

# Finite Element Analyses of Pavements

J. M. DUNCAN, C. L. MONISMITH, and E. L. WILSON,  
University of California, Berkeley

This paper describes an application of the finite element technique to the analysis of systems representative of pavement structures. Use is made of a digital computer program which generates suitable finite element configurations for axisymmetric structures and accommodates approximations of nonlinearity which appear appropriate to represent the behavior of granular base and cohesive subgrade materials under conditions corresponding to moving traffic.

Examples are presented for systems with linear material properties showing comparisons between displacements and stresses computed using the finite element technique and those computed using elastic half-space and layered system analyses to establish criteria for boundary conditions in the finite element procedure.

For the elastic half-space subjected to a uniform circular load, displacements and stresses computed by the finite element technique compare favorably with those determined from the Boussinesq solution where the nodal points in the finite element procedure are fixed at a depth of 18 radii for the bottom boundary and constrained from moving radially on the vertical boundary at a distance of about 12 radii from the center.

To obtain a reasonable comparison between the two procedures for a three-layered system however, it was necessary to move the fixed boundary in the finite element procedure to a depth of about 50 radii while maintaining the same radial constraints as for the half-space analysis.

Two analyses are presented for deflection determinations for an in-service pavement near Gonzales, Calif., one for a condition where the asphalt concrete was at a comparatively high temperature (stiffness modulus in the range 120,000 to 280,000 psi), and the other with the material at a low temperature (stiffness modulus approximately 1,500,000 psi). Nonlinear material properties, determined from the result of repeated load triaxial compression tests, were used to represent the behavior of the untreated granular base and subbase and the fine-grained subgrade soil.

Deflections predicted by the finite element procedure are in the same range as those measured with the California traveling deflectometer indicating that the method has potential to simulate actual pavement behavior to a reasonable degree.

The analysis also indicates that when the extensional strains in the asphalt concrete are large for this pavement (i. e., where the stiffness of the asphalt bound material is low), the granular material exhibits a very low modulus under the loaded area and a large proportion of the surface deflection can be attributed to deformations within this material. On the other hand, when the asphalt layer is stiff, the majority of the surface deflection is contributed by the subgrade.

An analysis is also presented for the results of plate load tests on a two-layer prototype pavement consisting of granular base and a cohesive subgrade soil using the same nonlinear characterization for material properties as for the in-service pavement. The computed deflections were

found to converge much more slowly than those for the Gonzales Bypass pavement, apparently because of the dominant role in the behavior of the system which is played by the granular material, and the fact that the resilient modulus of this material is very strongly stress-dependent.

The nonlinear analyses are considered to be preliminary, in that the method of analysis represents only one of a number of techniques which might be employed in conjunction with the finite element method.

•THE finite element method of analysis provides an extremely powerful technique for solving problems involving the behavior of structures subjected to accelerations, loads, displacements or changes in temperature. Problems involving the behavior of heterogeneous, anisotropic structures with complex boundary conditions may be handled.

By combining finite element computer programs for analysis of axisymmetric structures with experimental data concerning the behavior of pavement materials under repeated loading test conditions, it appears to be feasible to extend analyses of pavement structures to include nonlinear material behavior in an approximate manner. With sufficient attention devoted to determination of appropriate physical property values and to simulation of actual boundary and loading conditions, the finite element method of analysis promises to afford improved understanding of the behavior of pavement structures under load.

#### FINITE ELEMENT METHOD

For analysis by the finite element technique, the body to be analyzed, such as the cylinder shown in Figure 1, is divided into a set of elements connected at their joints or nodal points. On the basis of an assumed variation of displacements within elements together with the stress-strain characteristics of the element material, the stiffness of each nodal point of each element is computed. For each nodal point in the system, two equilibrium equations may be written expressing the nodal point forces in terms of the nodal point displacements and stiffnesses. These equations are then solved for the unknown displacements. With the displacements of all nodal points known, strains and stresses within each element are then computed. Detailed descriptions of the method and its application to a wide variety of problems are contained in a number of publications (1, 2, 3, 4, 5). Analysis of realistic systems commonly requires formulation and solution of several hundred simultaneous equations. For this reason the technique is only practicable when formulated for high-speed digital computers.

The digital computer program used for the present study is described elsewhere (4). Modifications have been made to generate automatically suitable finite element configurations for analysis of axisymmetric pavement structures and to accommodate types of modulus dependency on stress which would appear appropriate to represent the behavior of granular base and cohesive subgrade materials under conditions corresponding to moving traffic (13, 14, 15).

For analysis with this computer program, the structure to be analyzed is divided into a series of quadrilaterals and/or triangles. (Only quadrilaterals were used in the present study.) Each quadrilateral is subsequently divided into four triangles by the computer program (Fig. 1). Displacements are assumed to vary linearly within each triangle; this assumption insures that no gaps will develop in the deformed structure and that displacements will be compatible throughout the structure as well as at the nodal points.

Besides the finite element configuration to be used, additional items of input consist of specifying loads or displacements for each nodal point and material properties (Young's modulus and Poisson's ratio) for each element. In the nonlinear analyses, an initial gravity stress (corresponding to no applied load on the pavement) was

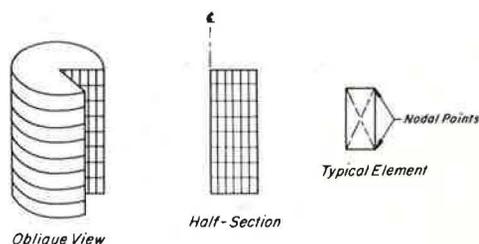


Figure 1. Finite element idealization of a cylinder.

