Settlement of an Embankment Resting on a Semi-Infinite Elastic Soil

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This paper is concerned with the theoretical determination of the settlement of an embankment. The ground is assumed to be a semi-infinite mass of elastic isotropic homogeneous soil with a plane upper boundary. The load applied by the embankment on the ground surface is approximated by a linearly distributed pressure.

The settlement coefficients have been calculated for seven points of the contact area between the embankment and the soil mass by integration of the Boussinesq formula for the settlement due to a single normal load. The numerical values of these coefficients have been listed in tables and plotted in charts on which the parameters are the geometrical dimensions of the embankment. Practical examples are given.

•THE settlement of a load resting on a soil mass is usually determined by plotting the stress distribution curve and computing the settlement of each soil layer according to the results of consolidation tests. However, if the soil is homogeneous, the settlement is given directly by formulas derived from the theory of elasticity. For example, the settlement of a normal load uniformly distributed on a circular area at the surface of a semi-infinite elastic soil mass (Fig. 1) is given by the well-known formulas

$$w_1 = \frac{1 - \nu^2}{E} 2pR \text{ at the center} \tag{1}$$

$$w_2 = \frac{1 - \nu^2}{E} \frac{4}{\pi} pR \text{ at the edge} \tag{2}$$

in which

- w = settlement (meters or feet);
- p = load (bars or lb/sq ft);
- R = radius of the loaded area (meters or feet);
- E = Young's modulus of elasticity (bars or lb/sq ft); and
- ν = Poisson's ratio (dimensionless).

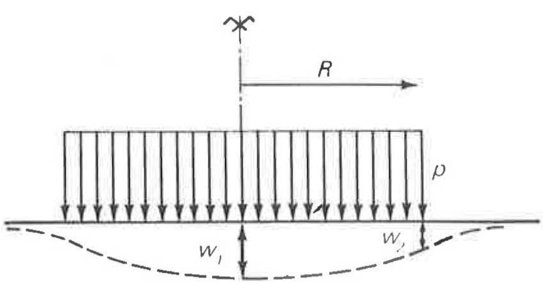

Figure 1. Settlement of a uniformly loaded circular area.

These relations, established by Boussinesq (2, p. 140), are based on the assumptions that (a) the soil mass is semi-infinite with a plane horizontal upper boundary; (b) the soil behaves as an isotropic homogeneous continuum

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which follows Hooke's law (linear elasticity with small strains); (c) the load is perfectly flexible and normal and there is no friction between it and the boundary of the soil mass; and (d) body forces are set equal to zero.

The same assumptions will be used here to obtain the expressions for the settlement of an embankment resting on a horizontal ground. It is pointed out that the formulas thus obtained are likely to be used in the case of a soil slightly heterogeneous if one may estimate the mean values of the modulus E and the ratio ν.

PRINCIPLE OF COMPUTATIONS

The settlement of a point of the ground surface due to a single normal load (Fig. 2) is also given by a formula established by Boussinesq (2, p. 100):

$$w = \frac{1 - \nu^2}{E} \frac{N}{\pi r} \tag{3}$$

in which

N = concentrated normal load (newtons or pounds); and
r = horizontal distance from load N to point P where the settlement is expressed (meters or feet).

The settlement due to a normal load distributed over an area of the ground surface is obtained by integrating Eq. 3. For example, Eq. 1 is given by

$$w = \frac{1 - \nu^2}{E} \frac{p}{\pi} \int_0^{2\pi} d\theta \int_0^{R} \frac{r dr}{r} = \frac{1 - \nu^2}{E} 2pR \tag{4}$$

in which θ is the angle in polar coordinates.

The same process will be used for the computation of the settlement due to the load of an embankment; it will be accomplished with integrals similar to the one of Eq. 4 but, owing to the complexity of the load distribution, the computations are much longer.

THE EMBANKMENT LOAD

Let us consider the embankment shown in Figure 3a. The load applied to the ground surface can be approximated by a linearly distributed normal pressure as shown in Figure 3b. This pressure increases from zero along the edge of the loaded rectangular area to the uniform pressure p on the inner rectangle. The value of p is given by

$$p = \gamma h = \rho g h \tag{5}$$

in which

p = pressure on the inner rectangle (bars or lb/sq ft);
γ = unit weight of soil in embankment (newton/m^3 or lb/cu ft);
ρ = density of soil in embankment (kg/m^3);
h = height of the embankment (meters or feet); and
g = acceleration of gravity (9.81 m/s^2).

Actually, the contact stresses between the embankment and the ground surface are not exactly normal (Fig. 4). But one may argue that the computation based on the assumption of normal pressure yields a good approximation since

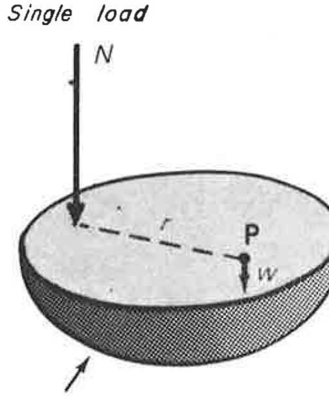

Figure 2. Settlement at point P due to a single normal load N at distance r.

