# Modulus of Soil Reaction as Determined from Triaxial Shear Test

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In 1942 Spangler published his classical theory for the deflection of underground conduits. This theory has received only limited use, partially because of the inability to determine the proper numerical value of the modulus of soil reaction. Several different attempts have been made to determine this modulus in the laboratory, but each has met with only limited success. Burns and Richard (1964) solved the problem of the buried conduit, using the theory of elasticity by assuming ideal elastic conditions. Their equations have been used to calculate the modulus of soil reaction. The ensuing equation became very difficult to evaluate; therefore, the computer was used to determine an approximate relationship for the necessary variables and the modulus of soil reaction. The modulus has also been shown to be determinable from the triaxial shear test. The modulus is dependent on Poisson's ratio and the modulus of elasticity. Both values can be determined from a single triaxial shear test. Data obtained from a triaxial shear test are used to calculate the modulus of soil reaction which is then compared with the value measured in the device constructed by Watkins and Nielson called the modpares device.

•IN 1942, Spangler (3) published his classical theory for the prediction of deflection of circular underground pipe. He postulated the following equation:

$$\Delta X = \frac{KW_c r^3}{EI + 0.061E'r^3} \tag{1}$$

where

$\Delta X$ = change in horizontal diameter of pipe under load $W_C$,
$K$ = bedding constant (0.083 for 180-deg bedding),
$r$ = mean radius of pipe,
$EI$ = pipe wall stiffness,
$E'$ = modulus of passive resistance which equals er, and
$e$ = modulus of passive resistance.

In the derivation of Eq. 1 the value er was used instead of $E'$. The value of e is analogous to the modulus of subgrade reaction obtained from the plate bearing test. The value of e was assumed to be a constant for the soil under given conditions. Later Watkins and Spangler (5) showed that e was not constant, but the proper constant was $er = E'$.

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Eq. 1 has received only limited use because of the inability to determine a numerical value for the modulus of soil reaction, $E'$. Spangler (3) presented values which were determined by calculating $E'$ from measured deflections of actual field installations.

To measure the modulus of soil reaction, Watkins and Nielson (6) constructed a device which simulated the side of a pipe being forced into the side fill. Satisfactory results were obtained, but the complexity of the test and the requirement of special equipment limit the device's use. It would be desirable to devise a method of measuring the modulus of soil reaction by some equipment which is common to most soil mechanics laboratories.

## THEORY

Watkins and Nielson (6) have shown the modulus of soil reaction to be

$$E' = \frac{h}{\frac{\Delta X}{D}} \tag{2}$$

where

$E'$ = modulus of soil reaction,
$h$ = pressure at side of pipe caused by forcing side of pipe into side fill,
$\Delta X$ = change in horizontal diameter of pipe under loading, and
$D$ = original diameter of pipe.

Eq. 2 has the same form as the equation for the modulus of elasticity for a metal, i.e.,

$$E = \frac{P}{\frac{\Delta L}{L}} \tag{3}$$

where

$E$ = modulus of elasticity,
$P$ = applied pressure,
$\Delta L$ = deflection, and
$L$ = length.

Because the equations are similar and each relates pressure and deflection, there should exist a relationship between the modulus of soil reaction, $E'$, and the modulus of elasticity of the soil, $E$.

Figure 1 shows a buried pipe. An infinitesimal cube, A, is shown on the right side of the pipe at the horizontal diameter. Burns and Richard (2) derived the following equations for the pressure $\sigma_1$ and $\sigma_3$ and the deflection $\Delta X/2$ as shown on the pipe, using the theory of elasticity:

$$\sigma_1 = P\,\{B[1 - a_0^*(R/r)^2] - C[1 - 3a_2^*(R/r)^4 - 4b_2^*(R/r)^2]\} \tag{4}$$

$$\sigma_3 = P\,\{B[1 + a_0^*(R/r)^2] + C[1 - 3a_2^*(R/r)^4]\} \tag{5}$$

$$\frac{\Delta X}{2} = \frac{PR}{M^*}\,\tfrac{1}{2}\{[1 - (B/C)a_0^*(R/r)^2] - [1 + a_2^*(R/r)^4 + (2/B)b_2^*(R/r)^2]\} \tag{6}$$