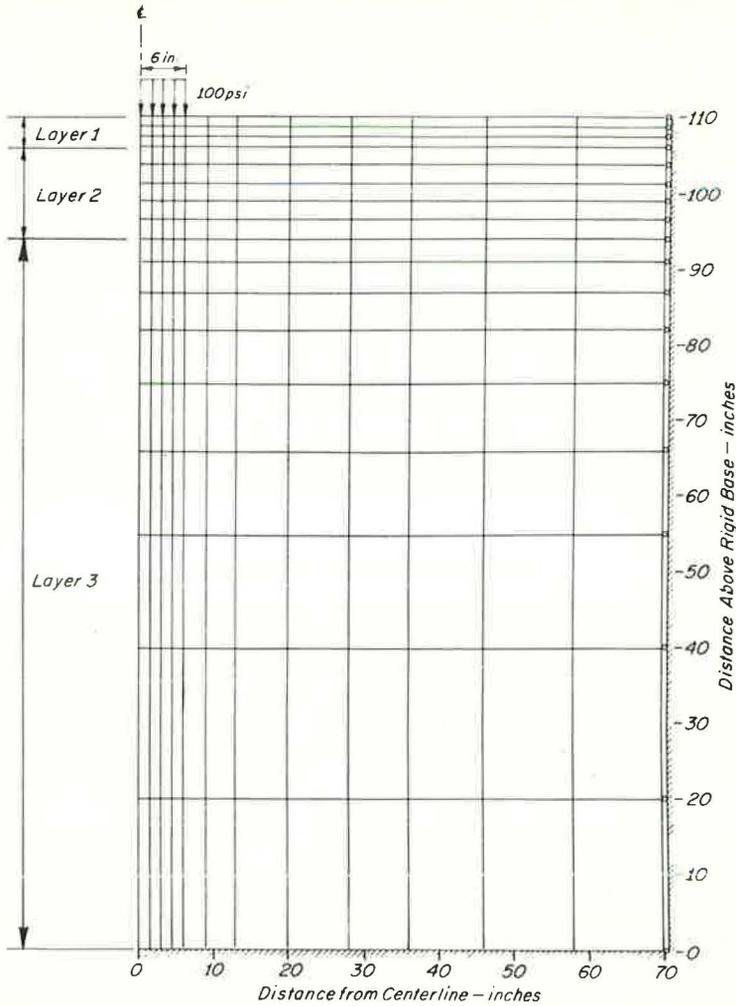


Figure 2. Finite element configuration used for analysis of homogeneous and layered systems.

calculated for each element, requiring that the density of each material be specified as well. The computer output consists of radial and axial displacements at each of the nodal points exterior to the quadrilateral elements and the complete state of stress at the centroid of each quadrilateral. Quadrilateral stresses are computed as the average of the stresses in the four triangles.

### LINEAR ANALYSES

The amount of computer time required to solve a finite element problem depends on the number of nodal points (or elements) used to represent the system. For efficient operation, these should be kept to the minimum necessary for accurate representation of the system being studied. Although some judgment is inevitably required in choosing a system of finite elements, the number of elements required is strongly dependent on the criteria for element sizes and shapes and on the size of the region which must be represented for valid simulation of the actual problem. Thus, it is desirable to establish criteria for boundary conditions by examining finite element solutions to problems for which other solutions are readily available.

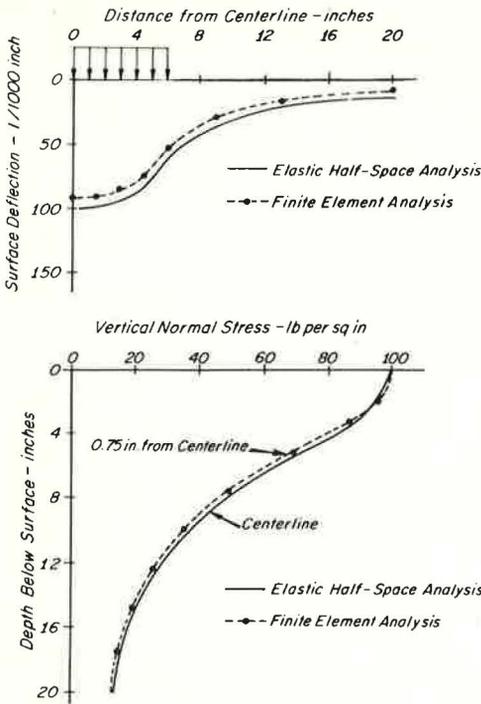


Figure 3. Comparison of deflections and stresses in uniform subgrade by finite element and elastic half-space analysis.

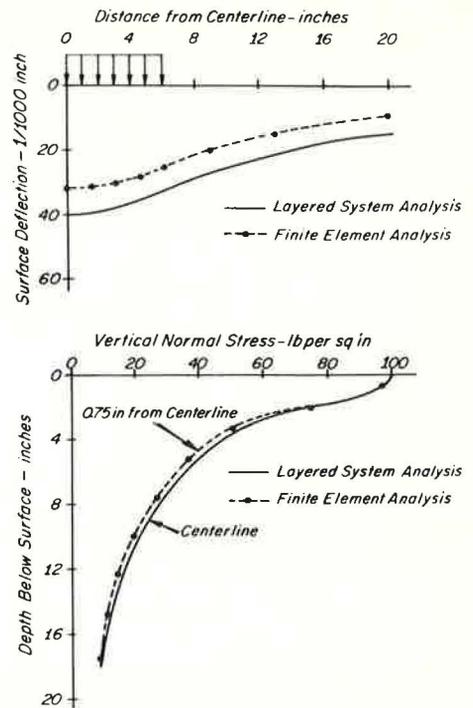


Figure 4. Comparison of deflections and stresses in layered system by finite element and layered system analysis.

For this purpose the deflections and stresses beneath a uniformly loaded circular area were studied using the finite element configuration (Fig. 2). The nodal points on the bottom boundary were fixed, whereas those on the right boundary were constrained from moving in the radial direction. The nodal points on the centerline (left boundary) can only move vertically because of the symmetry of the problem. The nodal points beneath the loaded area are subjected to vertical forces which form the static equivalent of the pressure shown; the magnitudes of these forces are calculated by the computer program from the specified pressure. All other nodal points in the system are subjected to no forces or geometric constraints.

The first problem studied was that of a uniformly loaded circular area on the surface of a homogeneous subgrade, i. e., layers one, two, and three were all assigned the same values of Young's modulus,  $E = 10,000$  psi and Poisson's ratio,  $\nu = 0.4$ . The surface deflections and vertical normal stresses beneath the load are shown in Figure 3. Also shown are the deflections and stresses in an elastic half-space subjected to the same loading; these values were computed using a digital computer program for layered elastic systems developed by the Chevron Research Company (6, 7). Both the deflections and stresses compare very favorably. Because the compressible layer extended only to a depth of about 18 radii in the finite element analysis, the surface deflections computed by the finite element method are somewhat smaller than those computed by the half-space analysis. The difference in deflection beneath the center of the loaded area amounts to about 7 percent.

The stresses in Figure 3 for the finite element analysis are those computed at the centers of the furthest left elements in Figure 2, 0.75 in. from the centerline. Even though the stresses in Figure 3 correspond to slightly different positions, it is evident that the agreement between the two methods is good, indicating that the choice of element sizes and shapes is acceptable.

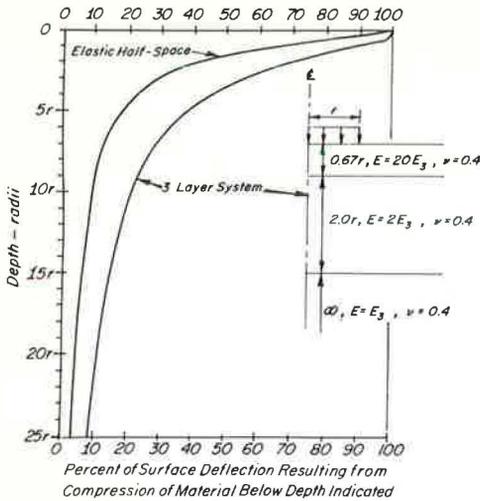


Figure 5. Comparison of surface deflections of elastic half-space and layered system.

deflections computed by the layered system analysis are some 25 percent larger than those computed by the finite element method. Thus, although the finite element configuration (Fig. 2) is adequate to simulate a homogeneous system, it is not adequate to simulate a layered system where the upper layer is much stiffer than those below. In a layered system the upper, stiffer layer contributes only a small amount to the surface deflection and the contribution of the deeper, softer layers is relatively much more important than in a homogeneous system.

