Comparison of Laboratory and Field Values Of c_v for Boston Blue Clay

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Laboratory values of c_{vs} (the coefficient of consolidation for swelling) for Boston blue clay are compared with the value back-figured from piezometer observations at a large building excavation. The field value is shown to be six times larger than the average laboratory value. Possible reasons for the large discrepancy between the laboratory and field values are sample disturbance, errors in laboratory test procedures, errors in field measurements, and three-dimensional consolidation effects. Of these four items, errors in laboratory test procedures, particularly the difficulty of measuring c_{vs} from oedometer tests, can contribute significantly to the discrepancy. In addittion, it is shown that three-dimensional consolidation effects can account for the differences between laboratory and field values.

•SETTLEMENT predictions, although one of the most common tasks of the soil engineer, are subject to very large errors. Estimating rate of settlement is generally even more hazardous than estimating total settlement. This paper compares the laboratory value of c_{vs} (the coefficient of consolidation for swelling) for Boston blue clay with the value back-figured from piezometer observations at a large building excavation. The field data were taken under a program known as FERMIT (Foundation Evaluation and Research, M.I.T.), which is sponsored by the M.I.T. Office of Physical Plant. The building under study is the Julius A. Stratton Building, the M.I.T. Student Center.

DESCRIPTION OF FOUNDATION AND SUBSOILS

The M.I.T. Student Center is a five-story reinforced concrete frame structure. It was constructed on the west side of the campus during 1963-64. The location is shown in Figure 1. The building has a floating foundation and rests on a 3 to 10-ft thick concrete mat constructed on a sand-gravel layer.

Figure 2 shows the average soil profile at the site. The Boston blue clay, which is 60 to 75 ft thick in this area, is overconsolidated at the top (OCR~ 6). The amount of overconsolidation decreases with depth, and the bottom half of the clay is normally consolidated.

The results of extensive soil tests on the clay have been reported by Ladd and Luscher (1). The soil parameter of interest to this paper is the coefficient of consolidation for swelling, c_{vs}. Values of c_{vs} are shown in Figure 2 and are summarized in Table 1. Also given in Table 1 are values of c_{vc}, the coefficient of consolidation for compression. The values in Table 1 are averages of values computed by both the log time and the square root of time-fitting methods from 40 tests.

The average value of c_{vs} for the first stress decrement from the maximum past pressure, $\bar{\sigma}_{vm}$, is $60 \pm 30 \times 10^{-4}$ cm²/sec. In addition, c_{vs} decreases with increasing OCR. For an OCR = 4, the average value of $c_{vs} = 30 \pm 10 \times 10^{-4}$ cm²/sec. Thus, the upper layer of Boston blue clay, which is overconsolidated, should have a lower value of c_{vs}

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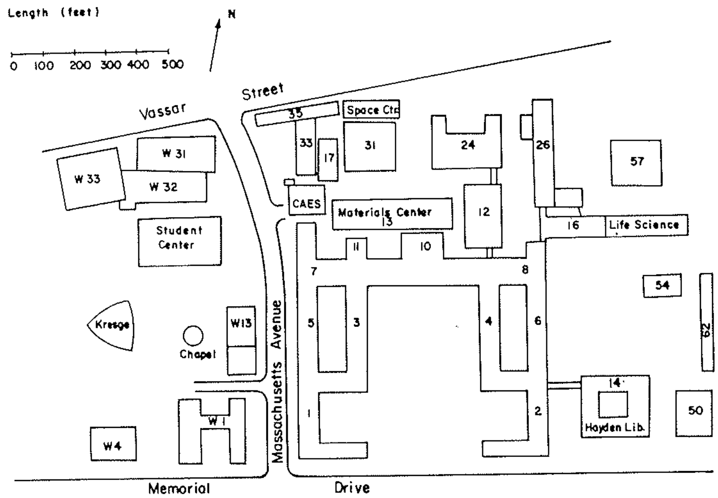

Figure 1. Plan of central M.I.T. campus.

than the normally consolidated clay. This variability in laboratory values of c_{vs} has not been considered in the anȧlysis of the field data. The point is, however, that the large variability of laboratory values will in itself preclude the possibility of accurate rate predictions based on a few laboratory tests.

CONSTRUCTION HISTORY

A plan view of the foundation excavation is shown in Figure 3. The excavation was made in two stages:

1. Stage 1, from Elev. +22 to Elev. +15, during the period September 21-30, 1963. No dewatering was required during this excavation.
2. Stage 2, from Elev. +15 to Elev. +7.5, during the period October 7-16, 1963. Steady pumping from a well-point system located in the sand-gravel layer above the clay began on October 4 and continued throughout the stage 2 excavation.

The total vertical stress release from the excavation (Elev. +22 to Elev. +7.5) was about 0.82 kg/cm^2. The foundation mat, which constitutes about one-third of the structural dead load, was poured in four sections between November 23, 1963, and February 15, 1964.

