Volume Warrants for Left-Turn Storage Lanes At Unsignalized Grade Intersections

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This paper describes the derivation of volume warrants and design charts for left-turn storage lanes at unsignalized grade intersections on four-lane and two-lane highways. The design charts are based on a theoretical analysis and on a series of field studies of traffic behavior at intersections.

The analysis is based on a queuing model in which arrival and service times are assumed to follow a negative exponential distribution. The arrival rates are determined by the volumes of left-turning, through or "advancing," and opposing traffic, and by the time interval required by the left-turning vehicle to clear the advancing lane. The service rates are determined by the volume of opposing traffic, and by the time interval required to make a left-turn maneuver.

Field studies of traffic behavior conducted at seven unsignalized Ontario intersections provided average values of the time interval required by a left-turning vehicle to make a left turn and to clear the advancing lane, the delay experienced by a left-turning vehicle because of opposing traffic, gap acceptance and rejection behavior, and actual arrival rates and headway distributions at various volume levels.

•THIS study was undertaken because of the lack of consistent volume warrants for left-turn storage lanes at unsignalized intersections. The usual method of analyzing such intersections individually on the basis of past experience, accident records, complaints from the traveling public, and engineering judgment has led to inconsistency from location to location.

It was felt that the volume warrants developed should be consistent in their evaluation of traffic parameters from location to location; they should provide reasonable recommendations for specific intersections; and they should be based on traffic and operational considerations, rather than on a benefit-cost analysis, because of the difficulty of translating the benefits received to a monetary value on a suitable rational basis.

The study contained three phases: a theoretical analysis, a series of field studies of traffic behavior, and analysis of a series of questionnaires completed for specific intersections by Department of Highways regional traffic engineers.

THEORETICAL ANALYSIS

Queuing theory may be used to analyze operational flow problems where the state of the system changes from time to time and which have elements that follow this basic behavior: A sequence of units arrives at some facility which services each unit and eventually discharges it (1). In our problem, a sequence of left-turning vehicles arrives at some intersection that permits each left-turning vehicle to proceed if and when there is a suitable gap in the opposing traffic stream, and then discharges the vehicle from the intersection. Morse (1) explains that instead of trying to predict in detail how the state of the system changes with time, we can calculate the probabilities that the system is in each of the possible states.

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In this analysis it was assumed that the arrivals of left-turning vehicles follow a Poisson distribution (random or negative exponential distribution) and that the service time distribution is also negative exponential, i.e., the probability of prolongation of service is independent of how long ago the service started. It may be shown (1) that the state of such a service system is dependent only upon the average arrival rate λ, and the average service rate μ, and the state probabilities are independent of time. The ratio $\rho = \lambda/\mu$ is called the "utilization factor"; steady-state solutions may be determined only if $\rho < 1$.

It may further be shown (1) that for the steady-state system with negative exponential arrival-time and service-time distributions, and where every arriving unit joins the queue,

$$P_n = (1 - \rho)\rho^n \tag{1}$$

and

$$Q_n = \rho^n \tag{2}$$

where P_n is the probability of n units in the system (both queue and service) and Q_n is the probability of n or more units in the system (queue plus service). The volume warrants were based on these two relationships.

The derivation of the warrant was based on the following conditions:

1. On four-lane highways, it is the presence of a left-turning vehicle extending into the through lanes that will affect safety and capacity; the probability of this occurrence should not exceed 0.005 for divided highways or 0.03 for undivided highways. (Divided highways are those with sufficient median width for the storage of at least one left-turning vehicle; undivided highways are those with less or no median width.) It is assumed that there is sufficient through or "advancing" traffic for such an occurrence to be undesirable.

On two-lane highways, it is the arrival of advancing, through vehicles behind a stopped left-turning vehicle that will affect safety and capacity (an arriving through vehicle is one that has been stopped or brought to creep speed by a left-turning vehicle in the advancing lane); the probability of this occurrence should not exceed 0.020 for design speed = 50 mph, operating speed (v) = 40 mph; 0.015 for design speed = 60 mph, operating speed (v) = 50 mph; and 0.010 for design speed = 70 mph, operating speed (v) = 60 mph.

These probability levels were determined from preliminary investigations, from the judgments of various highway department engineers and, particularly in the case of two-lane highways, from highway capacity considerations.

2. Both arrival-time and service-time distributions are negative exponential.
3. There is strict queue discipline: "first come, first served," and every arriving unit must join the queue.
4. No left-turning vehicle can begin its maneuver until the previous vehicle has completed its left turn.
5. On four-lane highways, the average time t_1 required for making a left turn is 4.0 sec. On two-lane highways, t_1 is 3.0 sec. These values were determined from field studies.
6. The required critical headway G_c in the opposing traffic stream for a left-turn maneuver is 6.0 sec on four-lane highways and 5.0 sec on two-lane highways. These values were determined from field studies.
7. On a two-lane highway, the average time t_e required for a left-turning vehicle to clear itself or "exit" from the advancing lane is 1.9 sec, as determined from field studies.

For four-lane highways, the average arrival rate is

$$\lambda = V_L \tag{3}$$

where V_L is the number of vehicles per hour making left turns.

For two-lane highways, the problem situation is that in which a left-turning vehicle is followed by a through vehicle. From the theorem of compound probability, the probability of this occurrence is L(1 - L), where L is the proportion of left turns in the total advancing traffic stream of through and left-turning vehicles. This is not the probability of an arrival, however, for the left-turning vehicle may have completed its turn before the following through vehicle arrives. Whether or not the following vehicle arrives before the turn is completed largely depends on three factors:

1. The average time t_W that a left-turning vehicle must wait for a suitable gap in the opposing traffic stream (t_W expressed in seconds). As developed by Adams (2):

$$t_W = \frac{3600}{V_o e^{-\frac{V_o G_c}{3600}}} - \frac{3600}{V_o} - G_c \tag{4}$$

2. The time interval t_e defined earlier; t_e = 1.9 sec.
3. The median time interval (headway) between vehicles in the advancing stream. The median headway was selected rather than the mean headway because of the high frequency of headways less than the mean. From theoretical considerations and field tests (3):

$$t_{median} = \frac{2}{3} t_A = \frac{2}{3} \cdot \frac{3600}{V_A} = \frac{2400}{V_A} \tag{5}$$

where V_A = advancing volume (through, left-turning, and right-turning vehicles, vph), and t_A = mean headway in V_A.

On two-lane highways, then, the mean arrival rate is the number of arrivals per hour of through vehicles behind left-turning vehicles:

$$\lambda = [L(1 - L)\, V_A] \frac{t_W + t_e}{\frac{2}{3} t_A} \tag{6}$$

where $L = V_L/V_A$ as defined earlier.

For both four-lane and two-lane highways, the average service rate μ is the number of left turns that can be made in one hour. This parameter is a function of:

1. The volume of traffic in the opposing lane (s) V_o, and hence the amount of time per hour in which left turns can be made (unblocked time). The amount of unblocked time per hour during which left turns are possible may be computed by deducting from the total time (a) all time in the opposing stream composed of headways less than G_c, and (b) a certain proportion of the time when the left-turning vehicle is less than G_c sec from an oncoming vehicle in those opposing stream headways greater than G_c (4). Because of the random traffic arrivals and the possibility of a vehicle arriving at a time anywhere in the blocked period of less than G_c sec, this proportion was taken to be 0.5, i.e., the average amount of blocked time in a usable gap is $G_c/2$. The amount of unblocked time may then be determined from graphs of observed headways for various volume conditions on both four-lane and two-lane highways. The graphs used in this analysis, which agreed well with observed distributions, are shown in Figures 7 to 9 (4).
2. The average time t_1 taken to make the left-turn maneuver.

The mean service rate

$$\mu = \frac{\text{Unblocked Time/Hr (sec)}}{t_1 \quad \text{(sec)}} \tag{7}$$

For four-lane divided highways, the warranting traffic volumes were determined from Eq. 1 and the probability level of 0.005. Thus,

$$P_0 + P_1 \nless 0.995$$

Now

$$P_0 = (1 - \rho)\ 1 = 1 - \rho$$
$$P_1 = (1 - \rho)\ \rho = \rho - \rho^2$$

Adding,

$$P_0 + P_1 = 1 - \rho^2 \nless 0.995$$
$$\rho^2 \nless 0.005$$
$$\rho \nless 0.0707$$

For a left-turn storage lane to be warranted, $\rho \geq 0.0707$. For any opposing traffic volume V_{oi} and its corresponding service rate μ_i,

$$\lambda_i = \rho\mu_i = 0.0707\mu_i$$

The warrant curve is shown in Figure 1 (all figures are in the Appendix). Volume conditions above and to the right of the curve warrant a left-turn storage lane.