The difference in behavior between a homogeneous and a layered system is shown in Figure 5, where the contributions to surface deflection are functions of depth beneath the surface. Compression of the material below a depth of two radii contributes only 40 percent to the surface deflection of an elastic half-space, whereas it contributes 70 percent to that of the layered system. Similarly, at any other depth, the contribution of the material below is always much more important in the deflection of the layered system than the half-space. Thus it appears that the finite element deflections (Fig. 4) are appreciably less than the layered system deflections because the depth to which the bottom layer extends is too small to approximate the behavior of a semi-infinite layer.

The same layered system was reanalyzed by the finite element method using the configuration shown in Figure 6, in which the bottom rigid boundary is 50 radii beneath the surface. The layers were assigned the same values of Young's modulus (200,000, 20,000, and 10,000 psi) as in the previous analysis, and Poisson's ratio was again 0.4 for all layers. The configurations shown in Figures 2 and 6 contain the same number of nodal points (234) and elements (204); in fact, the sizes and shapes of the elements above 66 in. in Figure 2 are exactly the same as those above 256 in. in Figure 6. In the reanalysis, only the heights of the bottom four rows of elements were changed. The vertical normal stresses, computed using the configuration in Figure 6, were, for practical purposes, the same as those computed using the configuration in Figure 2. The surface deflections in Figure 7 were much larger, however, being only about 4 percent smaller than for an infinitely deep bottom layer.

These analyses indicate that the thickness of the bottom layer or subgrade has an important effect on the magnitude of the surface deflections of a stiff pavement. The importance of the subgrade has also been emphasized by Peattie and Jones (8).

The radial external boundary, as well as the bottom boundary, will have some influence on the magnitude of the computed surface deflections. On the basis of the preceding analyses, however, this influence appears to be rather small. For practical

Previous experience has indicated that quadrilateral element stresses will be accurate provided that the length-to-width ratio for the elements does not exceed five to one. Thinner elements may be used to increase the distance from the boundaries if accuracy in stresses in these regions is not important. Elongated elements have been used at the upper right and lower left of the configuration (Fig. 2).

The same configuration was also used to analyze a three-layer elastic system. For this analysis each element in layer one was assigned  $E = 200,000$  psi; layer two, 20,000 psi; and layer three, 10,000 psi. Poisson's ratio,  $\nu = 0.4$  was used for all three layers. The surface loading was again 100 psi distributed over a 12-in. diameter circular area. The computed deflections and stresses are shown in Figure 4, together with those computed using the elastic layer computer program.

The stresses computed by the two methods are again very similar, but the surface de-

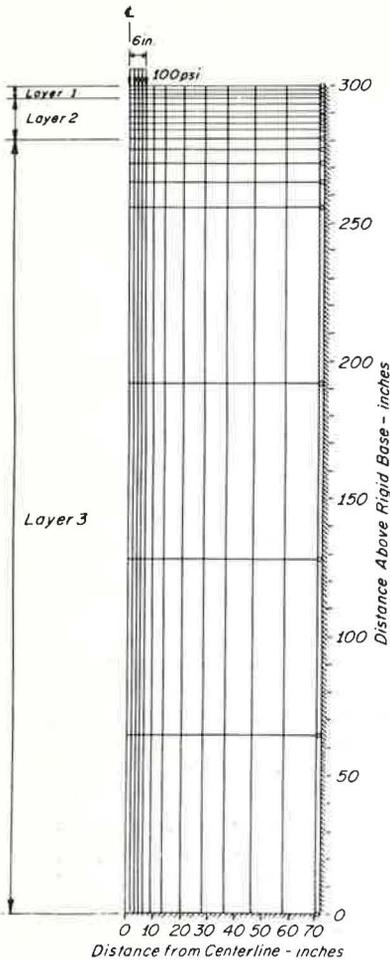


Figure 6. Finite element configuration used for analysis of layered system—deep bottom boundary.

elastic modulus values which are used in these analyses reflect the resilient behavior of the pavement materials as determined by means of repeated loading tests simulating the durations and magnitudes of the expected field loadings.

For granular base and cohesive subgrade materials, modulus values are determined by dividing the value of the repeatedly applied stress by the recoverable or resilient strain, and are termed the "resilient moduli" of the materials. For asphalt concrete, the corresponding modulus, which is determined in the same way, is called the "stiffness" of the material in accordance with the terminology suggested by van der Poel (17). To avoid confusion between the two different meanings of the term stiffness as used by asphalt technologists in describing pseudoelastic material behavior and by structural analysts to express a force induced by unit displacement, the factor relating applied stress

purposes sufficient accuracy may be obtained if this boundary is introduced at a distance of 12 radii or more from the centerline.

For analysis of systems with linear material properties, the finite element method offers little or no advantage over layered system analyses, for which digital computer programs are also readily available. However, because it is feasible to incorporate nonlinear material behavior in finite element analyses, the technique may be used for obtaining solutions to problems which cannot be solved by other available methods at the present time (1967).

## NONLINEAR ANALYSES

### Analysis of Recoverable Deformations

Asphalt concrete, granular base-course materials, and cohesive subgrade materials have all been found to undergo both permanent and recoverable deformations under repeated load conditions (9, 10, 11). In recent years, considerable effort has been directed toward determination of these recoverable deformations since there is considerable evidence to indicate that the transient or resilient deformations which occur under moving wheel loads may eventually result in fatigue failure of the asphalt concrete (12, 13).

Considerable effort has already been devoted to developing techniques for predicting resilient pavement deflections using laboratory test data and layered systems analyses (14, 15). In these analyses, the applied load is treated as if it were a static loading; since inertial effects have a small influence, at least under slower moving wheel loadings (16), their effects are neglected in the analyses. The problem of computing the magnitude of recoverable or resilient deformations is thereby reduced to the analysis of the behavior of an elastic pavement structure under static applied load. The pseudo-

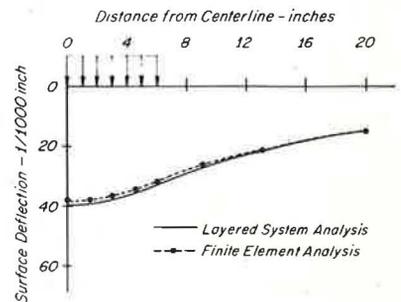


Figure 7. Comparison of deflections in layered system by layered system analysis and finite element analysis with deep bottom boundary.