TABLE 1

SUMMARY OF COEFFICIENT OF CONSOLIDATION DATA FOR BOSTON BLUE CLAY[a]

Coefficient	c_v, 10^{-4}cm^2/sec
1. Compression, c_{vc}, at $\bar{\sigma}_{vo}$	
Overconsolidated	40 ± 20
Normally consolidated	20 ± 10
2. Swelling, c_{vs}	
From $\bar{\sigma}_{vm}$ to $\frac{1}{2}\bar{\sigma}_{vm}$	60 ± 30
From $\frac{1}{2}\bar{\sigma}_{vm}$ to $\frac{1}{4}\bar{\sigma}_{vm}$	30 ± 10
From $\frac{1}{4}\bar{\sigma}_{vm}$ to $\frac{1}{8}$ or $\frac{1}{16}\bar{\sigma}_{vm}$	20 ± 10

[a]From Ladd and Luscher (1).

FOUNDATION INSTRUMENTATION

The foundation instrumentation at the Student Center included eight piezometers (Casagrande type), six heave roads, and a permanent benchmark. The locations of these instruments are shown in Figure 3. Several additional heave and settlement points were installed during later stages of construction. An observation well is located 100 ft north of the site.

CALCULATION OF FIELD c_{vs}

The measured pore pressures from piezometers 1 through 4 have been

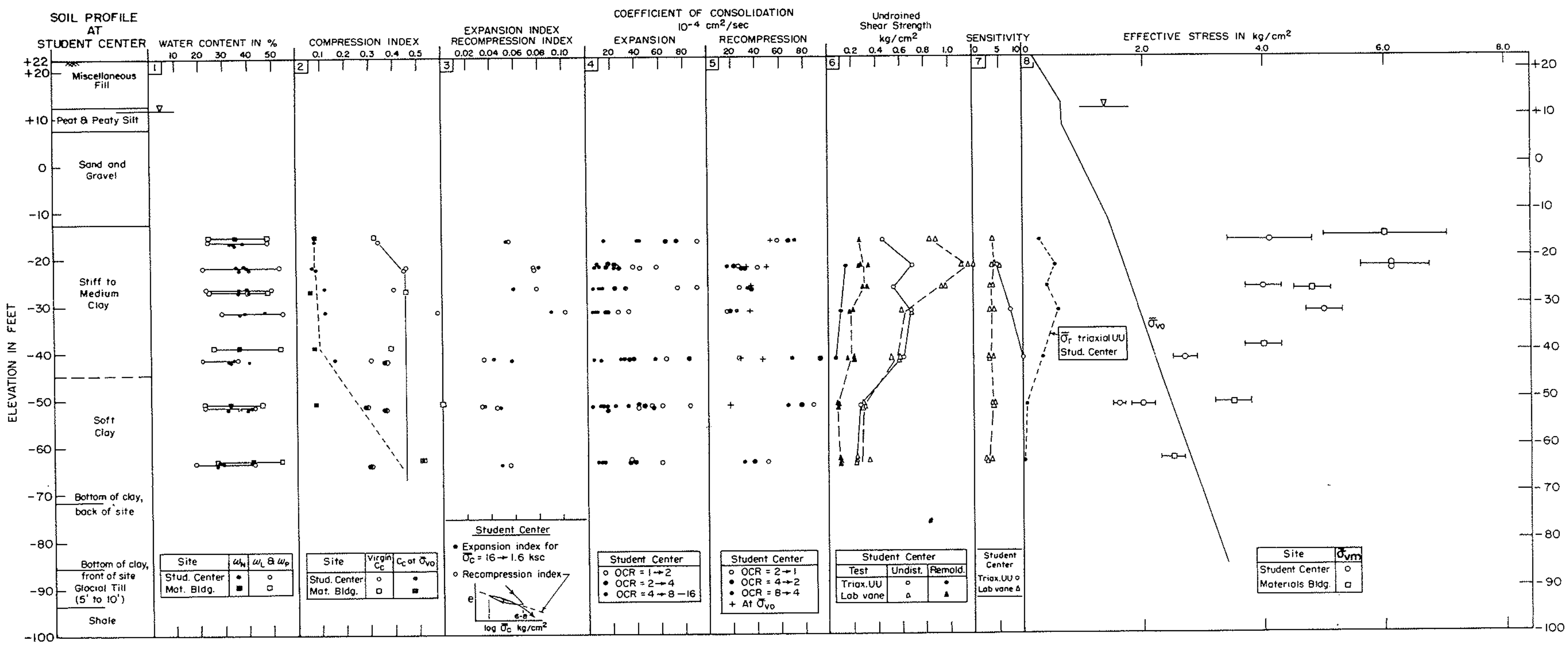

Figure 2. Typical soil test data for M.I.T. campus.