Along the warrant curve itself, $Q_2 = \rho^2 = 0.005$ (from Eq. 2) and the probability of 3 or more units in the system, $Q_3 = \rho^3 = 0.000354$. If $\rho^3 > 0.000354$, a longer storage length, S = 75 ft, is required (assuming 25 ft per vehicle). Extending this principle, for any storage length S, with storage capacity of n vehicles, the probability of n + 1 or more vehicles in the system should not exceed 0.000354. Thus, the probability of exceeding the capacity of the storage lane by one or more vehicles will be the same for each length. The design charts shown in Figure 1 were constructed on this basis. For example: S = 75 ft, n = 3, $Q_4 = \rho^4 = 0.000354$, $\rho = 0.137$, $V_L = \lambda = 0.137\mu$.

For four-lane undivided highways:

$$P_0 = 1 - \rho \nless 0.970$$
$$\rho \nless 0.30$$

A left-turn storage lane is warranted when $\rho \geq 0.030$. The warrant curve $\lambda = 0.030\mu$ is shown in Figure 1. For $V_o < 400$ vph, no left-turn lane should be provided unless $V_A > 400$ vph because of the advancing vehicles' freedom to maneuver at lower volumes. For this reason, the four-lane undivided highway curve is shown as a dashed line for $V_o < 400$ vph. Once volume conditions reach a point above and to the right of the warrant curve for the divided case, the undivided and divided cases are in effect the same, and the required storage lengths shown apply to both.

For two-lane highways:

$$P_0 = 1 - \rho \nless 0.980\ (v = 40\text{ mph})$$
$$\nless 0.985\ (v = 50\text{ mph})$$
$$\nless 0.990\ (v = 60\text{ mph})$$

Therefore, $\rho \nless 0.020$, 0.015, and 0.010 for v = 40, 50 and 60 mph respectively. The warrant curves were determined for each speed condition from $\lambda = \rho u$; for a given V_o, the warranting V_A for a given left-turn proportion and operating speed was determined from:

$$\lambda = [L(1 - L)V_A] = \frac{t_w + t_e}{\frac{2}{3} \cdot t_A} = \rho u$$

TABLE 1

STORAGE LENGTH TO BE ADDED TO CHART VALUES OF LEFT-TURN LANE STORAGE LENGTHS (Length in Feet)

CHART VALUE OF STORAGE LENGTH REQUIRED	% T_L = % TRUCKS IN V_L					
	0%	10%	20%	30%	40%	50%
75'	0	25'	25'	25'	50'	50'
100'	0	25'	25'	50'	50'	50'
125'	0	25'	25'	50'	50'	75'
150'	0	25'	50'	50'	75'	75'
175'	0	25'	50'	75'	75'	100'
200'	0	25'	50'	75'	100'	100'
250'	0	25'	50'	75'	100'	125'
300'	0	50'	75'	100'	125'	150'
350'	0	50'	75'	125'	150'	175'
400'	0	50'	100'	125'	175'	200'
450'	0	50'	100'	150'	200'	225'
500'	0	50'	100'	150'	200'	250'

Therefore

$$[L(1 - L)V_A^2] (t_w + t_e) = 2400\rho\mu$$

$$V_A^2 = \frac{2400\rho}{L(1 - L)} \cdot \frac{\mu}{(t_w + t_e)}$$

The curves are shown in Figures 2 through 19. The storage length S of the lane was determined using the same principles and methods as for four-lane highways: for any storage length S with storage capacity of n vehicles, the probability of n + 1 or more units in the system should not exceed 0.000008, 0.000003375, or 0.000001 for v = 40, 50, or 60 mph respectively.

Assuming an average required storage length per truck of 50 ft (5), the additional storage length required because of trucks may be determined from Table 1.

Figure 20 shows a typical design of left-turn storage lanes on two-lane Ontario highways. The design distances also apply to four-lane highways, except that the advancing lanes need not be tapered out to provide a shield for left-turning vehicles on divided highways because of the median.

FIELD STUDIES

Field studies were conducted at seven Ontario intersections to determine the values of parameters used in the analysis (3). The parameters measured were:

1. The average time interval t_e required for a left-turning vehicle on a two-lane highway to "exit" from the advancing lane (the lane from which the left-turn is made). A sample of 150 measurements gave t_e = 1.9 sec.
2. The average time interval t_l required to make the left-turn maneuver. For two-lane highways, t_l was found to be 3.0 sec. For four-lane highways, t_l was taken to be 4.0 sec, assuming that 1.0 additional sec is required to cross the additional lane.

TABLE 2

COMPARISON BETWEEN THEORETICAL AND OBSERVED ARRIVAL RATES
TWO-LANE HIGHWAYS

Intersection	V_L (vph)	V_A (vph)	$L = \frac{V_L}{V_A}$	V_o (vph)	t_w (sec.)	t_e (sec.)	t_a (sec.)	Theoretical $\lambda = [L(1-L)V_A] \frac{t_w + t_e}{\frac{2}{3} t_a}$ (vph)	Observed λ (vph)
Hwy. No. 7 and 2nd Line E.	75 88	580 600	0.130 0.147	628 658	3.00 3.10	1.9 1.9	6.21 6.00	78 96	80 87
Hwy. No. 2 and Altona Road	60	484	0.124	218	0.80	1.9	7.44	29	13
Keele St. and York Univ. Ent.	112	451	0.248	551	2.53	1.9	7.98	70	74
Hwy. No. 2 and Liverpool Road	133	271	0.491	220	0.84	1.9	13.30	21	16
Hwy. No. 2 and Brock Road	108	261	0.414	171	0.60	1.9	13.80	17	14

3. The critical gap G_c, i.e., the size gap which has the property that the number of accepted gaps shorter than G_c, is the same as the number of maximum rejected gaps longer than G_c. From an analysis of accepted and rejected gaps, G_c was found to be 5.0 sec for two-lane highways and 6.0 sec for four-lane highways.

4. The average time interval t_w which a left-turning vehicle must wait for a suitable gap in the opposing traffic stream. In the analysis, t_w was computed by using Eq. 4. For given V_o conditions, the observed t_w values were compared with the theoretical. Sizable fluctuations occurred when t_w was averaged over short time periods such as 15 min, but the fluctuations were considerably smoothed out when t_w was averaged over a time period of 45 to 60 min (3). As shown in Figure 21, the results for two-lane roads show reasonable agreement with the theoretical curve. Although t_w was not used in the four-lane analysis, it is interesting to note that the results for four-lane roads show closer agreement with the theoretical two-lane curve than with the four-lane curve.

5. Volume counts.

6. λ, the number of through vehicles which arrived behind vehicles waiting to make left turns and were delayed by them (two-lane highways). The observed λ values were compared with the theoretical values computed from Eq. 6 (Table 2). The agreement between the theoretical and observed λ values is quite good, except for the intersection of Highway No. 2 and Altona Road where an unusually small number of left-turning vehicles was delayed.

The field studies were conducted by a crew of five, using only prepared forms, two synchronizable wristwatches with sweep second hands, and a stopwatch. Team A (an observer and recorder) noted and recorded the time of arrival at the intersection of each vehicle in the advancing lane, whether that vehicle was a left-turning vehicle, and the number of arriving through vehicles delayed by waiting left-turning vehicles. Team B noted and recorded the time of arrival at the observer station of each vehicle in the opposing lane. The fifth observer was stationed with Team A and, using the stopwatch, noted and recorded t_e and t_w for left-turning vehicles. Values of t_l were determined separately.

TABLE 3

JUDGMENTS OF REGIONAL TRAFFIC ENGINEERS ON SUITABILITY OF CHART APPLICATION

	No. of Cases
In Agreement with Chart Recommendation	67
Lane not Warranted by Chart (i. e., traffic volumes), but required because of poor visibility and/or accident record.	8
Lane not Warranted by Chart (i. e., traffic volumes), but required because of character of route	1
Not in Agreement with Chart Recommendation (in all cases, a lane was felt to be warranted although the chart did not indicate it).	4
Total	80

ANALYSIS OF QUESTIONNAIRES

As requested, highway department regional traffic engineers completed detailed questionnaires on chart application at 80 specific Ontario intersections covering a wide range of traffic conditions (3). The engineers were asked to supply the following information for each intersection: explicit intersection data; an evaluation of visibility, suitability of storage lane length (if in operation), sideroad traffic interference, and congestion conditions; the reason(s), if known, for construction of an existing left-turn lane; whether a storage lane was warranted by the charts; whether the rater considered the chart recommendation a reasonable one; and the reason for agreement or disagreement with the chart recommendation. Table 3 summarizes the questionnaire replies. These results were interpreted to mean that, in general, where traffic volume was the governing factor, the volume warrants and charts provided reasonable solutions.

It is recognized that intersections with poor visibility and/or a bad accident record may require the designer to exercise his judgment when volume conditions alone do not warrant a storage lane. It is also recognized that the analytical "models" could be considerably improved and refined. Nevertheless, it is believed that the charts, based on a theoretical analysis (although crude) as well as on field observation and the judgment of design and control engineers, provide a basis for design that is more consistent and reasonable than those previously used.

REFERENCES

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2. Adams, W. F. Road Traffic Considered as a Random Series. Jour. Inst. of Civil Eng., London, 1936. The derivation of the formula quoted in this paper is also presented in "Statistics with Applications to Highway Traffic Analyses," Eno Foundation, 1952.

