Determination of Modulus of Soil Reaction From Standard Soil Tests

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In 1942, Spangler (6) presented an equation for determining the deflection of flexible metal pipe embedded in soil. The formula involved a soil modulus called the modulus of soil reaction. To date, no practical means have been presented for determining this modulus for design purposes. The objective of this research program was to find a correlation between the modulus of soil reaction (E′) and the results obtained from the CBR test, Hveem's stabilometer test and standard soil properties such as density, compaction, moisture content, and plasticity index. The theory of elasticity was used to determine the relationship between E′ and the CBR value. Laboratory tests show that the relationship obtained yields results satisfactory for most design situations. Suggested design values based on the CBR test are also given.

The relationship between E′ and the R-value determined from Hveem's stabilometer test was determined by using an empirical correlation presented by the California Division of Highways between the R-value and the plate-bearing test. The theory of elasticity was then used to relate the plate-bearing test to the modulus of soil reaction. Slight empirical adjustments had to be made in the California correlation; however, laboratory results indicate a satisfactory correlation. For the situation where the installation is to be small and laboratory tests such as the ones just described are not feasible, the modulus of soil reaction has been related to relative compaction of the soil. Other items included in the dimensional analysis were unit weight, plasticity index, moisture content and the difference in density between the AASHO Designation T180 and T99 standard compaction tests. The results of the study indicate that approximate value for the modulus of soil reaction can be obtained from a simple compaction test.

FLEXIBLE pipes are being widely used in the construction of culverts and other underground conduits, partly because of their comparatively light weight and ease of transportation and construction. The design of flexible pipe should be based on the deflection of the pipe as well as the ring compression or stress in the pipe wall.

The ultimate horizontal deflection of a flexible pipe under a fill can be predicted by Spangler's formula (6):

$$\Delta X = \frac{K\, W_C\ r^3}{EI + 0.61\, E'\ r^3} \tag{1}$$

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where

ΔX = horizontal deflection of pipe, in.;
K = bedding constant;
W_c = load on culvert, lb/in.;
r = mean radius of pipe, in.;
E = Young's modulus of elasticity of pipe, psi;
I = moment of inertia of pipewall, in.^{4}/in.;
E' = modulus of soil reaction = er, psi; and
e = modulus of passive resistance psi/in.

Eq. 1 involves a soil modulus called the modulus of soil reaction which can also be defined as

$$E' = h/\Delta X/D \tag{2}$$

where h = maximum pressure at the horizontal diameter of the pipe, psi. Other variables are as defined for Eq. 1.

It is generally recognized that the weakness of the Iowa formula is in the evaluation of the modulus of soil reaction. Many different attempts have been made to determine this modulus in the laboratory; but, to the present time, no convenient method has been presented. The Modpares device constructed by Watkins and Nielson (9) appears to give satisfactory results, but the complexity of the test and the time required to perform the test limits its usefulness. Nielson (5) has also shown that the modulus of soil reaction can be determined from the triaxial shear test, but the triaxial shear test is not normally performed on materials used in highway construction. The suitability of a material for use in a highway is usually determined by the California Bearing Ratio (CBR) test or the Hveem stabilometer (R-value) test. This paper presents the relationship between the modulus of soil reaction and CBR value, R-value, consolidation test, and other soil properties.

Nielson (5) has established a relationship between the modulus of soil reaction, E' and the modulus of elasticity of the soil, E, by using the theory of elasticity. He concluded that the modulus of soil reaction, E', can be approximated by

$$E' = 1.5\ M^* = 1.5\ \frac{E_s\ (1 - u_s)}{(1 + u_s)\ (1 - 2u_s)} \tag{3}$$

where

E' = modulus of soil reaction, psi;
E_s = modulus of elasticity of soil, psi;
u_s = Poisson's ratio of soil; and
M^* = constrained modulus of elasticity of soil, psi, $= [E_s\ (1 - u_s)]/[(1 + u_s)\ (1 - 2u_s)]$.

Eq. 3 shows that the modulus of soil reaction is dependent only on Poisson's ratio and the modulus of elasticity of a given soil.

CBR TEST

The CBR is a penetration test developed by the California Division of Highways for the evaluation of the strength of subgrade soils. The test consists of measuring the load required to cause a plunger of standard size to penetrate a specimen of soil at a specified rate.