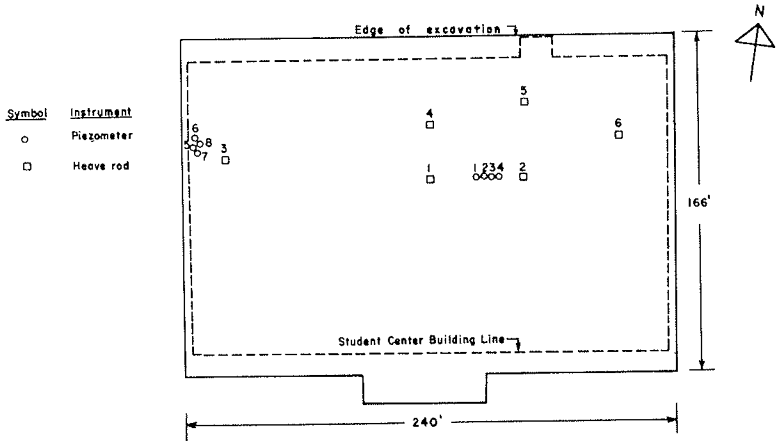

Figure 3. Dimensions of excavation.

used to compute a field value of c_{VS}. Figure 4 shows the location of these instruments in the clay layer. Figure 5 shows the measurements of pore pressures and vertical movements during foundation construction.

Figure 6 shows a plot of the measured negative excess pore pressures in the clay layer 15 and 105 days after excavation. The excavation and the dewatering were assumed to have occurred on October 7, the middle of the actual excavation period.

Also shown in Figure 6 are values of the initial excess pore pressures (t = 0) obtained by extrapolating the measured pore pressures at the end of excavation back to the middle of the excavation period. The values of initial excess pore pressure are quite close to $\Delta\sigma_V$, the change in vertical stress computed from elastic theory with $\nu = 1/2$.

Knowing the initial pore pressures at t = 0, and the equilibrium pore pressures at $t = \infty$, the measured values of pore pressure at any intermediate time can be used to compute a point value of U_Z, the percent consolidation. [The equilibrium pore pressures were taken as those resulting from steady state upward seepage due to pumping. The water table actually varied somewhat during excavation (as shown by the well-point data in Fig. 5). The average head drop in the sand-gravel layer due to pumping was -13 ft for t = 15 days and -10 ft for t = 105 days.] Using standard one-dimensional solutions (3), the time factor, T_S, can then be obtained. The time factor is related to the coefficient of consolidation by the following equation:

$$c_{VS} = \frac{T_S H^2}{t}$$

The effect of errors in the assumed initial pore pressure distribution can be minimized by computing c_{VS} using the following equation:

$$c_{VS} = \frac{T_{S2} - T_{S1}}{t_2 - t_1} H^2$$

where T_{S2} and T_{S1} are the time factors at times t_2 and t_1, respectively.

This procedure led to an average field value of $c_{VS} = 350 \pm 50 \times 10^{-4}$ cm²/sec. The agreement between the theoretical isochrone computed

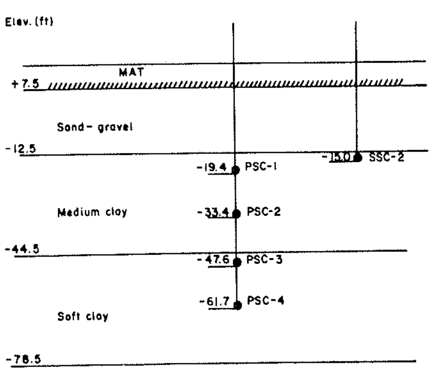

Figure 4. Location of piezometers.

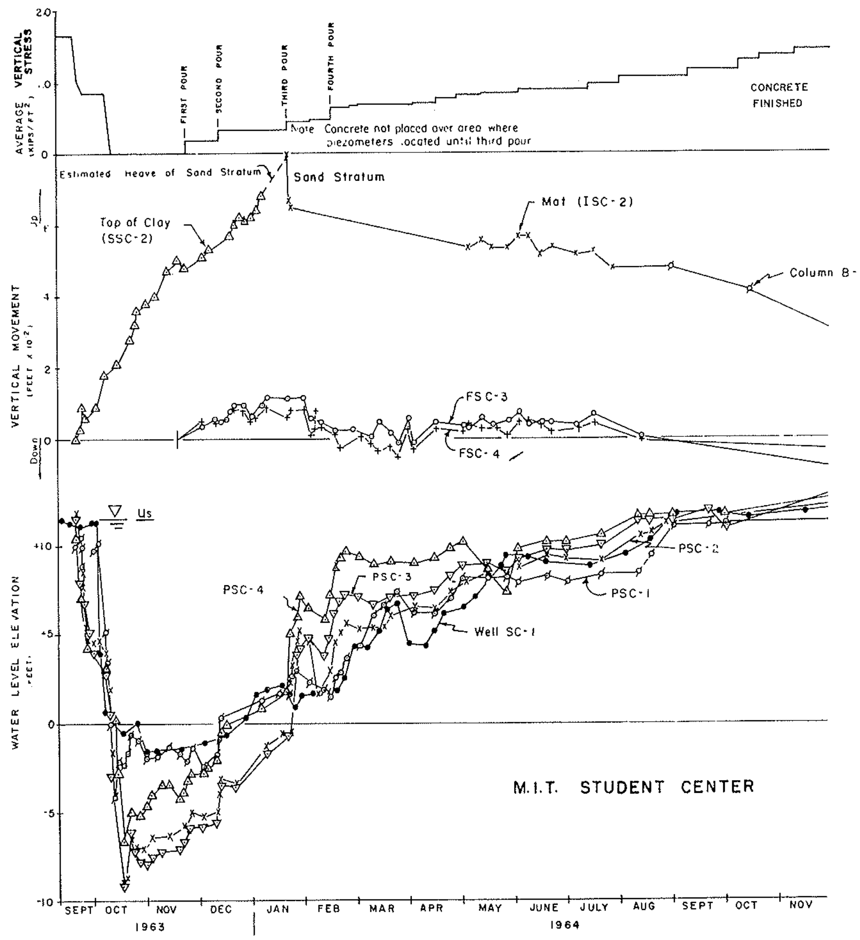

Figure 5. Pore pressures and vertical movements.