3. Harmelink, M. D. Volume Warrants for Left-Turn Storage Lanes at Unsignalized Grade Intersections. Research Report RR 122, Department of Highways, Ontario, 1967.
4. Matson, T. M., Smith, W. S., and Hurd, F. W. Traffic Engineering. McGraw-Hill, 1955.
5. AASHO. A Policy on Geometric Design of Rural Highways. 1954.

Appendix

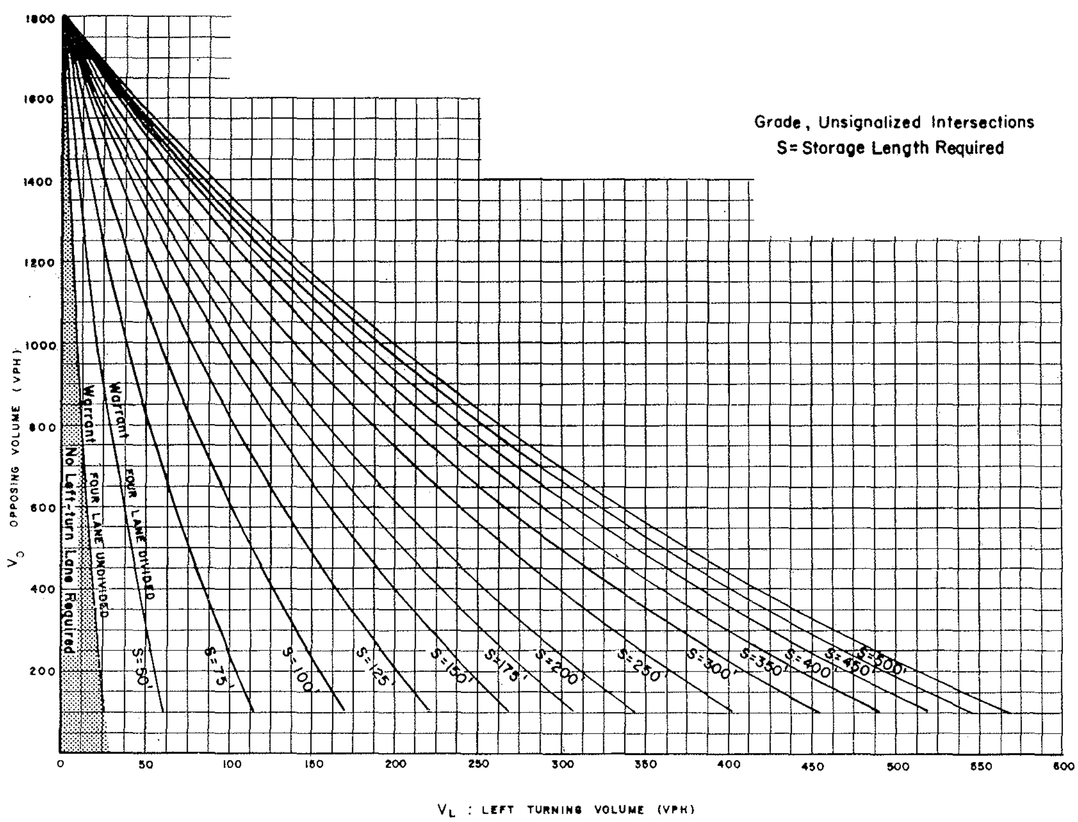

Figure 1. Warrant for left-turn storage lanes on four-lane highways.

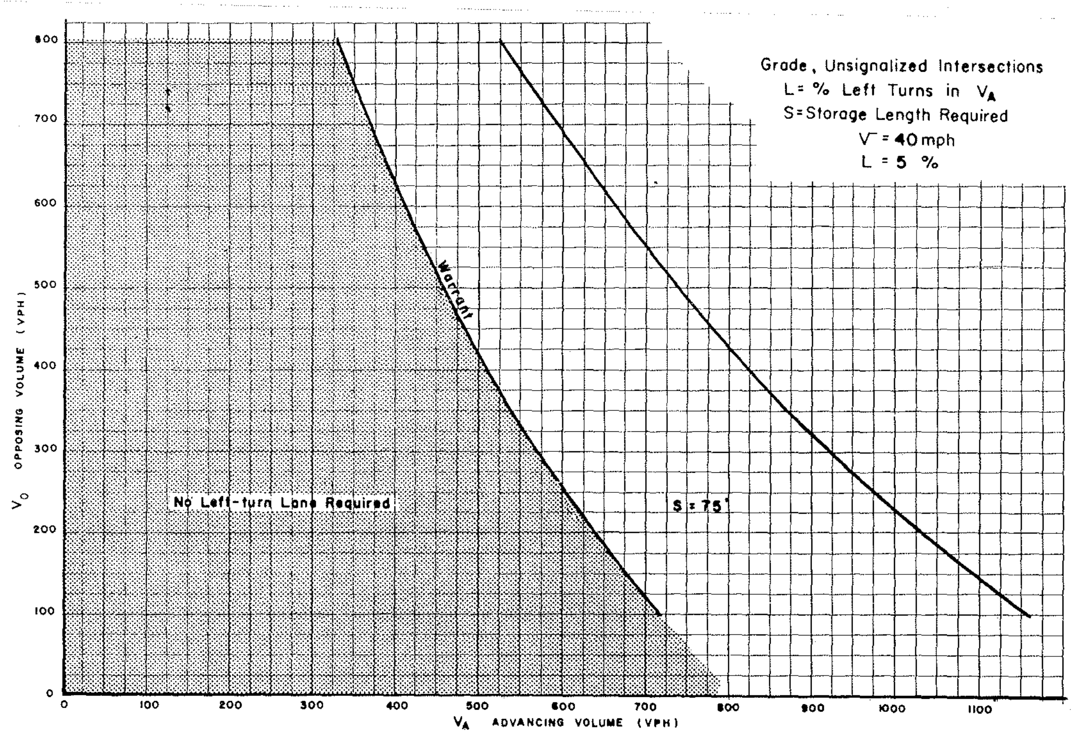

Figure 2. Warrant for left-turn storage lanes on two-lane highways.

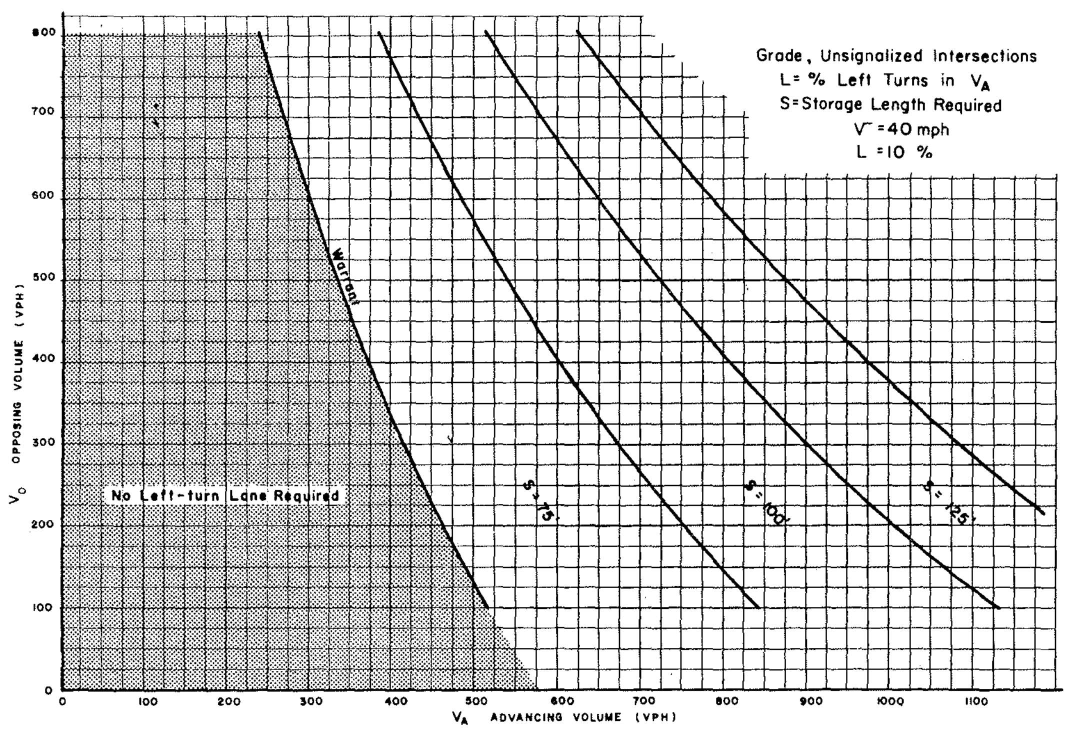

Figure 3. Warrant for left-turn storage lanes on two-lane highways.