Test procedures for laboratory analysis can be found elsewhere (1). Theoretically, the behavior of soil under a stress condition similar to the CBR test can be predicted by applying the theory of elasticity. The soil must be assumed to be homogeneous and isotropic.

Timoshenko and Goodier (8) presented a formula to evaluate the displacement of an absolutely rigid piston pressed against the plane boundary of a semi-infinite elastic

solid. The displacement is considered to be constant over the circular base of the die and its value is given by

$$\Delta = \pi \frac{(a)p\ (1 - u_s^2)}{2E_s} \tag{4}$$

where

Δ = displacement of the die, in.;
a = radius of the die, in.;
p = unit load on die, psi;
u_s = Poisson's ratio of soil; and
E_s = modulus of elasticity of soil, psi.

The CBR test is performed quite similarly. The CBR piston can be referred to as an absolutely rigid die because of its high stiffness in comparison with the soil. It is assumed that the boundary condition of the CBR mold does not affect the displacement of the plunger since the diameter of the mold is much larger than the diameter of the plunger. Consequently, Eq. 4 can be used to establish a theoretical relationship between load, displacement, Poisson's ratio, and the modulus of elasticity of a given soil by the CBR test.

The modulus of elasticity can be determined from Eq. 4 as

$$E_s = \frac{\pi\ a\ (1 - u_s^2)\ P}{2\Delta} \tag{5}$$

For an ideal elastic material, E will be constant. This is not the case with soil because the stress-strain relationship is nonlinear. In other words, the modulus of elasticity of soil does not remain constant. However, its value at a certain load or deformation can be approximated from the CBR test by substituting the secant slope of the CBR curve (P/Δ) into Eq. 5.

When Eq. 5 is substituted into Eq. 3, the resulting equation relates the modulus of soil reaction and the slope of the CBR curve (P/Δ) as follows:

$$E' = \frac{0.75\ \pi a\ (1 - u_s)^2\ P}{(1 - 2u_s)\Delta} \tag{6}$$

Let $K = P/\Delta$, then

$$E' = \frac{0.75\ \pi a\ (1 - u_s)^2\ K}{(1 - 2u_s)} \tag{7}$$

The value of K can be written in terms of the CBR-value. The CBR-value is the load, in pounds per square inch, required to force the plunger into the soil a specified depth. It is expressed as a percentage of the load required to force the plunger the same depth into a standard sample of crushed stone.

$$\text{CBR}\ (\%) = \frac{\text{unit load on plunger (psi)}}{\text{standard unit load (psi)}} \times 100 \tag{8}$$

The standard unit loads for crushed stone are as follows: 0.1-in. penetration, 1,000 psi; 0.2-in., 1,500 psi; 0.3-in., 1,900 psi; 0.4-in., 2,300 psi; and 0.5-in., 2,600 psi.

The unit load generally used for highway design is the load at 0.1-in. penetration which yields

$$\text{CBR}\ (\%) = \frac{P}{1000}\ 100 \quad \text{or} \quad P = 10\ \text{CBR}\ (\%) \tag{9}$$

TABLE 1

BASIC PROPERTIES OF SOIL SAMPLES

Soil	AASHO Class.	Specific Gravity	T-180 Max. Dry Density (pcf)	T-180 Opt. Moisture Content (%)	Plasticity Index
Fine sand	A-2-4	2.61	115.8	9.60	None
Silty sand	A-2-6	2.50	112.0	14.50	17.1
Sand-gravel with fines	A-1-b	2.63	124.0	8.50	None
Coarse sand	A-2-4	2.66	105.8	7.05	None
Well-graded soil mixture	A-1-a	2.70	134.1	6.45	None

The slope of the CBR curve, K, at 0.1-in. penetration, can be expressed as follows:

$$K = \frac{P}{\Delta} = \frac{P}{0.1} = \frac{10}{0.1} \text{ CBR} = 100 \text{ CBR } (\%) \tag{10}$$

By substituting Eq. 10 into Eq. 7, the relationship between the modulus of soil reaction and the CBR value at 0.1-in. penetration can be obtained as

$$E' = \frac{0.75 \pi a (1 - u_s)^2}{(1 - 2u_s)} 100 \text{ CBR } (\%) \tag{11}$$