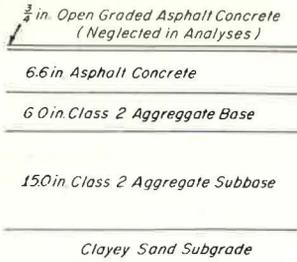


Figure 8. Structural section, Gonzales Bypass pavement.

to recoverable strain for asphalt concrete will be termed the "stiffness modulus" of the material.

Repeated load tests on granular and cohesive materials have shown that their resilient moduli are stress-dependent, i. e., these materials exhibit nonlinear behavior under the stresses to which they are subjected in in-service pavements (14). In these analyses, nonlinear material properties were approximated in the solution by solving the same problem repeatedly, choosing for successive solutions a value of resilient modulus for each element corresponding to the state of stress calculated for that element in the preceding cycle. The success of the technique was judged on the basis of the tendency for

the surface deflections to converge to a stable value within a reasonable number of cycles.

Use of this technique thus involves representing the actual nonlinear pavement structure by a linear, inhomogeneous structure in which the modulus values assigned to any element are related to previously computed values of stress for that element. The usefulness of this approximate procedure may be judged by the agreement between computed deflections with those measured in actual pavements, and by the extent to which the actual behavior of the pavement is represented by the analysis.

Gonzales Bypass

Using this procedure, two analyses have been made of the resilient deflections of the Gonzales Bypass pavement (19), one corresponding to cold ("winter") and one to warm ("summer") conditions. The structural section for this pavement is shown in Figure 8. Repeated load tests were performed to determine the stiffness modulus of the asphalt concrete and the resilient moduli of both the granular base and the cohesive subgrade materials. Repeated flexure tests were conducted on 1.5 by 1.5 by 15-in. beam specimens of the asphalt concrete sawed from a slab of the existing pavement obtained by the staff of the Materials and Research Department of the California Division of Highways. Tests were conducted at 40 F and 68 F from which the relationship between temperature and stiffness modulus was constructed (Fig. 9). The temperature profiles through the asphalt concrete surface which were used to represent the summer and winter conditions are shown in Figure 10, and the corresponding ranges in values of stiffness modulus are indicated in Figure 9; these ranges are 120,000 to 280,000 psi for the summer condition and 1,420,000 to 1,550,000 psi for the winter condition. Although the ranges of temperatures chosen do not represent realistic seasonal extremes at Gonzales, the corresponding analyses are illustrative of the pavement behavior under divergent temperature conditions.

Repeated load tests were performed on 3.9-in. diameter by 7.8-in. high triaxial specimens of the base course material which were compacted in the laboratory to conditions simulating the in-situ water content and density of the base. These tests, which were conducted using a frequency of 20 applications per minute and a load duration of 0.1 sec., indicate that the resilient modulus varies with confining pressure according to the expression

$$M_R = 15,000 \sigma_3^{0.48} \quad (1)$$

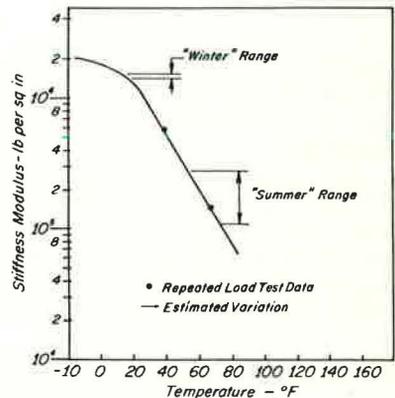


Figure 9. Relationship between stiffness modulus of asphalt concrete surface and temperature used for analysis of Gonzales Bypass pavement.

in which the units of  $M_R$  and  $\sigma_3$  are both in pounds per square inch. Although no tests were conducted on the subbase material, experience with similar granular materials indicates that the relationship between resilient modulus and confining pressure would be represented by an expression of the same form as Eq. 1. Because the subbase is of lower quality than the base, it seems likely that at any confining pressure its resilient modulus would be somewhat smaller than that of the base. Using tests on both the base and subbase of the Morro Bay pavement as a guide (15), it was estimated that the resilient modulus of the Gonzales Bypass subbase would vary approximately in accordance with the following expression

$$M_R = 10,000 \sigma_3^{0.4} \quad (2)$$

The relationships between resilient modulus and confining pressure expressed by Eqs. 1 and 2 are shown in Figure 11. The values of  $M_R$  for both the base and the subbase vary quite markedly with  $\sigma_3$ , especially at small values of  $\sigma_3$ , indicating the importance of the stress conditions in determining the resilient behavior of these materials. The relationships only apply to values of the principal stress ratio ( $\sigma_1/\sigma_3$ ) which will not cause failure of the material during test. Test results show, however, that the resilient behavior of granular materials may be represented by a single relationship of the form of Eqs. 1 and 2 up to values of  $\sigma_1/\sigma_3$  as high as 10 (14). The relationships represented by Eqs. 1 and 2 only have physical meaning for positive values of  $\sigma_3$  (compression). Although no test data are available for values of  $\sigma_3$  equal to or less than zero, it is assumed in the analyses that the resilient modulus is, for practical purposes, zero in this range of stresses.

The resilient modulus values of granular materials might also be related to the stress conditions (14) by

$$M_R = L (\sigma_m)^x \quad (3)$$

in which  $\sigma_m$  is the mean normal stress,  $\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$ . The values of mean stress calculated during the analyses indicate that in most cases when the value of  $\sigma_3$  is negative (tensile), the value of  $\sigma_m$  is also negative. On this basis, therefore, it seems likely that relating  $M_R$  to  $\sigma_m$ , as indicated by Eq. 3, would lead to results which would be much the same as those obtained by relating  $M_R$  to  $\sigma_3$ , as indicated by Eq. 1. The following analyses were all conducted using the relationship expressed by Eq. 1; it is planned in further analyses, however, to examine the usefulness of the relationship indicated by Eq. 3.

To determine the resilient characteristics of the subgrade, repeated load tests were also performed on 1.4-in. diameter by 3.4-in. high tri-axial specimens of the undisturbed clayey sand subgrade material taken from the Gonzales Bypass pavement. These tests were conducted using a lateral pressure of 1.4 psi and the same frequency (20 applications per minute) and load duration (0.1 sec.) as used in the tests on the granular base material. During the initial stages, it was observed that the resilient modulus increased with increasing

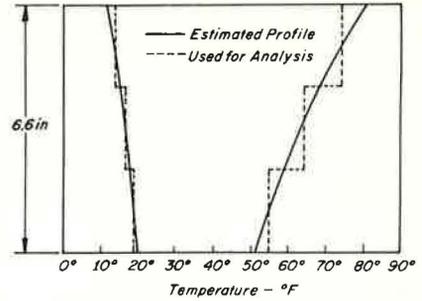


Figure 10. Temperature profiles used in analysis of Gonzales Bypass pavement.