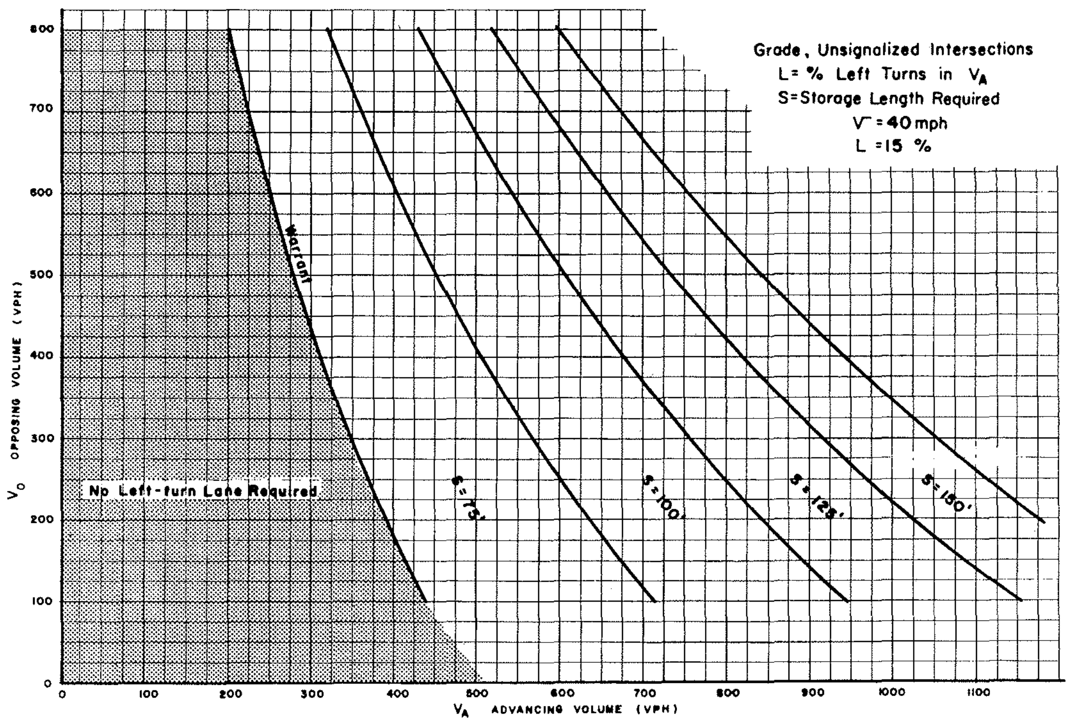

Figure 4. Warrant for left-turn storage lanes on two-lane highways.

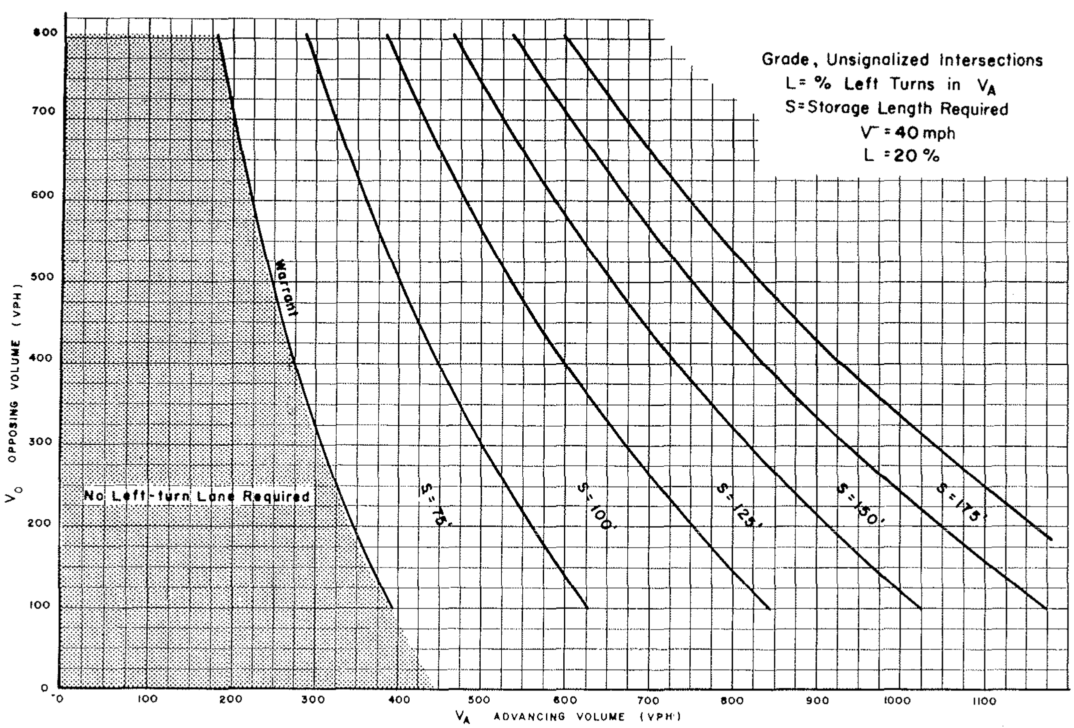

Figure 5. Warrant for left-turn storage lanes on two-lane highways.

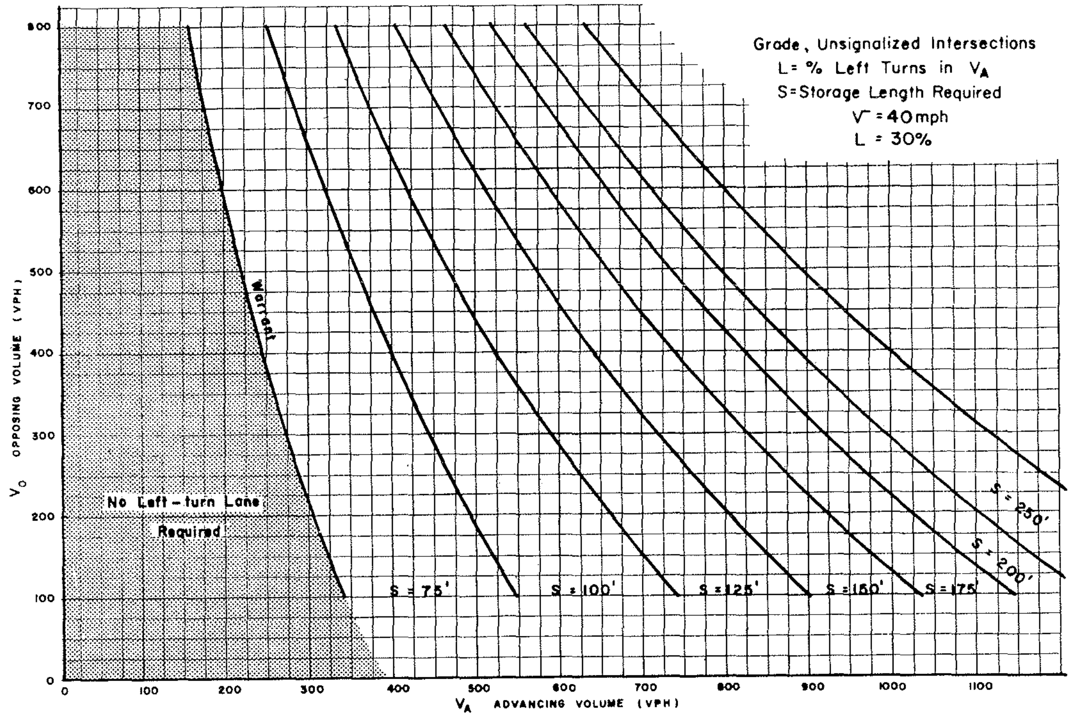

Figure 6. Warrant for left-turn storage lanes on two-lane highways.

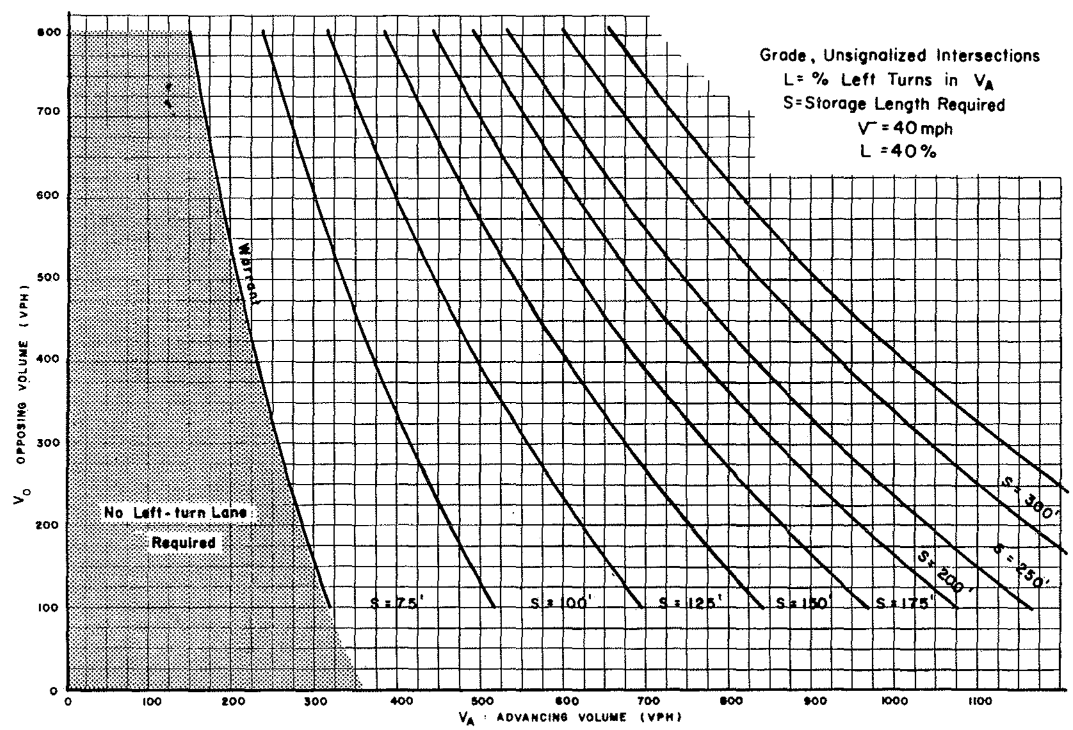

Figure 7. Warrant for left-turn storage lanes on two-lane highways.