A value of 0.25 for Poisson's ratio was used to correlate laboratory data and is recommended for design work. If Poisson's ratio equals 0.25 and a = 0.975 in., Eq. 11 can be reduced to

$$E' = 260 \text{ CBR } (\%) \tag{12}$$

To verify Eq. 12, a laboratory study was made to compare the modulus of soil reaction, as determined by the CBR test, with the values obtained from the Modpares device (9). Five different soils obtained in the vicinity of Las Cruces, New Mexico, were used. Their basic properties are given in Table 1 and the grain size distribution curves are shown in Figure 1. AASHO procedures were used in obtaining the data.

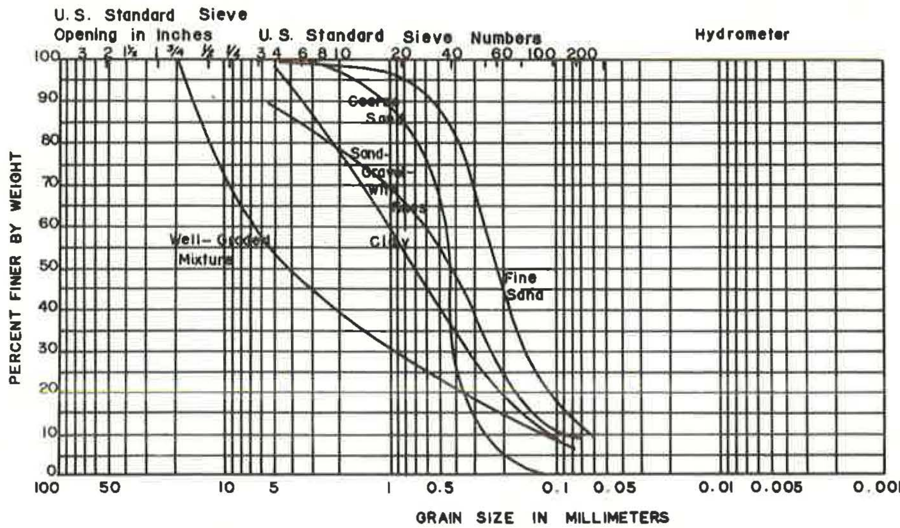

Figure 1. Grain size classification of the soil samples.

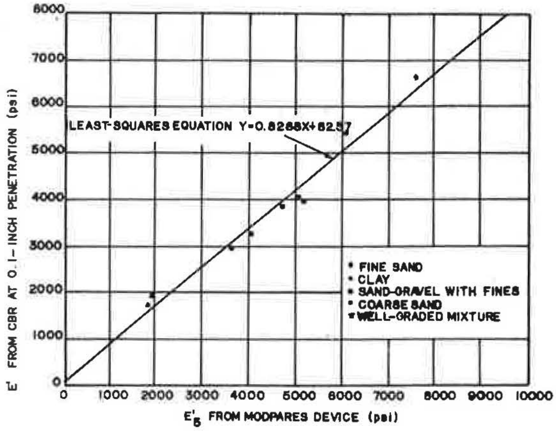

Figure 2. Relation of E′ from Modpares device and CBR test.

To make the results from the Modpares test comparable with results from the CBR test, it was necessary to make the moisture content, dry density, and compactive effort used to compact the soil the same for each test. The numerical results for nine sets of tests are given in Table 2 and shown in Figure 2.

The modulus of soil reaction obtained from the CBR test is only about 83 percent of the value obtained from the Modpares device. There are two possible explanations for this difference. Perhaps the wrong value of Poisson's ratio was used. As can be seen from Eq. 11, E′ depends on the value of Poisson's ratio. If a larger value of Poison's ratio had been used, the curve would have been much closer to a 45-deg line.

The second possibility, and the one which seems to be more likely, is the nonlinearity of the CBR load penetration curve. Nielson (5) has shown that the percent deflection of the pipe is $\Delta X/D = 2\epsilon$, where ϵ is the strain in the soil occurring at the horizontal axis of the pipe. To get an estimate of the strain occurring under the penetration piston, the equation for strain under a circular plate as presented by Wu (11) for a uniformly distributed load is used:

$$\epsilon_Z = \frac{1}{E_S}\left\{P\left[1 - \frac{Z^3}{(R^2 + Z^2)^{3/2}}\right] - u_S P\left[1+2u_S - \frac{2(1+u_S)Z}{(R^2 + Z^2)^{1/2}} + \frac{Z^3}{(R^2 + Z^2)^{3/2}}\right]\right\} \qquad (13)$$

where R = radius of loaded area, and Z = depth below surface.