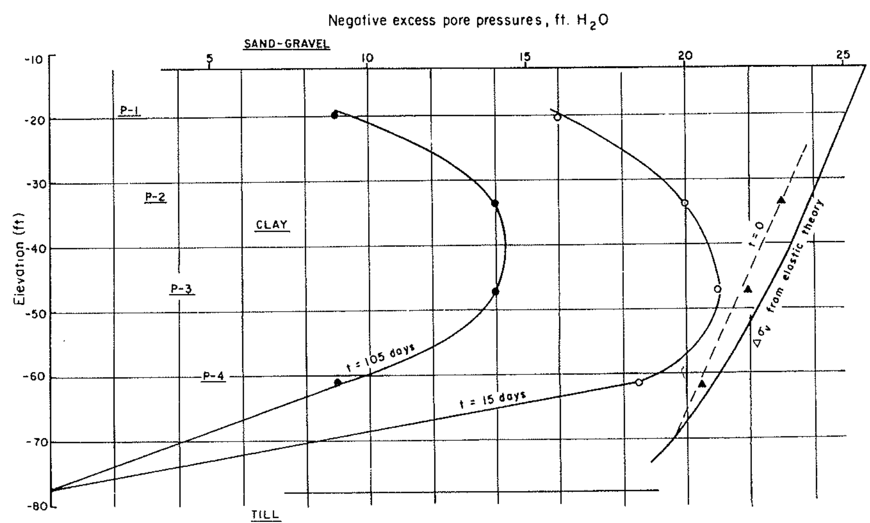

Figure 6. Measured pore pressures at 15 and 105 days.

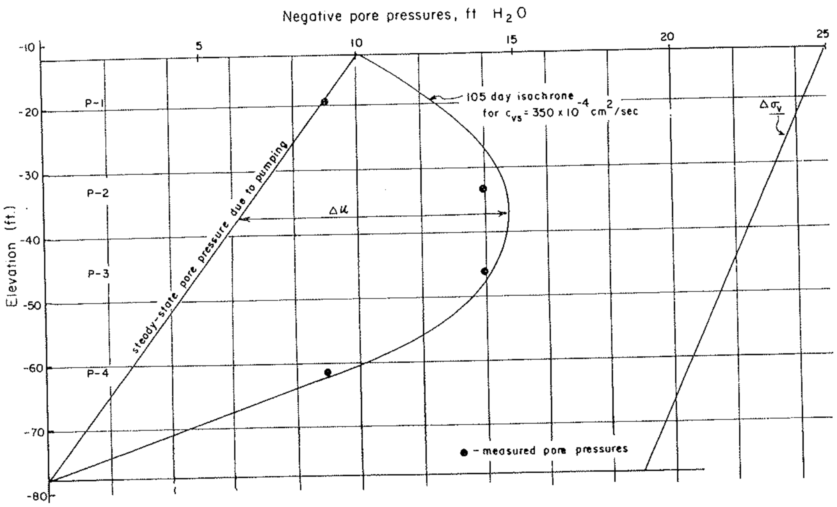

Figure 7. Measured and computed pore pressures after 105 days.

using this value of c_{vs} and the measured pore pressures at t = 105 days is shown in Figure 7. The agreement is quite good for the lower three piezometers. The top piezometer is much more sensitive to errors in the assumed steady-state pore pressures, which is probably why it does not show good agreement. The apparent field value of c_{vs} is about 6 times the average laboratory value. The remainder of the paper will discuss possible reasons for this discrepancy.

POSSIBLE CAUSES OF DISCREPANCY BETWEEN LAB AND FIELD VALUES

Possible reasons for the large difference between the average lab value of c_{vs} (60×10^{-4}) and the value back-figured from field data (350×10^{-4}) include the following:

1. Sample disturbance,
2. Laboratory test procedures,
3. Errors in field measurements, and
4. Use of one-dimensional consolidation theory.

Effect of Sample Disturbance

Disturbed soil samples give lower values of c_v. Taylor (2) showed a 40-fold decrease in c_v after remolding a sample of Boston blue clay. Sample disturbance is undoubtedly responsible for some of the scatter in c_v values shown in Figure 2.

A value of c_v corrected for disturbance can be obtained by estimating the effect of disturbance on k and a_v. The major change is the decrease in k resulting from the lower void ratio at any given consolidation pressure. A plot of e vs k can be used to estimate the magnitude of this effect. A corrected compression curve (4) can be used to obtain a better value of a_v. The new values of k and a_v can then be used to recompute an undisturbed c_v:

$$(c_v)_u = (c_v)_d \; \frac{k_u}{k_d} \frac{(a_v)_d}{(a_v)_u}$$

where subscript u refers to undisturbed conditions and subscript d refers to the disturbed laboratory sample.