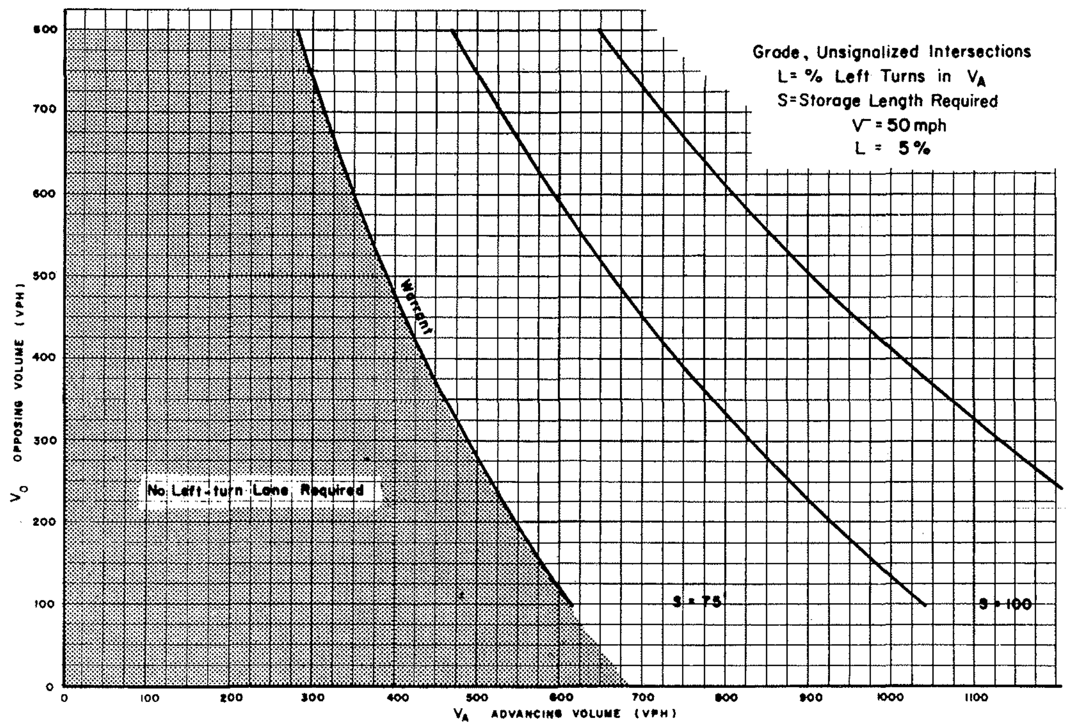

Figure 8. Warrant for left-turn storage lanes on two-lane highways.

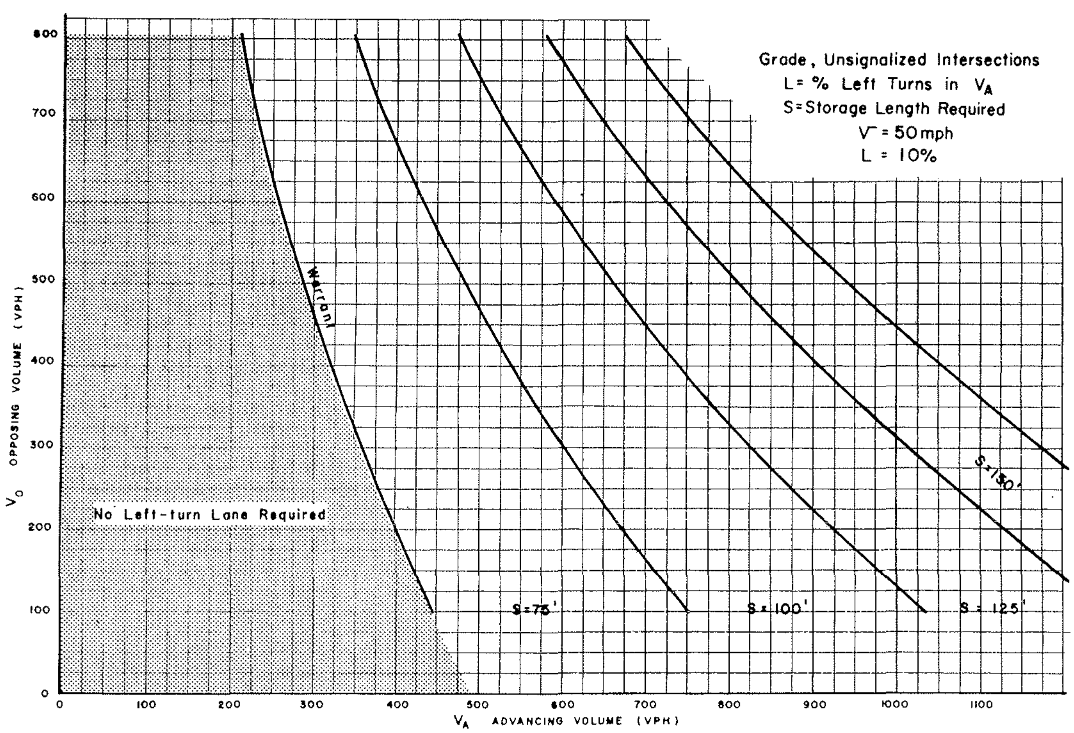

Figure 9. Warrant for left-turn storage lanes on two-lane highways.

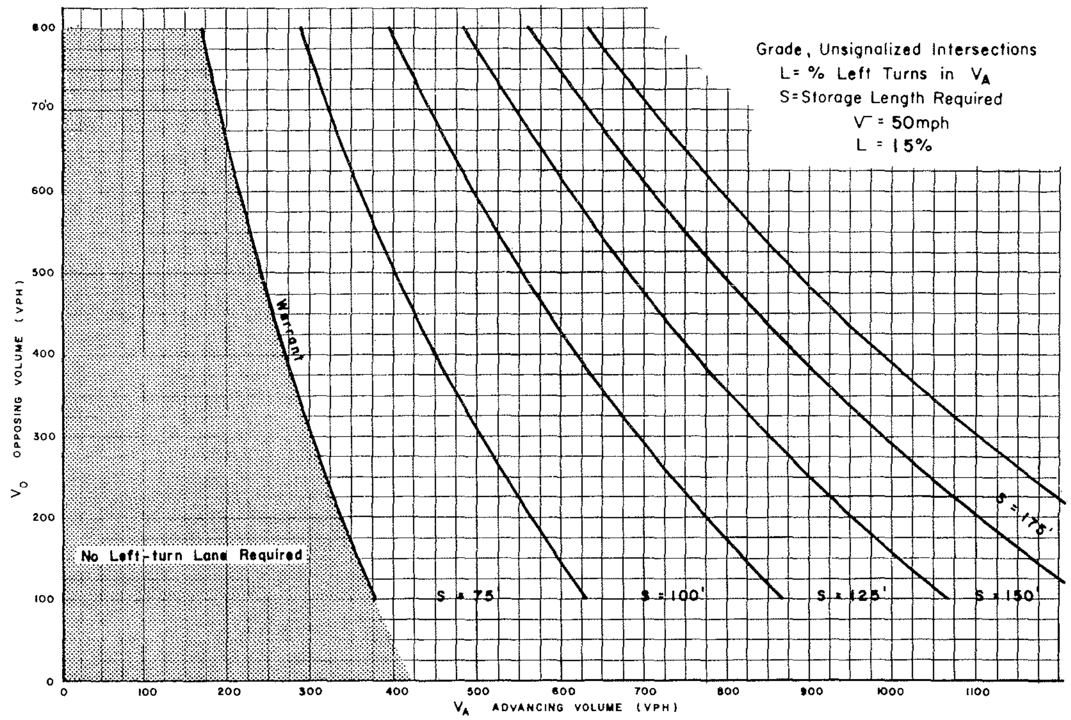

Figure 10. Warrant for left-turn storage lanes on two-lane highways.