If other variables are as previously defined with $u = 0.25$ and $Z = 0$, then the strain ϵ_Z can be formed. But $\Delta X/D = 2\epsilon$, therefore, by assuming that the strain produced by a uniform load is representative of the strain produced by the pipe, the deflection of the pipe can be reduced to a function of the pressure acting on the soil and the soil properties E_S and u_S. Solving for

$$\frac{\Delta X}{D} = \frac{2P}{E_S}(1 - 2u_S)(1 + u_S) \qquad (14)$$

or if $u = 0.25$ and $\frac{\Delta X}{D} = 0.05$ (corresponding to 5 percent deflection of the pipe)

$$p = 0.04\ E_S \qquad (15)$$

TABLE 2

COMPARISON OF E' FROM CBR TEST AND MODPARES DEVICE

CBR Test, E' (psi)	Modpares Device, E's (psi)	CBR Test, E' (psi)	Modpares Device, E's (psi)
1788	1821	4019	5054
1971	1863	3953	5184
2986	3683	5422	6053
3293	4032	6625	7568
3899	4717		

TABLE 3

SUGGESTED VALUES OF MODULUS OF SOIL REACTION FOR DIFFERENT CBR-VALUES BASED ON EQUATION 18

CBR (%)	Modulus of Soil Reaction at 5% Deflection, E'5 (psi)			
	u = 0.20	u = 0.25*	u = 0.30	u = 0.35
1	201	212	231	266
2	498	525	572	657
3	794	827	912	1049
4	1090	1150	1252	1440
5	1387	1462	1593	1831
6	1683	1775	1933	2222
7	1979	2088	2273	2613
8	2276	2400	2614	3005
9	2572	2713	2954	3396
10	2868	3025	3294	3787
11	3165	3338	3634	4178
12	3461	3650	3975	4570
13	3757	3963	4315	4961
14	4054	4275	4655	5352
15	4350	4588	4996	5743
16	4646	4900	5336	6135
17	4942	5213	5676	6526
18	5239	5525	6017	6917
19	5535	5838	6357	7308
20	5831	6150	6697	7700

*Poisson's Ratio, u = 0.25 is recommended for design work.

If Eq. 15 is substituted into Eq. 4,

$$\Delta = \frac{\pi a\,(0.04)\,E_s\,(1 - u_2^2)}{2E_s} \qquad (16)$$

Substituting values,

$$\Delta = 0.54 \text{ in.}$$

The assumption that the strains induced by a uniformly distributed load on the surface is representative of the strains induced by the pipe is probably not a realistic one; however, the results indicate that the penetration of the CBR plunger used to determine the CBR value should be considerably less than the 0.1 in. used. Data obtained by Yeh (10) show that modulus of soil reaction for 0.54 in. penetration is higher than value obtained at 0.1-in. penetration.

In order to avoid the problem of determining the CBR value at 0.54-in. penetration, the relationship between the modulus of soil reaction, as determined from the CBR test at 0.1-in. penetration, and the Modpares test was evaluated by the method of least squares. The equation for the straight line shown in Figure 2 was found to be

$$E'_{CBR} = 82.57 + 0.829\,E'_{Modpares} \qquad (17)$$

If the modulus of soil reaction, as determined by the Modpares device, is assumed to be the correct one, then it can be related to the CBR value by substituting Eq. 12 into Eq. 17. Thus, it follows that

$$E' = 312\ \text{CBR} - 100 \qquad (18)$$

or neglecting the 100 which is probably experimental error

$$E' = 312\ \text{CBR} \qquad (19)$$

Table 3 and Figure 3 are suggested design values based on equations similar to Eq. 18.