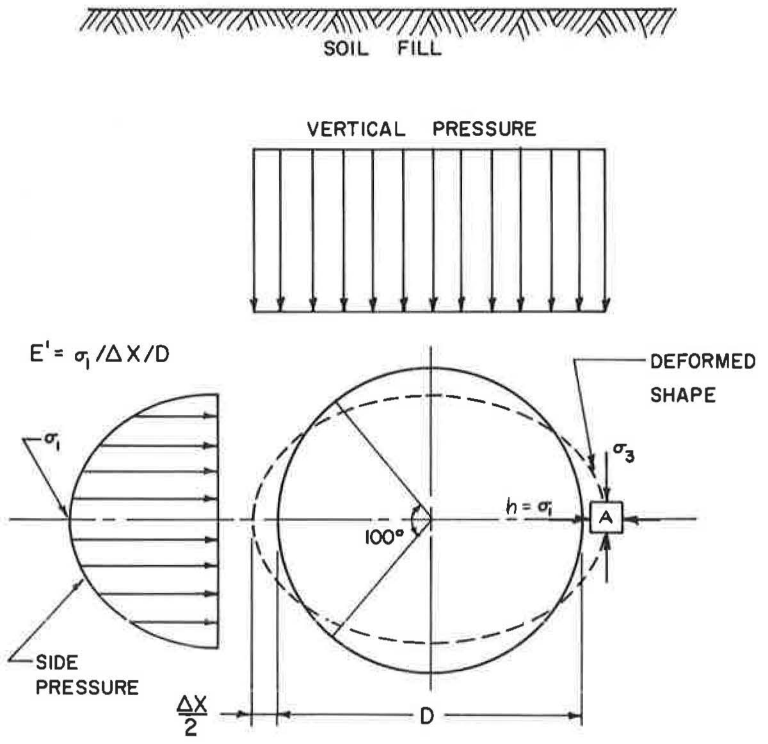


Figure 1. Buried pipe.

where

$M^* = \dfrac{E^*(1 - u)}{(1 + u)(1 - 2u)}$ which represents constrained modulus of elasticity of soil,

$r$ = radius from center of pipe to point in fill,

$E^*$ = modulus of elasticity of soil,

$u$ = Poisson's ratio,

$R$ = radius of pipe,

$C = \frac{1}{2}\left(\dfrac{1 - 2u}{1 - u}\right)$,

$B = \frac{1}{2}\left(\dfrac{1}{1 - u}\right)$,

$UF = \dfrac{2B\,M^*R}{EA}$,

$E$ = modulus of elasticity of pipe wall materials,

$A$ = area per unit length of pipe wall material,

$I$ = moment of inertia of pipe wall per unit length,

$VF = 2C\,\dfrac{M^*R^3}{EI}$,

$a_0^* = \dfrac{UF - 1}{UF + B/C}$,

$a_2^* = \dfrac{C(1 - UF)\,VF - (C/B)\,UF + 2B}{(1 + B)\,VF + C(VF + 1/B)\,UF + 2(1 + C)}$, and

$b_2^* = \dfrac{[B + C(UF)]\,VF - 2B}{(1 + B)\,VF + C(VF + 1/B)\,UF + 2(1 + C)}$.