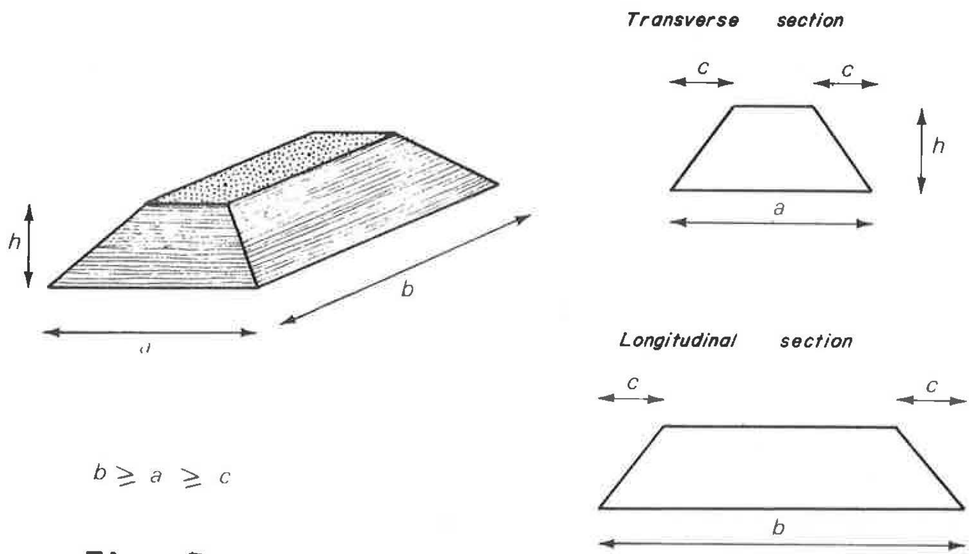

Fig. 3a

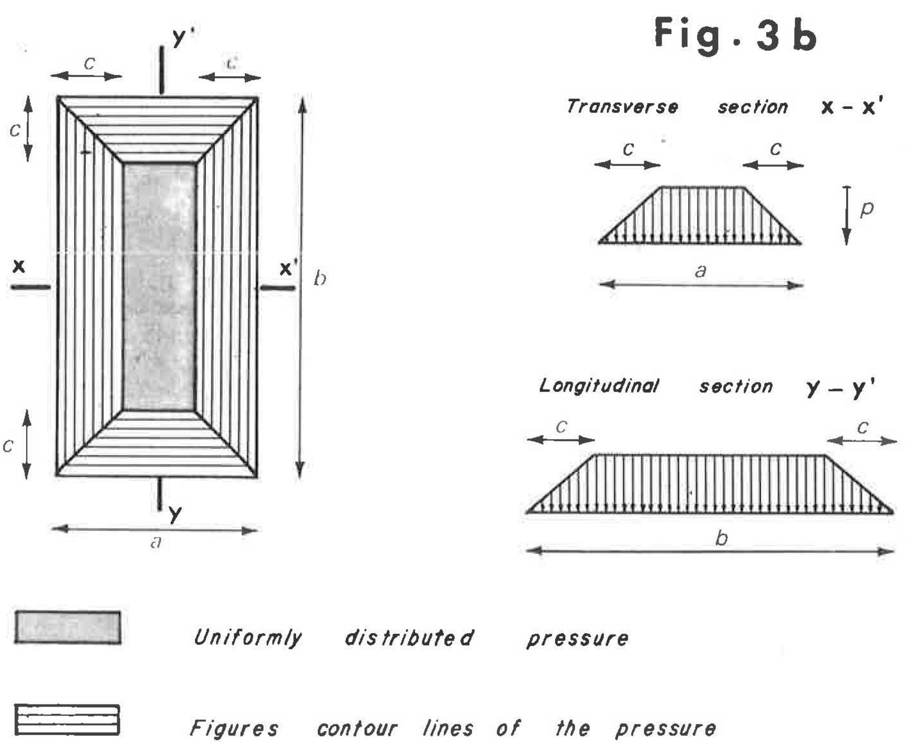

Figure 3. The embankment is defined in Figure 3a and the load applied by it on the ground surface is shown in Figure 3b (a = width of embankment, b = length of embankment, c = width of slope).

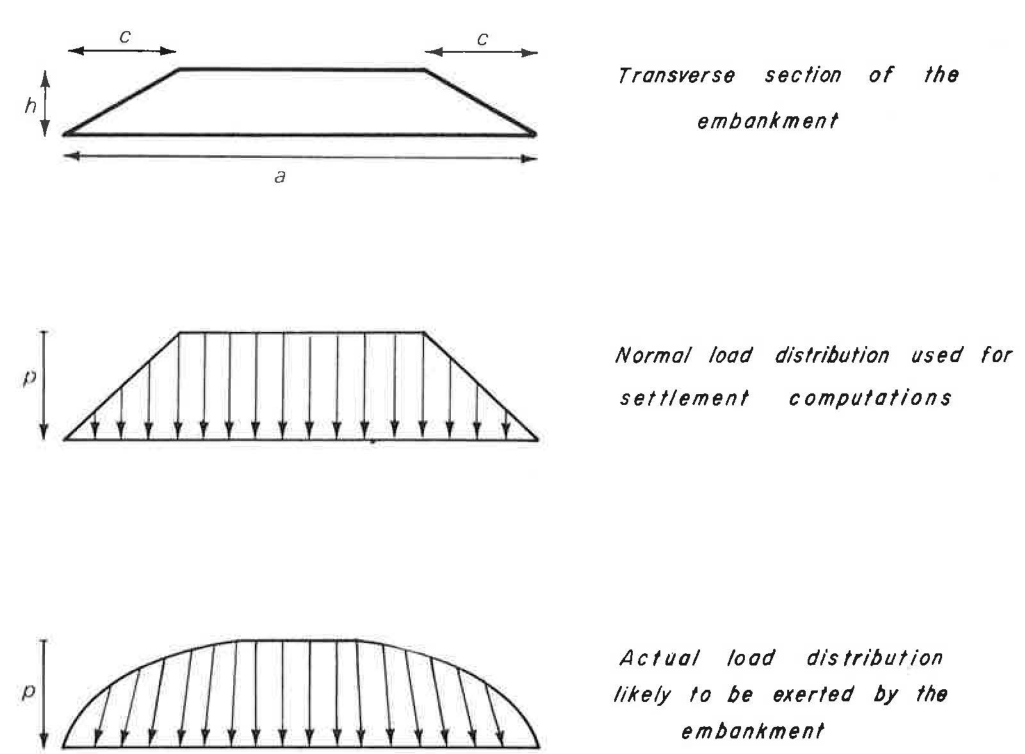

Figure 4. Comparison between the normal load distribution used for the computations and the load distribution likely to be exerted by the embankment on the ground surface.

1. The shearing contact stresses are probably small. For example, calculations for special cases give a maximum value of about p/10 according to Bishop (1, p. 34) and p/7 according to Christensen (3, p. 75).

2. Settlements due to tangential loads are smaller than settlements due to normal loads. For example, at the corner of a square area uniformly loaded:

$$w = \frac{1 - \nu^2}{E} \, a p \; 0.56 \quad \text{for a normal load p}$$

$$w = \frac{(1 + \nu)(1 - 2\nu)}{E} \, a t \; 0.18 \quad \text{for a tangential load t}$$

according to Vogt (5, p. 24). Hence, for $\nu = 0.3$,

$$w = \frac{ap}{E} \; 0.51$$

$$w = \frac{at}{E} \; 0.09$$

in which a is the side of the square.

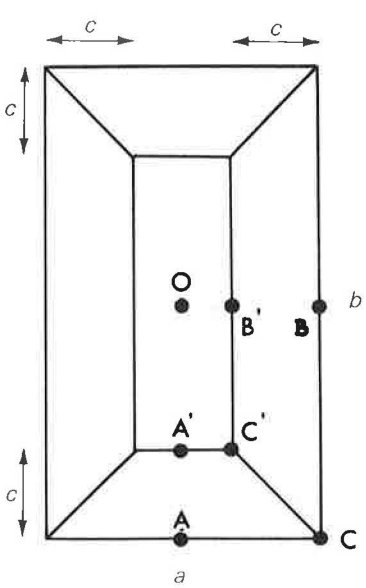

Figure 5. Location of points where the settlement is computed.

PRACTICAL UTILIZATION OF RESULTS

The settlements have been computed in seven points of the loaded area (Fig. 5). Thus seven coefficients have been obtained; their formal expressions are given later. The numerical values of these coefficients

were determined by computer and are shown in Charts 1 to 7 in the Appendix. The settlement of a point is given by:

$$w = \frac{1 - \nu^2}{E} \, p\,a\,K \tag{6}$$

in which

E = Young's modulus of elasticity (bars or lb/sq ft);
ν = Poisson's ratio (dimensionless);
p = uniform normal load over the central rectangular area (bars or lb/sq ft);
a = width of the embankment (meters or feet); and
K = settlement coefficient related to the considered point (dimensionless).

For example, at point B:

$$w_B = \frac{1 - \nu^2}{E} \, p\,a\,K_B \tag{7}$$

(the values of K_B being given by Chart 3 in the Appendix).

Let us notice that an embankment of infinite length resting on a semi-infinite mass yields an infinite settlement as predicted by the plane strain theory of elasticity.

NUMERICAL EXAMPLES

Let us consider an embankment 144 m (480 ft) long, 96 m (320 ft) wide, 12 m (40 ft) high and with a lateral slope of 26.5°. The unit weight of the material in fill is 1.78 t/m^3 (110 lb/cu ft) and the elastic properties of soil are E = 500 bars = 1.05×10^6 lb/sq ft and $\nu = 0.3$. What settlements are to be expected?

First, compute c, the width of the slope:

$$c = \frac{h}{\tan 26.5^\circ} = 2h = 24 \text{ m (80 ft)}$$

Hence, c/a = 0.25 and b/a = 1.5. Furthermore, p = 2.1 bars (4400 lb/sq ft) as calculated from Eq. 5.

The settlement at any point is given by Eq. 6:

$$w = \frac{1 - 0.09}{500} \times 2.1 \times 96 \times K = \frac{1 - 0.09}{1.05 \times 10^6} \times 4400 \times 320 \times K$$

$$= 0.367 \text{ K (m)} = 1.22 \text{ K (ft)}$$

$$= 36.7 \text{ K (cm)} = 14.6 \text{ K (in.)}$$

Hence, by decreasing values:

$K_O = 1.064$	$w_O \simeq 39$ cm $\simeq 15.5$ in.
$K_{B'} = 0.954$	$w_{B'} \simeq 35$ cm $\simeq 14$ in.
$K_{A'} = 0.882$	$w_{A'} \simeq 32$ cm $\simeq 13$ in.
$K_{C'} = 0.803$	$w_{C'} \simeq 29$ cm $\simeq 11.5$ in.
$K_B = 0.580$	$w_B \simeq 21$ cm $\simeq 8.5$ in.
$K_A = 0.515$	$w_A \simeq 19$ cm $\simeq 7.5$ in.
$K_C = 0.387$	$w_C \simeq 14$ cm $\simeq 5.5$ in.

