# Invariant Properties of a Sheet-Asphalt Mixture

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In this work properties of a sheet-asphalt mixture are obtained which are believed to have greater quantitative significance than those usually used to describe bituminous mixtures. The results are based on Newton's equations of motion under conditions of plane strain. As these equations are two in number but contain five unknowns, laboratory tests were conducted to obtain three additional independent expressions relating the unknowns. Only those parts of the relationships found reproducible and independent of time, temperature, and conditions of test were considered material properties. Four basic material constants were obtained as opposed to the more usual three constants: modulus of elasticity, Poisson's ratio, and coefficient of thermal expansion.

•NEWTON'S equations of motion for a two-dimensional system are

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{yz}}{\partial y} = \gamma + \rho \frac{\partial^2 w}{\partial t^2} \tag{1a}$$

$$\frac{\partial \tau_{yz}}{\partial z} + \frac{\partial \sigma_y}{\sigma_y} = \rho \frac{\partial^2 v}{\partial t^2} \tag{1b}$$

where $\sigma_z$ and $\sigma_y$ are normal stresses and w and v are displacements in the vertical (z) and horizontal (y) directions, respectively; $\tau_{yz}$ is the shear stress in the plane under consideration; $\gamma$ is the unit weight of the mixture, and $\rho$ is its mass unit weight ($\rho = \gamma/g$).

Eqs. 1 contain five unknowns: $\sigma_z$, $\sigma_y$, $\tau_{yz}$, w, v. Hence, three more independent expressions relating these are needed to render the system solvable. To achieve this balance, recourse was made to simple laboratory tests conducted under inputs varying with time and temperature in which pertinent relationships may be obtained from relevant observations.

Any parameters relating the unknowns that are found to be independent of the type of test or conditions of loading are material properties of the bituminous mixture studied. Underlying this concept is the requirement that for any material property to have quantitative significance it must, of necessity, reflect the action of the material in situ. Hence, true properties must remain unchanged (invariant) under transformations from laboratory to field conditions. For properties to be invariant under transformations from a simple laboratory test to very complicated field conditions, they must of necessity also remain constant under different laboratory tests. Using this condition of necessity, in combination with Eqs. 1, different types of simple laboratory tests with known boundary conditions were conducted, and the parameters remaining constant were identified as material properties.

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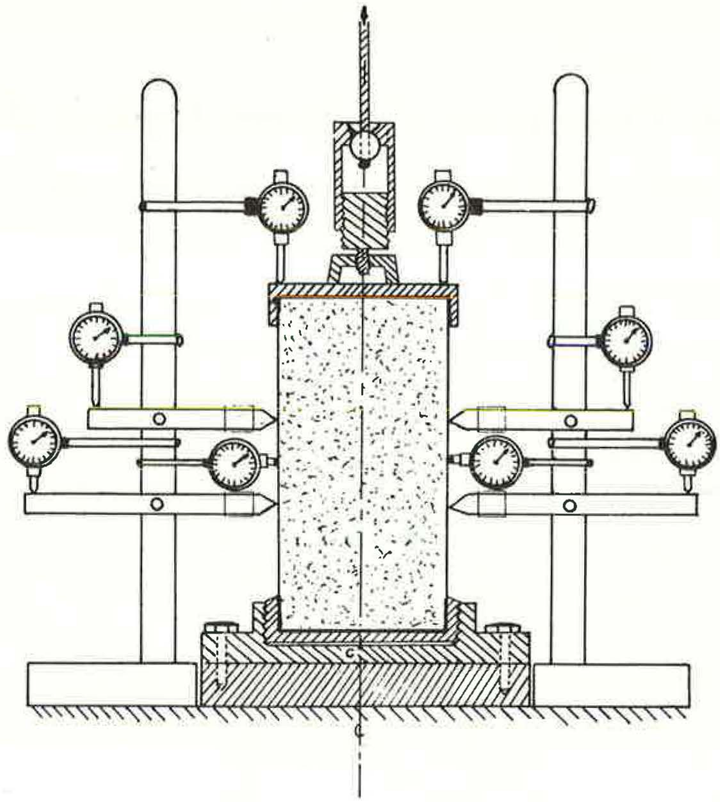

Figure 1. Instrumentation for uniaxial tension test.

## METHODS OF TESTING

To obtain the three equations required for solving the foregoing two-dimensional deformable system, relevant experimental data had to be obtained from different types of tests. For this purpose, uniaxial tension and simple shear tests were chosen. To verify the material constants as obtained from these tests, an axial compression test was utilized. The considerations for the choice of these tests and the experimental techniques employed therein are discussed in this paper.

### Uniaxial Tension Tests

To evaluate the material constants expected in the relationships between normal stress and strains along and at right angles to the direction of application of load, a uniaxial tension test was performed. In this test the shear stresses are zero in these directions and the material may be considered as subject to normal tensile stresses only.