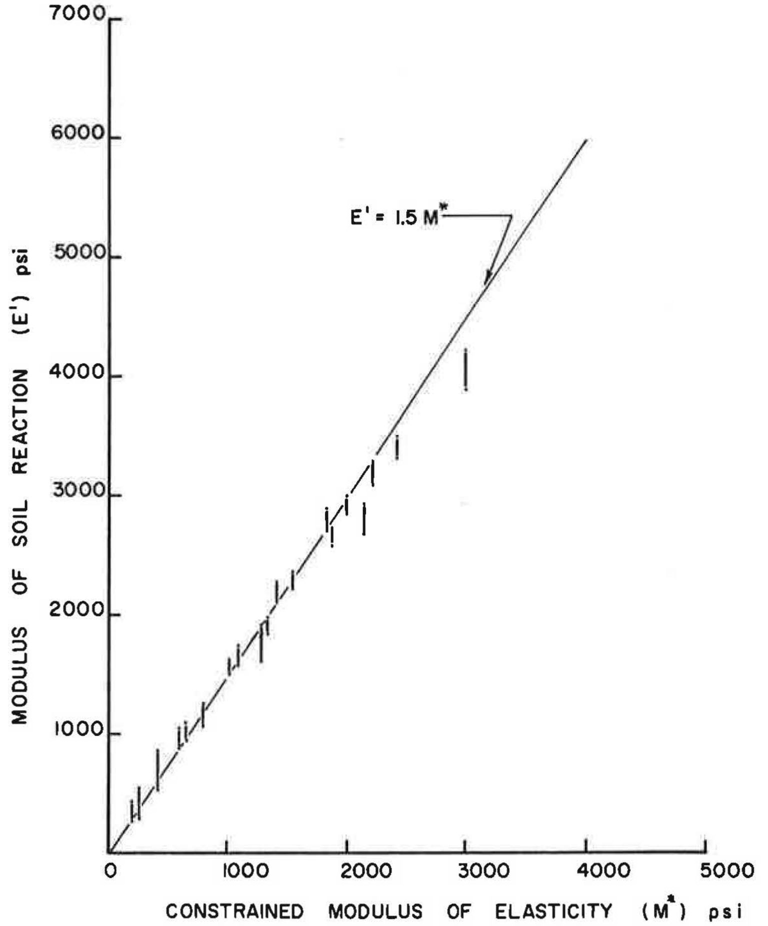


Figure 2. Comparison of modulus of soil reaction and constrained modulus of elasticity as determined from theory of elasticity.

Substituting Eqs. 4 and 6 into Eq. 2 and letting r = R, the resulting equation relates the modulus of soil reaction, $E'$, and the modulus of elasticity, E, as follows:

$$E' = \frac{2M^*[B(1 - a_0^*) - C(1 - 3a_2^* - 4b_2^*)]}{[1 + (B/C)a_0^*] - [1 - a_2^* + (2/B)b_2^*]} \tag{7}$$

Eq. 7 is rather difficult to solve; therefore, the computer was used to establish the relationship between $E'$ and $M^*$. A wide range of values for each variable was read into the computer program. Figure 2 shows a curve of the results obtained. The vertical lines represent limits for the modulus of soil reaction obtained by varying pipe radius, Poisson's ratio, pipe wall stiffness, etc. The modulus of soil reaction $E'$ can now be approximated by:

$$E' = 1.5M^* = 1.5\frac{E(1 - u)}{(1 + u)(1 - 2u)} \tag{8}$$

where

$E'$ = modulus of soil reaction,
$E$ = modulus of elasticity,
$u$ = Poisson's ratio, and
$M^*$ = constrained modulus of elasticity.

## TRIAXIAL SHEAR TEST

An approximation of the modulus of elasticity E and Poisson's ratio u can be determined from the triaxial shear test. Most textbooks in strength of materials give the stress-strain relationship for three-dimensional stress as:

$$\epsilon_1 = \frac{1}{E}(\sigma_1 - u\sigma_2 - u\sigma_3) \tag{9}$$

where

$\epsilon_1$ = maximum principal strain,
$\sigma_1$ = maximum principal stress,
$u$ = Poisson's ratio,
$\sigma_2$ = intermediate principal stress,
$\sigma_3$ = minor principal stress, and
$E$ = modulus of elasticity.

In the normal triaxial shear test the intermediate principal stress $\sigma_2$ and the minor principal stress $\sigma_3$ are equal and constant. For the triaxial shear test, Eq. 9 can be rewritten as:

$$\epsilon_1 = \frac{\sigma_1}{E} - C \tag{10}$$

where

$$C = \frac{(+ u\sigma_2 + u\sigma_3)}{E} = \frac{+2\ u\sigma_3}{E} = \epsilon_0$$

This substitution assumed Poisson's ratio and the modulus of elasticity to be constant, which is probably not valid for soils.