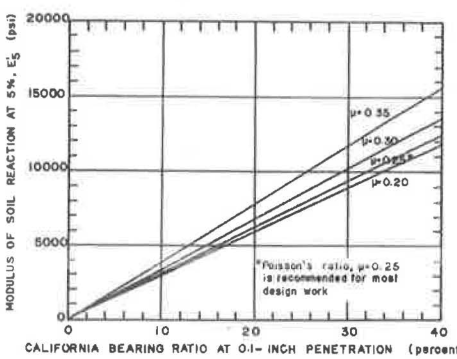

Figure 3. Relationship between CBR and modulus of soil reaction.

Laboratory data show that granular soil with large amounts of clay binder, which produce extremely high CBR values, do not fit the data presented. Table 3 and Figure 3 should not be used for this type soil as the values predicted will be much too high.

HVEEM STABILOMETER TEST

The relation between the R-value from the Hveem stabilometer test and the (K-value) plate bearing test is shown in a California Division of Highways Report (3). The report presents three different curves relating K-value and R-value (Fig. 4). The tentative empirical curve was selected to relate the K-value and R-value. To make results from the Hveem stabilometer test comparable with

the Modpares device, the lower part of the curve for clayey soils and the upper part of the curve for granular material had to be empirically adjusted as shown in Figure 4. A least squares fit of a polynomial equation for the adjusted curve yields a relationship relating K-value and R-value as

$$K = 0.401 + 2.546R - 0.042R^2 + 0.0008R^3 \qquad (20)$$

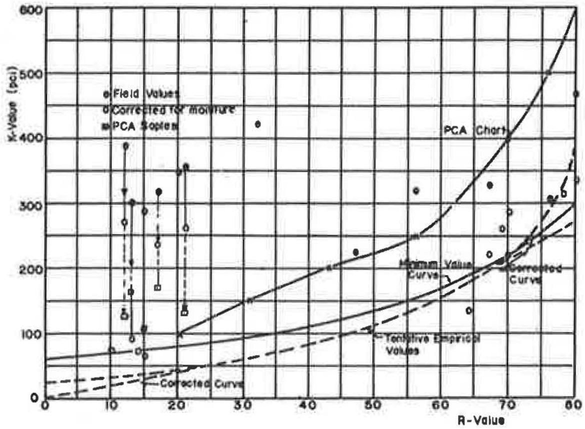

Figure 4. Relationship of K-value and R-value.

Timoshenko and Goodier (8) have shown from the theory of elasticity that, when a distributed load is applied over the circular area, the deflection at the edge can be expressed as

$$w = \frac{4(1 - u^2)\,pa}{\pi E} \qquad (21)$$

where

w = deflection at edge of load area, in.;
u = Poisson's ratio;
p = uniform load, psi;
a = radius of circular area = 15 in.; and
E = modulus of elasticity, psi.

It is assumed that Eq. 20 can be applied to the plate bearing test because in the plate bearing test, loads are distributed on the steel circular plate, and deflections are measured near the edge of the plate.

The modulus of subgrade reaction (k-value), as obtained from the plate bearing test, can be expressed as:

$$k = 1/\Delta \qquad (22)$$

where

k = modulus of subgrade reaction, psi/in.;
p = uniform pressure on the plate, psi; and
Δ = deflection of the plate, in.

Eq. 21 can be rewritten as:

$$E_S = \frac{4(1 - u_p^2)\,pA}{w\,\pi} \qquad (23)$$

in which $p/w = p/\Delta = k$.

Therefore, Eq. 23 can be reduced to

$$E_S = \frac{4(1 - u_p^2)\,k\,a}{\pi} \qquad (24)$$

Substituting E_S from Eq. 24 and Eq. 20 for k into Eq. 3 yields:

$$E' = \frac{6\,a\,(1 - u_S)^2\,(-0.401 + 2.646R - 0.042R^2 + 0.0008R^3)}{(1 - 2u_S)} \qquad (25)$$

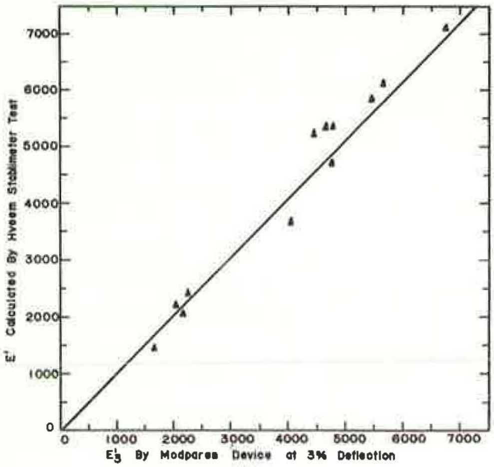

Figure 5. Comparison of E′ calculated by Hveem stabilometer test and Modpares device at 3 percent deflection.

