The Axisymmetric Deformation of a Homogeneous, Cross-Anisotropic Elastic Half Space

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The sources of anisotropy in the structure of soils are traced and it is suggested that many practical solutions from the homogeneous cross-anisotropic elastic half space can be used to predict the effect of the structure of an earth mass on its response to loading.

Based on previously developed methods, solutions are produced for the axisymmetric deformation of a homogeneous, cross-anisotropic elastic half space acted upon by the following loading conditions: (a) uniform vertical pressure, (b) uniform vertical displacement, (c) linearly increasing radial shear stress, (d) linearly increasing torsional shear stress, and (e) linearly increasing torsional shear displacement. In all cases the loaded areas are circular in shape and the solutions indicate the variation of all displacement, strain, and stress components throughout the half space.

•THE widespread existence of anisotropy patterns in the structure of soils has been largely overlooked until recent times. In general, soils can be grouped according to genesis as being either weathered in situ (residual) or deposited. For the former group it is highly likely that any anisotropy existing in the structure of parent rocks will be inherited. The general occurrence of anisotropic structures in both sedimentary and igneous rocks throughout the world can be seen by reference to the literature (2, 9, 19, 27, 28, 37, 40).

The concept that the symmetries of a rock fabric are reflections of the symmetries of the movements involved during the rock forming processes (19) can also be applied to relate the symmetries of the patterns of the anisotropic structures of deformed soils and the symmetries of the deformations. This has been verified by a variety of analytical and experimental methods for processes such as the deformation of residual and deposited soils, the deposition of soils, and the remolding of soils.

Examples of such deformations are (a) soils located on instable slopes (20, 21), (b) soils subject to non-equal principal stresses (25), (c) compacted soils and pavement materials (16, 33, 29), (d) soils deposited by the vertical dropping of particles (13, 35), (e) soils sheared in a preferred direction (7), (f) wind-deposited soils (3), (g) soils situated in wheel tracks and deformed by traffic action (17), and (h) clays subject to rotational rolling (38). From the preceding it can be seen that for a wide range of practical situations the soils involved will have anisotropic structures which should be taken into account when considering their mechanical behavior.

In this paper the axisymmetric deformation of a homogeneous elastic, cross-anisotropic half space is considered. [Solutions for some deformations possessing a vertical plane of symmetry are also now available (14).] Such a material is a simplification of the most general anisotropic state because it has an axis of symmetry of rotation so that all directions in the planes normal to the axis are equivalent with respect to the elastic properties. The axis of symmetry is taken as being vertical and the half space is bounded by a horizontal plane. The number of elastic parameters

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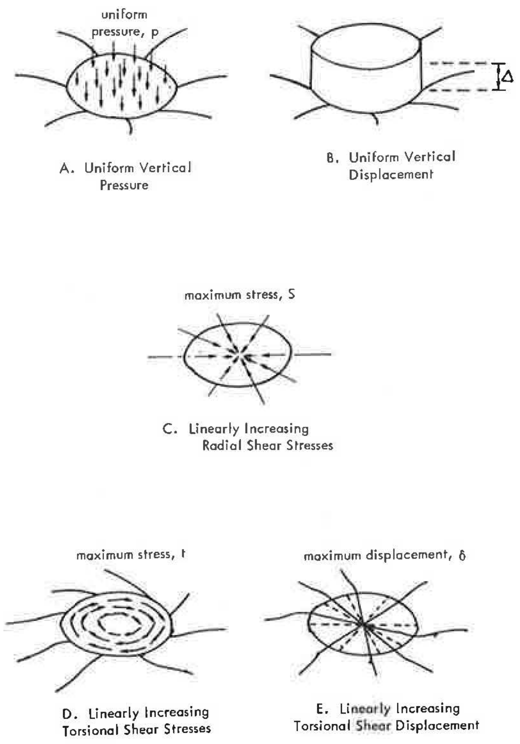

Figure 1. Loading conditions.

involved with a cross-anisotropic material is five compared with two for an isotropic material. Several workers have attempted to analyze soils and rocks as being cross-anisotropic elastic materials and have fully or partially measured the parameters involved (6, 13, 31, 32, 35, 37). Although soils are not usually homogeneous or elastic in the classical sense, the solutions provided will allow engineers to estimate the main differences in response to loading of soils with cross-anisotropic structures as compared with soils that have isotropic structures.