A cylindrical specimen 2 in. in diameter and 4 in. high was subjected to constant tensile stress at constant temperature, and both elongation per inch and decrease in radius per inch with time were determined. To reduce end-effects, the measurements of strain were confined to the middle portion of the specimen. Also, to minimize errors due to any possible eccentricity during the application of load, the strains were measured at locations 180 deg apart in the middle portion of the specimen. In this way, by measuring the elongation of the middle 1-in. portion and the reduction in diameter at the middle, plots of axial strain vs time and lateral strain vs time, at constant tensile stress, were obtained. By repeating the foregoing procedures for different temperatures, the relationships between normal stress and the axial and lateral strains were obtained as functions of time and temperature.

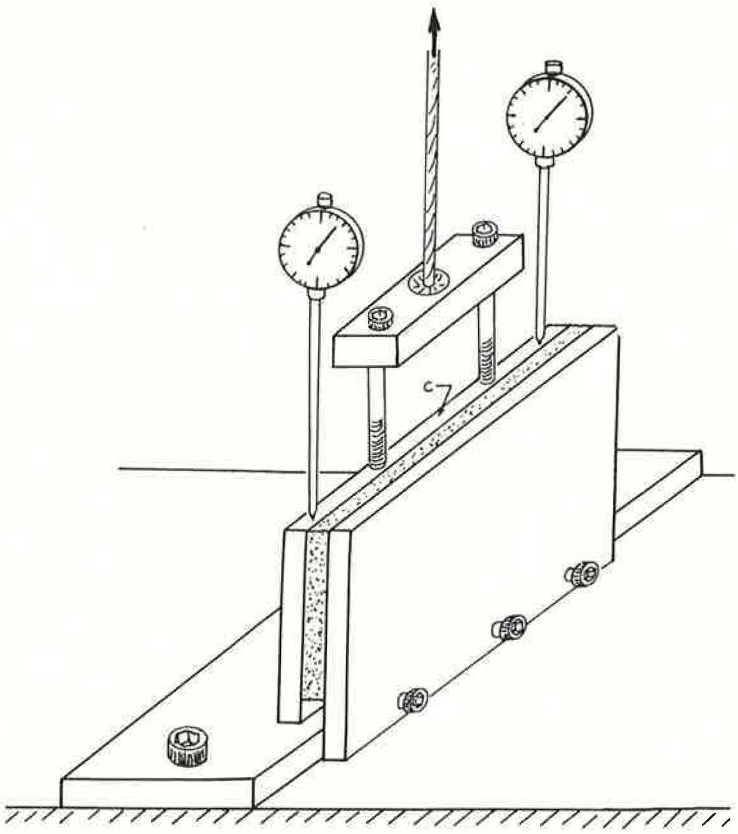


Figure 2. Instrumentation for simple shear test.

Figure 1 shows the instrumentation for this test. A simple mechanical device was developed which greatly enhanced the strain measuring technique. This consisted of levers with pointed ends which made contact with the specimen. Deformation of the specimen at the point of contact with the pointed end of the lever was recorded by a dial indicator attached to the lever end away from the specimen. Axial strain in the middle portion of the specimen was determined by the difference in deformations recorded by dial indicators attached to the levers 1 in. apart vertically. The readings of the dial indicators were estimated to 0.00005 in. Change in diameter at mid-height of specimen was recorded by dial indicators with their extensions, machined to fit the curved surface, resting directly on the specimen surface.

## Simple Shear Tests

The relationship between shear stress and shear strain was determined from a simple shear test in which the normal stresses may be taken to be zero in the considered planes.

A simple and direct means of determining shear strain as a function of time under constant stress and at constant temperature was achieved by forming a specimen of appropriate thickness, fixing one face and pulling the other parallel face under constant load (Fig. 2). In deciding on the thickness of specimens for these simple shear tests, the following points were considered:

1. Minimum amount of bending while the specimen is being subjected to simple shearing stress.
2. Non-interference of particles within the specimen.