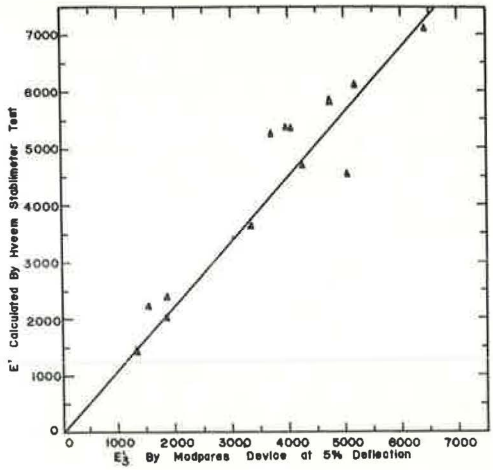

Figure 6. Comparison of E′ calculated by Hveem stabilometer test and Modpares device at 5 percent deflection.

The results from a series of tests (2) are shown in Figures 5 and 6. Eq. 25 seems to fit the modulus of soil reaction at 3 percent deflection (Fig. 5) better than it does the value at 5 percent (Fig. 6). Figure 7 is a suggested design chart based on Eq. 25.

CONSOLIDATION TEST

Most textbooks on strength of materials give the stress-strain equations for triaxial loading as

$$\epsilon_1 = \frac{\sigma_1}{E} - \frac{u\sigma_2}{E} - \frac{u\sigma_3}{E} \qquad (26a)$$

$$\epsilon_2 = \frac{\sigma_2}{E} - \frac{u\sigma_1}{E} - \frac{u\sigma_3}{E} \qquad (26b)$$

$$\epsilon_3 = \frac{\sigma_3}{E} - \frac{u\sigma_1}{E} - \frac{u\sigma_2}{E} \qquad (26c)$$

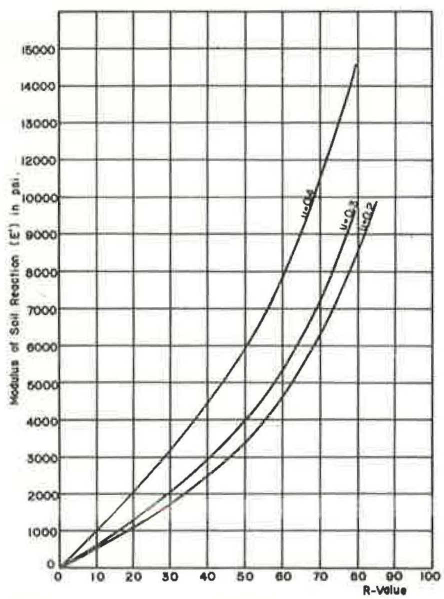

Figure 7. Relationship between R-value and E′-value.

where

E = modulus of elasticity;
u = Poisson's ratio;
ϵ_1, ϵ_2, ϵ_3 = principal strains; and
σ_1, σ_2, σ_3 = principal stresses.

If one applies Eqs. 26 to the consolidation tests, ϵ_1 is the strain in the vertical direction, and ϵ_2 and ϵ_3 are zero because of the rigid confining ring (Fig. 8). Also, because of the symmetry of the loading, σ_2 is equal to σ_3.

Applying these conditions, Eqs. 26 reduce to

$$\epsilon_1 = \frac{\sigma_1}{E} - \frac{2u\sigma_2}{E} \qquad (27a)$$

$$0 = \sigma_2 (1 - u) - u\sigma_1 \qquad (27b)$$

Solving Eq. 27b for σ_2 and substituting it back into Eq. 27a yields

$$\epsilon_1 = \frac{\sigma_1}{E} - \frac{2u}{E}\,\frac{u\sigma_1}{(1-u)}$$

or

$$\frac{\sigma_1}{\epsilon_1} = \frac{E\,(1-u)}{(1-u)\,(1-2u)} = M^* = \text{slope of curve} \qquad (28)$$

