravities tend also to have larger fracture lives for controlled-stress loading.

COMPOUND-LOADING CONSIDERATIONS

Normally, design of any structural element to preclude fatigue distress under in-service compound loading necessitates the performance of some type of laboratory fatigue tests. (The possibility of full-scale, in-service, fatigue tests before final design selection is excluded from this discussion.) Such tests may: (a) attempt to simulate the load histories anticipated in service or (b) provide data for use by a suitable compound-loading hypothesis. Practical difficulties often encountered in producing simulated load histories in the laboratory, the desirability of being able to predict the compound-loading fatigue behavior for a large range of load histories without requiring a correspondingly large number of laboratory compound-loading tests, and the ultimate desirability of quantification of the compound-loading fatigue behavior demonstrate the potential advantages of the second procedure. It is largely because of these factors that this study has focused on the possible application of compound-loading hypotheses.

Compound-Loading Hypotheses

A compound-loading hypothesis is simply a mathematical technique by which data obtained from relatively simple laboratory fatigue tests can be used to predict fatigue behavior under more complex load histories. The following represent certain inherently desirable features of compound-loading hypotheses that are useful as a set of criteria with which the most desirable hypothesis among possible alternatives can be selected.

1. As a working design tool, a compound-loading hypothesis should possess procedural simplicity and must be mathematically tractable.
2. A theoretical basis is desirable to serve as a firm foundation for the extension of basic principles.
3. Minimum data requirements, preferably of a simple-loading nature, facilitate the analysis.
4. A wide range of applicability to different types of compound loading including loading in which test variables other than load level are varied is desirable.
5. The utility of the predicted variables depends primarily on the nature of the design information required, although predictions of the stochastic distribution of the compound-loading service life are of maximum potential utility.

6. The accuracy with which the various predictions can be made is perhaps the most important feature of a compound-loading hypothesis.

Only recently have investigators considered the possible applicability of compound-loading hypotheses to the fatigue of asphalt materials, even though Bradbury (44) applied a classical hypothesis called the linear summation of cycle ratios concept to the analysis of the fatigue behavior of portland cement concrete pavements as early as 1938. One of the first considerations of the compound-loading fatigue behavior of asphalt mixtures was the suggestion that this same linear summation of cycle ratios hypothesis (also called Miner's hypothesis) might prove to be valid for asphalt-concrete (18). Shelley (20) conducted a limited number of repeated-block tests on asphalt-concrete specimens in the laboratory and, on the basis of the data obtained, tentatively concluded that the linear summation of cycle ratios hypothesis might well be valid for asphalt mixtures. Recent personal communications with Heukelom of the Shell Laboratory in Amsterdam and Pell of the University of Nottingham revealed that these researchers have likewise experimented with the application of compound-loading to asphalt-concrete test specimens.

These few attempts are indicative of the current state of knowledge relative to the compound-loading fatigue behavior of asphalt-concrete. At the same time, extensive experience has been accumulated in a related field, that of metallic materials. Numerous compound-loading hypotheses have been advanced in this field, and extensive laboratory verification of these hypotheses has been attempted. Most of them are intended to predict only some measure of central tendency of the compound-loading service life, commonly the mean or the median, or the service life corresponding to a q-percent survival. In general, little concern has been shown toward predicting the dispersion of the compound-loading service life, although there is at least one noteworthy effort in this regard (45). Likewise, there has been little effort to predict the stochastic distribution function of the compound-loading service lives, although assumptions have been made (46) that the distribution function of the compound-loading service life is identical in form to that of the simple-loading service life.

The linear summation of cycle ratios hypothesis is the most well known and possibly the most commonly accepted hypothesis used to predict the compound-loading fatigue behavior of metallic specimens. This hypothesis is said (47) to have been advanced originally by Palmgren in 1924 (48). In this country, however, it was first proposed by Langer (49) in 1937, although credit is most often given to Miner who advanced a similar proposal in 1945 (50).

Basically the hypothesis states that at failure the linear summation of cycle ratios of the individual load conditions equals one or

$$\sum_i (n_i/N_s^i) = 1 \qquad (2)$$

where

N_s^i = service life which would have been observed under simple loading of load condition i, and

n_i = number of applications of load condition i.

The compound-loading service life, N_s, which is derived from Eq. 2 is

$$N_s = 1/\sum_i (p_i/N_s^i) \qquad (3)$$

where

p_i = applied percentage of load condition i.

Eq. 3 has been used for predictions of various types of mean service life, and the service life corresponding to q-percent survival. The linear summation of cycle ratios hypothesis is not readily adaptable to predicting either the stochastic distribution of the service life or any measure of the dispersion of these lives.

TABLE 5

NUMBER OF TEST SPECIMENS ASSIGNED TO COMPOUND-LOADING TEST SERIES[a]

Test Series	Load History	Applied Percentage of Stress Level[b]			No. of Specimens[c]
		128.5 psi	113.5 psi	98.5 psi	
F	Two-level, increasing-sequence	Variable		Variable	24
	Two-level, decreasing-sequence	Variable		Variable	25
G	Two-level, repeated-block	5	95		4
		10	90		4
		25	75		4
		50	50		4
		5		95	4
		10		90	4
		25		75	4
		50		50	4
			5	95	4
			10	90	4
			25	75	4
			50	50	4
H	Three-level, repeated-block	10	30	60	6
		25	50	25	6
		60	30	10	6
I	Three-level, random	10	30	60	6
		25	50	25	6
		60	30	10	6

[a]These tests utilized a load duration of 0.1 sec and a rate of loading of 100 applications per minute and were performed at 75 F using a controlled-stress mode of loading.
[b]The tabulated values for the random load history refer to the probability of application expressed in percentage form.
[c]The total number of specimens subjected to compound loading was 133.

In addition to the linear summation of cycle ratios hypothesis, hypotheses of Shanley (51), Liu and Corten (52), Fuller (53), and others were employed in the formulation of techniques evaluated in this paper. Inasmuch as sample sizes of this study were limited (Tables 4 and 5), no effort was made to evaluate the fracture-life distribution, and only limited consideration of the dispersion of the fracture lives, as reflected by their standard deviations, was thought to be warranted. Therefore, the primary emphasis of this investigation of various compound-loading hypotheses is placed on those hypotheses that predict a mean, compound-loading fracture life. Table 6, which summarizes the hypotheses evaluated in this study, includes techniques for predicting root-mean-square, arithmetic-mean, geometric-mean, and harmonic-mean fracture lives.

With respect to the summary of Table 6, the Y's are predicted means for the compound-loading fracture lives with the subscripts referring to the type of mean predicted. The Z^i's are measured simple-loading means for load condition i with the subscript 1 denoting average squared fracture life, 2 denoting average fracture life, 3 denoting average logarithm of fracture life, and 4 denoting average reciprocal of fracture life. The S's represent various applied stress levels and the p_i's represent the applied percentages of load condition i for the repeated-block tests or the preset probabilities of application for the random tests. Other variables are adequately described by Deacon (23).