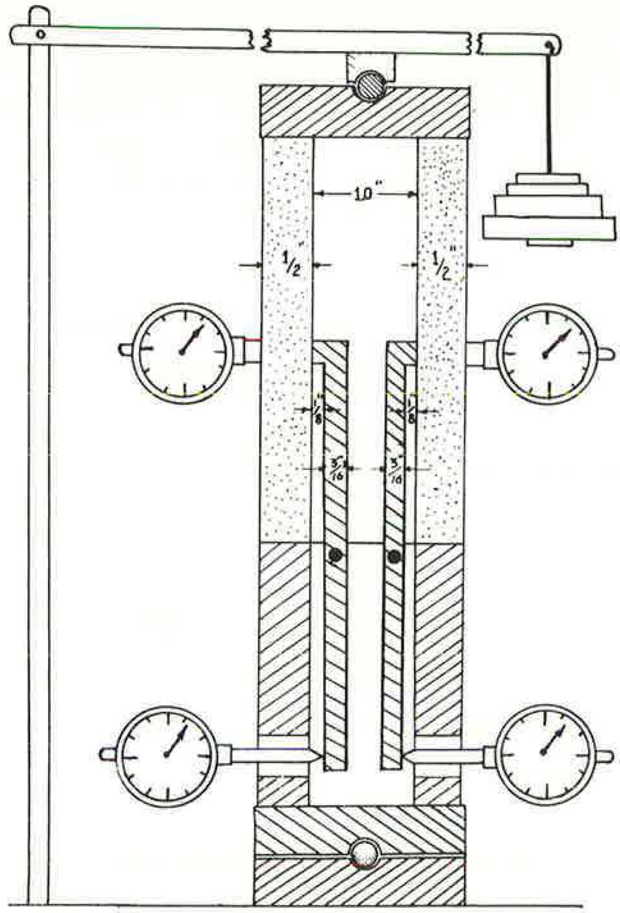


Figure 3. Instrumentation for change in thickness measurements in axial compression test.

3. Practicability of fabricating specimens with uniformity or homogeneity of compacted materials.
4. Ability to record the dilation of the specimen while undergoing shearing strain.

After trying several thicknesses with the foregoing points considered, a thickness of ¼ in. and a specimen size of 4 by 2 in. were chosen.

## AXIAL COMPRESSION TESTS

A hollow-cylinder compression test was performed to verify the material constants obtained from the uniaxial tension and simple shear tests. In this test, deformations of the material were observed in both the axial and the lateral direction by noting the deformations on the inside as well as the outside of the hollow cylinder. Again, the tests were performed under constant load and at constant temperature. The instrumentation developed for measuring axial deformations and change in external diameter in the uniaxial tension tests was applicable for these tests also. For measuring change in inside diameter, a modification of this lever system using dial indicators was used (Fig. 3).

The hollow cylindrical specimen, having a 2-in. external diameter, 1-in. internal diameter, and 4-in. height, was placed on a hollow steel cylinder fitted with two hinged levers. The upper parts of the hinges were machined to correspond to the inside curved

TABLE 1

AXIAL STRAIN-TIME RELATIONSHIPS FOR UNIAXIAL TENSION TESTS

| Applied Tensile Stress (psi) | Axial Strain at 1 Min (0.0001 in./in.) | Slope of Axial Strain vs Time Plot (log-log) |
|---|---|---|
| | (a) 40 F | |
| 18.43 | 2.55 | 1:1.95 |
| 30.43 | 4.40 | 1:1.90 |
| 40.43 | 6.60 | 1:1.85 |
| 52.43 | 9.60 | 1:1.80 |
| | (b) 77 F | |
| 1.70 | 11.5 | 1:2.80 |
| 2.43 | 15.0 | 1:2.50 |
| 4.43 | 28.5 | 1:2.00 |
| 5.43 | 39.0 | 1:1.80 |
| | (c) 100 F | |
| 0.75 | 7.2 | 1:4.40 |
| 1.07 | 18.5 | 1:3.70 |
| 1.43 | 26.0 | 1:3.20 |
| 2.43 | 60.0 | 1:2.40 |

TABLE 2

CIRCUMFERENTIAL STRAIN-TIME RELATIONSHIPS FOR UNIAXIAL TENSION TESTS

| Applied Tensile Stress (psi) | Circum-ferential Strain at 1 Min (0.0001 in./in.) | Slope of Circum-ferential Strain vs Time Plot (log-log) |
|---|---|---|
| | (a) 40 F | |
| 18.43 | 1.18 | 1:2.00 |
| 30.43 | 1.75 | 1:1.95 |
| 40.43 | 2.75 | 1:1.90 |
| 52.43 | 3.75 | 1:1.85 |
| | (b) 77 F | |
| 1.70 | 4.25 | 1:2.40 |
| 2.43 | 5.50 | 1:2.30 |
| 4.43 | 8.50 | 1:2.20 |
| 5.43 | 11.0 | 1:2.12 |
| | (c) 100 F | |
| 0.75 | 3.25 | 1:3.00 |
| 1.07 | 6.5 | 1:2.85 |
| 1.43 | 10.0 | 1:2.75 |
| 2.43 | 20.0 | 1:2.50 |

surface of the specimen, and the lower ends were contacted by extensions of the dial indicators. Changes in the diameter of the hole were recorded with time. Changes in outside diameter were recorded with time by dial indicators resting directly on the surface. Changes in unit thickness of specimen, or lateral strain, were thus calculated from the difference in changes of outside and inside diameters.

## SUMMARY OF TEST RESULTS

To obtain the experimental data required for this study, uniaxial tension, simple shear, and axial compression tests were performed. Complete test results for uni-axial tension and simple shear tests and data for tests at 77 F, including those for the axial compression tests, are presented.