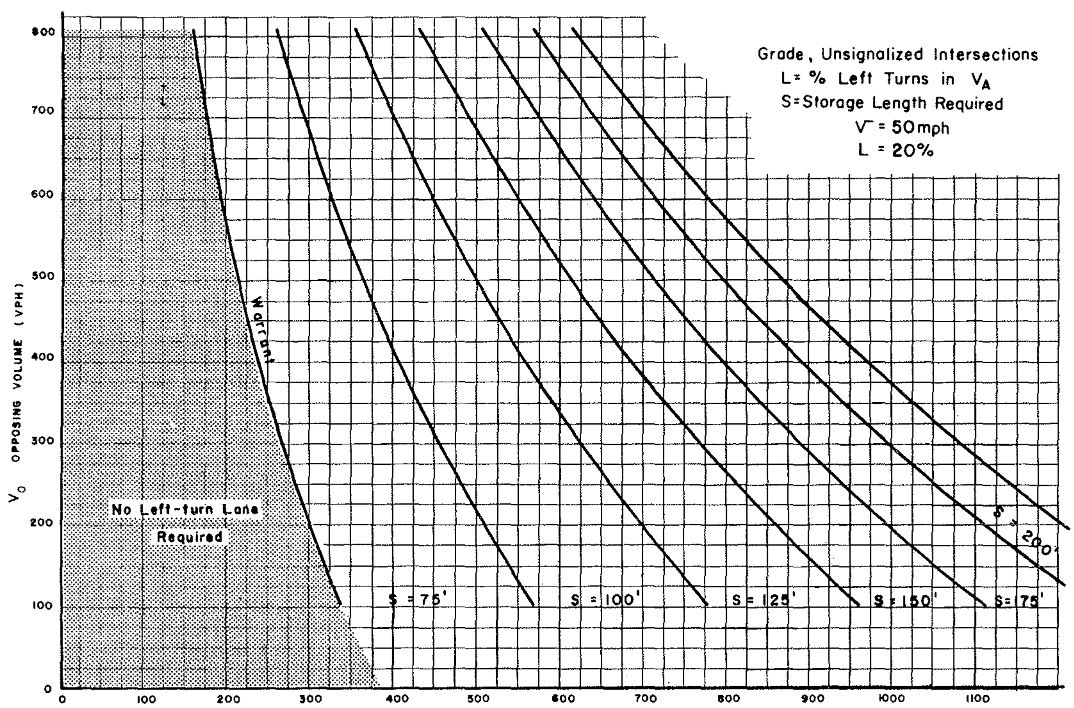

Figure 11. Warrant for left-turn storage lanes on two-lane highways.

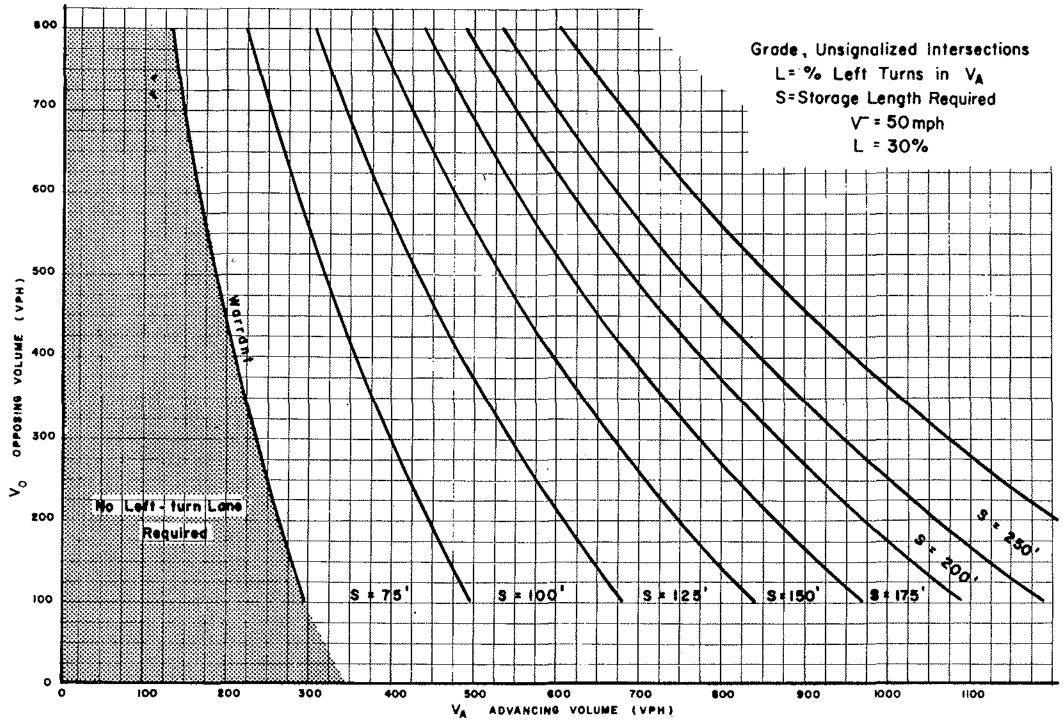

Figure 12. Warrant for left-turn storage lanes on two-lane highways.

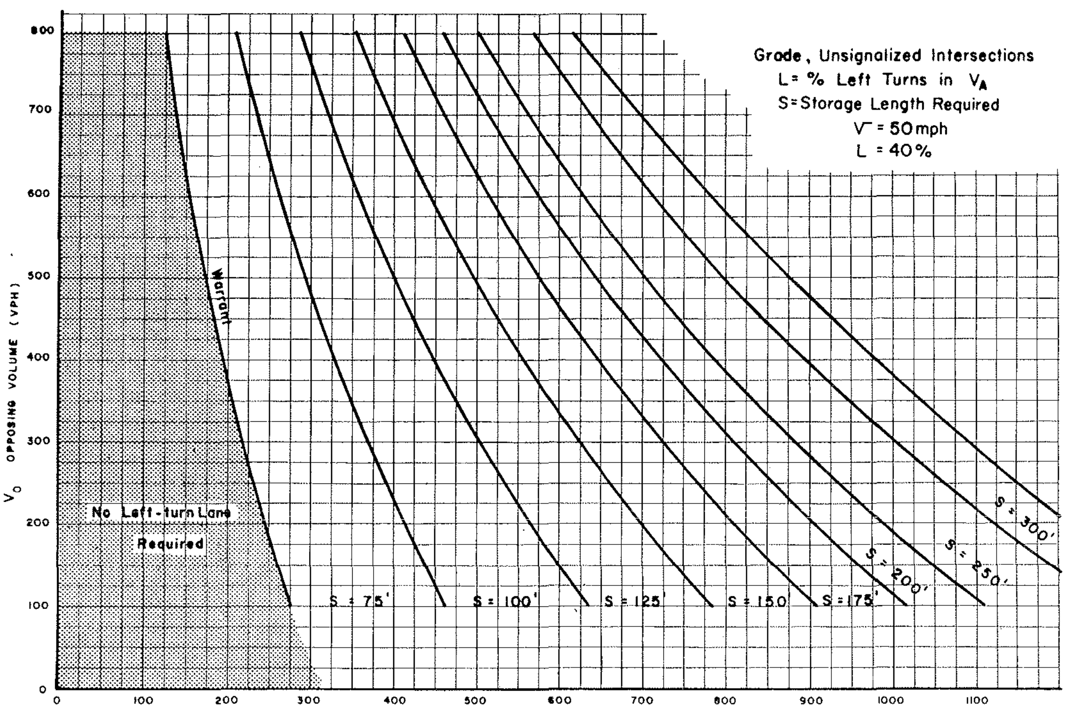

Figure 13. Warrant for left-turn storage lanes on two-lane highways.

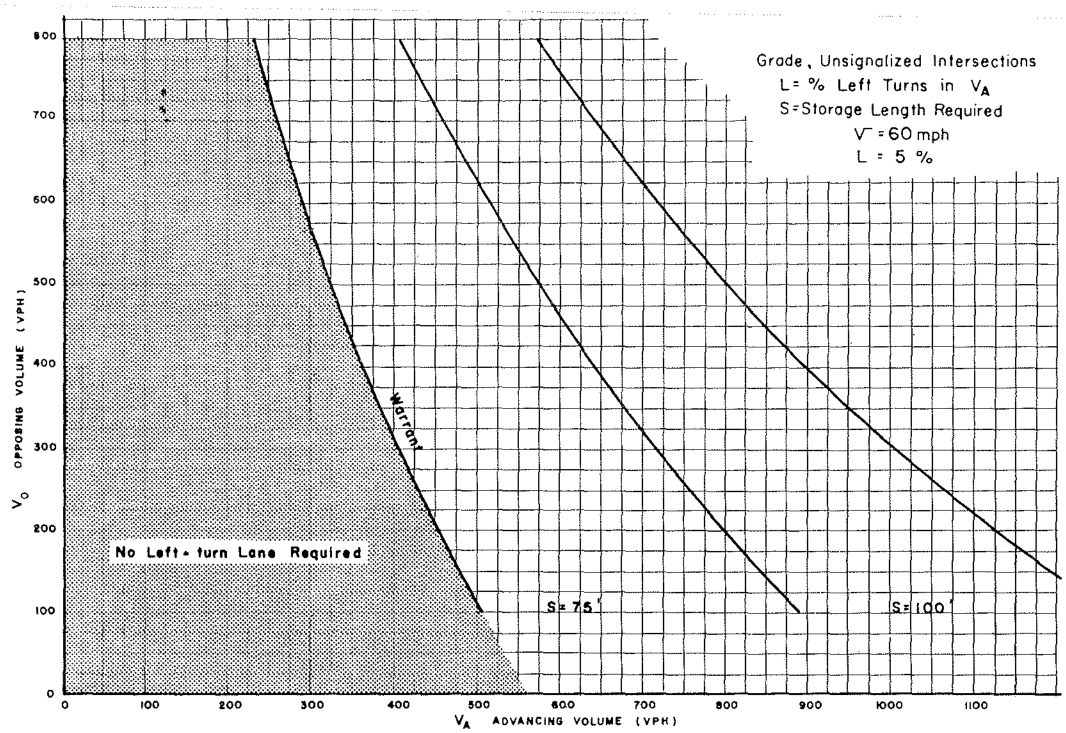

Figure 14. Warrant for left-turn storage lanes on two-lane highways.