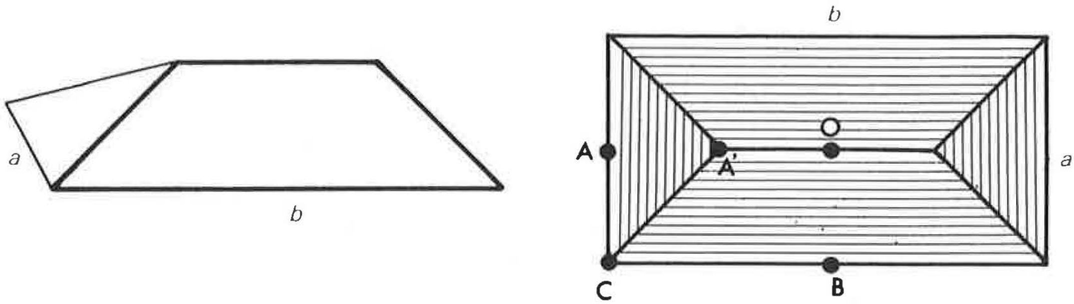

Figure 6. Shape of the embankment in the special case, a = 2c, and location of the points where the settlement is computed.

If the same embankment reaches the height of 24 m (80 ft), its shape is shown in Figure 6. In this case c/a = 0.5 and p = γh = 4.2 bars = 8800 lb/sq ft. The settlement is given by

$$w = \frac{1 - 0.09}{500} \times 4.2 \times 96 \times K = \frac{1 - 0.09}{1.05 \times 10^6} \times 8800 \times 320 \times K$$

$$= 73.4\ K\ (cm) = 29.3\ K\ (in.)$$

The values thus calculated are

$K_O = K_{B'} = 0.748$	$w_O = w_{B'} \simeq 55$ cm $\simeq 22$ in.
$K_{A'} = K_{C'} = 0.711$	$w_{A'} = w_{C'} \simeq 52$ cm $\simeq 21$ in.
$K_B = 0.352$	$w_B \simeq 26$ cm $\simeq 10$ in.
$K_A = 0.307$	$w_A \simeq 22$ cm $\simeq 9$ in.
$K_C = 0.232$	$w_C \simeq 17$ cm $\simeq 7$ in.

EXPRESSIONS OF SETTLEMENT COEFFICIENTS

The calculations are too lengthy to be reproduced in detail here, but since the main steps and the main results may be of interest to investigators dealing with similar problems they have been summarized in a Supplement. * The formal expressions of the settlement coefficients whose numerical values are found in Charts 1 through 7 in the Appendix (see Fig. 5 for location of points O, A, B, C, A′, B′ and C′) are as follows:

$$\alpha = \frac{b}{a} \qquad \beta = \frac{c}{a}$$

*Supplement (approximately 16 pages) is available from the Highway Research Board at cost of handling and reproduction. Refer to Supplement XS-13, Highway Research Record 223.

$$K_O = \frac{1}{2\pi}\left[\frac{1}{\beta}\log\left(\alpha+\sqrt{1+\alpha^2}\right)+\frac{\alpha^2}{\beta}\log\frac{1+\sqrt{1+\alpha^2}}{\alpha}\right.$$

$$-\frac{(1-2\beta)^2}{\beta}\log\frac{\alpha-2\beta+\sqrt{(1-2\beta)^2+(\alpha-2\beta)^2}}{1-2\beta}$$

$$-\frac{(\alpha-2\beta)^2}{\beta}\log\frac{1-2\beta+\sqrt{(1-2\beta)^2+(\alpha-2\beta)^2}}{\alpha-2\beta}$$

$$\left.-\frac{(\alpha-1)^2}{\sqrt{2}\,\beta}\log\frac{\sqrt{2\,(1+\alpha^2)}+\alpha+1}{\sqrt{2\,(1-2\beta)^2+2\,(\alpha-2\beta)^2}-4\beta+\alpha+1}\right] \tag{8}$$

$$K_A = \frac{1}{2\pi}\left[\frac{1}{2\beta}\log\left(2\alpha+\sqrt{1+4\alpha^2}\right)+\frac{2\alpha^2}{\beta}\log\frac{1+\sqrt{1+4\alpha^2}}{2\alpha}\right.$$

$$-\frac{1}{2\sqrt{2}\,\beta}\log\frac{\sqrt{2\,(1-2\beta)^2+8\beta^2}+4\beta-1}{\sqrt{2}-1}$$

$$-\frac{(2\alpha-1)^2}{2\sqrt{2}\,\beta}\log\frac{\sqrt{2\,(1-2\beta)^2+8\,(\alpha-\beta)^2}+4\beta-2\alpha-1}{\sqrt{2\,(1+4\alpha^2)}-(1+2\alpha)}$$

$$-\frac{(1-2\beta)^2}{2\beta}\log\frac{\sqrt{4\beta^2+(1-2\beta)^2}-2\beta}{\sqrt{(1-2\beta)^2+4\,(\alpha-\beta)^2}+2\,(\beta-\alpha)}$$

$$-\frac{2\,(\alpha-\beta)^2}{\beta}\log\frac{2\,(\alpha-\beta)}{\sqrt{(1-2\beta)^2+4\,(\alpha-\beta)^2}+2\beta-1}$$

$$\left.-2\beta\log\frac{2\beta}{\sqrt{(1-2\beta)^2+4\beta^2}+2\beta-1}\right] \tag{9}$$

$$K_B = \frac{1}{2\pi}\left[\frac{\alpha^2}{2\beta}\log\frac{2+\sqrt{\alpha^2+4}}{\alpha}+\frac{2}{\beta}\log\frac{\alpha+\sqrt{\alpha^2+4}}{2}\right.$$

$$-\frac{\alpha^2}{2\sqrt{2}\,\beta}\log\frac{\sqrt{2\,(\alpha-2\beta)^2+8\beta^2}+4\beta-\alpha}{\alpha\,(\sqrt{2}-1)}$$

$$-\frac{(2-\alpha)^2}{2\sqrt{2}\,\beta}\log\frac{\sqrt{2\,(\alpha-2\beta)^2+8\,(1-\beta)^2}+4\beta-\alpha-2}{\sqrt{2\alpha^2+8}-(\alpha+2)}$$

$$-\frac{(\alpha-2\beta)^2}{2\beta}\log\frac{-2\beta+\sqrt{4\beta^2+(\alpha-2\beta)^2}}{2\,(\beta-1)+\sqrt{(\alpha-2\beta)^2+4\,(1-\beta)^2}}$$

$$- \frac{2(1-\beta)^2}{\beta} \log \frac{2(1-\beta)}{2\beta - \alpha + \sqrt{(\alpha - 2\beta)^2 + 4(1-\beta)^2}}$$

$$\left. - 2\beta \log \frac{2\beta}{2\beta - \alpha + \sqrt{(\alpha - 2\beta)^2 + 4\beta^2}} \right] \tag{10}$$

$$K_C = \frac{1}{2\pi} \left[\frac{1}{\beta} \log (\alpha + \sqrt{1+\alpha^2}) + \frac{\alpha^2}{\beta} \log \frac{1 + \sqrt{1+\alpha^2}}{\alpha} \right.$$

$$+ \beta \log \frac{\beta - \alpha + \sqrt{\beta^2 + (\alpha-\beta)^2}}{1 - \beta + \sqrt{\beta^2 + (1-\beta)^2}} - \frac{1}{\sqrt{2}\,\beta} \log \frac{\sqrt{2\beta^2 + 2(1-\beta)^2} + 2\beta - 1}{\sqrt{2} - 1}$$