Results of the uniaxial tension tests are given for axial and circumferential strain, respectively (Tables 1 and 2). Figure 4 shows that the axial strain of specimens under constant stress and constant temperature, when plotted as ordinate against time on log-log scales, gave a straight-line relationship in the pre-failure region. As failure started to take place with the appearance of minute cracks in the middle portion of specimen, the straight line on the log-log plot tended to curve upward. It was found convenient to characterize the straight-line portion of the plot by its slope and axial strain at one minute (it being a log scale). Within the range of temperatures and stress levels tested, the following points of interest were observed from the uniaxial tension test results.

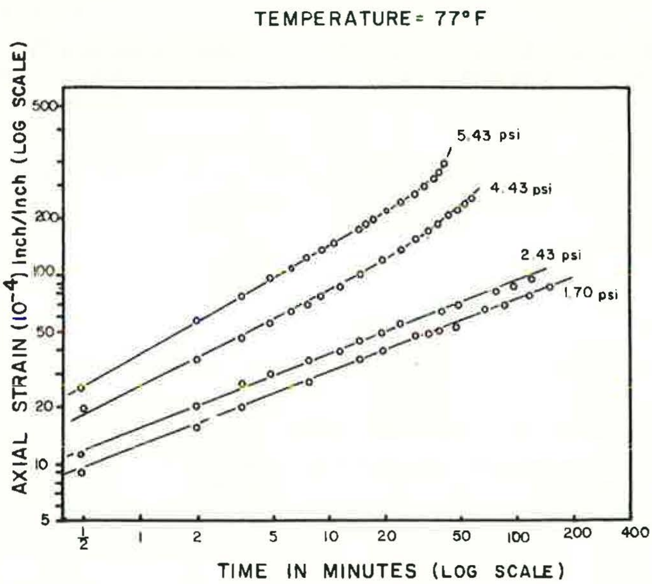


Figure 4. Uniaxial tension test: axial strain vs time curves.

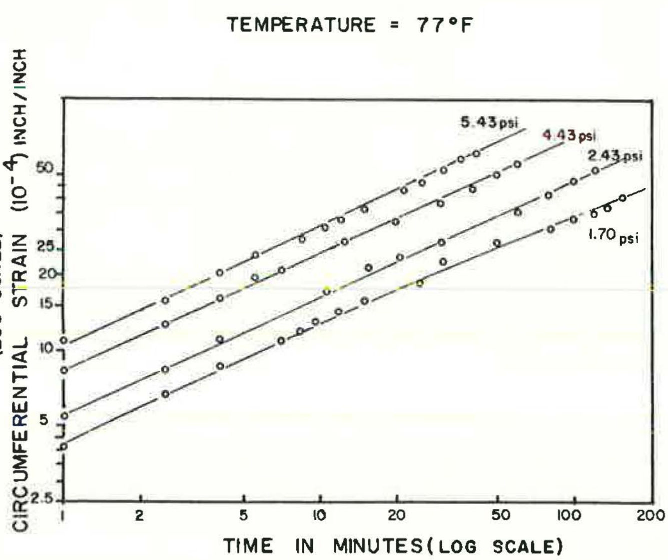


Figure 5. Uniaxial tension test: circumferential strain vs time curves.

At constant temperature, the axial strain at one minute did not vary proportionally with applied stress. The deviation from proportionality increased with increasing temperature as well as with increasing applied stress. The slopes of the straight-line portions of the log-log plots varied with the applied stress and temperature. The slopes became steeper with increase in applied stress at constant temperature. For an incremental change in stress, the corresponding change in slope was greater at higher temperatures.

TABLE 3

SHEAR STRAIN-TIME RELATIONSHIPS FOR SIMPLE SHEAR TESTS

| Applied Shear Stress (psi) | Shear Strain at 1 Min (0. 0001 in./in.) | Slope of Shear Strain vs Time Plot (log-log) |
|---|---|---|
| | (a) 40 F | |
| 5. 64 | 8. 0 | 1:2. 55 |
| 10. 05 | 13. 6 | 1:2. 50 |
| 18. 3 | 24. 4 | 1:2. 45 |
| 23. 82 | 32. 0 | 1:2. 40 |
| | (b) 77 F | |
| 1. 73 | 30 | 1:3. 00 |
| 3. 32 | 56. 0 | 1:2. 80 |
| 4. 83 | 80. 0 | 1:2. 60 |
| 6. 45 | 114. 0 | 1:2. 50 |
| 8. 02 | 146. 0 | 1:2. 30 |
| | (c) 100 F | |
| 0. 563 | 34. 0 | 1:4. 50 |
| 0. 950 | 56. 0 | 1:4. 00 |
| 1. 35 | 88. 0 | 1:3. 50 |
| 1. 732 | 110. 0 | 1:3. 38 |