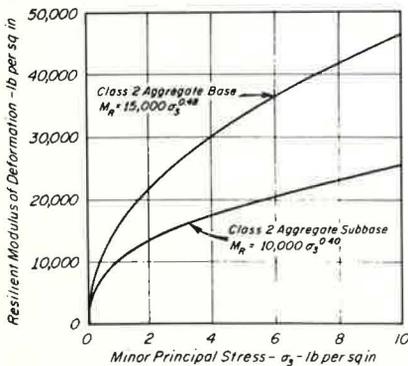


Figure 11. Variation of resilient moduli of base and subbase with confining pressure used in analysis of Gonzales Bypass pavement.

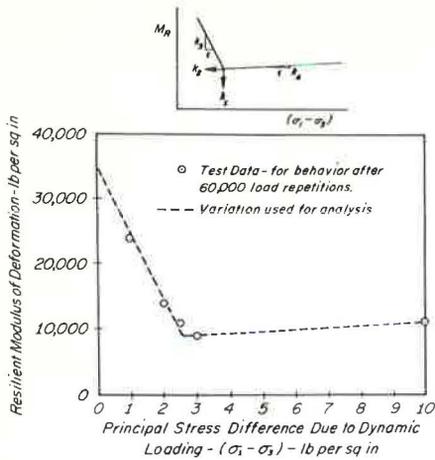


Figure 12. Variation of resilient modulus of clayey sand subgrade with principal stress difference used in analysis of Gonzales Bypass pavement.

ment components (Figs. 9, 11 and 12) were incorporated in analyses of the Gonzales Bypass pavement through repeated solution of the problem. The variations of resilient modulus for the base and subbase materials were introduced by means of algebraic expressions of the form

$$M_R = K \sigma_3^x \quad (4)$$

For each granular material, appropriate values of the constant  $K$  and the exponent  $x$  were read in as data. Prior to each solution of the problem the computer program evaluated an expression of the form of Eq. 4 and assigned a new value of  $M_R$  to each element of granular material, using the value of  $\sigma_3$  calculated for that element in the previous solution. Within the computer program, the assigned value of  $M_R$  was treated as if it were a value of Young's modulus and the material as if it were linearly elastic. A value of Poisson's ratio,  $\nu = 0.4$ , was assigned to the granular base and subbase materials.

The variation of resilient modulus for the subgrade material was introduced by means of two expressions of the form

$$M_R = K_2 + K_3 \left[ K_1 - (\sigma_1 - \sigma_3) \right], \quad K_1 > (\sigma_1 - \sigma_3) \quad (5a)$$

and

$$M_R = K_2 + K_4 \left[ (\sigma_1 - \sigma_3) - K_1 \right], \quad K_1 < (\sigma_1 - \sigma_3) \quad (5b)$$

The relationship of the constants  $K_1$ ,  $K_2$ ,  $K_3$ ,  $K_4$  to the measured properties is shown in Figure 12. These four values were read in as input data. As in the case of the granular materials, the resilient modulus of each element of the cohesive subgrade was reevaluated automatically by the computer before each new solution of the problem, using the value of  $(\sigma_1 - \sigma_3)$  calculated for that element in the previous solution. A value of Poisson's ratio,  $\nu = 0.47$ , was assigned to the cohesive subgrade.

The relationship between stiffness modulus of the asphalt concrete and temperature was read in as a series of pairs of values describing the curve shown in Figure 9. The temperature of each nodal point in the asphalt concrete was also specified in the input data (these temperatures were the same for all nodal points at the same level). Element temperatures were determined by the computer as the average of the nodal point

number of load repetitions; after a large number of load repetitions, however, the resilient modulus was found to be sensibly independent of the number of loading cycles, and the values determined after 60,000 repetitions of applied load were selected as being representative of the subgrade behavior for the in-service pavement.

The repeated load tests were conducted using various values of principal stress difference  $(\sigma_1 - \sigma_3)$ , which represents the repeated axial stress applied to the specimen. The relationship between the principal stress difference and resilient modulus after 60,000 applications is shown in Figure 12. This form of variation, in which the resilient modulus first decreases with increasing repeated stress, and then increases slightly, is similar to the behavior observed previously in repeated load tests on other cohesive materials (18). The dotted line indicates the approximation to the actual behavior which was used in the analysis.

The resilient characteristics of the pavement components (Figs. 9, 11 and 12) were incorporated in analyses of the Gonzales Bypass pavement through repeated solution of the problem. The variations of resilient modulus for the base and subbase materials were introduced by means of algebraic expressions of the form

temperatures immediately above and below, and a value of stiffness modulus was assigned to each element by semilogarithmic interpolation. A value of Poisson's ratio,  $\nu = 0.4$  was assigned to the asphalt concrete. Because the stiffness modulus of the asphalt concrete was considered to be independent of stress, the values of modulus for the surface varied only with depth (since the temperature varied with depth) and were the same in successive solutions.

A simplified flow diagram for the computer program is shown in Figure 13. In order to make it possible to evaluate the stress-dependent moduli for the first solution, an approximate stress distribution was calculated prior to the first solution using the load-spread concept with a one-to-one load spread. To these stresses, and to the stresses calculated in each of the successive solutions, the initial gravity stress distribution was added. The vertical normal gravity stress was calculated as the weight per unit area of the overlying material, and the horizontal normal gravity stress was taken to be three-fourths of the vertical. The gravity stress component was included in the value of  $\sigma_3$  used to evaluate the resilient moduli of the elements in granular material, but only the load-induced stress component was included in the value of  $(\sigma_1 - \sigma_3)$  used in evaluating the moduli of elements in cohesive material.

Using the temperature profiles (Fig. 10), the Gonzales Bypass pavement was analyzed for the summer and winter conditions. Each of these analyses comprised four solution cycles; the values of surface deflection under the center of the loaded area computed for each cycle are shown in Figure 14. For both the summer and the winter conditions the surface deflections appear to reach reasonably stable values after three cycles or fewer. Also shown in Figure 14 are the deflections measured using the California traveling deflectometer. The 7500-lb load on dual wheels used with the deflectometer was represented by a uniform 70-psi pressure on a 12-in. diameter circular area (7900-lb load) in the analysis. The pavement temperature was not recorded at the time of the deflectometer measurements, but it is known that the mean air temperature at that time was about 55 F. It seems likely, therefore, that the actual pavement temperature was slightly lower than temperatures associated with the summer analysis. Since the stiffness of the asphalt concrete increases with decreasing temperature and the surface deflections decrease, it appears that the calculated surface deflections agree at least reasonably well with the measured deflections. On this basis, it