$$- \frac{\alpha^2}{\sqrt{2}\,\beta} \log \frac{\sqrt{2\beta^2 + 2(\alpha-\beta)^2} + 2\beta - \alpha}{\alpha(\sqrt{2} - 1)}$$

$$- \frac{(\alpha-1)^2}{\sqrt{2}\,\beta} \log \frac{\sqrt{2(1-\beta)^2 + 2(\alpha-\beta)^2} + 2\beta - \alpha - 1}{\sqrt{2(1+\alpha^2)} - (1+\alpha)}$$

$$- \frac{(1-\beta)^2}{\beta} \log \frac{\sqrt{\beta^2 + (1-\beta)^2} - \beta}{\sqrt{(1-\beta)^2 + (\alpha-\beta)^2} + \beta - \alpha} - \frac{(\alpha-\beta)^2}{\beta} \log \frac{\sqrt{\beta^2 + (\alpha-\beta)^2} - \beta}{\sqrt{(1-\beta)^2 + (\alpha-\beta)^2} + \beta - 1}$$

$$\left. + 2\beta \log (1 + \sqrt{2}) \right] \tag{11}$$

$$K_{A'} = \frac{1}{2\pi} \left[- \frac{(1-2\beta)^2}{2\beta} \log \frac{2(\alpha - 2\beta) + \sqrt{(1-2\beta)^2 + 4(\alpha-2\beta)^2}}{1 - 2\beta} \right.$$

$$- \frac{2(\alpha-2\beta)^2}{\beta} \log \frac{1 - 2\beta + \sqrt{(1-2\beta)^2 + 4(\alpha-2\beta)^2}}{2(\alpha - 2\beta)}$$

$$- \frac{(1 - 2\alpha + 2\beta)^2}{2\sqrt{2}\,\beta} \log \frac{\sqrt{2 + 8(\alpha-\beta)^2} + 2\alpha - 2\beta + 1}{\sqrt{2(1-2\beta)^2 + 8(\alpha-2\beta)^2} + 2\alpha - 6\beta + 1}$$

$$- \frac{(1-2\beta)^2}{2\sqrt{2}\,\beta} \log \frac{\sqrt{2 + 8\beta^2} + 2\beta + 1}{(1-2\beta)(\sqrt{2}+1)} + 2\beta \log \frac{1 + \sqrt{1 + 4\beta^2}}{2\beta}$$

$$\left. + \frac{2(\alpha-\beta)^2}{\beta} \log \frac{1 + \sqrt{1 + 4(\alpha-\beta)^2}}{2(\alpha-\beta)} + \frac{1}{2\beta} \log \frac{\sqrt{1 + 4(\alpha-\beta)^2} + 2(\alpha-\beta)}{\sqrt{1 + 4\beta^2} - 2\beta} \right] \tag{12}$$

$$K_{B'} = \frac{1}{2\pi}\left[-\frac{(\alpha - 2\beta)^2}{2\beta}\log\frac{2(1-2\beta)+\sqrt{(\alpha-2\beta)^2+4(1-2\beta)^2}}{\alpha-2\beta} \right.$$

$$-\frac{2(1-2\beta)^2}{\beta}\log\frac{\alpha-2\beta+\sqrt{(\alpha-2\beta)^2+4(1-2\beta)^2}}{2(1-2\beta)}$$

$$-\frac{(\alpha+2\beta-2)^2}{2\sqrt{2}\beta}\log\frac{\sqrt{2\alpha^2+8(1-\beta)^2}+\alpha-2\beta+2}{\sqrt{2(\alpha-2\beta)^2+8(1-2\beta)^2}+\alpha-6\beta+2}$$

$$-\frac{(\alpha-2\beta)^2}{2\sqrt{2}\beta}\log\frac{\sqrt{2\alpha^2+8\beta^2}+\alpha+2\beta}{(\alpha-2\beta)(\sqrt{2}+1)} + 2\beta\log\frac{\sqrt{\alpha^2+4\beta^2}+\alpha}{2\beta}$$

$$\left. +\frac{2(1-\beta)^2}{\beta}\log\frac{\alpha+\sqrt{\alpha^2+4(1-\beta)^2}}{2(1-\beta)} + \frac{\alpha^2}{2\beta}\log\frac{2(1-\beta)+\sqrt{\alpha^2+4(1-\beta)^2}}{-2\beta+\sqrt{\alpha^2+4\beta^2}} \right] \qquad (13)$$

$$K_{C'} = \frac{1}{2\pi}\left[\frac{(1-\beta)^2}{\beta}\log\frac{\alpha-\beta+\sqrt{(1-\beta)^2+(\alpha-\beta)^2}}{-\beta+\sqrt{(1-\beta)^2+\beta^2}} \right.$$

$$+\frac{(\alpha-\beta)^2}{\beta}\log\frac{1-\beta+\sqrt{(1-\beta)^2+(\alpha-\beta)^2}}{-\beta+\sqrt{(\alpha-\beta)^2+\beta^2}}$$

$$+\beta\log\frac{\alpha-\beta+\sqrt{\beta^2+(\alpha-\beta)^2}}{\beta(\sqrt{2}-1)} + \beta\log\frac{1-\beta+\sqrt{\beta^2+(1-\beta)^2}}{\beta(\sqrt{2}-1)}$$

$$-\frac{(1-2\beta)^2}{\beta}\log\frac{\alpha-2\beta+\sqrt{(1-2\beta)^2+(\alpha-2\beta)^2}}{1-2\beta}$$

$$-\frac{(\alpha-2\beta)^2}{\beta}\log\frac{1-2\beta+\sqrt{(1-2\beta)^2+(\alpha-2\beta)^2}}{\alpha-2\beta}$$

$$-\frac{(1-2\beta)^2}{\beta\sqrt{2}}\log\frac{\sqrt{2\beta^2+2(1-\beta)^2}+1}{(1-2\beta)(\sqrt{2}+1)} - \frac{(\alpha-2\beta)^2}{\beta\sqrt{2}}\log\frac{\sqrt{2\beta^2+2(\alpha-\beta)^2}+\alpha}{(\alpha-2\beta)(\sqrt{2}+1)}$$

$$\left. -\frac{(\alpha-1)^2}{\beta\sqrt{2}}\log\frac{\sqrt{2(1-\beta)^2+2(\alpha-\beta)^2}+1+\alpha-2\beta}{\sqrt{2(1-2\beta)^2+2(\alpha-2\beta)^2}+1+\alpha-4\beta} \right] \qquad (14)$$

EXPRESSION OF THE COEFFICIENTS IN THE SPECIAL CASE a = 2c

The shape of the embankment is shown for this case in Figure 6. The settlement coefficients have been directly calculated at points O, A, B, C and A′ and we checked that the same expressions were obtained by setting $\beta = 0.5$ in Eqs. 8 through 14.

$$\alpha = \frac{b}{a}$$

$$K_O(a = 2c) = K_{B'}(a = 2c) = \frac{1}{2\pi}\left[2 \log(\alpha + \sqrt{1+\alpha^2}) + 2\alpha^2 \log\frac{1+\sqrt{1+\alpha^2}}{\alpha} - \sqrt{2}(\alpha - 1)^2 \log\frac{\sqrt{2+2\alpha^2}+1+\alpha}{(\alpha-1)(\sqrt{2}+1)}\right] \tag{15}$$

$$K_A(a = 2c) = \frac{1}{2\pi}\left[\log(2\alpha + \sqrt{1+4\alpha^2}) + 4\alpha^2 \log\frac{1+\sqrt{1+4\alpha^2}}{2\alpha} - \frac{(2\alpha-1)^2}{\sqrt{2}} \log\frac{(2\alpha-1)(\sqrt{2}-1)}{\sqrt{2+8\alpha^2}-1-2\alpha} - \sqrt{2}\log(1+\sqrt{2})\right] \tag{16}$$