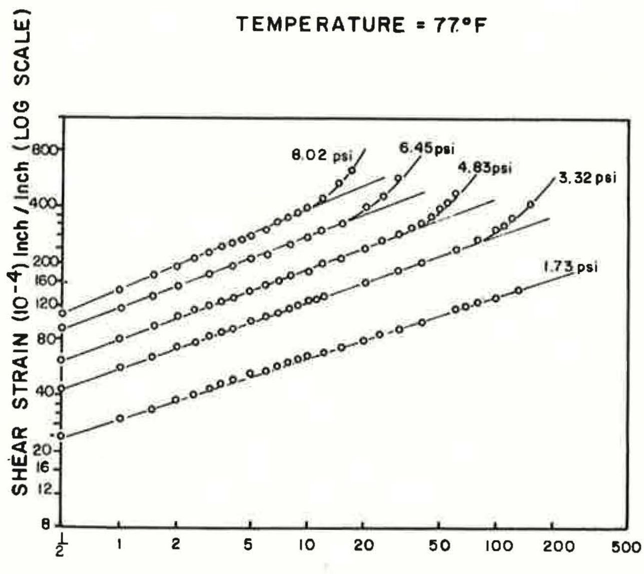


Figure 6. Simple shear test results: shear strain vs time curves.

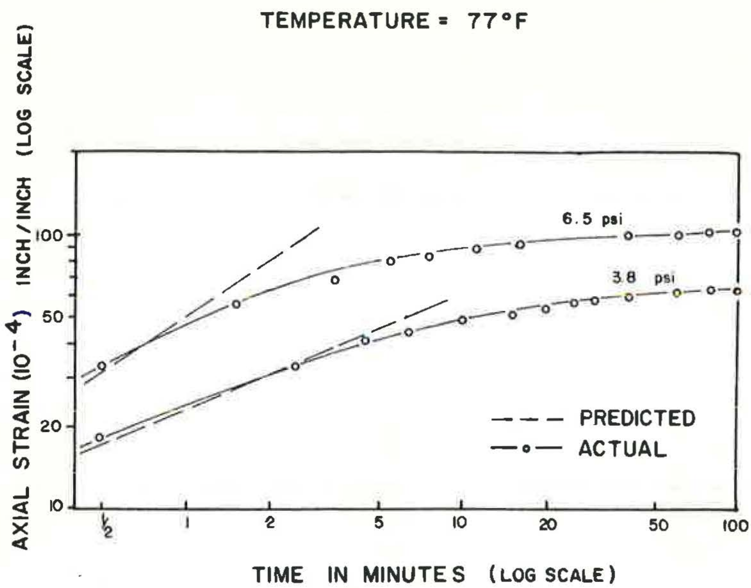


Figure 7. Axial compression test results: axial strain vs time curves.

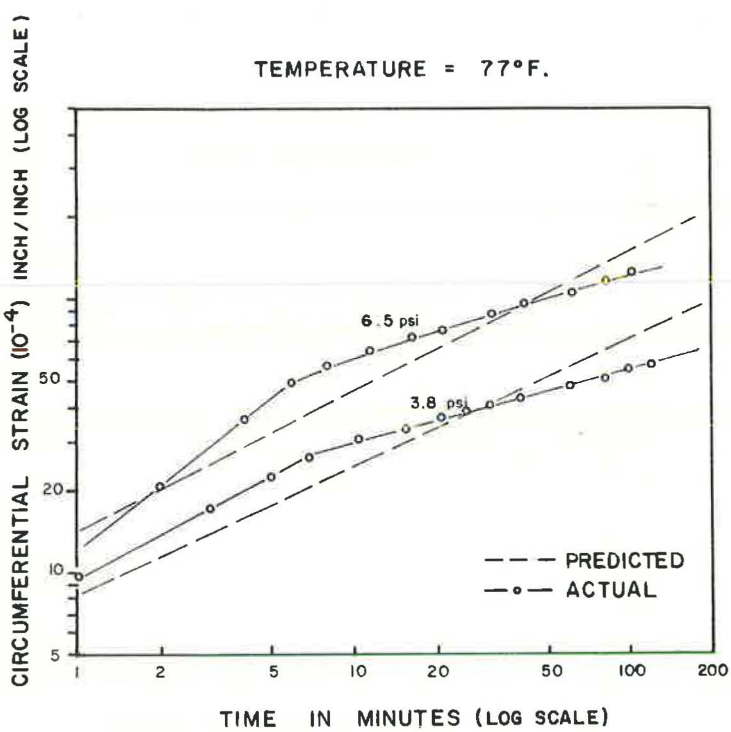


Figure 8. Axial compression test results: circumferential strain vs time curves.