Calculations using this procedure indicated that the effect of sample disturbance on the lab values of c_v is within the range of experimental error given in Table 1.

Laboratory Test Procedures

Lambe (5) discussed the differences in stress paths between laboratory tests and actual field conditions. In order to predict field behavior accurately, laboratory tests should in general follow the field stress path as closely as possible. However, this procedure will not, in general, provide accurate c_v data. For example, Lambe (6) described another M.I.T. building excavation (Center for Advanced Engineering Study), located across Massachusetts Avenue from the Student Center (see Fig. 1), where an oedometer test that duplicated the field vertical stress release gave a value of c_{vs} = 3×10^{-4} cm²/sec. This value is a factor of 100 less than the value back-figured from field data.

Standard oedometer tests use a stress-increment ratio ($\Delta\bar{\sigma}/\bar{\sigma}$) of one. It has been shown that for smaller values of stress-increment ratio the Terzaghi consolidation theory does not apply (2, 7). However, the actual value of $\Delta\bar{\sigma}/\bar{\sigma}$ in the field is frequently less than one, and it varies with depth. A technique for correcting laboratory data to take this effect into account is not currently available. Hence, it is necessary to use a stress-increment ratio of one in laboratory tests in order to obtain c_v values that appear reasonable.

Another factor that influences the laboratory value of c_v is the curve-fitting method used. The $\sqrt{t}$ fitting method generally gives a value of c_v about 50 percent higher than the log t method for Boston blue clay (1). Values from the two methods were generally averaged to give the c_v values used in this paper.

The calculation of c_{vs} from oedometer tests is frequently difficult, because swelling curves tend to deviate much more from the theoretical time curves than do compression curves. Perhaps an equally valid procedure would be to calculate c_{vs} from measured values of c_{vc}, a_{vc}, and a_{vs}. If all of the values are chosen at the same average void ratio, then it can be assumed that k is the same for both compression and swelling without introducing a significant error. Then c_{vs} can be calculated as follows:

$$c_{vs} = c_{vc} \left(\frac{a_{vc}}{a_{vs}}\right)$$

Application of this procedure to the laboratory data on Boston blue clay yields an average value of $c_{vs} = 100 \pm 50 \times 10^{-4}$ cm²/sec, almost two times higher than the average value calculated from time-swelling curves.

Errors in Field Measurements

The field value of c_{vs} relies on accurate piezometer data. The estimated accuracy of the measured pore pressures is ± 0.25 ft. The time lag for the piezometers is less than 24 hr for 90 percent equalization of pore pressure.

The assumed initial pore pressures may have been in error, although the distribution used is more likely to have underestimated the field c_{vs} than overestimated it.

Use of One-Dimensional Consolidation Theory

Most settlement problems are solved by use of the Terzaghi one-dimensional theory. Significant errors in rate of settlement predictions can result from this assumption. Rapid advances, using numerical methods, are being made in the application of three-dimensional consolidation theory to settlement problems. In the near future, the soil engineer may routinely use these three-dimensional analyses for settlement predictions. Solutions currently available can be used to estimate how much of the sixfold discrepancy between predicted and observed rates of pore pressure dissipation at the Student Center can be attributed to three-dimensional effects.

The importance of two- or three-dimensional drainage will increase as (a) the ratio of horizontal permeability to vertical permeability increases (k_h/k_v), and (b) the ratio of vertical drainage path to radius of loaded area increases (H/a).

The field permeability of Boston blue clay, measured by falling-head tests on piezometers, is $2\text{-}3 \times 10^{-7}$ cm/sec, which is about 6 to 8 times greater than the vertical permeability measured on laboratory samples. To some extent the measured field permeability probably reflects the existence of thin silt seams that act as drainage layers.

Davis and Poulos (8) recommend using an average value of permeability to calculate a three-dimensional c_v:

$$k_{av} = \sqrt[3]{k_h^2 \, k_v}$$

The resulting equation for computing $(c_{vs})_3$ from measured oedometer values of $(c_{vs})_1$ is

$$(c_{vs})_3 = \frac{1}{3}\left(\frac{1 + \nu}{1 - \nu}\right)\frac{k_{av}}{k_v}\,(c_{vs})_1$$

Assuming $\nu = 1/3$, $k_h = 6\,k_v$, and $(c_{vs})_1 = 60 \times 10^{-4}$ cm^2/sec; this gives an estimated in situ $(c_{vs})_3 = 130 \times 10^{-4}$ cm^2/sec.

In actuality, the Student Center excavation would not at first glance appear to have significant three-dimensional effects. The vertical drainage path is 33 ft and the equivalent radius is 112 ft, giving a value of $H/a = 0.29$. However, curves given by Gibson et al (9) indicate that the rate of settlement, even with these dimensions, may be twice as fast as predicted by one-dimensional theory. This calculation assumes that $k_h = k_v$. When the effect of anisotropic permeability is added, an "equivalent" one-dimensional c_{vs} of about 250×10^{-4} cm^2/sec is obtained. Considering the assumptions required to obtain this result, the rather close agreement with the field value must be considered fortuitous. However, it does indicate that three-dimensional consolidation effects may be able to account for the large discrepancy between laboratory and field values of c_{vs} at the Student Center.