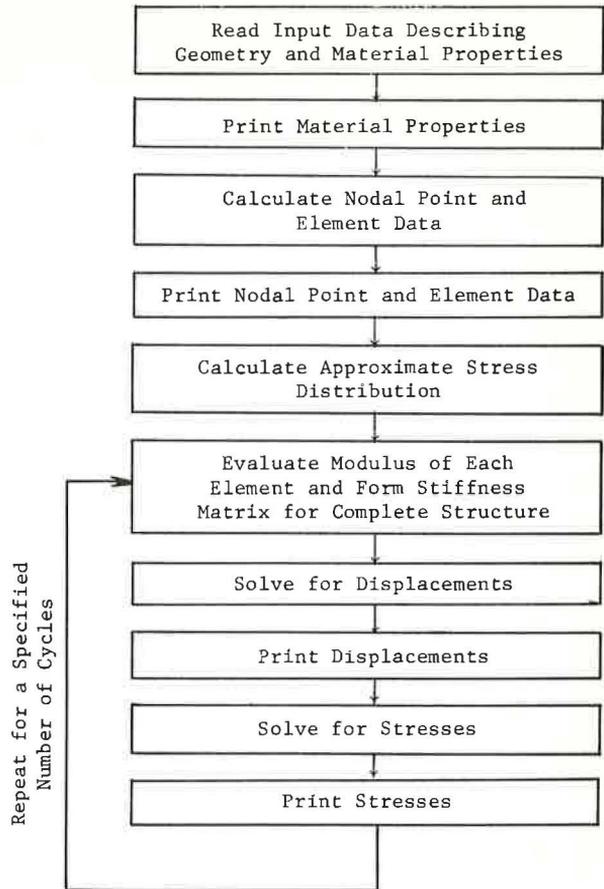


Figure 13. Simplified flow diagram.

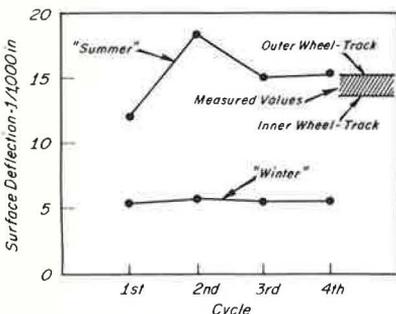


Figure 14. Convergence of surface deflections of Gonzales Bypass pavement.

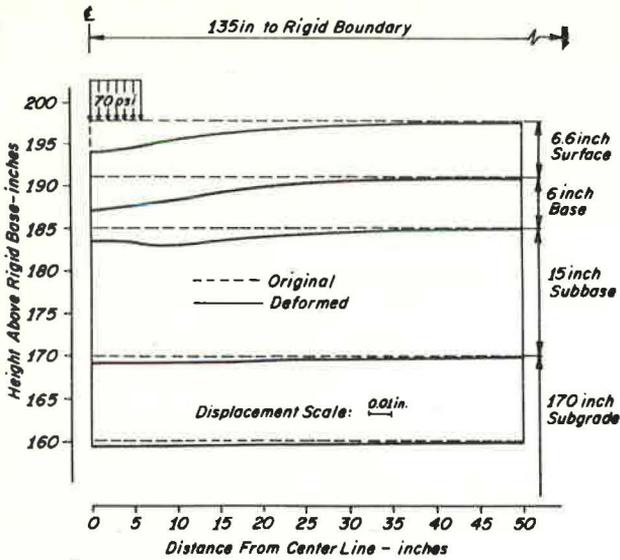


Figure 15. Deformed shape of Gonzales Bypass pavement—summer.

tioned previously, when tensile stresses developed in the granular material the modulus was reduced to a small value for the subsequent solution (in these analyses the modulus was reduced to 10 psi). Thus, because of the tendency for tension in the base course immediately beneath the asphalt concrete, a zone developed within which the modulus was effectively zero.

Contours of modulus within the base and subbase during the fourth cycle of analysis are shown in Figure 17. The modulus values jumped abruptly from 10 psi (within the zone of incipient tension) to about 10,000 psi in the next row of elements. The variation of modulus in the base for the summer condition may be contrasted with that determined for the winter condition in which there was no tendency for tension to develop. This difference in behavior must be attributed to the differences in the stiffness of the asphalt concrete surface since all other factors were the same in both analyses. It seems likely that when the stiffness modulus of the asphalt concrete is low, as in summer, fairly large radial extensional strains develop at the bottom of the pavement that induce similar strains in the granular material. At the same time, the load carried by the granular material tends to cause it to compress in the vertical direction and extend in the radial direction. The radial strains at the bottom of the asphalt concrete and at the top of the base must of course be compatible. Thus, depending

may be concluded that the analytical technique is capable of simulating the actual pavement behavior to a reasonable degree.

The deformed shapes of the pavement for the summer and winter analyses are shown in Figures 15 and 16, respectively. The surface deflections for the summer condition increase much more sharply near the loaded area than do those for the winter condition. Also, the vertical deflection of the bottom of the base is less near the centerline than beyond the edge of the loaded area, indicating that the portion of the base directly beneath the loaded area is subjected to considerable deformation in the summer condition. Within this zone there was a tendency for tensile stresses to develop in the granular material. As men-

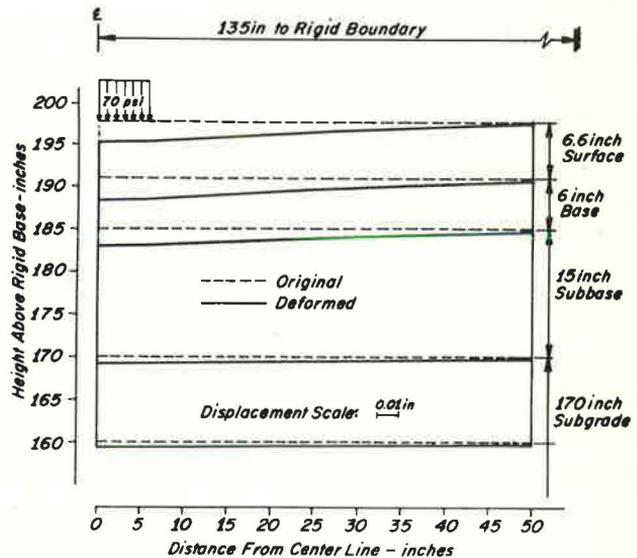


Figure 16. Deformed shape of Gonzales Bypass pavement—winter.