TABLE 6

SUMMARY OF COMPOUND-LOADING TECHNIQUES FOR PREDICTING MEAN FRACTURE LIFE

Technique No.	Type of Mean Predicted	Equation	Equation No.
1	Y_1, root mean square	$Y_1 = \left[1/\sum_i p_i/Z_1^i\right]^{1/2}$	4
2	Y_2, arithmetic mean	$Y_2 = \left[1/\sum_i p_i/(Z_2^i)^2\right]^{1/2}$	5
3	Y_2, arithmetic mean	$Y_2 - 1/\sum_i (p_i/Z_2^i)$	6
4	Y_2, arithmetic mean	$Y_2 = Z_2^k/\sum_i p_i\ (S^k/S^i)^b$	7
5	Y_2, arithmetic mean	$Y_2 = Z_2^1/\sum_i p_i\ (S^i/S^1)^r$	8
6	Y_2, arithmetic mean	$Y_2 = (Z_2^{max})^B/(Z_2^{min})^{B-1}$	9
7	Y_2, arithmetic mean	$Y_2 = Z_2^k \left[\pi_i (S^i/S^k)^{bp_i}\right]$	10
8	Y_3, geometric mean	$Y_3 = \text{antilog}\left[Z_3^k + \sum_i p_i \log (S^i/S^k)^{b'}\right]$	11
9	Y_4, harmonic mean	$Y_4 = \left[\sum_i p_i\ Z_4^i\right]^{-1}$	12

The standard deviation of the compound-loading fracture life can also be predicted using appropriately modified forms of three of the techniques presented, i. e., techniques 4, 5, and 7 of Table 6. The equations for predicting the standard deviations by these techniques are, respectively:

$$s\left[N_f\right] = s\left[N_f^k\right] \left| 1/\sum_i p_i\ (S^k/S^i)^b \right| \tag{13}$$

$$s\left[N_f\right] = s\left[N_f^1\right] \left| 1/\sum_i p_i\ (S^i/S^1)^r \right| \tag{14}$$

and

$$s\left[N_f\right] = s\left[N_f^k\right] \left| \pi_i (S^i/S^k)^{bp_i} \right| \tag{15}$$

where

$s\left[\ \right]$ = the standard deviation,

S^k = any standard stress level,

S^1 = maximum applied stress level, and

N_f^1 = simple-loading fracture life corresponding to S^1.

These equations are theoretically applicable only for repeated-block loading (small block size) in which the applied percentages of the various stress levels are constant.

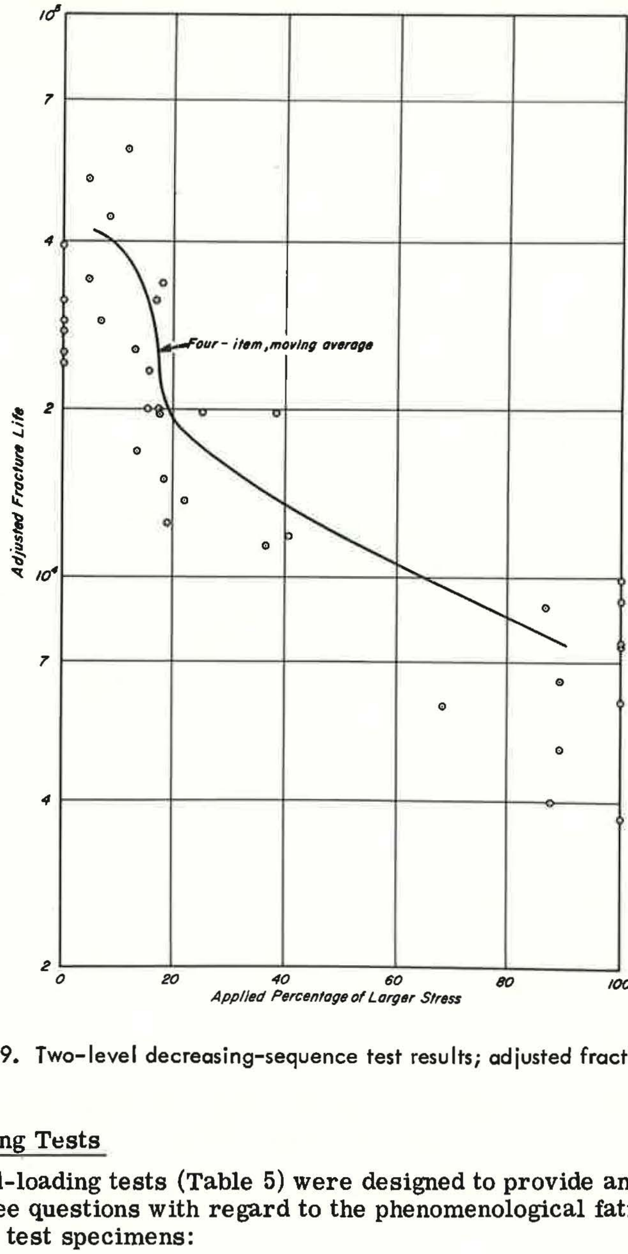


Figure 9. Two-level decreasing-sequence test results; adjusted fracture lives.

Compound-Loading Tests

The compound-loading tests (Table 5) were designed to provide answers to each of the following three questions with regard to the phenomenological fatigue behavior of asphalt-concrete test specimens:

1. Is the fatigue behavior affected by the order or sequence of load application?
2. Does the fatigue behavior observed under a random load history differ significantly from that observed under a repeated-block load history providing that the block size for the latter load history is kept small?
3. Which of the nine techniques of Table 6 is most appropriate for predicting a mean compound-loading fracture life and for which of the three load histories is it applicable?

The following test conditions were selected for all of the compound-loading tests: temperature, 75 F; rate of loading, 100 applications per minute; load duration, 0.10 sec; and mode of loading, controlled-stress. Continuous measurements of dynamic deflection under load were made for all of the specimens subjected to compound loading.

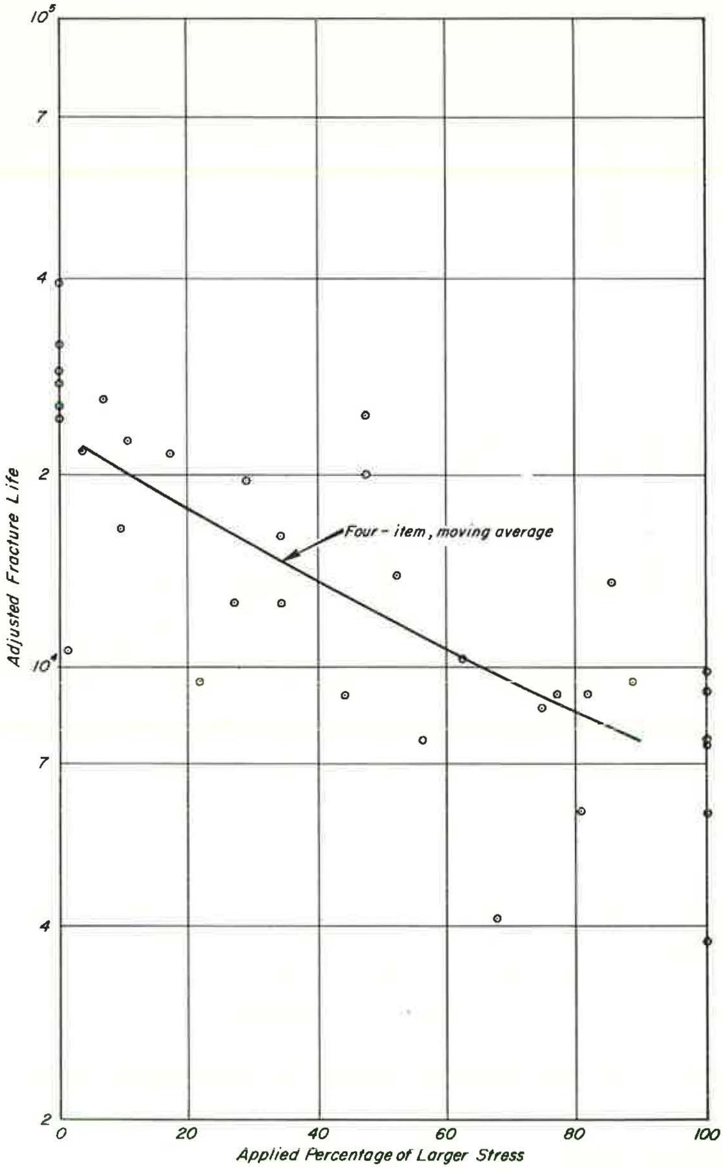


Figure 10. Two-level increasing-sequence test results; adjusted fracture lives.