However, by making the assumption that Poisson's ratio is constant, a value for the modulus of elasticity and Poisson's ratio can be approximated. Figure 3 shows a typical stress-strain curve obtained from a triaxial shear with constant lateral pressure. Eq. 9 shows that the axial strain occurring in a triaxial specimen during the application of the chamber pressure $\sigma_3$, at which time $\sigma_1 = \sigma_2 = \sigma_3$, is

$$\epsilon_1 = \frac{\sigma_3}{E} - u\frac{\sigma_3}{E} - u\frac{\sigma_3}{E} \tag{11}$$

$$\epsilon_1 = \frac{\sigma_3}{E}(1 - 2u) \tag{12}$$

or

$$u = \frac{\sigma_3 - \epsilon_1 E}{2\sigma_3} \tag{13}$$

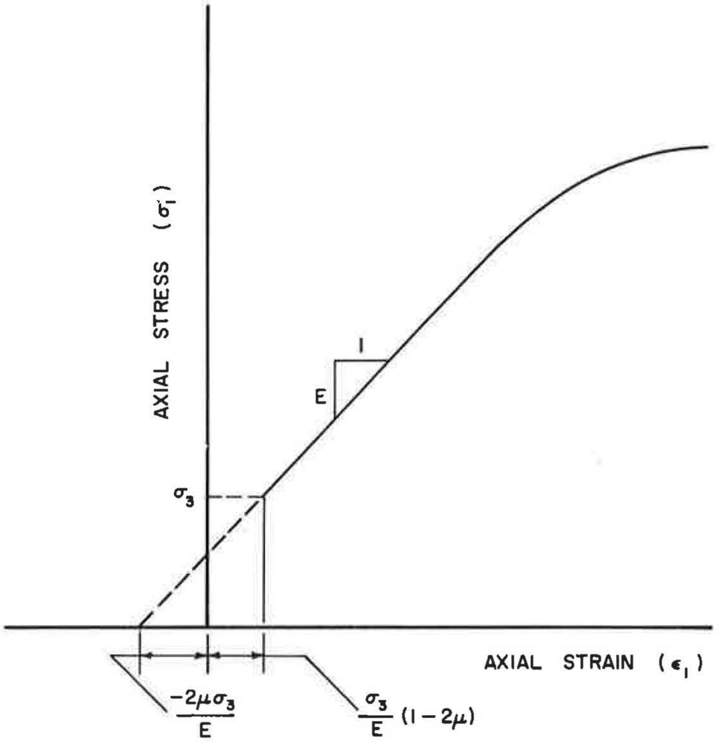


Figure 3. Triaxial shear test with constant lateral pressures.

Eq. 13 shows that an approximation of Poisson's ratio can be determined by measuring the axial strains $\epsilon_1$ or the decrease in length that occurs in the sample during the application of the chamber pressure, $\sigma_3$. E is assumed to be the slope of the stress-strain curve above the value of $\sigma_1 = \sigma_3$. Typical values of u for soils with $\sigma_3$ ranging from 15 to 45 psi show E varying from 1000 to 7000 psi and u ranging from 0.25 to 0.35. Values of Poisson's ratio obtained by Barkan (1) indicate that it ranges between 0.3 and 0.35 for sands. Tsytouich (4) recommends the value of Poisson's ratio for sands as u = 0.15 to 0.25, for clays with sand and silt 0.30 to 0.35, for clays, 0.35 to 0.40.

If the value change occurring in the triaxial shear sample is measured, Poisson's ratio can be determined by another method. The volume change $\Delta V$ that occurs during testing divided by the original volume is the cubical dilatation and is approximately given by:

$$\frac{\Delta V}{V} = \epsilon_1 + \epsilon_2 + \epsilon_3 \tag{14}$$

where

$\Delta V$ = volume change of specimen,
$V$ = original volume of specimen, and
$\epsilon_1, \epsilon_2, \epsilon_3$ = strains.