$$K_B(a = 2c) = \frac{1}{2\pi}\left[\alpha^2 \log\frac{2+\sqrt{\alpha^2+4}}{\alpha} + 4\log\frac{\alpha+\sqrt{\alpha^2+4}}{2} - \frac{\alpha^2}{\sqrt{2}}\log\frac{2-\alpha+\sqrt{2(\alpha-1)^2+2}}{\alpha(\sqrt{2}-1)} - \frac{(2-\alpha)^2}{\sqrt{2}}\log\frac{-\alpha+\sqrt{2(\alpha-1)^2+2}}{-\alpha-2+\sqrt{2\alpha^2+8}} + 2\log\left(1-\alpha+\sqrt{1+(\alpha-1)^2}\right)\right] \tag{17}$$

$$K_C(a = 2c) = \frac{1}{2\pi}\left[2\log(\alpha+\sqrt{1+\alpha^2}) + 2\alpha^2\log\frac{1+\sqrt{1+\alpha^2}}{\alpha} + \log(1-2\alpha+\sqrt{1+(2\alpha-1)^2}) - (\sqrt{2}-1)\log(1+\sqrt{2}) - (\alpha-1)^2\sqrt{2}\log\frac{2+2\alpha+2\sqrt{2+2\alpha^2}}{2\alpha+\sqrt{2+2(2\alpha-1)^2}} - \alpha^2\sqrt{2}\log\frac{2\alpha(\sqrt{2}+1)}{2(\alpha-1)+\sqrt{2+2(2\alpha-1)^2}}\right] \tag{18}$$

$$K_{A'(a=2c)} = K_{C'(a=2c)} = \frac{1}{2\pi}\left[(2\alpha - 1)^2 \log\frac{1+\sqrt{1+2\alpha-1)^2}}{2\alpha-1} + \log\frac{2\alpha - 1 + \sqrt{1+(2\alpha-1)^2}}{(\sqrt{2}-1)^2} - 2\sqrt{2}(\alpha-1)^2\log\frac{2\alpha+\sqrt{2+2(2\alpha-1)^2}}{2(\alpha-1)(1+\sqrt{2})}\right] \quad (19)$$

EXPRESSION OF THE COEFFICIENTS IN THE SPECIAL CASE c = 0

In this case the embankment load is reduced to a uniformly distributed pressure over a rectangular area. Hence Eqs. 8 through 14 become the well-known formulas

$$K_O = \frac{2}{\pi}\left[\log(\alpha+\sqrt{1+\alpha^2}) + \alpha\log\frac{1+\sqrt{1+\alpha^2}}{\alpha}\right] \quad (20)$$

$$K_{AA'} = \frac{1}{\pi}\left[\log(2\alpha+\sqrt{1+4\alpha^2}) + 2\alpha\log\frac{1+\sqrt{1+4\alpha^2}}{2\alpha}\right] \quad (21)$$

$$K_{BB'} = \frac{1}{\pi}\left[2\log\frac{\alpha+\sqrt{4+\alpha^2}}{2} + \alpha\log\frac{2+\sqrt{4+\alpha^2}}{\alpha}\right] \quad (22)$$

$$K_{CC'} = \frac{1}{\pi}\left[\log(\alpha+\sqrt{1+\alpha^2}) + \alpha\log\frac{1+\sqrt{1+\alpha^2}}{\alpha}\right] \quad (23)$$

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Appendix

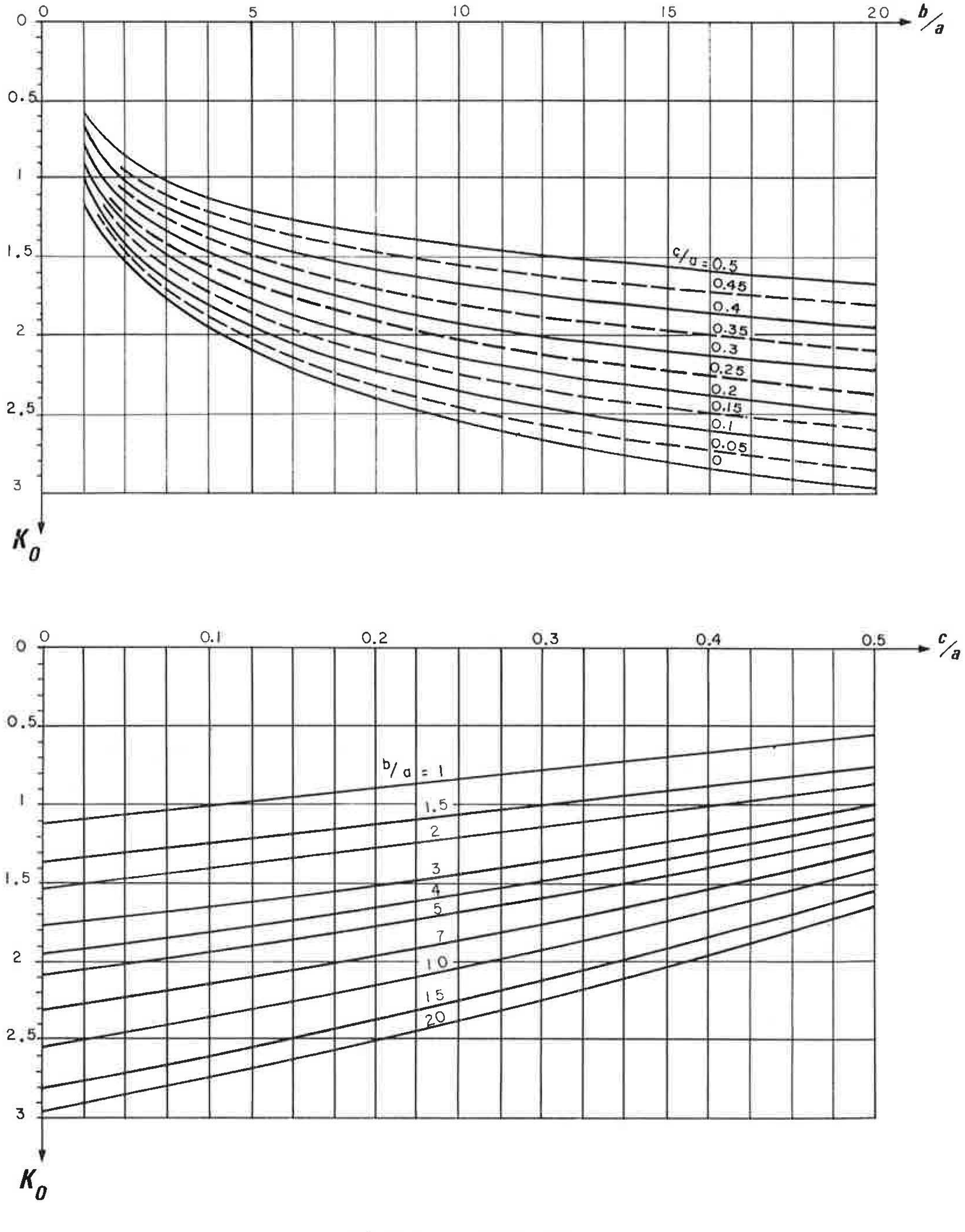


Chart 1. Coefficient K_O.