The first question can be answered by examining the data of the two-level sequence tests. These tests were of two types: (a) increasing-sequence tests in which applications of the 98. 5-psi stress preceded those of the more destructive 128. 5-psi stress, and (b) decreasing-sequence tests in which the order of application of the same two stress levels was reversed. The results of these tests are shown in Figures 9 and 10, in which there are also curves representing four-item, moving-average fracture lives. These two average curves are compared in Figure 11.

These data indicate that the load sequence does affect the observed fatigue behavior at least for small applied percentages of the larger stress level. For such conditions, the mean fracture lives for two-level decreasing-sequence tests exceed those for two-level increasing-sequence tests. More extensive data are required to discover with any

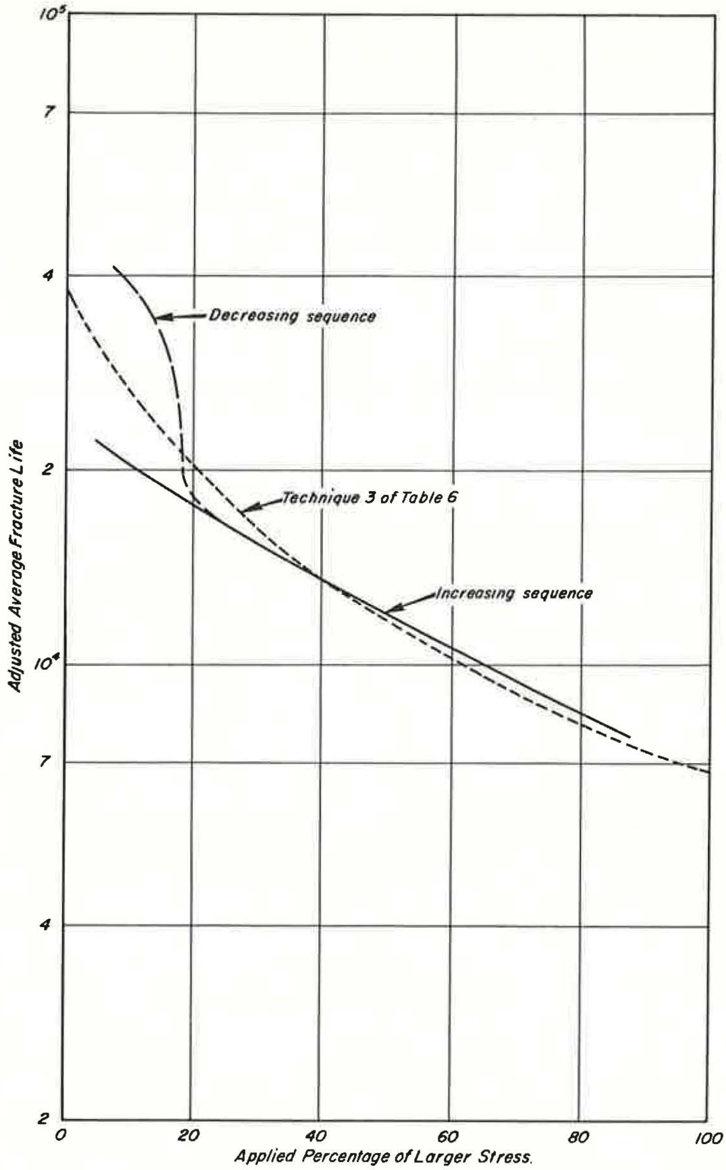


Figure 11. Comparison of average fracture lives (four-item moving average) for two-level sequence tests.

degree of certainty if comparable effects exist for larger applied percentages of the more destructive load condition. None of the nine compound-loading techniques in Table 6 can be used to predict a mean fracture life for sequence tests for all possible applied percentages, because none is capable of treating the sequence of loading.

This failure of all of the compound-loading hypotheses to apply to sequence loading of the type employed in this study is not of much practical concern, because this form of loading is not normally considered to be encountered in service. There is, however, some sequence effect in service due, for example, to seasonal climatic variations, that deserves additional evaluation in future investigations.

Test series H and I (Table 5) provided data to investigate the second question, i. e., whether fatigue behavior observed under random loading differs significantly from that observed under repeated-block loading (small block size). The mean fracture lives and

TABLE 7

COMPARISON OF MEAN FRACTURE LIVES—RANDOM VS REPEATED-BLOCK LOADING

Test[a]	Random		Repeated-Block	
	Mean Fracture Life	Mean Initial Stiffness Modulus (1, 000 psi)	Mean Fracture Life	Mean Initial Stiffness Modulus (1, 000 psi)
A	26, 500	263	15, 800	245
B	13, 500	250	9, 600	241
C	8, 600	237	11, 200	258

[a]Approximate applied percentages:
Test A: 10 percent of 128.5 psi, 30 percent of 113.5 psi, 60 percent of 98.5 psi;
Test B: 25 percent of 128.5 psi, 50 percent of 113.5 psi, 25 percent of 98.5 psi; and
Test C: 60 percent of 128.5 psi, 30 percent of 113.5 psi, 10 percent of 98.5 psi.

TABLE 8

COMPARISON OF VARIABILITY OF FRACTURE LIFE—RANDOM VS REPEATED-BLOCK LOADING

Test[a]	Random		Repeated-Block	
	Std. Dev. of Fracture Life	Coeff. of Variation (%)	Std. Dev. of Fracture Life	Coeff. of Variation (%)
A	17, 600	66	10, 700	68
B	8, 300	62	4, 100	43
C	6, 200	72	5, 800	52

[a]Approximate applied percentages:
Test A: 10 percent of 128.5 psi, 30 percent of 113.5 psi, 60 percent of 98.5 psi;
Test B: 25 percent of 128.5 psi, 50 percent of 113.5 psi, 25 percent of 98.5 psi; and
Test C: 60 percent of 128.5 psi, 30 percent of 113.5 psi, 10 percent of 98.5 psi.

TABLE 9

DATA FROM RANDOM TESTS[a]

Fracture Life	Pre-set Probability of Application (%)			Actual Applied Percentage of Stress Level		
	128. 5 psi	113. 5 psi	98. 5 psi	128. 5 psi	113. 5 psi	98. 5 psi
51, 592	10	30	60	10. 0	30. 1	59. 9
45, 449	10	30	60	10. 4	29. 6	60. 0
16, 491	10	30	60	10. 1	29. 6	60. 3
22, 200	10	30	60	10. 3	28. 8	60. 9
14, 432	10	30	60	—	—	—
9, 074	10	30	60	—	—	—
27, 348	25	50	25	24. 1	51. 6	24. 3
12, 313	25	50	25	24. 8	50. 9	24. 3
17, 658	25	50	25	24. 5	49. 4	26. 1
3, 684	25	50	25	24. 3	52. 6	23. 0
7, 937	25	50	25	25. 2	51. 2	23. 6
11, 897	25	50	25	24. 2	49. 4	26. 4
20, 829	60	30	10	60. 2	29. 6	10. 2
3, 032	60	30	10	59. 6	31. 3	9. 1
6, 700	60	30	10	60. 5	29. 9	9. 6
7, 423	60	30	10	60. 2	29. 8	10. 0
6, 139	60	30	10	59. 5	30. 6	9. 9
7, 691	60	30	10	60. 5	30. 1	9. 4

[a]Eighteen specimens tested.

the variabilities of fracture life for these two types of load histories are compared in Tables 7 and 8, respectively. For valid comparisons, the actual applied percentages of the three stress levels for the two types of histories should be nearly identical. Table 9 shows the achieved agreement between the actual applied percentages for the random tests and the pre-set probabilities of application. This agreement is adequate to enable valid comparisons.