The original volume of the specimen is known and $\Delta V$ and $\epsilon_1$ can be measured. By the nature of the triaxial shear test $\epsilon_2 = \epsilon_3$, therefore, the only unknown is $\epsilon_3$. If the principal stresses and strains are known, Poisson's ratio can be solved by using Eq. 9 and the similar equation for $\epsilon_2$.

Indications are that Poisson's ratio can be assumed to be approximately 0.25 for most design work. This does not increase the error significantly more than error

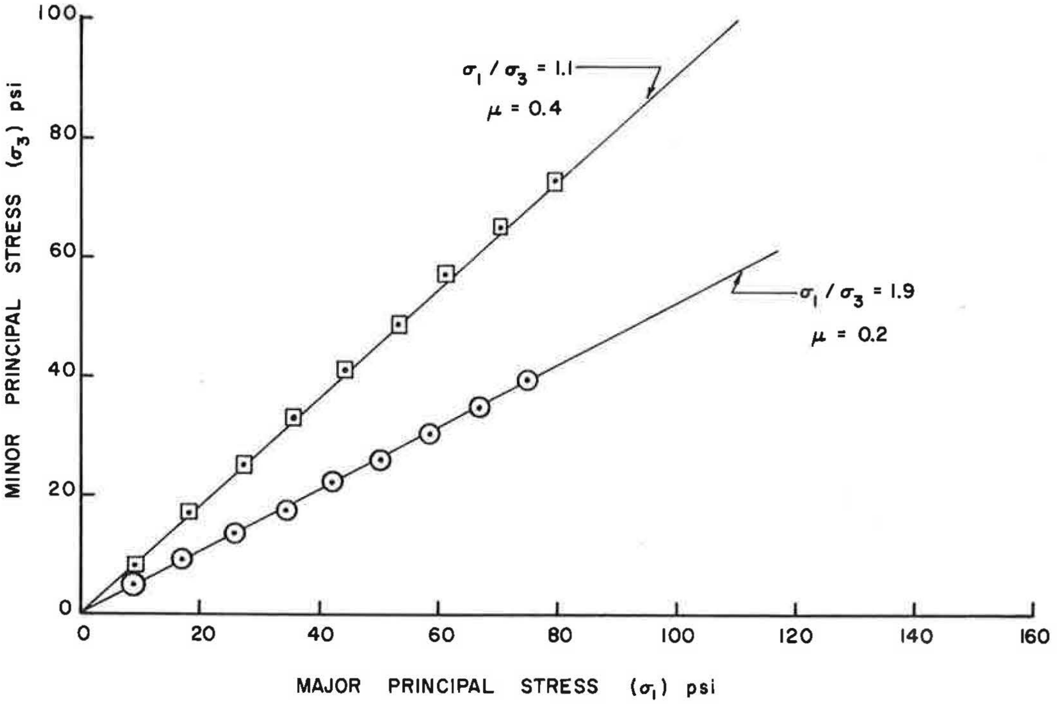


Figure 4. Relationship between major and minor principal stresses for various Poisson's ratio as determined from the theory of elasticity.

already present in the measuring procedures because of the nonlinear stress-strain diagram.

The proper lateral pressure $\sigma_3$ in the triaxial chamber has to be determined so that correct stress-strain curve can be obtained. The value of $\sigma_3$ should change with the value of $\sigma_1$ to simulate the stress conditions shown in Figure 1. The effects of the intermediate principal stress are assumed negligible. Evaluation of Eqs. 4 and 5 shows that the ratio of $\sigma_1/\sigma_3$ is approximately 1.1 for Poisson's ratio = 0.4 and 1.9 for Poisson's ratio = 0.2. Figure 4 shows the relationship of $\sigma_1/\sigma_3$ as calculated from Eqs. 4 and 5 for two of the different values of Poisson's ratio.

The ratio of 1.1 is not realistic for soils. Load-deflection curves observed in the field are not possible unless the ratio of $\sigma_1/\sigma_3$ approaches $\tan^2$ [45- ($\Phi$/2)] for a cohesionless soil.