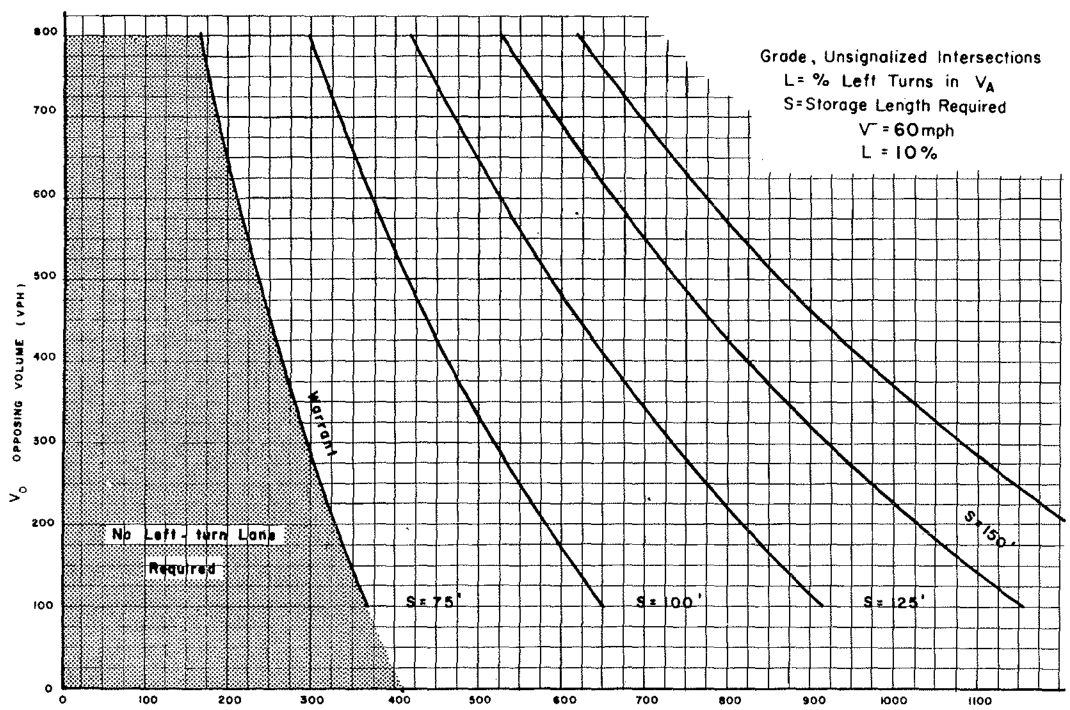

Figure 15. Warrant for left-turn storage lanes on two-lane highways.

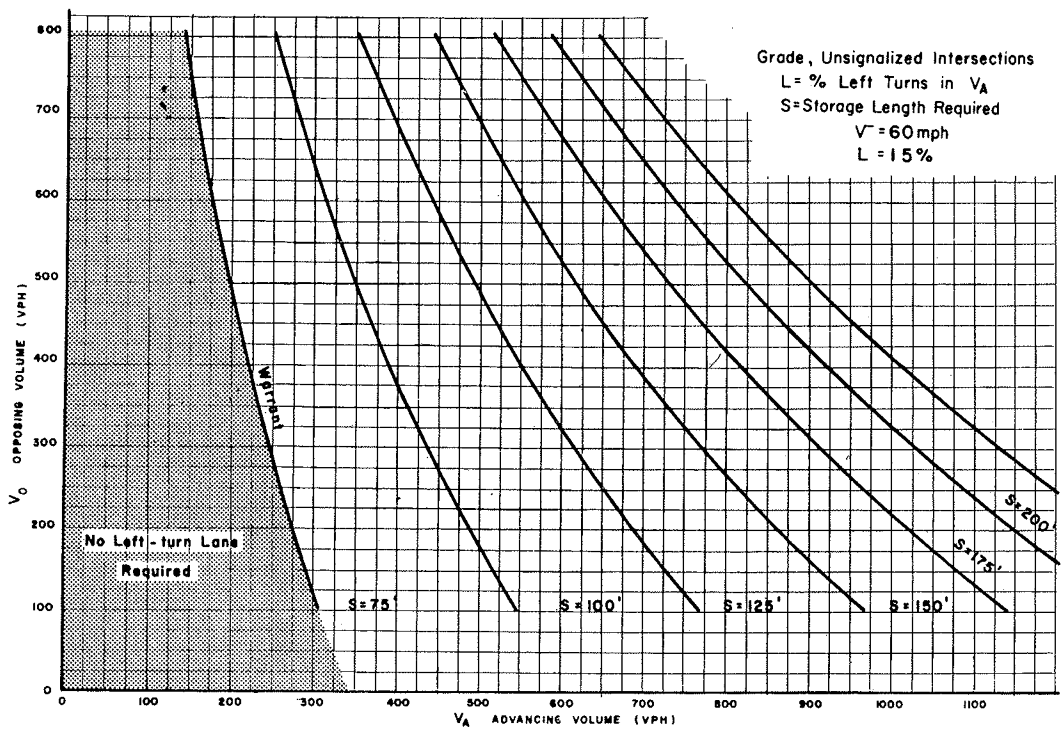

Figure 16. Warrant for left-turn storage lanes on two-lane highways.

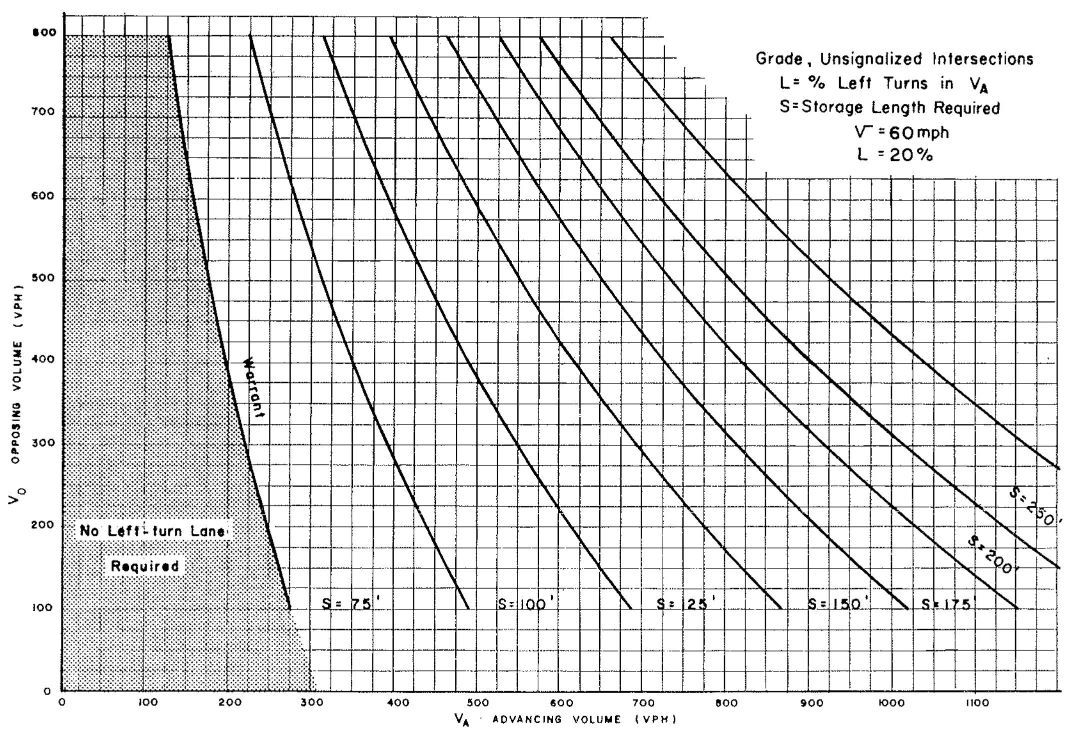

Figure 17. Warrant for left-turn storage lanes on two-lane highways.