A comparison of the mean fracture lives for the two load histories can be obtained from Table 7. Although differences in the mean lives are readily apparent, they are inconsistent and may be explained in terms of specimen variability and the influence of the initial stiffness modulus. To show the effect of variability, consider the mean fracture lives of the repeated-block series for tests B and C. Test C is the more destructive of the two tests and should, therefore, cause a smaller mean fracture life. The experimentally observed mean life for test C was larger, however, than that for test B, a fact that can be explained only on the basis of the small sample size and the large variability inherent in the fatigue data. The effect of initial stiffness modulus is shown by the fact that the larger mean fracture life in every case corresponds to those specimens exhibiting the larger mean initial stiffness modulus. This observation is in agreement with the contention that a larger fracture life generally corresponds with a larger initial stiffness modulus. It supports the premise that the observed mean fracture lives would have been identical if the mean initial stiffness moduli had likewise been the same.

On the basis of this rationale, it is concluded that the mean fracture lives for the random load history are identical to those for the repeated- block load history if the block size of the repeated-block history is sufficiently small and if the pre-set probabilities of application for the random history equal the applied percentages (expressed in decimal form) for the repeated-block history.

One of the primary sources of variability in observed fracture life for both the random and repeated- block loading is obviously that of variability among specimens. The random tests, however, incorporate an additional source of variability, that of the variability in the applied percentages of the load conditions caused by the stochastic nature of the loading. It may be reasoned, on this basis, that the variability of fracture life (as evidenced by both the standard deviations and the coefficients of variation) should be larger for the random loading than for the repeated-block loading.

This was exactly what occurred (Table 8) with one exception. For test A, the coefficient of variation is slightly larger for the repeated-block loading than for the random loading. This inconsistency is attributed to two factors. The first is the rather small sample size, which decreases the accuracy of the estimates. The second is the fact that, for larger fracture lives, the actual applied percentages for the random loading should converge on the pre-set probabilities of application, thereby reducing that portion of variability attributed solely to the random loading.

It is concluded, therefore, that the variability of the fracture life is larger for random loading than for comparable repeated-block loading. The relative difference in variability should decrease, however, as the fracture life increases.

The final question for consideration is the possible applicability of one or more of the nine techniques of Table 6 to predictions of a mean compound- loading fracture life. It has previously been concluded that none of these techniques is applicable without reservation to sequence tests; therefore, only the repeated- block and random test data warrant further analysis. The nine techniques were compared first with regard to the criterion of predictive accuracy. Primarily because of the different sample sizes for the two-level and the three-level tests, it was convenient to separate the test data into two groups depending on the number of stress levels employed.

To evaluate the predictive accuracy criterion in a quantitative manner the following procedure was used. First the deviation of the predicted mean fracture life from the measured mean fracture life was calculated for each technique (Table 6) and each set of test conditions (Test Series G, H, and I of Table 5). These deviations were expressed as percentages of the measured means according to the following equation:

$$\text{Dev} = \frac{(\text{Predicted Y - Measured Y})\ 100}{\text{Measured Y}} \qquad (16)$$

where

Dev = deviation of predicted mean life from measured mean life expressed as a percentage of measured mean life.

Both the deviations and the squared deviations were next summed for all of the test conditions for each technique. The best technique on the basis of predictive accuracy was taken to be that with the smallest average squared deviation. If two techniques yielded essentially the same average squared deviations, then that technique producing the more conservative estimates (as evidenced by a smaller average deviation) was taken to be the superior of the two.

The data from the two-level, repeated-block tests used in the evaluation of the arithmetic-mean fracture lives are shown in Figures 12 through 15. The broken lines

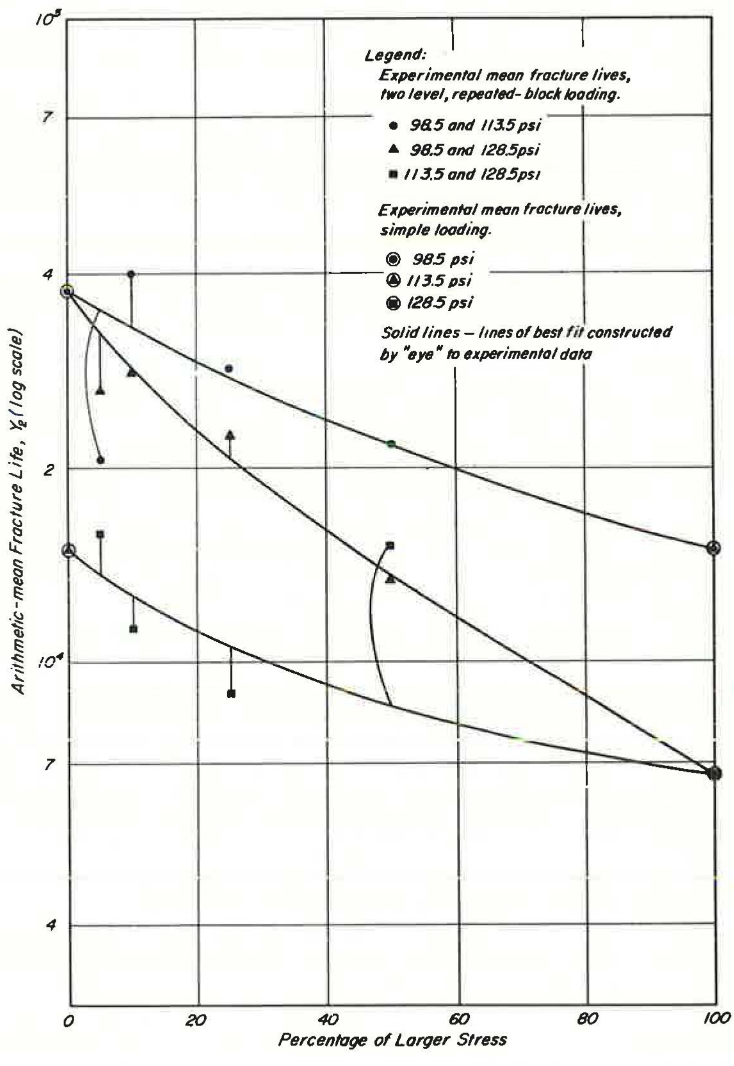


Figure 12. Experimental arithmetic-mean fracture lives, Y_2; two-level repeated-block loading.