For the sake of simplicity, with the understanding that results are only approximate, it is recommended that the lateral pressure used in making the triaxial shear test be set at $^3/_8$ of the actual weight of the completed fill above the mid-height of the pipe. Evaluation of Eq. 5 shows that the lateral pressure should be approximately $^3/_4$ of the vertical load, P. A value of lateral pressure $\sigma_3$ equal to $^3/_8$ of the weight of soil is the average value of the $\sigma_3$ during the construction of the fill.

When a pipe is strutted before the fill is placed over it, the lateral pressure $\sigma_3$ must be increased to approximately $^3/_4$ the weight of the vertical column of fill. No deflection (or negligible deflection) is allowed to take place before the struts are removed. Therefore, the maximum lateral pressure of approximately $^3/_4$ the weight of the fill would be exerted before any deflection was allowed.

## DEFLECTION-STRAIN RELATIONSHIP

It is necessary to get a relationship between the deflection of the pipe and the strain in the soil. The strain in the soil can be obtained by differentiating Eq. 6 with respect to r and then letting r = R. A wide range of values was again substituted into the

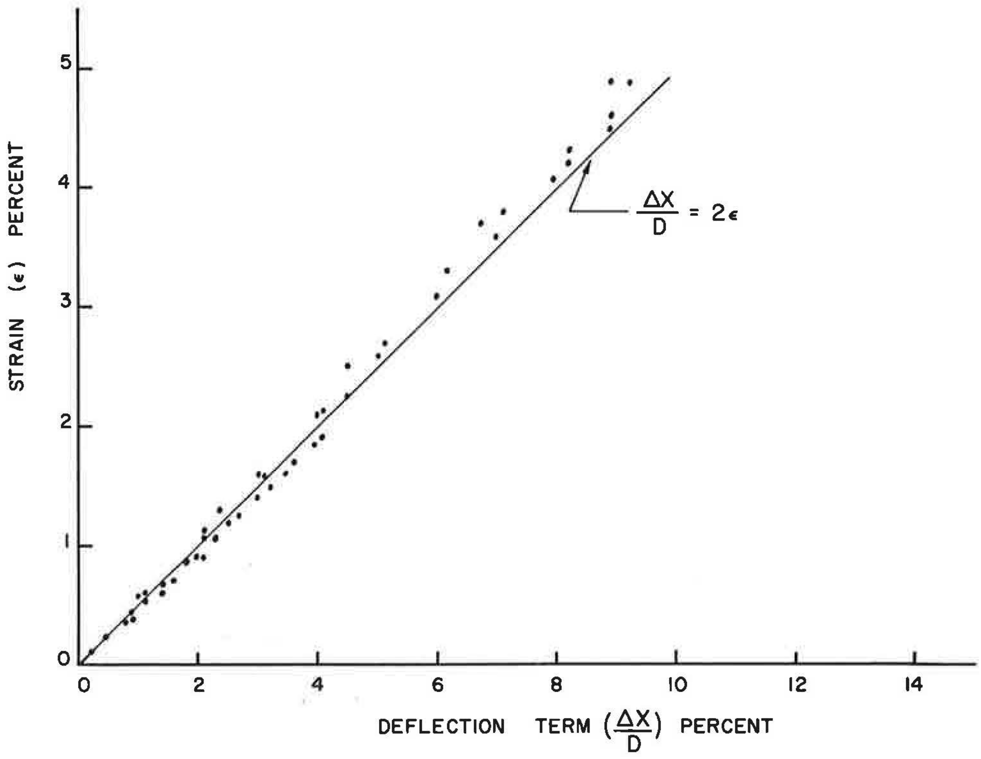


Figure 5. Comparison between soil strain and percent deflection of pipe.

computer. Figure 5 shows the results obtained for typical soil values for each variable. Figure 5 shows that

$$\frac{\Delta X}{D} = 2\epsilon \tag{15}$$

## OBTAINING THE MODULUS OF SOIL REACTION

Equations 2 and 8 give the value for h as

$$h = 1.5\ M^{*} \frac{\Delta X}{D} \tag{16}$$

Because

$$\Delta X/D = 2\epsilon$$

$$h = 1.5 M^{*}\ 2\epsilon = 3 \frac{(1 - u)\ E\ \epsilon}{(1 + u)\ (1 - 2u)} \tag{17}$$