Comparing Eq. 28 with Eq. 3, the modulus of soil reaction is equal to

$$E' = 1.5\,\frac{(\sigma_1)}{\epsilon_1} \qquad (29)$$

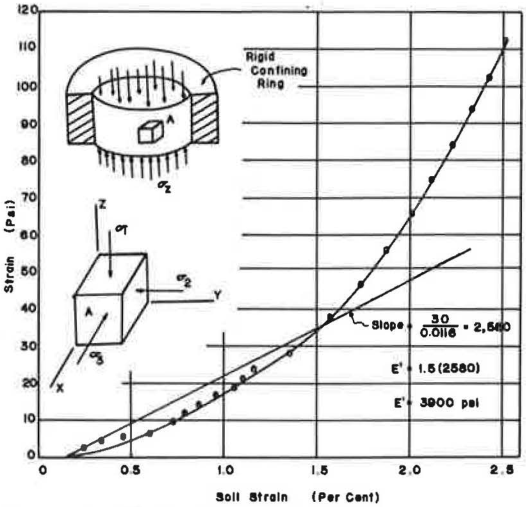

Figure 8. Confined stress-strain curve for determining modulus of soil reaction.

Figure 8 shows a set of data obtained on an arroyo sand. The slope of the curve beyond about 1.5 percent strain should not be used to determine the modulus of soil reaction since the increase in slope is due to a "locking up" of the soil grains resulting from the confined nature of the test which is not exhibited in buried pipe installation. This method can only be used to determine E$'$ for deflections less than approximately 3 percent deflection.

OTHER SOIL PROPERTIES

Various soil properties are known to affect the modulus of soil reaction. The principles of similitude can be used to determine the relationship between E$'$ and these variables.

Inasmuch as the modulus of soil reaction is a function of the properties of a soil sample, the assumed mathematical model can be expressed

$$E' = f\,(\gamma_d,\ \gamma_{180},\ \gamma_{dif},\ r,\ PI,\ W) \qquad (30)$$

where

E' = modulus of soil reaction, psi;
γ_d = the density of soil sample used in the experimental test, pcf;
γ_{180} = the density of AASHO Designation T180 compaction, pcf;
W = the moisture content of a soil sample, dimensionless;
γ_{dif} = the difference between the density of AASHO Designation T180 and T99 compaction, pcf;
r = mean radius of the pipe, in.; and
PI = plasticity index, dimensionless.

There are seven fundamental variables and two dimensions; therefore, according to Buchingham's π-theorem, the number of π - terms should be $7 - 2 = 5$. These five π-terms may be expressed as follows:

$$\pi_1 = \frac{E'}{r\,\gamma_{180}} \qquad (31)$$

$$\pi_2 = \frac{\gamma_d}{\gamma_{180}} \qquad (32)$$

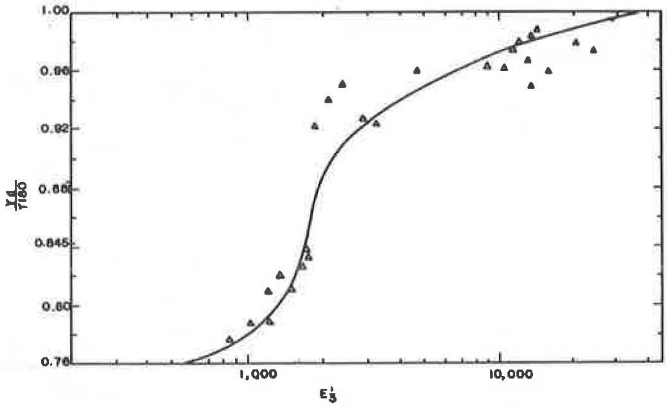

Figure 9. E_3' versus γ_d/γ_{180}.

$$\pi_3 = \frac{\gamma_d}{\gamma_{dif}} \tag{33}$$

$$\pi_4 = W \tag{34}$$

$$\pi_5 = PI \tag{35}$$

The π-terms can be arranged into a functional equation form as follows:

$$\pi_1 = f\ (\pi_2,\ \pi_3,\ \pi_4,\ \pi_5) \tag{36}$$

From the data obtained from experimental tests (4) the modulus of soil reaction seemed to have little correlation with π_3, π_4, or π_5.