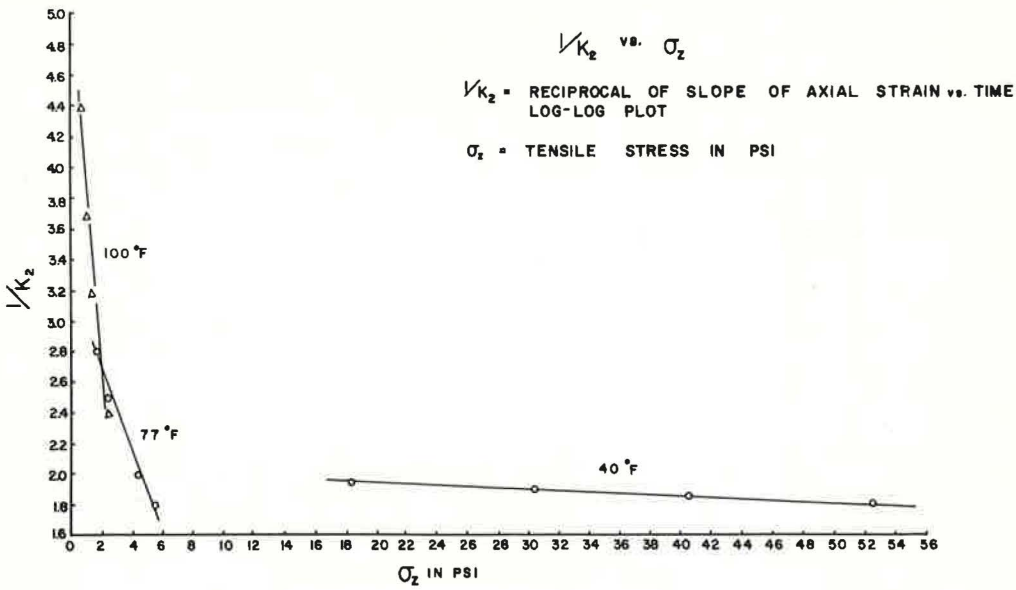


Figure 9. Uniaxial tension test results.

The circumferential strain when plotted as ordinate against time on log-log scales also gave a straight-line relationship (Fig. 5). The same trends as observed for axial strains were observed for circumferential strains.

A study of Poisson's ratio for the material as determined from the uniaxial tension test data showed it to be a function of applied stress, time, and temperature. Whereas at 40 F its value was almost independent of applied stress, as would be the case for an elastic material, at higher temperatures it decreased with increasing applied stress.

Results for the simple shear tests are given in Table 3. The shearing strain in the specimen under constant shear stress and constant temperature, when plotted as ordinate against time on log-log scales, gave a straight-line relationship in the pre-failure region (Fig. 6). The shearing strain-time relationships showed the same trends with regard to temperature and applied stress as the axial strain-time relationships determined by the uniaxial tension tests, indicating that the same basic material properties were reflected in the two types of tests.

For the axial compression tests, the log-log plots of axial strain vs time (Fig. 7) were not continuous straight lines for the entire range. The plots were straight lines up to a certain percentage of deformation, after which they curved downward to lesser slopes. Figure 8 shows that a similar result for circumferential strains was recorded in the axial compression tests. The initial straight-line portions of the axial compression test plots showed the same trends with regard to applied stress and temperature as in the axial strain-time plots from the uniaxial tension tests. The axial compression test results also showed that strains up to about 0. 4 percent can be quite satisfactorily predicted from the uniaxial tension tests.

## DERIVATION OF STRESS-STRAIN EXPRESSIONS

The derivations of stress-strain expressions from uniaxial tension and simple shear test results are based on the following observations: (a) the strain-time plots in the pre-failure regions on log-log scales were straight lines; and (b) the slopes of these straight lines varied with the applied stress and temperature.

Derivation of the relationship between normal stress $\sigma_z$ and axial strain $\epsilon_z$ as a function of time and temperature is as follows. Axial strain-time plots on log-log

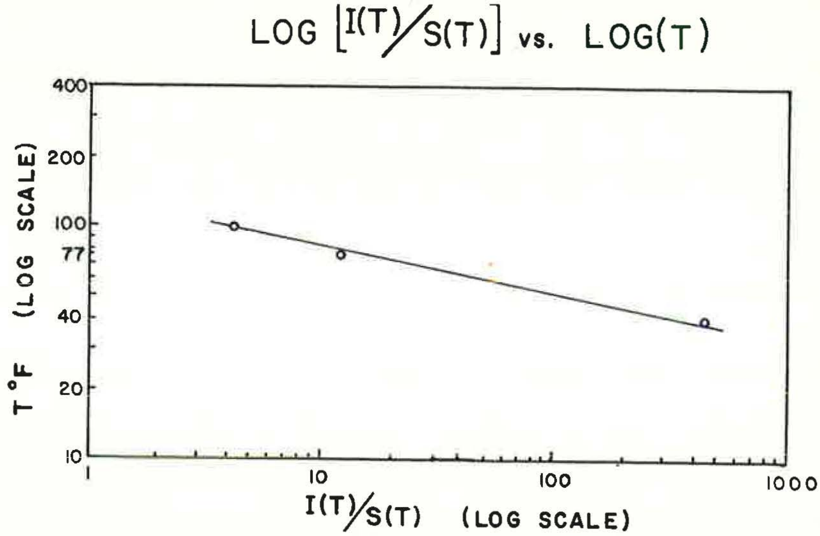


Figure 10. Uniaxial tension test results.