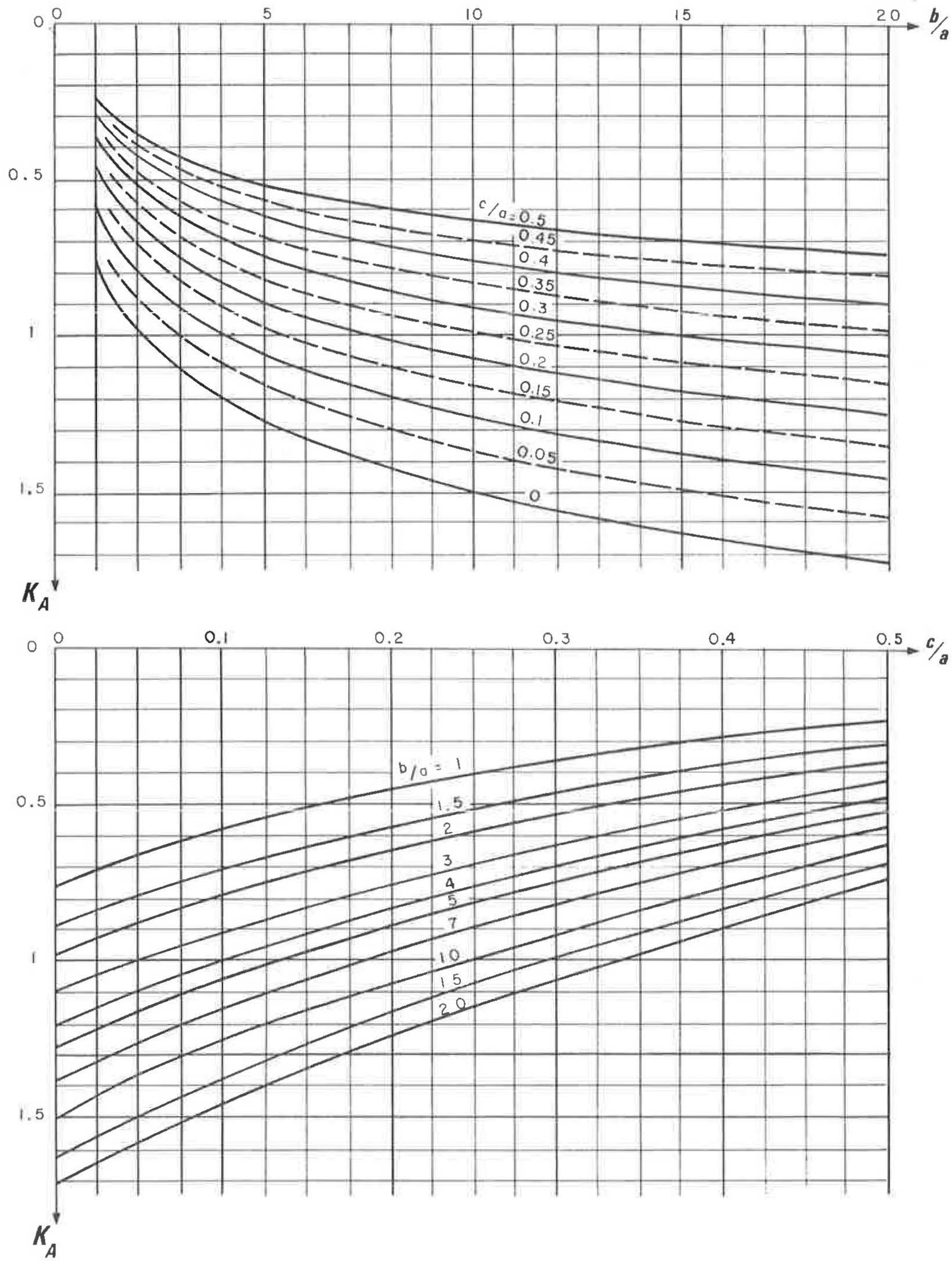
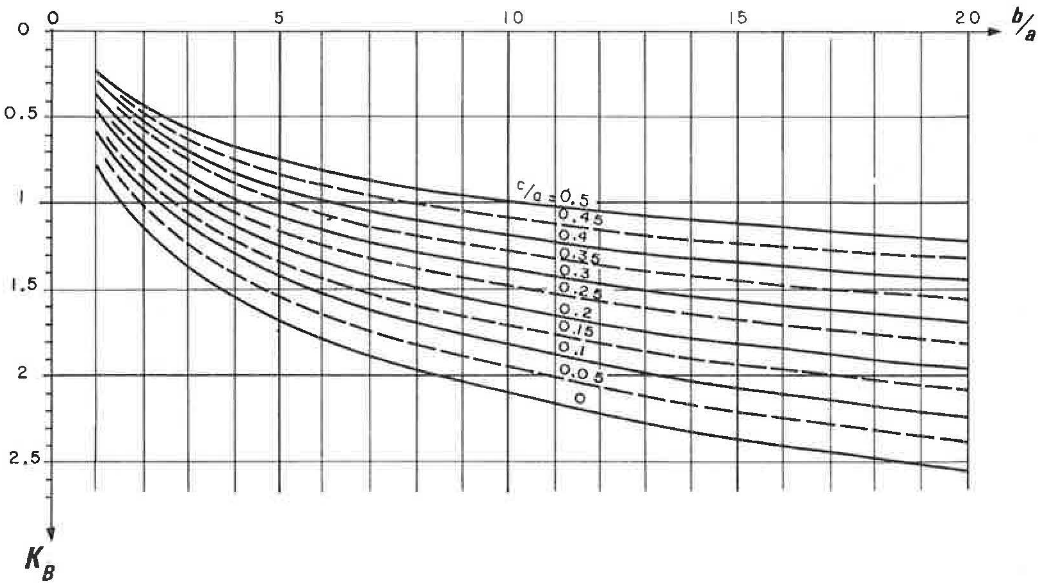


Chart 2. Coefficient K_A.



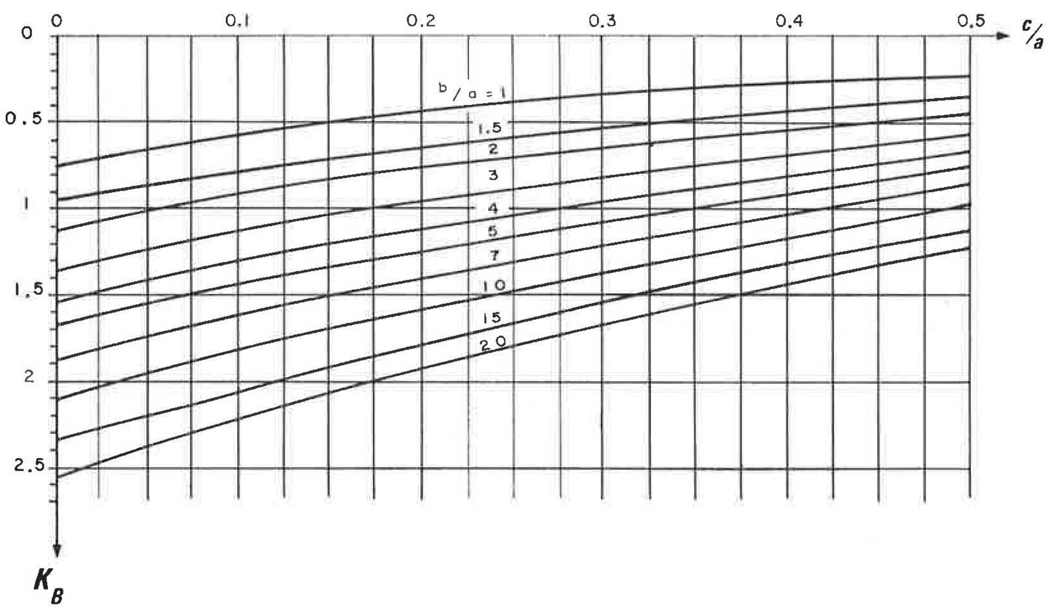
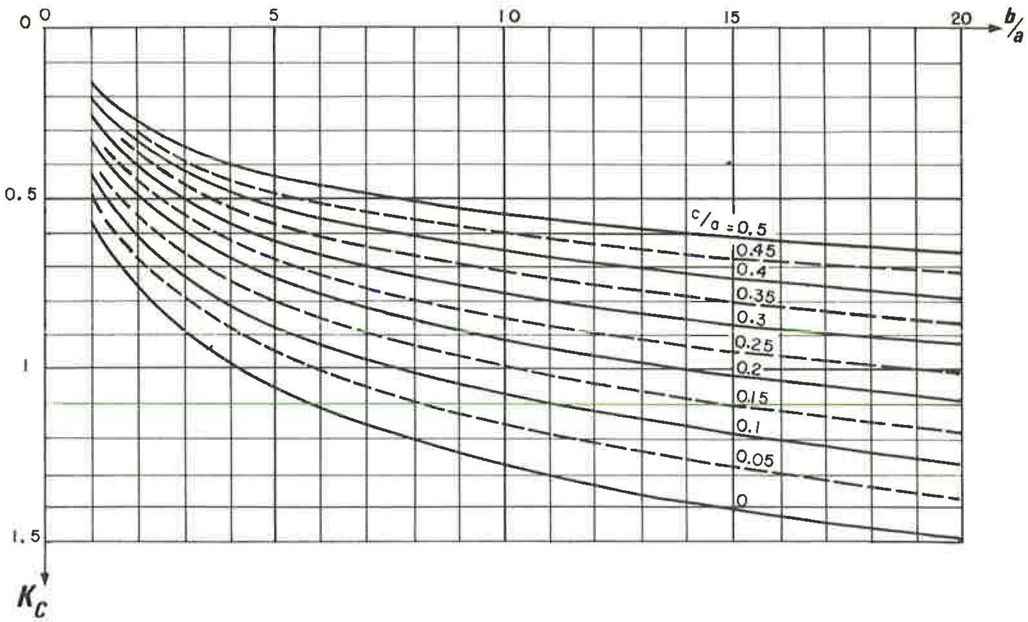