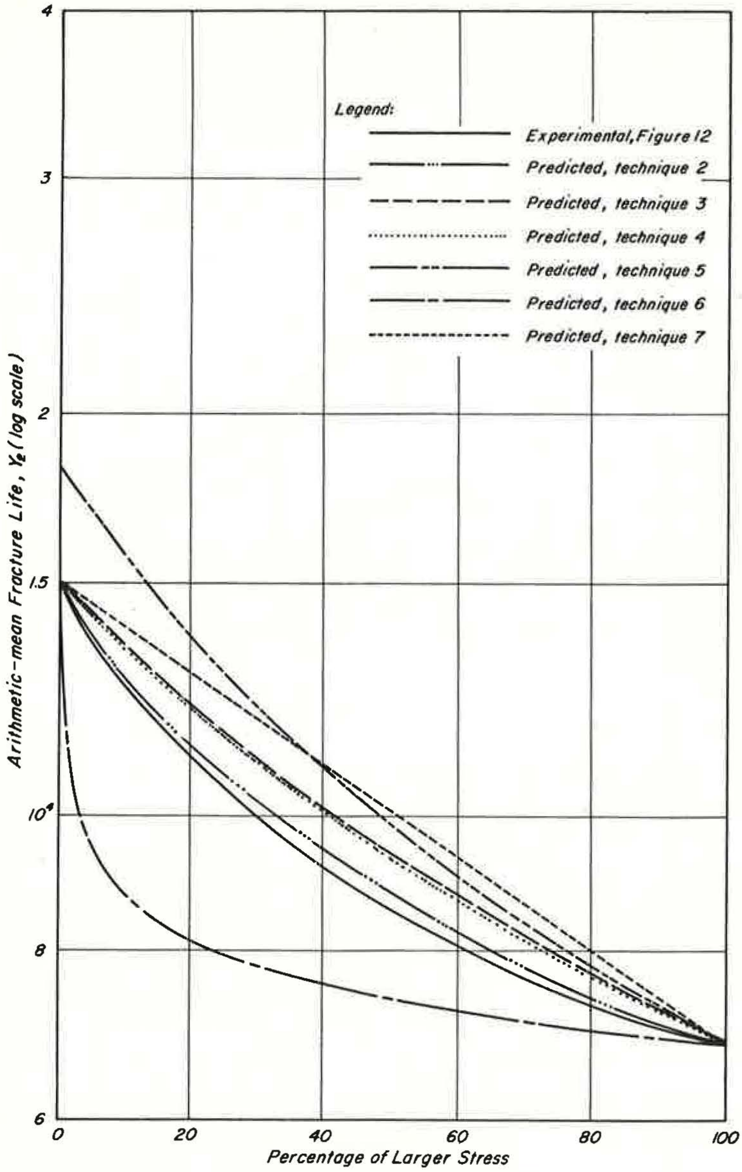


Figure 13. Experimental and predicted arithmetic-mean fracture lives, Y_2; two-level repeated-block loading with stress levels of 113.5 and 128.5 psi.

show the predicted means based on the various techniques in Table 6. The solid lines were constructed visually through the experimental means. These figures are helpful in assessing the applicability of the various techniques to these test data, but are not used in the quantitative comparisons that follow.

The quantitative comparison for the two-level repeated-block tests is summarized in Table 10. The average deviations and the average squared deviations were calculated from a total of 12 test observations, each observation consisting of the mean data for four test specimens.

It has been concluded that the mean fracture lives for the three-level repeated-block loading did not differ from those for the three-level random loading; the results of

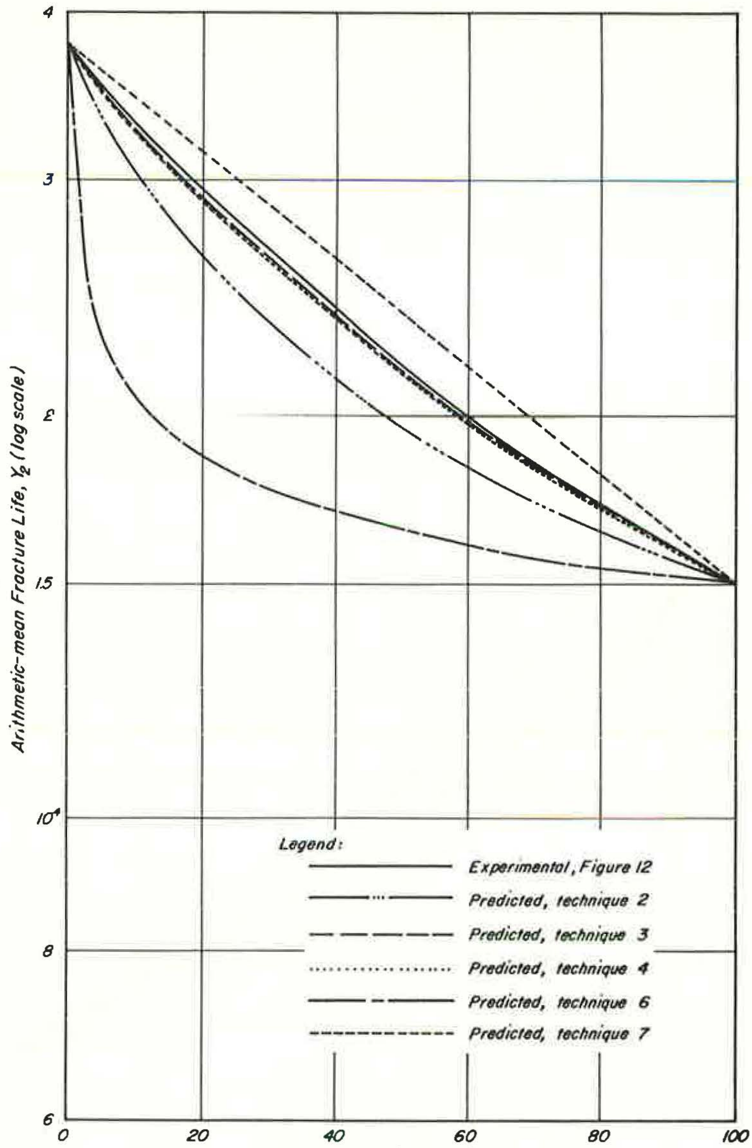


Figure 14. Experimental and predicted arithmetic-mean fracture lives, Y_2; two-level repeated-block loading with stress levels of 98.5 and 113.5 psi.

these two types of tests were combined to obtain better estimates of the mean fracture lives. The quantitative predictive-accuracy comparison for the three-level tests is summarized in Table 11.

A comparison of Tables 10 and 11 reveals that the nine techniques are not ranked in the same order for the two-level and three-level tests. To select a final ranking on the basis of all of the applicable compound-loading tests, the rank numbers of each technique from Tables 10 and 11 were once again ordered. Table 12 gives the results of this final predictive-accuracy comparison. Techniques having identical sums of the rank numbers were ordered according to the sequence obtained from the three-level tests.