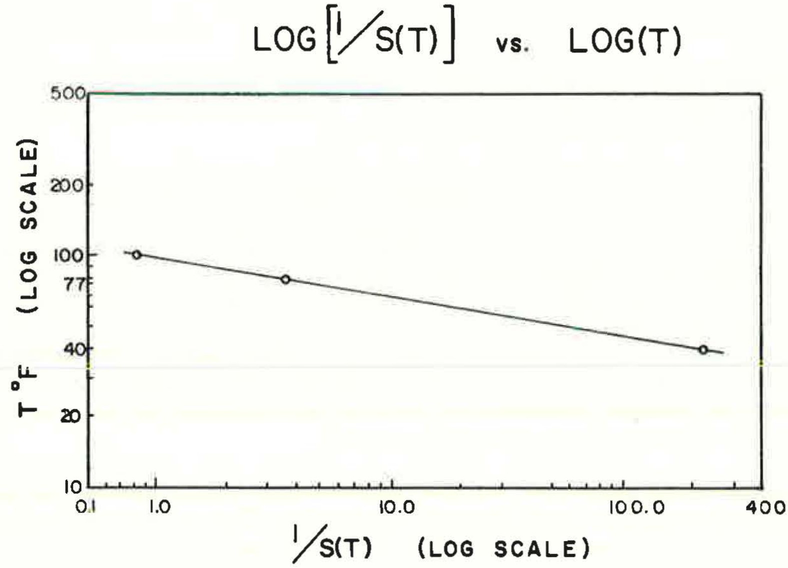


Figure 11. Uniaxial tension test results.

scales, being straight lines in pre-failure regions, can be represented as: $\log \epsilon_z = \log k_1 + k_2 \log t$ where t stands for time, $k_2$ is the slope of the straight line and $k_1$ is the axial strain at unit time. Differentiating with respect to t, gives

$$\frac{1}{\epsilon_z} \frac{\partial \epsilon_z}{\partial t} = \frac{k_2}{t} \tag{2}$$

The slopes $k_2$ of these straight lines varied linearly with stress in the test range (Fig. 9), and this consideration yields the relation

$$\sigma_z = \frac{I(T)}{S(T)} - \frac{1}{S(T)} \cdot \frac{1}{k_2} \tag{3}$$

where I(T) and S(T) are the intercepts on $1/k_2$ axis and slopes of the straight lines, respectively, as a function of temperature T.

Figure 10 shows on log-log scales a straight-line relationship between temperature T and the ratio I(T)/S(T), i. e., $I(T)/S(T) = \left[T/c_1\right]^{-c_2}$ where $c_1$ and $c_2$ are constants. Similarly, the log-log plot of Figure 11 shows $1/S(T) = \left[T/p_1\right]^{-p_2}$ where $p_1$ and $p_2$ are constants.

Substituting these values of I(T)/S(T) and 1/S(T) and also the value of $k_2$ from Eq. 2 in Eq. 3 gives

$$\sigma_z = \left[\frac{T}{c_1}\right]^{-c_2} - \left[\frac{T}{p_1}\right]^{-p_2} \frac{\epsilon_z}{t\,\frac{\partial \epsilon_z}{\partial t}}$$

where $c_1$, $c_2$, $p_1$, $p_2$ are material constants independent of time and temperature.

## DISCUSSION OF STRESS-STRAIN EXPRESSIONS

From uniaxial tension test results, the expression relating the normal tensile stress ($\sigma_z$) to axial strain ($\epsilon_z$) was found in the foregoing. An expression of the same form relating $\sigma_z$ to circumferential strain ($\epsilon_y$) was

$$\sigma_z = \left[\frac{T}{c_1'}\right]^{-c_2'} - \left[\frac{T}{p_1'}\right]^{-p_2'} \frac{\epsilon_y}{t\,\frac{\partial \epsilon_y}{\partial t}}$$

Assuming the material to be isotropic and homogeneous, the following equations can be written:

$$\sigma_y = \left[\frac{T}{c_1}\right]^{-c_2} - \left[\frac{T}{p_1}\right]^{-p_2} \frac{\epsilon_y}{t\,\frac{\partial \epsilon_y}{\partial t}} = \left[\frac{T}{c_1'}\right]^{-c_2'} - \left[\frac{T}{p_1'}\right]^{-p_2'} \frac{\epsilon_z}{t\,\frac{\partial \epsilon_z}{\partial t}}$$