Chart 3. Coefficient K_B.



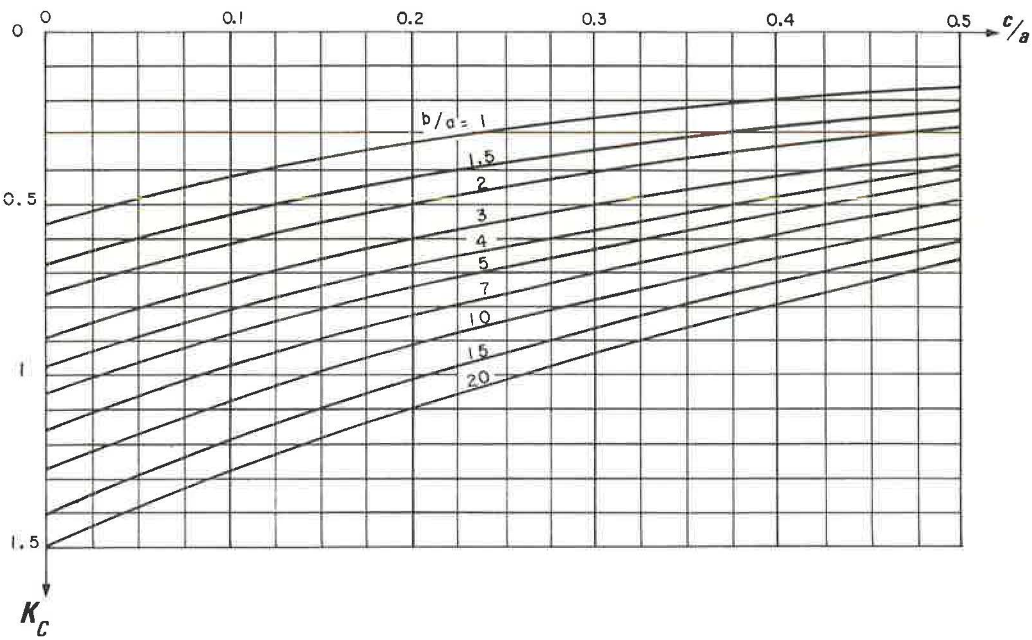


Chart 4. Coefficient K_C.

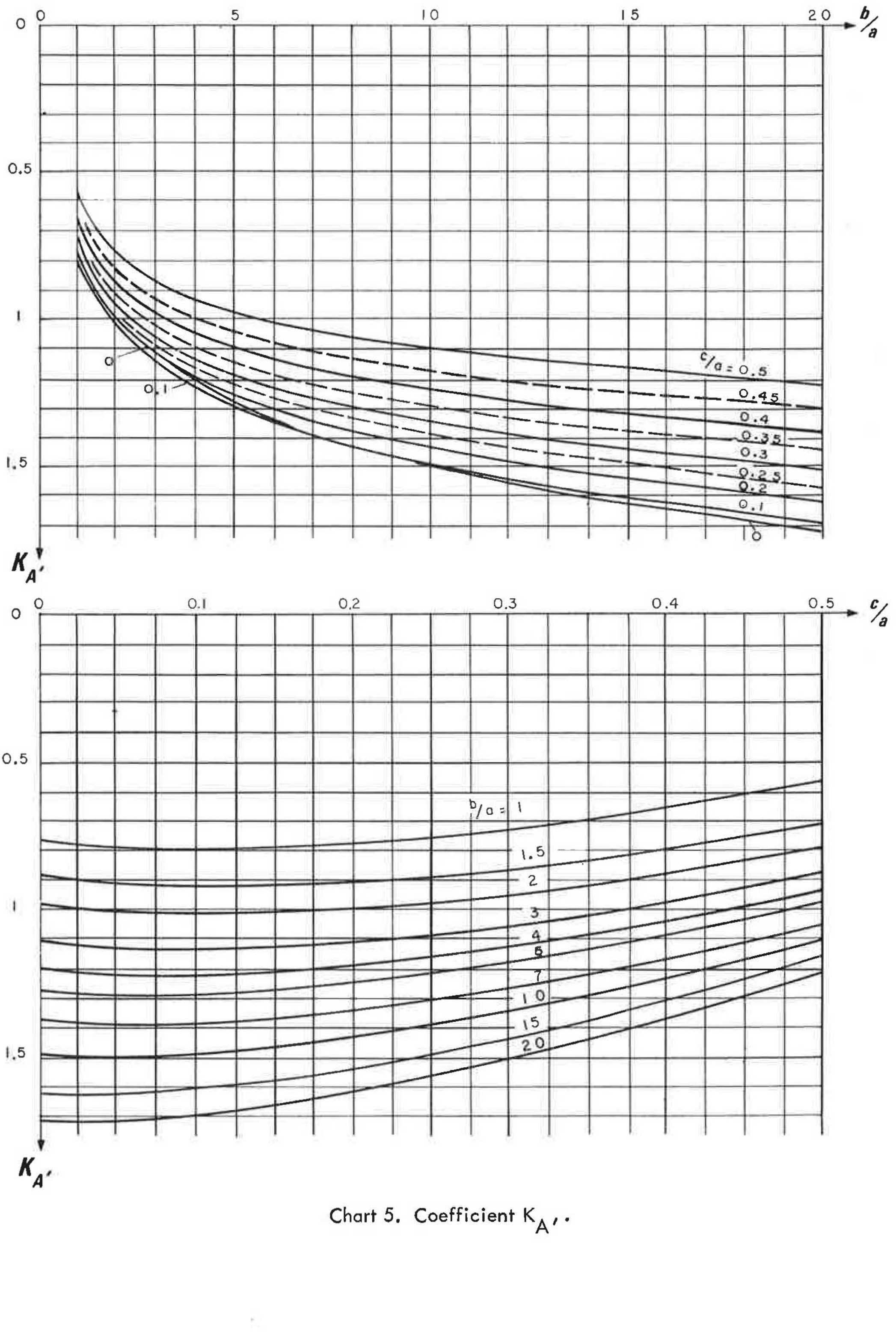
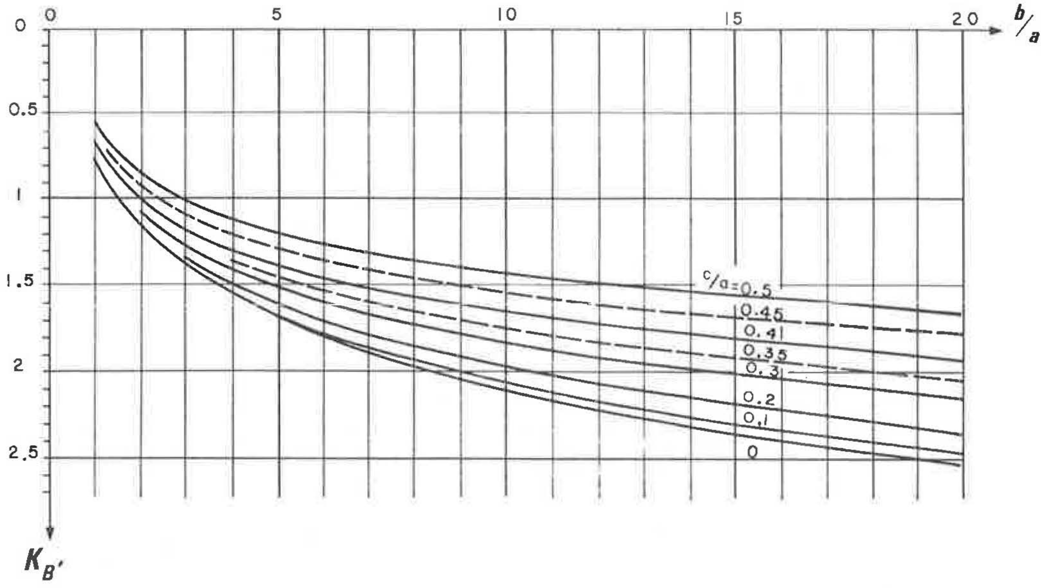