The π-term equation was reduced to

$$\pi_1 = f\ (\pi_2)$$

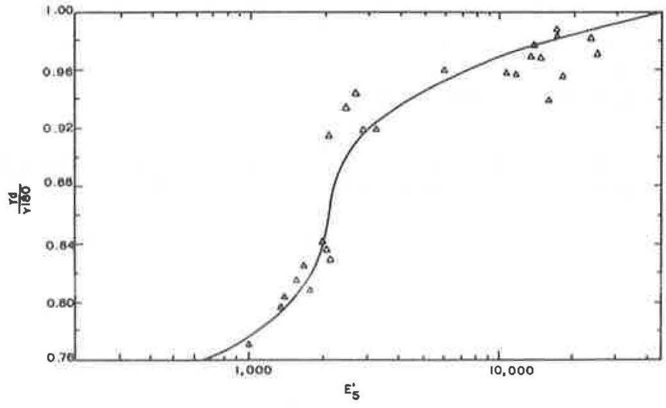

Figure 10. E' versus γ_d/γ_{180}.

TABLE 4

SUGGESTED DESIGN VALUES* OF MODULUS OF SOIL REACTION

γ_d/γ 180	E'₃	E'₅	γ_d/γ 180	E'₃	E'₅
0.75	500	400	0.88	2150	1900
0.76	600	500	0.89	2250	2000
0.77	800	700	0.90	2500	2300
0.78	1150	900	0.91	2800	2500
0.79	1400	1150	0.92	3100	2800
0.80	1550	1300	0.93	3600	3250
0.81	1700	1400	0.94	4300	4000
0.82	1800	1500	0.95	5500	5000
0.83	1900	1600	0.96	7200	6600
0.84	2000	1700	0.97	10300	9300
0.85	2050	1800	0.98	15000	14000
0.86	2100	1800	0.99	25000	22000
0.87	2100	1800	1.00	44000	38000

*These values should not be used unless more accurate means of analysis are not available, and then only with the knowledge that error as much as 100 percent may exist.

In other words,

$$\frac{E'}{r\,\gamma 180} = f\left(\frac{\gamma_d}{\gamma 180}\right) \tag{37}$$

The data gained from the Modpares device were analyzed by the least squares method to obtain the value of E′ at 3 and 5 percent horizontal deflection of the conduit. The orthogonal polynomial regression analysis subroutine on the IBM 1130 computer was employed to determine the best fit of all (E_3')'s and (E_5')'s (the value of E′ at 3 and 5 percent horizontal deflection of the conduit, respectively).

Based on the experimental data, the soil density was considered to be the most important variable in determining the modulus of soil reaction. The limited amount of data indicates the different optimum moisture contents of all soil samples tested and seems to have little influence on the modulus of soil reaction as long as it is approximately optimum.

According to the data obtained from the Modpares device, soil density had a significant influence on the modulus of soil reaction, the range of E′ varied widely between the low-density and the highly compacted soil samples. The range of E_5' varied from 815 psi to 28,700 psi. The difference between the low- and high-density soil samples is almost 35 times.

The following equation was obtained from IBM 1130 computer's orthogonal polynomial analysis subroutine which was used to determine the modulus of soil reaction:

$$\frac{E_3'}{r\,\gamma 180} = 254.7463 + 892.8421\left(\frac{\gamma_d}{\gamma_{180}}\right) - 1040.688\left(\frac{\gamma_d}{\gamma_{180}}\right)^2 + 404.8318\left(\frac{\gamma_d}{\gamma_{180}}\right)^3 \tag{38}$$

$$\frac{E_5'}{r\,\gamma 180} = -240.6943 + 843.4371\left(\frac{\gamma_d}{\gamma_{180}}\right) - 983.4282\left(\frac{\gamma_d}{\gamma_{180}}\right)^2 + 382.8650\left(\frac{\gamma_d}{\gamma_{180}}\right)^3 \tag{39}$$

where

E_3' and E_5' = modulus of soil reaction at 3 and 5 percent deflection of conduit, respectively;

γ_d = soil dry density used in experimental studies; and

γ_{180} = soil dry density of AASHO Designation T180 compaction test.

The data are shown in Figures 9 and 10. Suggested design values of the modulus of soil reaction are given in Table 4.

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