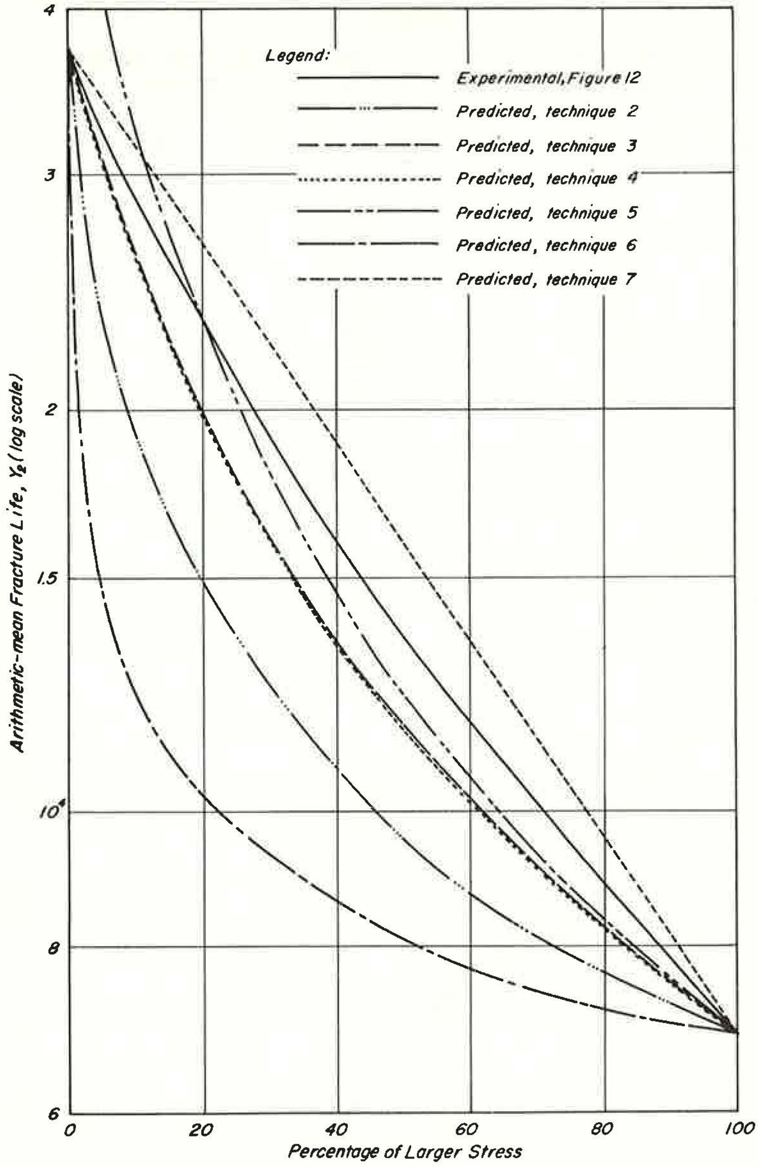


Figure 15. Experimental and predicted arithmetic-mean fracture lives, Y_2; two-level repeated-block loading with stress levels of 98.5 and 128.5 psi.

TABLE 10

PREDICTIVE-ACCURACY COMPARISON FOR MEAN FRACTURE LIFE (TWO-LEVEL TESTS)

Rank	Technique	Avg. Squared Dev. ($\%^2$)	Avg. Dev. (%)
1	4	780	0.9
2	3	831	1.6
3	7	916	13.1
4	9	947	16.0
5	2	962	-37.5
6	5	1,083	13.1
7	8	1,074	20.0
8	1	1,259	-12.0
9	6	1,646	-35.5

TABLE 11

PREDICTIVE-ACCURACY COMPARISON FOR MEAN FRACTURE LIFE (THREE-LEVEL TESTS)

Rank	Technique	Avg. Squared Dev. (%²)	Avg. Dev. (%)
1	4	101	- 1.2
2	3	98	- 0.8
3	2	214	-14.1
4	5	312	11.9
5	7	441	16.6
6	1	619	-22.7
7	6	1,380	-35.1
8	8	1,453	35.5
9	9	1,583	36.8

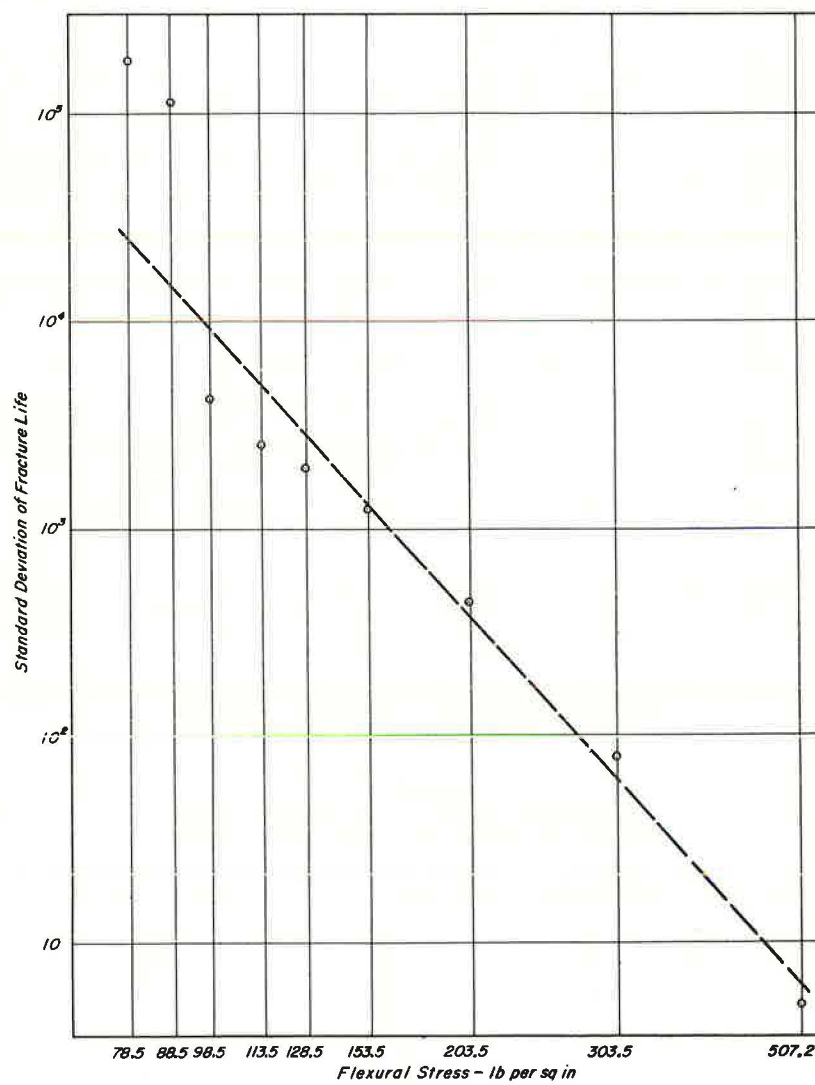


Figure 16. Standard deviation of fracture life as a function of stress level—simple loading.

The standard deviations of the compound-loading fracture life can be predicted using Eqs. 13, 14, and 15 which were derived from techniques 4, 5, and 7, respectively. Each of these equations specifies, at the boundary conditions, a linear relation between the logarithm of the standard deviation and the logarithm of the stress level for simple loading. Figure 16 shows that the simple-loading data tend to substantiate this linear relationship.

A comparison of the experimentally measured standard deviations and the standard deviations predicted by techniques 4, 5, and 7 for the three-level, repeated-block load history is made in Table 13. The data are not sufficiently extensive to select the most satisfactory among these three techniques. However, all appear to yield reasonably acceptable predictions considering the variability in the experimental standard deviations. The predictions from these equations are applicable only when the applied percentages of the various stress levels are constant.

Having ascertained the comparative acceptability of the various compound-loading hypotheses on the basis of predictive accuracy (Table 12), other comparative criteria must also be evaluated.

TABLE 12

PREDICTIVE-ACCURACY COMPARISON FOR MEAN FRACTURE LIFE (ALL TESTS)

Rank	Technique
1	4
2	3
3	2
4	7
5	5
6	9
7	1
8	8
9	6

TABLE 13

COMPARISON OF EXPERIMENTAL AND PREDICTED STANDARD DEVIATIONS OF FRACTURE LIFE—THREE-LEVEL, REPEATED-BLOCK LOADING

Item	Std. Dev. of Fracture Life[a]		
	A	B	C
Predicted Technique 4 (Eq. 13)	6, 400	4, 760	3, 620
Predicted Technique 5 (Eq. 14)	6, 070	4, 500	3, 440
Predicted Technique 7 (Eq. 15)	7, 030	5, 180	3, 870
Experimental	10, 700	4, 100	5, 800

[a]Applied percentages:
Test A: 10 percent of 128.5 psi, 30 percent of 113.5 psi, 60 percent of 98.5 psi;
Test B: 25 percent of 128.5 psi, 50 percent of 113.5 psi, 25 percent of 98.5 psi; and
Text C: 60 percent of 128.5 psi, 30 percent of 113.5 psi, 10 percent of 98.5 psi.