Chart 5. Coefficient $K_{A'}$.



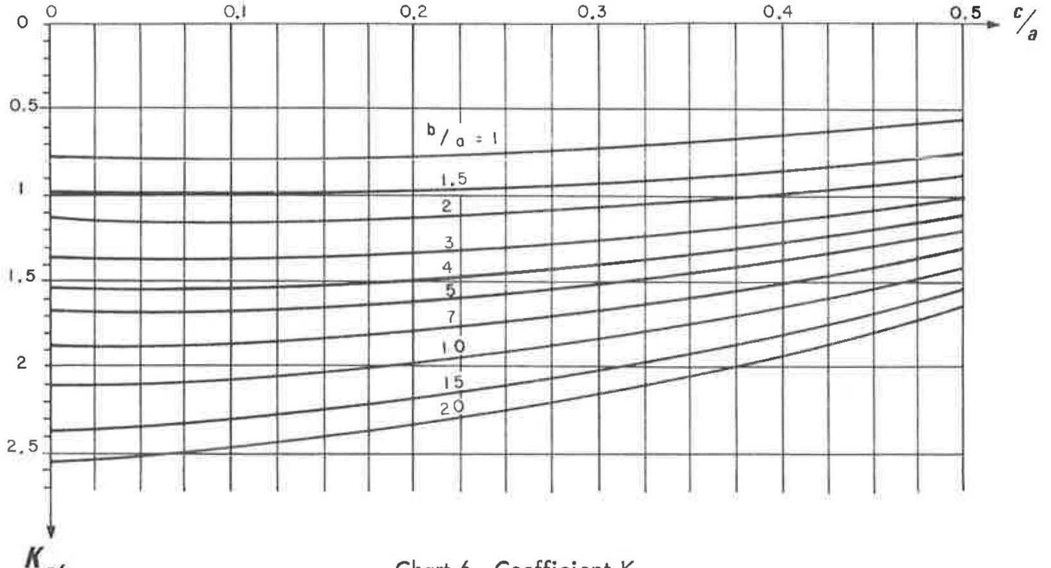
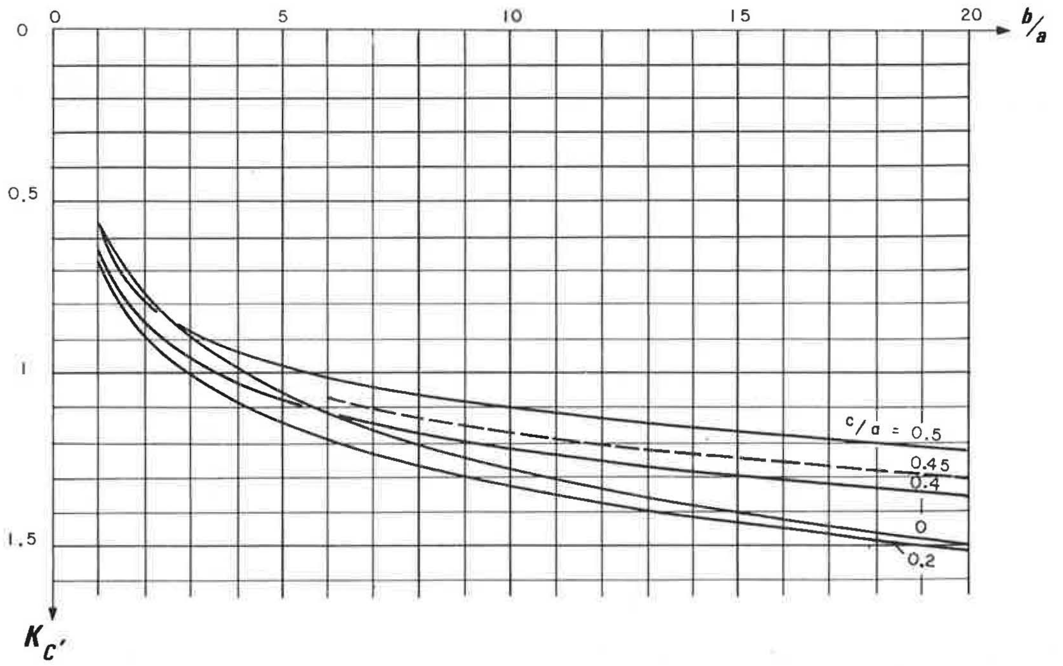


Chart 6. Coefficient $K_{B'}$.



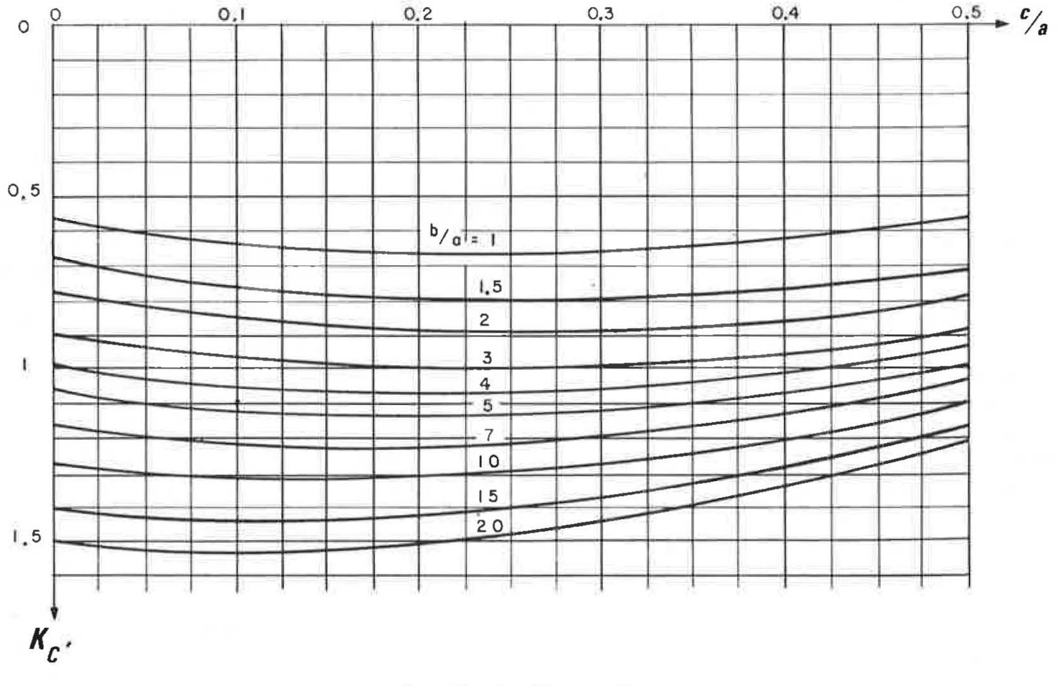


Chart 7. Coefficient $K_{C'}$.