Five loading systems, applied over circular areas, are considered in this paper (Fig. 1). The first of these, loading condition A, is a uniform vertical pressure p and is an idealization of static traffic loadings and is the usual approximation made regarding foundations on sand. Loading condition B, in the form of a uniform vertical displacement Δ with no surface shear, has application to loading by a rigid plate or rigid punch, e.g., plate bearing tests and foundations.

The findings of Marwick and Starks (24) that pneumatic tires exert inwardly acting shear stresses on pavements suggested loading condition C, which is a linearly increasing radial shear stress from a value of zero at the center of the load to a maximum value of s at the periphery. Lee (22) has highlighted the fact that inwardly acting shear stresses are also developed under foundations with rough bases. Hence, an approximate solution to this class of problems can be found by superimposing the solutions for loading condition C onto the solutions for loading conditions A or B.

Loading conditions D and E are both torsional in nature; they are, respectively, a linear increase in torsional shear stress from zero at the center to a maximum value t at the periphery, and a linear increase in torsional displacement from zero at the center to a maximum value δ at the periphery. This type of loading can be produced in foundations as a result of wind loading and also in the pavements of parking areas from the rotation of wheels about a vertical axis. It may also have application to the analysis of the early stages of vane shear tests.

In considering the five loading conditions, the aim is to provide complete solutions to all displacement, strain, and stress components at all points throughout the half space. This is necessary because, first, the most critically loaded points from a soil mechanics point of view may lie away from the load axis. Second, the assessment of the behavior of an earth mass under load often requires an estimate of the total field of stresses and/or strains and/or displacements, e.g., three-dimensional consolidation problems. For a homogeneous, isotropic material subjected to a circular load of uniform pressure, solutions for all stress, strain, and displacement components throughout a half space have been given by Ahlvin and Ulery (1).

Some of the loading conditions mentioned have been examined previously. One of the first reported solutions to the loading of a homogeneous, cross-anisotropic medium is that of Wolf (39), who considered a point load. However, the generality of his work is limited due to certain restrictions imposed on the values that can be assumed for the elastic parameters. Quinlan (26) has used Fourier-Bessel integral methods to examine the response of cross-anisotropic half spaces to various axisymmetric distributions of vertical stresses or displacements. Although these methods are developed for a general point within the elastic system, the solutions presented apply mainly to points either on the surface or on the load axis.

Koning (18) and the Delft Laboratory (8) have given the general solution for stresses and displacements in a homogeneous cross-anisotropic half space due to a point load and a rigid punch load. In addition, they have produced a solution for points on the surface and the load axis for the condition of a circular area of uniform vertical pressure. Similar problems have also been considered by Lekhnitskii (23), who has solved for the stress distributions due to a point load and a circular area of uniform vertical pressure.

A finite difference method to examine the axisymmetric distribution of stresses and displacements in a cross-anisotropic half space, which has properties varying with depth, has been presented by Gerrard and Mulholland (11).

Barber (4) has investigated the response of an isotropic, homogeneous half space to various axisymmetric distributions of inwardly acting (radial) shear stresses. However, in his solutions the only component given is the vertical direct stress. In a more recent publication, Gerrard (12) has outlined a method for the calculation of the complete pattern of stresses, strains and displacements in layered, cross-anisotropic systems. The loadings considered are distributed in an axisymmetric fashion and may include vertical direct stresses and/or radial shear stresses.

Some of the solutions given in this paper are an enlargement of those given earlier by the Delft Laboratory (8) and hence this latter work was used to provide a check on the solutions for some of the stress and displacement components. In every comparison made the two respective solutions were found to be equivalent.