No distinction can be made among the nine techniques on the bases of procedural simplicity, theoretical basis, and range of applicability.

Technique 5 is the only technique requiring other than simple-loading data for its application. In this regard it is less desirable than other compound-loading techniques unless it yields superior accuracy. Table 12 reveals that technique 5 does not excel in this regard.

All nine techniques can be employed to predict mean fracture lives. The desirability of predicting the various types of means (e. g., geometric means as compared with arithmetic means) cannot be ascertained here but depends rather on the use to which the various predictions are put. Only techniques 4, 5, and 7, however, can be used to predict the standard deviations of the compound-loading fracture lives. Hence these techniques are more desirable than the remaining six.

Technique 4 is superior on the basis of predictive accuracy, and as it can be used to predict the standard deviation of the compound-loading fracture life, it is selected as the most acceptable technique among those investigated, for predicting the compound-loading fatigue behavior of the asphalt-concrete test specimens. Technique 4 is simply a modification of the linear summation of cycle ratios hypothesis.

CONCLUSIONS

The following represent some of the most significant conclusions of this study.

1. Mode of loading has a profound influence on the observed fatigue behavior of asphalt-concrete specimens. For repetitive loading of a controlled-stress nature, specimens exhibiting the largest initial stiffness moduli tend to perform most satisfactorily as long as the mixture is nonbrittle and has a reasonable balance among the proportions of its constituent materials. The reverse appears to be true for controlled-strain loading.

2. Fatigue behavior is clearly a stochastic rather than a deterministic phenomenon. Therefore, techniques of analysis must be geared to an apt treatment of probability and statistical concepts. The importance of this observation to the design and analysis of future experiments can hardly be overemphasized.

3. One of the most desirable features of a compound-loading hypothesis is that of predicting the probability distribution of the compound-loading service lives. Other desirable features include (a) procedural simplicity, (b) a wide range of applicability to different types of compound loading, (c) minimum data requirements preferably of a simple-loading nature, (d) a theoretical basis, and (e) predictive accuracy. No hypothesis has been identified with respect to asphalt-concrete test specimens that possesses all of these desirable characteristics.

4. The mean fracture lives of specimens subjected to two-level decreasing-sequence tests exceed those for two-level increasing-sequence tests if the applied percentage of the larger stress level is small. The mean fracture lives, however, are approximately equal for a rather wide range in the applied percentage of the more destructive stress. Any compound-loading hypothesis applicable to sequence load histories must, therefore, be able to account for the order of application of the various load conditions.

5. The mean fracture lives for random and repeated-block (small block size) load histories are identical if the probabilities of application of the various stress levels for the random loading equal the corresponding applied percentages (expressed in decimal form) for the repeated-block loading. At the same time, the variability of fracture life for random tests exceeds that for comparable repeated-block tests. The relative difference in variability is thought to decrease as the fracture life increases.

6. The best technique found for predicting the arithmetic-mean fracture life for both random and repeated-block (small block size) loading is given by the following equation:

$$Y_2 = Z_2^k / \sum_i p_i \, (S^k/S^i)^b \qquad (17)$$

where

Y_2 = predicted, arithmetic-mean fracture life, compound loading,

Z_2^k = average fracture life at standard stress level, S^k, simple loading,
p_i = applied percentage of load condition i (for random loading, a sufficiently accurate approximation may be made by setting the applied percentages equal to probabilities of application),
S^k = any standard stress level, and
b = a constant taken to be the slope of the linear log $\overline{N}_f$ - log S relation for simple loading.

Although the applicability of this equation has been ascertained only for multilevel loading, future modified forms may prove useful for other types of compound loading as well. Eq. 17 is a modification of the linear summation of cycle ratios hypothesis.

7. This same technique has also been tentatively found to be applicable for predicting the standard deviation of the compound-loading fracture life for repeated-block loading having a small block size. Eq. 18 is employed in this regard.

$$s\left[N_f\right] = s\left[N_f^k\right] \left| 1/\sum_i p_i \; (S^k/S^i)^c \right| \tag{18}$$

where

$s\left[N_f\right]$ = predicted standard deviation of compound-loading fracture life,
$s\left[N_f^k\right]$ = measured standard deviation of the simple-loading fracture life at standard stress level, S^k,
c = a constant taken to be slope of linear log $s\left[N_f\right]$ - log S relation for simple loading.

ACKNOWLEDGMENTS

The authors wish to acknowledge the assistance of the staff of the Institute of Transportation and Traffic Engineering, University of California in the development of this research. George Dierking of the ITTE staff prepared the figures.

REFERENCES

1. Grumm, Fred J. Designing Foundation Courses for Highway Pavements and Surfaces. California Highways and Public Works, Vol. 20, No. 3, pp. 6-9, 20, March 1942.
2. Porter, O. J. Foundations for Flexible Pavements. HRB Proc., Vol. 22, pp. 100-143, 1942.
3. Hveem, F. N., and Carmany, R. M. The Factors Underlying the Rational Design of Pavements. HRB Proc., Vol. 28, pp. 101-136, 1948.
4. Nijboer, L. W., and Van der Poel, C. A Study of Vibration Phenomena in Asphaltic Road Constructions. Proc., Assoc. of Asphalt Paving Technologists, Vol. 22, pp. 197-237, 1953.
5. Hveem, F. N. Pavement Deflections and Fatigue Failures. HRB Bull. 114, pp. 43-87, 1955.
6. The WASHO Road Test, Part 2: Test Data, Analyses, and Findings. HRB Spec. Rept. 22, 1955.
7. Jimenez, R. A., and Gallaway, B. M. Preliminary Report of an Apparatus for the Testing of Asphaltic Concrete Diaphragms. Proc., Assoc. of Asphalt Paving Technologists, Vol. 31, pp. 477-506, 1962.
8. Thomas, T. W. The Testing of Asphalt Paving Specimens Upon a Flexible Spring Base. Proc., Assoc. of Asphalt Paving Technologists, Vol. 17, pp. 174-183, 1948.
9. Carey, W. C. Discussion appended to Ref. 26, pp. 207-208.
10. Chen, H. H., and Hennes, R. G. Dynamic Design of Bituminous Pavements. The Trend in Engineering, Vol. 2, No. 1, pp. 22-25, Jan. 1950.
11. Nijboer, L. W. Mechanical Properties of Asphalt Materials and Structural Design of Asphalt Roads. HRB Proc., Vol. 33, pp. 185-200, 1954.