The preceding analysis was made for one foundation, and is not intended to imply that the results are generally applicable. However, the authors hope that numerous such analyses, combined with advances in three-dimensional consolidation theory and advances in field measurements of soil properties will lead to better techniques for making settlement predictions.

CONCLUSIONS

1. Field rates of excess pore pressure dissipation measured in Boston blue clay gave a value of c_{vs} for swelling of 350×10^{-4} cm^2/sec, using a one-dimensional analysis.
2. Although the discrepancy cannot be definitely attributed to any one cause, three-dimensional drainage seems the most likely reason. The difficulty of obtaining an accurate value of c_{vs} from laboratory time-swelling curves appears also to be an important factor.

ACKNOWLEDGMENTS

The success of the FERMIT program is the result of numerous individual contributions. In particular, A. A. Gass (10) and K. C. de Fries-Von Arnim (11) studied the Student Center project for their graduate theses. Other present and former members of the M.I.T. staff who had prominent roles are Harry M. Horn (formerly Assistant Professor of Civil Engineering), Charles C. Ladd, L. A. Wolfskill, U. Luscher (formerly Assistant Professor of Civil Engineering), R. S. Ladd, W. R. Beckett, and N. F. Braathen. The M.I.T. Office of Physical Plant has provided continuous support and encouragement for FERMIT. Philip A. Stoddard, Vice-President, and William R. Dickson, Assistant Director of Physical Plant, have been especially helpful.

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Discussion

ROBERT L. SCHIFFMAN, University of Illinois at Chicago Circle—The authors are to be congratulated on pointing out the many factors that contribute to the discrepancy between laboratory and field values of c_v. They have noted that one of the reasons for differences between laboratory and field values of the coefficient of consolidation is the three-dimensional consolidation effect. It is to this general point that this discussion is directed.

The coefficient of consolidation, c_v, is defined (12) as

$$c_v = \frac{k}{\gamma_w m} \tag{1}$$

where k is a coefficient of permeability, m is the compressibility, and γ_w is the unit weight of water. For c_v to be theoretically the same in both the field and the laboratory, one of two conditions must be satisfied. First, if c_v were a fundamental soil property, its value would be independent of the measurement conditions. If c_v is not a fundamental soil property, then the same value can be expected theoretically only if the k and m were of the same magnitude in the laboratory and in the field. That is, if the boundary conditions in both the field and the laboratory were the same, one could expect k and m to theoretically have the same values. On the other hand, different boundary conditions would provide different values for k and m.

In the conventional oedometer test, the value of the coefficient of permeability is restricted to the vertical component k_v. For a natural soil, consolidated in a triaxial device, or in the field, there are at least two values of the coefficient of permeability: the vertical permeability, k_v, and the horizontal permeability, k_h. Unless the soil is isotropic with respect to permeability there are, due to this effect alone, two possible coefficients of consolidation:

$$c_v = \frac{k_v}{\gamma_w m} \tag{2a}$$

for vertical flow of water, and

$$c_h = \frac{k_h}{\gamma_w m} \tag{2b}$$

for lateral flow of water.

The compressibility, m, is a factor that depends on the geometry of the loading and the boundary conditions, independent of the permeability. To show this, consider the consolidation of a clay layer of thickness h when the loading extends indefinitely on the surface (9). The governing consolidation equation is

$$\frac{k}{\gamma_w} \frac{\partial^2 u}{\partial z^2} = - \frac{\partial e}{\partial t} \tag{3}$$

where k is the coefficient of permeability, u is the excess pore pressure, and e is the dilatation of the soil skeleton. The dilatation is defined as

$$e = e_{xx} + e_{yy} + e_{zz} \tag{4}$$

where e_{xx}, e_{yy}, and e_{zz} are the normal strains of the soil skeleton.

Assume that the soil skeleton is elastic. Then the effective stress-strain relations are

$$Ee_{xx} = \sigma'_{xx} - \nu(\sigma'_{yy} + \sigma'_{zz}) \tag{5a}$$

$$Ee_{yy} = \sigma'_{yy} - \nu(\sigma'_{xx} + \sigma'_{zz}) \tag{5b}$$

$$Ee_{zz} = \sigma'_{zz} - \nu(\sigma'_{xx} + \sigma'_{yy}) \tag{5c}$$

where σ'_{xx}, σ'_{yy}, and σ'_{zz} are the normal effective stress components, and E and ν are Young's modulus and Poisson's ratio, respectively, of the soil skeleton.