NOTATION

General

r, θ, z	cylindrical coordinates
u, v, w	displacements in corresponding coordinate directions
$\widehat{rr}, \widehat{\theta\theta}, \widehat{zz}$ $\widehat{rz}, \widehat{\theta z}, \widehat{r\theta}$	direct stress and shear stress components of stress tensor
$\epsilon_{rr}, \epsilon_{\theta\theta}, \epsilon_{zz}$ $\epsilon_{rz}, \epsilon_{\theta z}, \epsilon_{r\theta}$	direct strain and shear strain components of strain tensor
a, b, c, d, f	components of elasticity tensor
E_V	modulus of elasticity in the vertical direction
E_H	modulus of elasticity in the horizontal direction
ν_H	Poisson's ratio—effect of horizontal strain on complementary horizontal strain
ν_{HV}	Poisson's ratio—effect of horizontal strain on vertical strain
ν_{VH}	Poisson's ratio—effect of vertical strain on horizontal strain
E	modulus of elasticity—isotropic material
ν	Poisson's ratio—isotropic material

Quantities and Parameters Involved in Solutions

$$\alpha = \frac{\left[ad - c^2 - cf + f(ad)^{1/2}\right]^{1/2}}{(2fd)^{1/2}}$$

$$\beta = \frac{\left[ad - c^2 - cf - f(ad)^{1/2}\right]^{1/2}}{(2fd)^{1/2}}$$

$$\gamma = \left(\frac{a - b}{f}\right)^{1/2}$$

Δ = uniform vertical displacement (inches)
δ = maximum torsional displacement (inches)

$$\sigma = \operatorname{arsin}\left\{\frac{2}{[\psi^2+(1+r)^2]^{1/2}+[\psi^2+(1-r)^2]^{1/2}}\right\}$$

$$\lambda = \operatorname{artan}\frac{2\psi}{\psi^2+r^2-1}$$

$$\phi = \alpha - \beta$$

$$\rho = \alpha + \beta$$

$$\ell_1 = \frac{d\,\alpha^2 - \frac{f}{2}}{\left(c+\frac{f}{2}\right)\alpha}$$

$$\ell_2 = \frac{d\,\alpha^2 + \frac{f}{2}}{\left(c+\frac{f}{2}\right)\alpha^2}$$

$$m_1 = \frac{\left(c+\frac{f}{2}\right)(\alpha^2-\beta^2)}{f\beta} = \frac{c+\frac{f}{2}\;\rho\phi}{\frac{f}{2}\;(\rho-\phi)}$$

$$m_2 = \frac{c+\frac{f}{2}}{\frac{f}{2}}\;\frac{\alpha^2}{c+d\,\alpha^2}$$

$$m_3 = -\frac{2^{1/2}}{\pi^{1/2}}\;\frac{(c+d\,\rho^2)\,(c+d\,\phi^2)}{d\,(\rho^2-\phi^2)}$$

$$m_4 = \frac{2^{1/2}}{\pi^{1/2}}\;\frac{c+d\,\alpha^2}{2d\,\alpha}$$

$$\eta = [(\psi^2+r^2-1)^2+4\,\psi^2]^{1/4}$$

p = uniform vertical loading pressure (psi)

$$q_1 = \frac{d\,\phi^2 - \frac{f}{2}}{\left(c+\frac{f}{2}\right)\phi}$$

$$q_2 = \frac{d\,\rho^2 - \frac{f}{2}}{\left(c+\frac{f}{2}\right)\rho}$$

r = horizontal offset from load axis/r_0 (inch/inch)
r_0 = loaded radius (inches)
s = maximum radial shear stress load (psi)
t = maximum torsional shear stress load (psi)
z = depth from surface/r_0 (inch/inch)

STRESS-STRAIN RELATIONSHIPS

By definition an axisymmetric deformation is one in which all of the stresses, strains, and displacements do not vary with θ. In this paper the torsional and non-torsional components are dealt with separately and hence a general deformation in which these components are combined can be considered by superposition.