If Poisson's ratio equals 0.4

$$h = 4.29\ E\ \epsilon$$

For Poisson's ratio equals 0.3

$$h = 3.1\ E\ \epsilon \tag{18}$$

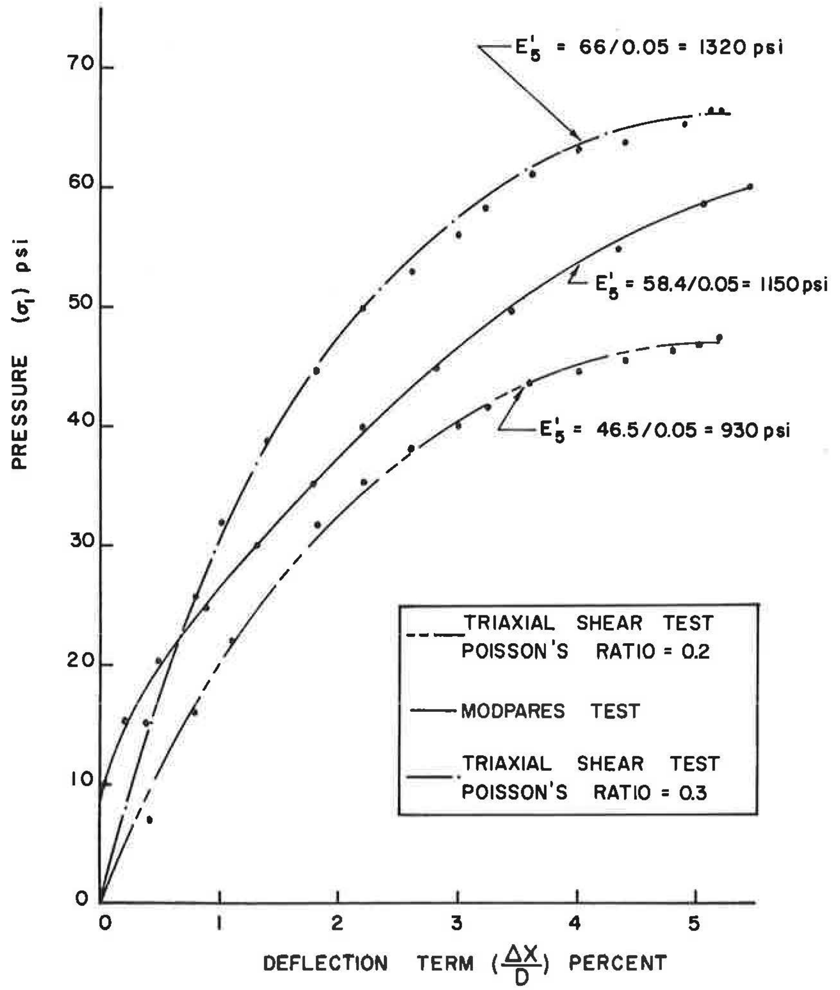


Figure 6. Modpares curve for determining modulus of soil reaction vs curves calculated from triaxial shear test for fine sand.

or if Poisson's ratio equals 0.2

$$h = 2.2 E \epsilon \tag{19}$$

The value of h can be reduced to

$$h = 4.29 \sigma \quad \text{for } u = 0.4$$

$$h = 3.1 \sigma \quad \text{for } u = 0.3$$

or

$$h = 2.2 \sigma \quad \text{for } u = 0.2 \tag{20}$$

where $\sigma$ equals vertical pressure on the triaxial specimen which equals P/A.

The modulus of soil reaction $E'$ can be solved by substituting one of the values from Eq. 20 and Eq. 15 into Eq. 2. Figure 6 shows a comparison of the curves for the modulus soil reaction calculated from a triaxial shear test and one measured in the device

constructed by Watkins and Nielson (6). The soil in each case has a density of approximately 87 percent AASHO T-180 density. The soil to be tested in the triaxial shear test must have the same density and moisture content as the sample compacted around the pipe. Data for Figure 6 are given in the Appendix.