The oedometer provides axially symmetric compression with frictionless sides. Thus, $\sigma'_{rr} = \sigma'_{zz}$. The stress-strain relations are

$$Ee_{rr} = (1 - \nu)\sigma'_{rr} - \nu\sigma'_{zz} \tag{6a}$$

and

$$Ee_{zz} = \sigma'_{zz} - 2\nu\sigma'_{rr} \tag{6b}$$

Since the lateral strain $e_{rr} = 0$, then

$$\sigma'_{rr} = \frac{\nu}{1 - \nu} \sigma'_{zz} \tag{7}$$

The dilatation, e, is equal to the vertical strain, e_{zz}, in the oedometer. Thus from Eqs. 6b and 7

$$e = \frac{(1 - 2\nu)\ (1 + \nu)}{E(1 - \nu)}\ \sigma'_{zz} \tag{8}$$

From the effective stress principle

$$\sigma_{zz} = \sigma'_{zz} + u \tag{9}$$

where σ_{zz} is the vertical total stress. Substituting the effective stress Eq. 9 into Eq. 8 and then into Eq. 3 results in

$$c_o \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \tag{10a}$$

where

$$c_o = \frac{kE\,(1 - \nu)}{\gamma_w(1 - 2\nu)\ (1 + \nu)} \tag{10b}$$

if the applied load intensity is time-independent. The coefficient c_o is the coefficient of consolidation for the oedometer.

The solution to Eq. 10a is of the form

$$u = u_o F\left(\frac{c_o t}{h^2}, \frac{z}{h}\right) \tag{11a}$$

where the initial excess pore pressure, u_o, is

$$u_o = \sigma_{zz} \tag{11b}$$

The vertical strain is

$$e_{zz} = \frac{(1 - 2\nu)\ (1 + \nu)}{E(1 - \nu)}\ \sigma_{zz} \left[1 - F\left(\frac{c_o t}{h^2}, \frac{z}{h}\right)\right] \tag{12}$$

and the settlement, ρ, is

$$e_{zz} = \frac{\partial \rho}{\partial z} \tag{13}$$

Then

$$\rho(t) = \frac{(1 - 2\nu)\ (1 + \nu)}{E\ (1 - \nu)}\ \sigma_{zz} \int_o^h (1 - F)dz \tag{14}$$

The degree of consolidation, U, is defined as

$$U = \frac{\rho(t)}{\rho(\infty)} \tag{15}$$

which becomes

$$U(t) = 1 - \frac{1}{h} \int_o^h F dz \tag{16}$$

Consider the consolidation under plane strain conditions. This is achieved by extending the width of a strip load indefinitely. Plane strain conditions are maintained in that the strain in the y direction is zero, or $e_{yy} = 0$. Then

$$\sigma'_{yy} = \nu(\sigma'_{xx} + \sigma'_{zz}) \tag{17}$$

The effective stress-strain relations become

$$Ee_{xx} = (1 - \nu^2)\sigma'_{xx} - \nu(1 + \nu)\sigma'_{zz} \tag{18a}$$

$$Ee_{zz} = (1 - \nu^2)\sigma'_{zz} - \nu(1 + \nu)\sigma'_{xx} \tag{18b}$$

The dilatation is

$$e = \frac{(1 + \nu)\ (1 - 2\nu)}{E}\ (\sigma'_{xx} + \sigma'_{zz}) \tag{19}$$

or, using the effective stress equation,

$$e = \frac{(1 + \nu)\ (1 - 2\nu)}{E}\ (\sigma_{xx} + \sigma_{zz} - 2u) \tag{20}$$

It can be shown that when the surface loading extends infinitely in all directions the total stress components are independent of time if the surface loading is static. Thus, substituting Eq. 20 into Eq. 3 results in

$$c_p \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \tag{21a}$$

where

$$c_p = \frac{kE}{2\gamma_w(1 - 2\nu)\ (1 + \nu)} \tag{21b}$$

The solution to Eq. 21a is

$$u = u_o F\left(\frac{c_p t}{h^2}, \frac{z}{h}\right) \tag{22}$$

where the initial excess pore pressure, u_o, is

$$u_o = \left[\frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3}\right]_{\nu = 1/2} \tag{23}$$

Assume that, under plane strain, the clay layer rests on a frictionless base. Then the lateral total stress, σ_{xx}, is zero. Thus,

$$u_o = \frac{\sigma_{zz}}{2} \tag{24}$$

and

$$u = \frac{\sigma_{zz}}{2}\ F\left(\frac{c_p t}{h^2}, \frac{z}{h}\right) \tag{25}$$

The consolidation settlement of the surface is

$$\rho(t) - \rho(0) = \frac{2(1 - 2\nu)\ (1 + \nu)}{E} \int_0^h \left[\frac{\sigma_{zz}}{2} - u\right] dz \tag{26}$$

and the degree of consolidation, $\overline{U}$, is defined as

$$\overline{U}(t) = \frac{\rho(t) - \rho(0)}{\rho(\infty) - \rho(0)} \tag{27}$$

Thus,

$$\overline{U}(t) = 1 - \frac{1}{h} \int_0^h F dz \tag{28}$$

Consider the consolidation of a layer under an axially symmetric loading. The effective stress-strain relations are given by Eqs. 6a and 6b. The dilatation is then

$$Ee = (1 - 2\nu)\ (\sigma_{zz} + 2\sigma_{rr} - 3u) \tag{29}$$

In this case, the load of indefinite extent is achieved by expanding the radius of a circular load to infinity. Thus Eq. 3 takes the form

$$c_r \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \tag{30a}$$

where

$$c_r = \frac{kE}{3\gamma_w (1 - 2\nu)} \tag{30b}$$

The solution is,

$$u = u_o F \left(\frac{c_r t}{h^2}, \frac{z}{h}\right) \tag{31}$$

For a smooth-based layer the lateral stress, σ_{rr}, is zero. Then the initial excess pore pressure, u_o, is

$$u_o = \frac{\sigma_{zz}}{3} \tag{32}$$

Thus the consolidation settlement of the surface is

$$\rho(t) - \rho(0) = \frac{3(1 - 2\nu)}{E} \int_0^h \left[\frac{\sigma_{zz}}{3} - u\right] dz \tag{33}$$