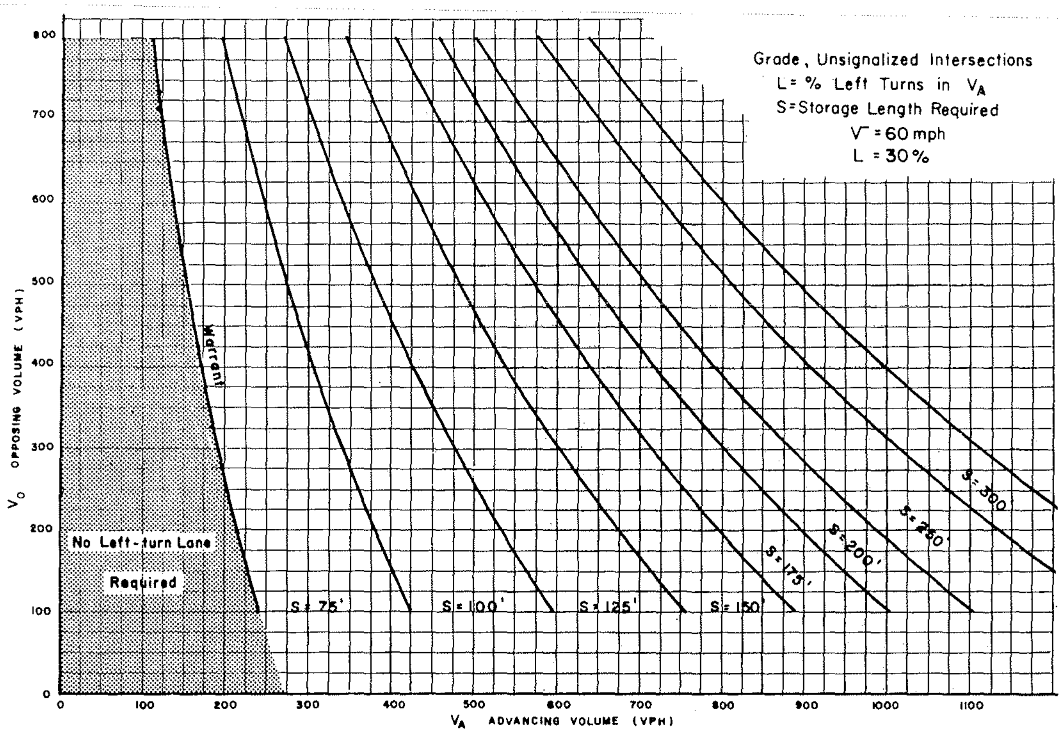

Figure 18. Warrant for left-turn storage lanes on two-lane highways.

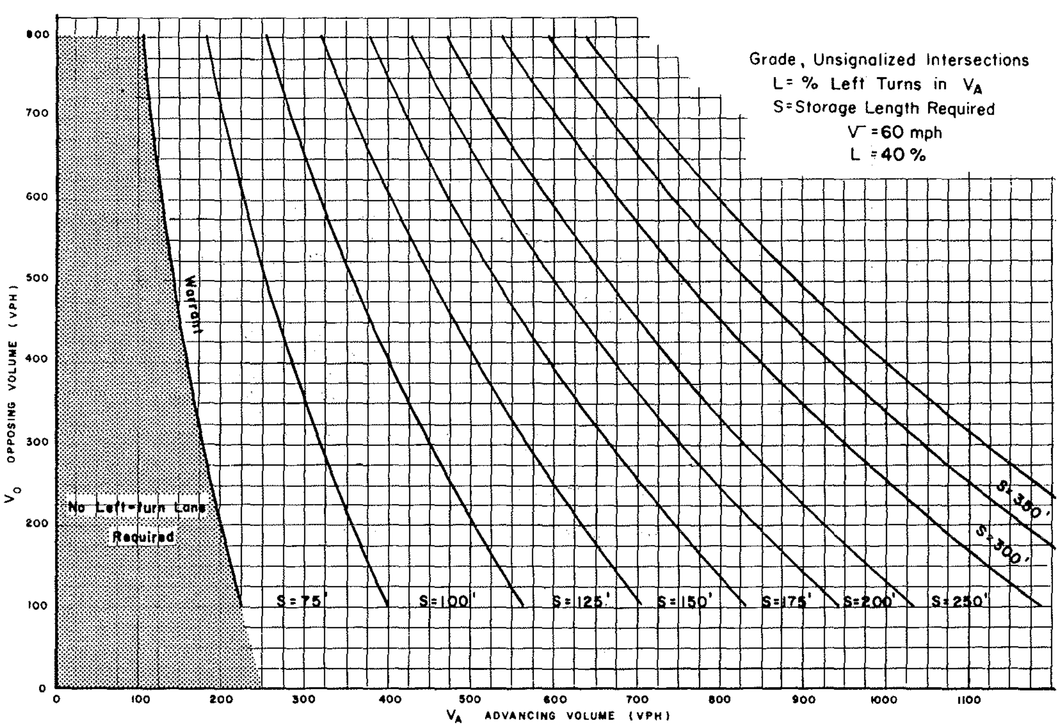

Figure 19. Warrant for left-turn storage lanes on two-lane highways.

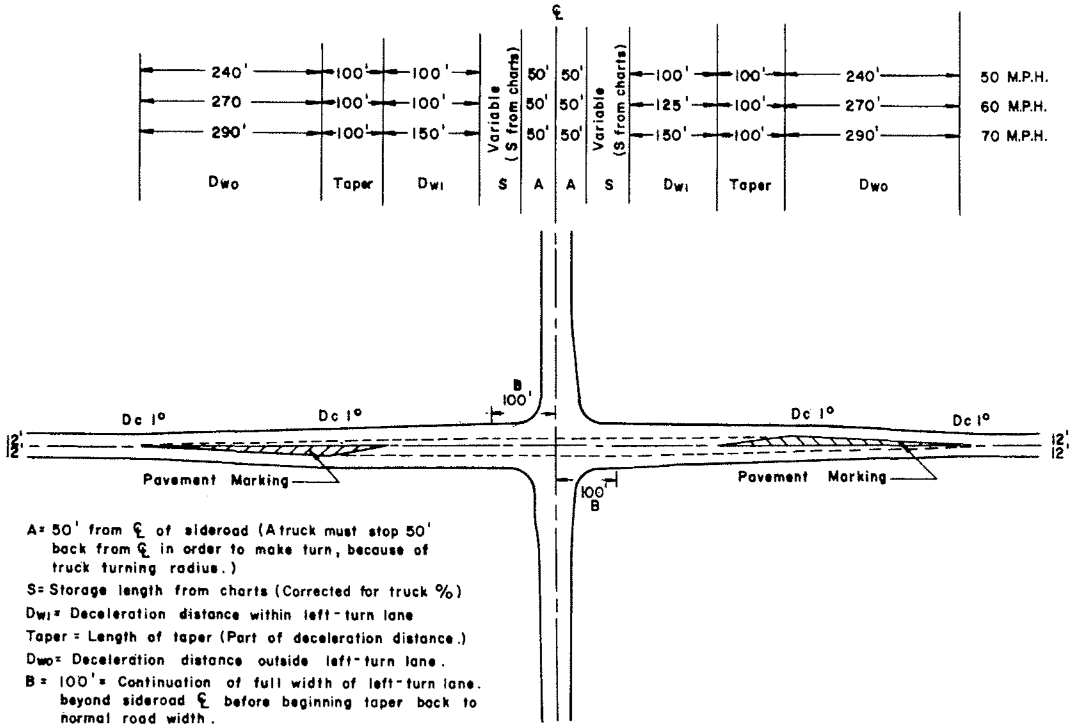

Figure 20. Cross-intersection with left-turn lanes of various design speeds (two-lane highway).

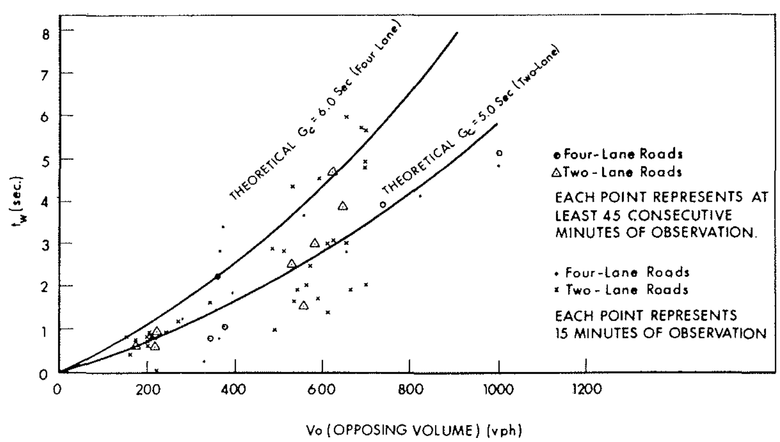

Figure 21. Relationship between t_w and V_o.