Non-Torsional Component

For a cross-anisotropic elastic material undergoing non-torsional axisymmetric deformation, the stresses can be expressed in terms of the strains by

$$\widehat{rr} = a \cdot \epsilon_{rr} + b \cdot \epsilon_{\theta\theta} + c \cdot \epsilon_{zz} \tag{1a}$$

$$\widehat{\theta\theta} = b \cdot \epsilon_{rr} + a \cdot \epsilon_{\theta\theta} + c \cdot \epsilon_{zz} \tag{1b}$$

$$\widehat{zz} = c \cdot \epsilon_{rr} + c \cdot \epsilon_{\theta\theta} + d \cdot \epsilon_{zz} \tag{1c}$$

$$\widehat{rz} = f \cdot \epsilon_{rz} \tag{1d}$$

The direct strains can be expressed in terms of the direct stresses by the following relations:

$$\epsilon_{rr} = \frac{\widehat{rr}}{E_H} - \frac{\nu_H \widehat{\theta\theta}}{E_H} - \frac{\nu_{VH} \widehat{zz}}{E_V} \tag{2a}$$

$$\epsilon_{\theta\theta} = -\frac{\nu_H \widehat{rr}}{E_H} + \frac{\widehat{\theta\theta}}{E_H} - \frac{\nu_{VH} \widehat{zz}}{E_V} \tag{2b}$$

$$\epsilon_{zz} = -\frac{\nu_{HV} \widehat{rr}}{E_H} - \frac{\nu_{HV} \widehat{\theta\theta}}{E_H} + \frac{\widehat{zz}}{E_V} \tag{2c}$$

From Eqs. 1 and 2 certain interrelationships between the elastic parameters can be determined:

$$a = \frac{E_H \left(1 - \nu_{HV} \nu_{VH}\right)}{\left(1 + \nu_H\right)\left(1 - \nu_H - 2\,\nu_{HV} \nu_{VH}\right)} \tag{3a}$$

$$b = \frac{E_H \left(\nu_H + \nu_{HV} \nu_{VH}\right)}{\left(1 + \nu_H\right)\left(1 - \nu_H - 2\,\nu_{HV} \nu_{VH}\right)} \tag{3b}$$

$$c = \frac{E_H \nu_{VH}}{1 - \nu_H - 2\,\nu_{HV} \nu_{VH}} \tag{3c}$$

$$d = \frac{E_H \nu_{VH} \left(1 - \nu_H\right)}{\nu_{HV}\left(1 - \nu_H - 2\,\nu_{HV} \nu_{VH}\right)} \tag{3d}$$

$$\frac{E_H}{E_V} = \frac{\nu_{HV}}{\nu_{VH}} \tag{3e}$$

The strain components are

$$\epsilon_{rr} = \frac{\partial u}{\partial r} \tag{4a}$$

$$\epsilon_{\theta\theta} = \frac{u}{r} \tag{4b}$$

$$\epsilon_{zz} = \frac{\partial w}{\partial z} \tag{4c}$$

$$\epsilon_{rz} = \frac{1}{2}\left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}\right) \tag{4d}$$

At all points on the load axis (i.e., r = 0) it can be shown that

$$\widehat{rr} = \widehat{\theta\theta} \quad \text{and} \quad \frac{u}{r} = \frac{\partial u}{\partial r} \tag{5}$$

Torsional Component

The relevant stress-strain relationships are

$$\widehat{\theta z} = f \cdot \epsilon_{\theta z} = f \,\frac{1}{2}\,\frac{\partial v}{\partial z} \tag{6a}$$

$$\widehat{r\theta} = (a - b)\,\epsilon_{r\theta} = (a - b)\,\frac{1}{2}\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right) \tag{6b}$$

METHOD OF SOLUTION AND PRESENTATION OF RESULTS

The method of solution of problems of this type involving cross-anisotropic materials has been outlined previously by Gerrard (12) and Gerrard and Mulholland (15) and is based on the integral transform techniques developed by Sneddon (30) and Tranter (34). For all five loading conditions, the solutions for the displacements, strains, and stresses involve integrals of products of Bessel functions. In order to simplify presentation these integrals have been abbreviated according to the following notation:

$$I(\eta, \tau, \chi, \psi) = \int_0^\infty J_\eta(k) \cdot J_\tau(kr) \cdot k^\chi \cdot e^{-\psi k} \cdot dk \tag{7}$$

The form of the solution obtained for non-torsional axisymmetric deformation has been shown to depend on parameters that are functions only of the elastic constants (8, 12). In the latter work these parameters are given as α^2 and β^2. Hence, for each of the loading conditions A, B, and C, three complete lists of solutions are given for the cases of

1. Cross-anisotropic: α^2 positive, β^2 positive;
2. Cross-anisotropic: α^2 positive, β^2 zero; and
3. Isotropic: this is a simplification of case 2 with $\alpha^2 = 1$, $\beta^2 = 0$.