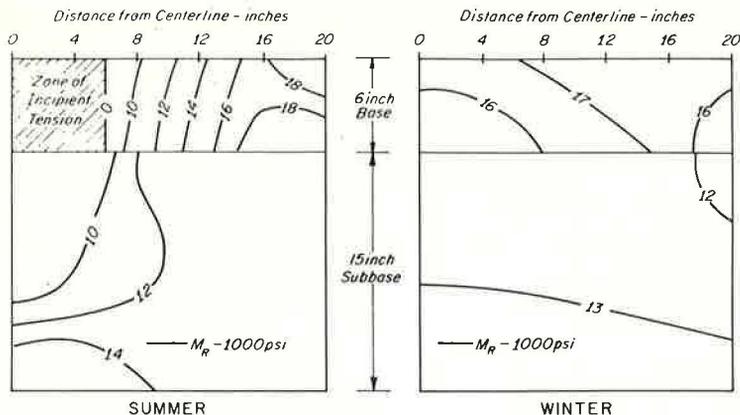


Figure 17. Contours of resilient modulus in the Gonzales Bypass base and subbase.

on the amount of extensional strain in the asphalt concrete and the amount of vertical compression in the base, either compressive or tensile radial stresses might tend to develop in the base. It appears that while tensile stresses tend to develop in the summer condition, there is no such tendency for the winter condition. It seems likely that the tendency for tension to develop would be greatest for thick pavements with low values of stiffness modulus, because it is under these conditions when high extensional strains at the bottom of the pavement would be accompanied by relatively small vertical deflections.

The contours of resilient modulus within the base and subbase (Fig. 16) indicate significant variations in the radial direction even outside the zone of incipient tension.

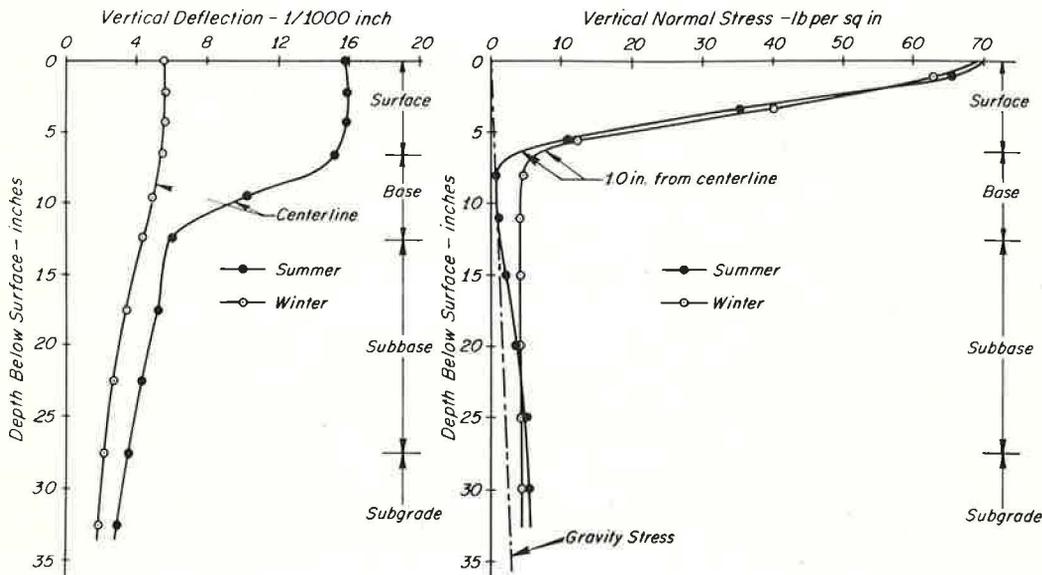


Figure 18. Computed deflections and stresses for Gonzales Bypass pavement.

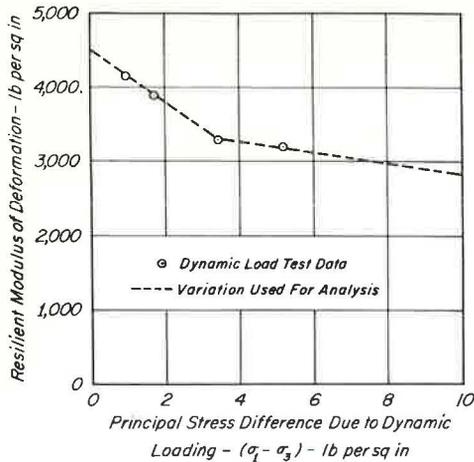


Figure 19. Variation of resilient modulus of subgrade with dynamic stress difference used in analysis of plate load test.

Thus, it would be very difficult to simulate this behavior by means of layered system theory, where modulus values can be varied only in the vertical direction. For the winter condition, the values of modulus are sensibly constant throughout both the base and the subbase. It is perhaps worthy of note that the values of modulus within the base and subbase were not as stable as the surface deflections; it is therefore doubtful that great significance can be attached to the precise values represented by the contours in Figure 17.

The influence of the zone of incipient tension on the performance of the pavement is shown in Figure 18. More than half of the compression contributing to deflection occurs within the base in the summer condition, and that the material within the zone of incipient tension carries only the gravity stress. Although significant compression occurs within the base under the winter condition, it comprises a much smaller portion of the total vertical deflection.

#### Plate Load Test on Granular Material

Using the same procedure, an analysis was made of a repeated plate load test conducted on a two-layer system, 12 in. of granular material overlying a cohesive subgrade (14). Repeated load triaxial tests were used to establish the resilience characteristics of base-course and subgrade materials. The relationship between resilient modulus and confining pressure for the base course was found to be

$$M_R = 7000 (\sigma_3)^{0.55} \quad (6)$$

in which  $M_R$  and  $\sigma_3$  are both measured in pounds per square inch. The relationship between resilient modulus and stress difference for the subgrade is shown in Figure 19; the finite element configuration used for the analysis, in Figure 20.

Values of vertical deflection calculated on successive cycles are shown in Figure 21. Whereas the measured deflection of the plate was only about 0.037 in., the calculated deflection for the eighth cycle was 0.087 in. Although it is possible that the calculated value would have converged to the measured value after a large number of cycles, the analytical method would still not be practical for this problem, because of the large amount of computer time required, about 1.8 minutes of 7094 computer time for each of the eight solution cycles.

It seems likely that the reason for the divergence, or slow convergence, of the surface deflections in this plate load test analysis is the dominant role in the behavior which is played by the granular base material, and the fact that the resilient modulus of this material is so strongly stress-dependent. The procedure used would be expected to converge rapidly only if the stress distribution, considered for the system as a whole, was not drastically altered by the variations in modulus from cycle to cycle. Whereas this may be true for the case where an asphalt concrete pavement overlies the base, it apparently is not true in the case of a plate load applied directly to the surface of the granular material.