From the simple shear test results, the expression relating the shear stress ($\tau_{yz}$) to shear strain ($\gamma_{yz}$), was of similar algebraic form, i. e.,

$$\tau_{yz} = \left[\frac{T}{c_1''}\right]^{-c_2''} - \left[\frac{T}{p_1''}\right]^{-p_2''} \frac{\gamma_{yz}}{t\,\frac{\partial \gamma_{yz}}{\partial t}}$$

where $c_1''$, $c_2''$, $p_1''$ and $p_2''$ are material constants. The values of the four material constants as determined from uniaxial tension test results are

$$c_1 = 130 \qquad c_2 = 5.15$$
$$p_1 = 98 \qquad p_2 = 6.00$$

The corresponding material constants as determined from the simple shear test results are

$$c_1'' = 150 \qquad c_2'' = 4.40$$
$$p_1'' = 108 \qquad p_2'' = 5.15$$

A comparison of these material constants as obtained from the two types of tests shows that they are quite close, considering the experimental limitations involved in the study. The stress-strain expressions show that there are at least four material constants independent of time and temperature. When used to predict strains in an axial compression test, these expressions gave reasonably good results for small

strains up to 0.4 percent only, as mentioned in the previous section. For strains greater than 0.4 percent it appears that a different deformation mechanism is operating in compression as compared to tension tests. However, pure compression was probably not achieved in the test performed, and the measurements made were less than ideal.

These expressions can be usefully employed in devising a laboratory test for bituminous mixtures which would evaluate the constants for the material. Such a test would be quantitative and not merely qualitative like the triaxial test used for testing bituminous mixtures.

The existence of at least four material constants, independent of time and temperature, as obtained from two different types of tests in this study, gives promise of more meaningful quantitative evaluation in the mixtures. The precise purpose of this study was to determine more meaningful properties for a sheet-asphalt mixture than those customarily used. This has been achieved with the three stress-strain expressions obtained from relevant experimental data, which, with the two equations of motion in two dimensions, give a set of five equations and five unknowns as follows:

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{yz}}{\partial y} = \gamma + \rho \frac{\partial^2 w}{\partial t^2} \tag{4}$$

$$\frac{\partial \tau_{yz}}{\partial z} + \frac{\partial \sigma_y}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2} \tag{5}$$

$$\sigma_z = \left[\frac{T}{c_1}\right]^{-c_2} - \left[\frac{T}{p_1}\right]^{-p_2} \frac{\frac{\partial w}{\partial z}}{t\,\frac{\partial^2 w}{\partial t\,\partial z}} = \left[\frac{T}{c_1'}\right]^{-c_2'} - \left[\frac{T}{p_1'}\right]^{-p_2'} \frac{\frac{\partial v}{\partial y}}{t\,\frac{\partial^2 v}{\partial t\,\partial y}} \tag{6}$$

$$\sigma_y = \left[\frac{T}{c_1}\right]^{-c_2} - \left[\frac{T}{p_1}\right]^{-p_2} \frac{\frac{\partial v}{\partial y}}{t\,\frac{\partial^2 v}{\partial t\,\partial y}} = \left[\frac{T}{c_1'}\right]^{-c_2'} - \left[\frac{T}{p_1'}\right]^{-p_2'} \frac{\frac{\partial w}{\partial z}}{t\,\frac{\partial^2 w}{\partial z\,\partial t}} \tag{7}$$

$$\tau_{yz} = \left[\frac{T}{c_1''}\right]^{-c_2''} - \left[\frac{T}{p_1''}\right]^{-p_2''} \frac{\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}}{t\left[\frac{\partial^2 w}{\partial t\,\partial y} + \frac{\partial^2 v}{\partial t\,\partial z}\right]} \tag{8}$$

However, the three stress-strain expressions obtained from experimental data are valid only for the range of temperatures and stress-levels for which the material was tested in this study.

## CONCLUSIONS

The following conclusions have been drawn from the experimental data obtained for the sheet-asphalt mixture, within the range of temperatures and stress levels for which it was tested.

1. Three independent stress-strain relationships exist as functions of time and temperature which together with the two-dimensional equations of motion, give a system of five equations containing five unknowns.

2. For the sheet-asphalt mixture tested, there exist at least four basic material constants independent of time and temperature as opposed to the usual modulus of elasticity and Poisson's ratio constants assumed in elastic theory. These four basic material constants exist in the tensile stress-axial strain expression derived from uniaxial tension test results and also in the shear stress-shear strain expression derived from simple shear test results. From the fact that the magnitude of the material constants as determined from two different types of tests, performed for a number of different conditions of time and temperature, were quite close to each other, it may be concluded that these material constants are independent of the type of test. As the results from axial compression tests corresponded reasonably well with those predicted from uniaxial tension test results for strains less than about 0.4 percent, it may be concluded that the derived expressions hold for both tension and compression of the material for very small strains.

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