The degree of consolidation, $\overline{U}$, is then

$$\overline{U}(t) = 1 - \frac{1}{h} \int_0^h F dz \tag{34}$$

Since Eqs. 16, 28, and 34 are identical in form, the relationships between the degrees of consolidation for the three cases considered follow from the relationship between c_o, c_p, and c_r. That is, the time factor for a given degree of consolidation can be adjusted for case to case by a simple ratio of coefficients, or

$$\frac{T_o}{T_p} = \frac{c_o}{c_p} \tag{35a}$$

or

$$\frac{T_o}{T_r} = \frac{c_o}{c_r} \tag{35b}$$

in which

$$T_o = \frac{c_o t}{h^2} \tag{36a}$$

$$T_p = \frac{c_p t}{h^2} \tag{36b}$$

$$T_r = \frac{c_r t}{h^2} \tag{36c}$$

where T_o, T_p, and T_r are the time factors at the same degree of consolidation for the oedometer, the smooth-based layer under plane strain loading, and the smooth-based layer under an axially symmetrical loading, respectively. In the latter two cases, the load is extended to infinity in all directions.

Thus, for Poisson's ratio, ν, at zero,

$$T_p = \frac{T_o}{2} \tag{37a}$$

$$T_r = \frac{T_o}{3} \tag{37b}$$

The only time when these factors would be equal is the special case when the soil skeleton is incompressible ($\nu = 0.5$).

The foregoing analysis is based on a "one-dimensional" loading which extends indefinitely in all directions. The effect of a three-dimensional loading is given in Table 2. These results are taken from the solution of the three-dimensional consolidation problem for a clay layer of limited thickness (9). The surface is loaded by a uniform circular load of radius a. The layer, of thickness h, is assumed to have a frictionless base. The values in Table 2 are the oedometer time factors, T_o, for the given degree of consolidation, $\overline{U}$, Poisson's ratio, ν, and geometric ratio, a/h. The degree of consolidation, $\overline{U}$, has been calculated at the center of the load. The last values in the table are the time factors, T_o, for the usual one-dimensional theory (12).

TABLE 2

TIME FACTORS FOR AXIALLY SYMMETRIC CONSOLIDATION OF A SMOOTH-BASED LAYER

a/h	μ	$\overline{U} = 0.5$	$\overline{U} = 0.9$
0.1	0	2.0×10^{-3}	4.0×10^{-2}
	0.2	1.1×10^{-3}	2.4×10^{-2}
	0.4	3.4×10^{-4}	7.5×10^{-3}
1.0	0	1.5×10^{-1}	8.0×10^{-1}
	0.2	8.0×10^{-2}	4.7×10^{-1}
	0.4	2.4×10^{-2}	1.6×10^{-1}
10	0	4.5×10^{-1}	1.6
	0.2	2.5×10^{-1}	1.0
	0.4	8.5×10^{-2}	3.4×10^{-1}
∞	0	5.9×10^{-1}	2.5
	0.2	3.9×10^{-1}	1.7
	0.4	2.5×10^{-1}	1.1
	0.5	2.0×10^{-1}	8.5×10^{-1}

The field evaluation of the coefficient of consolidation depends on the theoretical value of the time factor for a particular degree of consolidation. This value is a function of the geometric ratio, a/h. The time factor, T_O, for 50 percent consolidation and Poisson's ratio of 0.4 can vary by a factor of approximately 250 for a geometric ratio change of 100 (Table 2). For 90 percent consolidation the factor is approximately 45 for a geometric ratio change of 100. Thus, as expected, the higher the degree of consolidation, the closer the time factors. As pointed out by the authors, the three-dimensional effect will seriously influence the basis for evaluating the coefficient of consolidation.

This shows two of the many factors involved in the evaluation of the coefficient of consolidation. The first of these concerns the definition of this coefficient. It is seen that the coefficient of consolidation depends on the anisotropy of permeability and the definition of compressibility. This latter property depends on the nature of the surface loading and the boundary conditions of the clay layer. This means that the coefficient of consolidation is not a fundamental soil property. As such, values measured under different conditions will be different.

The second factor is the three-dimensional effect. That is, the curve-fitting at a specific degree of consolidation will have to be based on different time factors that depend on the effective stress-strain properties, the type of loading, the boundary conditions of the clay, and the load width to layer-thickness ratio.

All in all, even with perfect sampling, perfect field measurements, and perfect testing, there is no reason to expect that the coefficient of consolidation determined in the field and in the laboratory will be the same. In fact, they should be different.

Reference

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