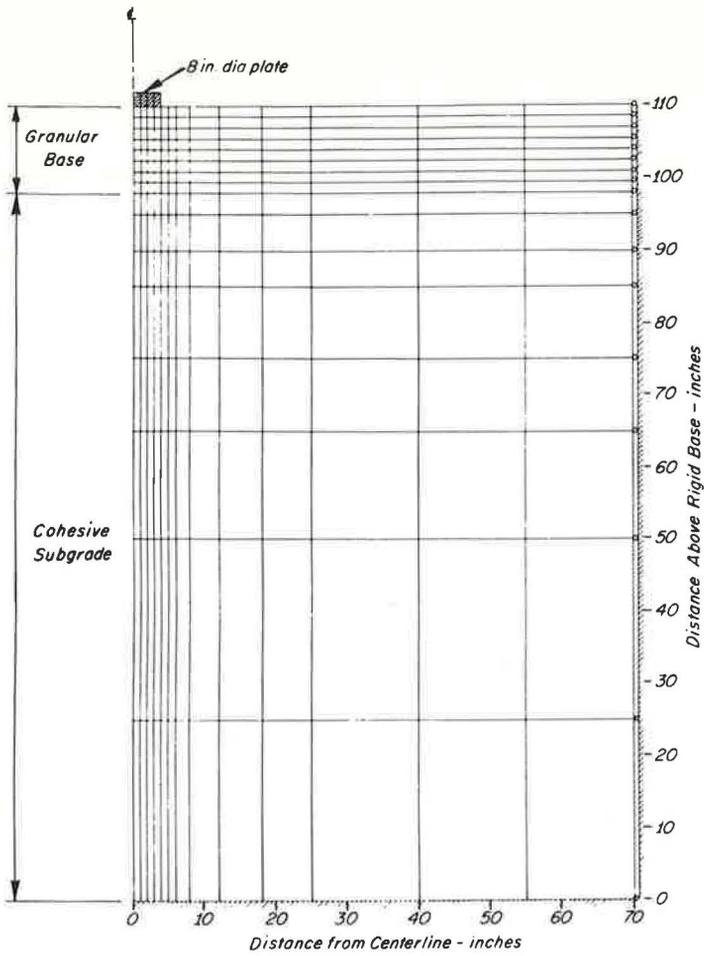
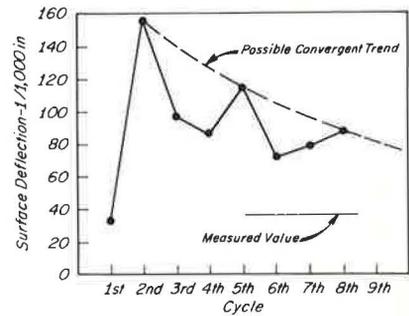


Figure 20. Finite element configuration used for analysis of plate load test.

Figure 21. Variation of calculated surface deflections of plate on granular base with number of iterative cycles.



## SUMMARY AND CONCLUSIONS

Axisymmetric finite element analyses may be readily applied to pavement problems. With appropriate attention to boundary conditions, accurate analyses of the behavior of pavement structures with linear material properties may be made using this technique.

This study also indicates that it is feasible to approximate nonlinear material properties in analyses of pavement structures. The nonlinear analyses which have been performed must be considered as preliminary, and the method used represents only one of a number of possible techniques. It is likely, however, that studies of this type may eventually afford improved understanding of the behavior of pavement structures under load.

## REFERENCES

1. Clough, R. W. The Finite Element Method in Plane Stress Analysis. ASCE Conf. on Electronic Computation, Proc., 2nd, Pittsburgh, Sept. 1960.
2. Wilson, E. L. Finite Element Analysis of Two-Dimensional Structures. Berkeley: Univ. of California, Dept. of Civil Engineering (Report No. 63-2), 1963.
3. Clough, R. W., and Rashid, Y. Finite Element Analysis of Axisymmetric Solids. Jour. of the Engineering Mechanics Division, ASCE, p. 71-85, Feb. 1965.
4. Wilson, E. L. A Digital Computer Program for the Finite Element Analysis of Solids With Non-Linear Material Properties. Berkeley: Univ. of California, Dept. of Civil Engineering, 1965.
5. Wilson, E. L. Structural Analysis of Axisymmetric Solids. AIAA Jour., Vol. 3, No. 12, p. 2269-2273, Dec. 1965.
6. Michelo, J. Analysis of Stresses and Displacements in an N-Layered Elastic System Under a Load Uniformly Distributed on a Circular Area. Internal Report, California Research Corp., Sept. 1963.
7. Warren, H., and Dieckmann, W. L. Numerical Computation of Stresses and Strains in a Multiple-Layer Asphalt Pavement System. Internal Report, California Research Corp., Sept. 1963.
8. Peattie, K. R., and Jones, A. Surface Deflections of Road Structures. Symposium on Road Tests for Pavement Design, Proc., Lisbon, 1962.
9. Monismith, C. L. Asphalt Mixture Behavior in Repeated Flexure. Berkeley: Univ. of California, Dept. of Civil Engineering (Report TE 65-9), 1965.
10. Haynes, J. H., and Yoder, E. J. Effects of Repeated Loading on Gravel and Crushed Stone Base Course Materials Used in the AASHO Road Test. Highway Research Record 39, p. 82-96, 1963.
11. Seed, H. B., and Chan, C. K. Effect of Duration of Stress Applications on Soil Deformation Under Repeated Loading. International Conference on Soil Mechanics and Foundation Engineering, Proc., 5th, Vol. 1, p. 340-345, 1964.
12. Hveem, F. N. Pavement Deflections and Fatigue Failures. HRB Bull. 114, p. 43-87, 1955.
13. Monismith, C. L. Symposium on Pavement Behavior as Related to Deflection, Pt. II: Significance of Pavement Deflections. AAPT Proc., 31st, p. 231-253, 1962.
14. Seed, H. B., Mitry, F. G., Monismith, C. L., and Chan, C. K. Factors Influencing the Resilient Deformations of Untreated Aggregate Base in Two-Layer Pavements Subjected to Repeated Loading. Highway Research Record 190, p. 19-57, 1967.
15. Monismith, C. L., Seed, H. B., Mitry, F. G., and Chan, C. K. Prediction of Pavement Deflections From Laboratory Tests. 2nd International Conference on the Structural Design of Asphalt Pavements, Univ. of Michigan, Aug. 1967.
16. Avremesco, A. Dynamic Phenomena in Pavements Considered as Elastic Layered Structures. 2nd International Conference on the Structural Design of Asphalt Pavements, Univ. of Michigan, Aug. 1967.

17. van der Poel, C. J. General System Describing Viscoelastic Properties of Bitumens and Its Relation to Routine Road Tests. Jour. of Applied Chemistry, Vol. 4, p. 221-236, May 1954.
18. Seed, H. B., Chan, C. K., and Lee, C. E. Resilience Characteristics of Sub-grade Soils and Their Relation to Fatigue Failures in Asphalt Pavements. International Conference on Structural Design of Asphalt Pavements, Proc., Univ. of Michigan, 1962.
19. Monismith, C. L. Asphalt Mixture Behavior in Repeated Flexure. Berkeley: Univ. of California, Dept. of Civil Engineering (Report No. TE 66-6), 1966.