12. Nijboer, L. W. Mechanical Properties of Bituminous Road Mixtures. Proc., Symposium on Vibration Testing of Roads and Runways. Koninklijke/Shell-Laboratorium, Amsterdam, April 20-24, 1959.
13. Gregg, L. E. Additional Observations on the Use of Rubber in Bituminous Paving Mixtures. Tech. Bull. No. 194, American Road Builders Assoc., pp. 19-30, 1953.
14. Gregg, L. E., and Alcoke, W. H. Investigations of Rubber Additives in Asphalt Paving Mixtures. Proc., Assoc. of Asphalt Paving Technologists, Vol. 23, pp. 28-63, 1954.
15. Monismith, C. L. Flexibility Characteristics of Asphaltic Paving Mixtures. Proc., Assoc. of Asphalt Paving Technologists, Vol. 27, pp. 74-106, 1958.
16. Monismith, C. L. Effect of Temperature on the Flexibility Characteristics of Asphaltic Paving Mixtures. Symp. on Road and Paving Materials, STP No. 277, ASTM, Philadelphia, pp. 89-108, 1960.
17. Monismith, C. L., and Secor, K. E. Thixotropic Characteristics of Asphaltic Paving Mixtures with Reference to Behavior in Repeated Loading. Proc., Assoc. of Asphalt Paving Technologists, Vol. 29, pp. 114-140, 1960.
18. Monismith, C. L., Secor, K. E., and Blackmer, E. W. Asphalt Mixture Behavior in Repeated Flexure. Proc., Assoc. of Asphalt Paving Technologists, Vol. 30, pp. 188-222, 1961.
19. Monismith, C. L. Symposium on Flexible Pavement Behavior as Related to Deflection, Part II—Significance of Pavement Deflections. Proc., Assoc. of Asphalt Paving Technologists, Vol. 31, pp. 231-260, 1962.
20. Shelley, E. A. Relationship Between Strain and Cycles to Failure in an Asphaltic Paving Mixture in Repeated Flexure. Master of Eng. rept., Univ. of California, Berkeley, June 1962.
21. Monismith, C. L. Asphaltic Mixture Behavior in Repeated Flexure. Rept. No. TE 63-2, Inst. of Eng. Res., Univ. of California, Berkeley, Nov. 1963.
22. Monismith, C. L. Asphalt Mixture Behavior in Repeated Flexure. Rept. No. TE 64-2, Inst. of Eng. Res., Univ. of California, Berkeley, Nov. 1964.
23. Deacon, John A. Fatigue of Asphalt Concrete. Graduate rept., The Institute of Transp. and Traffic Eng., Univ. of California, Berkeley, 1965.
24. Hicks, Russell Gary. Fatigue of Asphalt Concrete. Master of Eng. rept., Univ. of California, Berkeley, April 1965.
25. Chandrangsu, Sirilak. Effects of Rapid, Repeated Loads on Asphaltic Concrete Beams Resting on Elastic Supports. Master's thesis, Ohio State Univ., 1958; cited by H. S. Papazian and R. F. Baker (26).
26. Papazian, H. S., and Baker, R. F. Analysis of Fatigue Type Properties of Bituminous Concrete. Proc., Assoc. of Asphalt Paving Technologists, Vol. 28, pp. 179-210, 1959.
27. Saal, R. N. J., and Pell, P. S. Fatigue of Bituminous Road Mixes. Kolloid-Zeitschrift (Darmstadt), Vol. 171, pp. 61-71, 1960.
28. Pell, P. S., McCarthy, P. F., and Gardner, R. R. Fatigue of Bitumen and Bituminous Mixes. Internat. Journal of Mech. Sciences, Vol. 3, pp. 247-267, 1961.
29. Pell, P. S. Fatigue Characteristics of Bitumen and Bituminous Mixes. Proc., Internat. Conf. on the Structural Design of Asphalt Pavements, Braun-Brumfield, Inc., Ann Arbor, pp. 280, 282, 294-297, 310-323, 1963.
30. Pell, P. S. Fatigue of Bituminous Materials in Flexible Pavements. Proc., The Inst. of Civ. Eng., Vol. 31, pp. 283-312, July 1965.
31. Jimenez, R. A. An Apparatus for Laboratory Investigations of Asphaltic Concrete Under Repeated Flexural Deformations. Rept. to Texas Highway Dept., College Station, Texas Transportation Inst., 1962.
32. Jimenez, R. A., and Gallaway, B. M. Behavior of Asphaltic Concrete Diaphragms to Repetitive Loadings. Proc., Internat. Conf. on Structural Design of Asphalt Pavements, Braun-Brumfield, Inc., Ann Arbor, pp. 281-282, 295, 297, 339-344, 1963.

33. Hveem, F. N., Zube, Ernest, Bridges, Robert, and Forsyth, Raymond. The Effect of Resilience-Deflection Relationships on the Structural Design of Asphaltic Pavements. Proc., Internat. Conf. on Structural Design of Asphalt Pavements, Braun-Brumfield, Inc., Ann Arbor, pp. 649-666, 1963.
34. Jones, A. Tables of Stresses in Three-Layer Elastic Systems. HRB Bull. 342, pp. 176-214, 1962.
35. Peattie, K. R. Stress and Strain Factors for Three-Layer Elastic Systems. HRB Bull. 342, pp. 215-253, 1962.
36. Dorman, G. M. The Extension to Practice of a Fundamental Procedure for the Design of Flexible Pavements. Proc., Internat. Conf. on Structural Design of Asphalt Pavements, Braun-Brumfield, Inc., Ann Arbor, pp. 785-793, 1963.
37. Dorman, G. M., and Metcalf, C. T. Design Curves for Flexible Pavements Based on Layered System Theory. Highway Research Record No. 71, pp. 69-84, 1965.
38. ASTM. Revision of Section II, Manual on Fatigue Testing, STP No. 91, Philadelphia, 1959.
39. Weibull, W. Fatigue Testing and Analysis of Results. Pergamon Press, New York, 1961.
40. AASHO. Standard Specifications for Highway Materials and Methods of Sampling and Testing. Washington, 1961.
41. State of California, Department of Public Works, Division of Highways. Standard Specifications. Jan. 1960.
42. State of California, Department of Public Works, Division of Highways. Materials Manual, Testing and Control Procedures. 2nd ed., 1963.
43. Van der Poel, C. Road Asphalt. In Building Materials, Their Elasticity and Inelasticity, M. Reiner, Ed., Interscience Publishers, Inc., N. Y., 1954.
44. Bradbury, Royall D. Reinforced Concrete Pavements. Wire Reinforcement Institute, Washington, 1938.
45. Hayes, William S. Statistical Aspects of Cumulative Damage in Fatigue. Unpubl. Master's thesis in Eng., Univ. of California, Los Angeles, Jan. 1955.
46. Freudenthal, A. M. Fatigue of Structural Metals Under Random Loading. Symp. on Acoustical Fatigue, STP No. 284. ASTM, Philadelphia, pp. 26-44, 1961.
47. Heywood, R. B. Designing Against Fatigue. Chapman and Hall, Ltd., London, 1962.
48. Palmgren, A. Die Lebensdauer von Kugellagern. Zeitschrift Verein Deutscher Ingenieure, Vol. 68, No. 14, pp. 339-341, 1924; cited by R. B. Heywood (47).
49. Langer, B. F. Fatigue Failure From Stress Cycles of Varying Amplitude. Transactions, American Soc. of Mech. Eng., Vol. 59, pp. A160-A162, 1937.
50. Miner, Milton A. Cumulative Damage in Fatigue. Transactions, American Soc. of Mech. Eng. Vol. 66, pp. A159-A164, 1945.
51. Shanley, F. R. A Theory of Fatigue Based on Unbonding During Reversed Slip. P-350 and P-350 Supplement, Rand Corp., Santa Monica, Nov. 1952 and May 1953.
52. Liu, H. W., and Corten, H. T. Fatigue Damage During Complex Stress Histories, Technical Note D-256, National Aeronautics and Space Administration, Washington, Nov. 1959.
53. Fuller, J. R. Cumulative Fatigue Damage due to Variable-Cycle Loading. Noise Control, pp. 11-24, July-August 1961.