The form of the solution for torsional deformation does not depend on the value of parameters in the same way as for non-torsional deformation. Thus, for each of loading conditions D and E, two complete lists of solutions have been prepared, corresponding to cross-anisotropic behavior and isotropic behavior respectively.

The solutions for each loading condition contain a section dealing with the evaluation of the appropriate integrals. These sections are divided into three parts depending on whether the integrals are to be evaluated (a) at a general point within the system (i.e., $r \neq 0$, $\psi \neq 0$), (b) at a point on the load axis, (c) at a point on the surface.

With regard to the evaluation of the integrals at a general point, simple and direct results are obtained for the defined displacement conditions (B and E). However, this is not so for the defined stress conditions (A, C, and D) where the integrals were evaluated by numerical integration on a high-speed digital computer for a range of values of r and ψ. Altogether thirteen integrals were treated in this way.*

Some of these integrals have been previously tabulated by Eason, Noble, and Sneddon (10) for different ranges of values of r and ψ. For anisotropic materials, the parameter ψ is in the form of either ρz, ϕz, αz, γz and hence it is a function of the elastic properties as well as the depth. This means that when using the Appendix tables to calculate the stresses, strains, and displacements at a particular point within the system it will generally be necessary to interpolate between the tabulated points along the ψ coordinate.

At points on the load axis (i.e., $r = 0$) the integrals involved in all of the loading conditions can be evaluated simply and directly. On the other hand, for points on the surface (i.e., $\psi = z = 0$) the integrals for all loading conditions become discontinuous in nature in order to fulfill the loading conditions. As indicated, most of the integrals have simple results while the remainder yield results in the form of hypergeometric functions (30, 36). Integrals whose coefficients contain z' have not been considered since their products $(z \cdot I)$ are in general zero for points on the surface. The values of integrals given in this paper are based directly or indirectly on the results obtained by Watson (36), Sneddon (30), and Bateman Manuscript Project (5).

TOTAL LOAD-DEFINED DISPLACEMENT RELATIONSHIPS

For loading condition B it is possible to derive relationships between the defined surface displacement and the requisite total external load. The relationships, found by integrating the appropriate surface stress over the loaded area, are as follows:

Cross-anisotropic: α^2 positive, β^2 positive

$$\text{Total load} = -4\,\Delta\, r_0 \frac{\frac{f}{2}}{c + \frac{f}{2}} \frac{(c + d\rho^2)(c + d\phi^2)}{d\rho\phi(\rho + \phi)} \tag{8a}$$

Cross-anisotropic: α^2 positive, β^2 zero

$$\text{Total load} = -4\,\Delta\, r_0 \frac{\frac{f}{2}}{c + \frac{f}{2}} \frac{(c + d\alpha^2)^2}{2d\alpha^3} \tag{8b}$$

Isotropic

$$\text{Total load} = -4\,\Delta\, r_0 \frac{E}{2(1 - \nu^2)} \tag{8c}$$

*The results are given in Tables 1 through 13, which are included in a lengthy Appendix not published in this Record but which is available from the Highway Research Board at cost of handling and reproduction. Refer to Supplement XS-14, Highway Research Record 223.

Similarly, for loading condition E relationships can be found between defined surface displacement and the requisite total external torque. These relations are:
Cross-anisotropic:

$$\text{Total torque} = -\frac{8}{3} \cdot \delta \cdot r_0{}^2 \cdot f \cdot \gamma \tag{9a}$$

Isotropic:

$$\text{Total torque} = -\frac{8}{3} \cdot \delta \cdot r_0{}^2 \frac{E}{1+\nu} \tag{9b}$$

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