## CONCLUSIONS

The modulus of soil reaction can be determined by the triaxial shear test with sufficient accuracy for use in design work. The modulus is sensitive to the value of Poisson's ratio. For design work, a value of 0.25 is recommended.

## ACKNOWLEDGMENTS

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# *Appendix*

TABLE 1

MODPARES TEST[a]

| Membrane Pressure (psi) | Deflection (in. × $10^{-3}$) | Membrane Pressure (psi) | Deflection (in. × $10^{-3}$) |
|---|---|---|---|
| 0.0 | 0.0 | 35.1 | 35.5 |
| 5.0 | 0.0 | 40.0 | 43.8 |
| 10.2 | 1.3 | 45.2 | 56.0 |
| 15.0 | 3.6 | 50.0 | 68.3 |
| 20.0 | 9.5 | 55.2 | 87.3 |
| 25.2 | 17.0 | 60.0 | 108.6 |
| 30.1 | 27.3 | | |

[a]Soil sample: fine sand
Compactive effort: 3440 ft-lb/ft$^3$
Wet wt: 112.1 pcf
Dry wt: 99.2 pcf

Date: Jan. 10, 1966
Vertical pressure: 25 psi
Moisture content: 13.1%

TABLE 2

TRIAXIAL SHEAR TEST[a]

| Length Change (in.) | Proving Ring Dial (in. × $10^4$) | Strain | Area (sq in.) | Axial (lb) | P/A (psi) | $\sigma_1$ (psi) |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 6.158 | 0 | 0 | 10.000 |
| 0.010 | 17 | 0.002 | 6.169 | 21.658 | 3.511 | 13.511 |
| 0.020 | 36 | 0.004 | 6.180 | 45.864 | 7.421 | 17.421 |
| 0.030 | 50 | 0.005 | 6.191 | 63.700 | 10.288 | 20.288 |
| 0.040 | 62 | 0.007 | 6.203 | 78.988 | 12.734 | 22.734 |
| 0.050 | 71 | 0.009 | 6.214 | 90.454 | 14.556 | 24.556 |
| 0.060 | 79 | 0.011 | 6.226 | 100.646 | 16.166 | 26.166 |
| 0.070 | 84 | 0.013 | 6.237 | 107.016 | 17.157 | 27.157 |
| 0.080 | 89 | 0.015 | 6.249 | 113.386 | 18.145 | 28.145 |
| 0.090 | 93 | 0.016 | 6.261 | 118.482 | 18.925 | 28.925 |
| 0.100 | 97 | 0.018 | 6.272 | 123.578 | 19.702 | 29.702 |
| 0.110 | 100 | 0.020 | 6.284 | 127.400 | 20.274 | 30.274 |
| 0.120 | 102 | 0.022 | 6.296 | 129.948 | 20.641 | 30.641 |
| 0.130 | 105 | 0.024 | 6.307 | 133.770 | 21.208 | 31.208 |
| 0.140 | 107 | 0.026 | 6.319 | 136.318 | 21.572 | 31.572 |
| 0.150 | 108 | 0.027 | 6.331 | 137.592 | 21.732 | 31.732 |
| 0.200 | 113 | 0.037 | 6.391 | 143.962 | 22.525 | 32.525 |
| 0.250 | 116 | 0.046 | 6.452 | 147.784 | 22.903 | 32.903 |

[a]Sample: fine sand
Location: Lohman interchange
Specific gravity: 2.64

Specimen Dimensions

Diameter = 2.80 in.
Length = 5.47 in.

Proving ring no. = 1535
Calibration factor = 1.274 lb/div
Chamber pressure = 10.0 psi
Deformation rate = 0.0600 in./min
Void ratio = 0.59
Dry unit weight = 103.4 lb/CF
Max. stress = 32.90 at 4.571 percent strain

Test No.: special
Date: June 22, 1966
Tested by: Don Bell

Specimen Weights

Wt tare + soil = 1420.000 gm
Wt tare = 387.60 gm
Wt soil = 1420.00 gm

Moisture Content

Wt tare and wet soil = 926 gm
Wt tare and dry soil = 200 gm
Wt tare = 0 gm
Moisture content = 13 percent