Optimal Selection of a Progression for a Two-Way Arterial Street

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A procedure is demonstrated for selecting a progression on a two-way street which minimizes total steady-state vehicle transit time while taking into account constraints by driver car-following behavior.

•IT is common practice to coordinate the timing of the traffic signals on urban streets so as to achieve progressions. When this is done, it is possible to design the progression on arterial streets so that vehicles traveling at the design speed will encounter at the most one red light. The parameters of the progression, such as cycle length, splits, and offsets, can be determined graphically from a time-space diagram (Fig. 1) by trial and error. From such a diagram it is also possible to determine the bandwidth of the progression, i.e., the time headway between the first and last cars of the platoon (traveling at the design speed) that can pass through the green signal at a given intersection without encountering red signals at subsequent intersections.

The progression design is not unique for any given street in that there are many combinations of progression speeds and bandwidths that can be realized. If the signal timing is implemented by fixed timers at local controllers and if the street carries traffic in only one direction, perhaps the best choice is an average speed that will accommodate medium density traffic. However, when progression is applied to two-way streets, an increase in the bandwidth in one direction is usually accompanied by a decrease in bandwidth in the other direction. Several papers have been published on the trade-offs that might be made and on techniques for selecting bandwidths that are optimum in the sense that (a) both bandwidths are equal and maximum or (b) one is maximum subject to the restriction that the other does not drop below a preassigned minimum. Very little, if any, work has been reported on the design of progressions to meet given flow demands on the arterial street and its cross streets.

This paper develops a rational basis for realizing the optimal progression on two-way streets as a function of the prevailing flow demands. It is both economically and technically feasible to automatically change the timing of traffic signals, by means of a central digital controller, in accordance with the measurable demands at any given time. Initial efforts in this direction have already been undertaken in Toronto, San Jose, and Wichita Falls (1, 2, 3). These early efforts indicate that significant gains can be obtained by adjusting the timing of traffic signals to the prevailing demand in an on-line manner. Several cost studies have shown that even small improvements in the flow of traffic can result in large economic benefits (3, 4). The specific benefit depends largely on the criteria used by the central digital controller in deciding which stream of traffic should receive the green signal at any given instant of time.

In addition to matching the operation of the network to the demands on the system, the control algorithm must also minimize the effects of random disturbances in the system. The control algorithm presented in this paper assumes that changes in demands on a system are sufficiently slow so that optimization of the control action can be achieved technically by considering a steady-state model of the traffic network and

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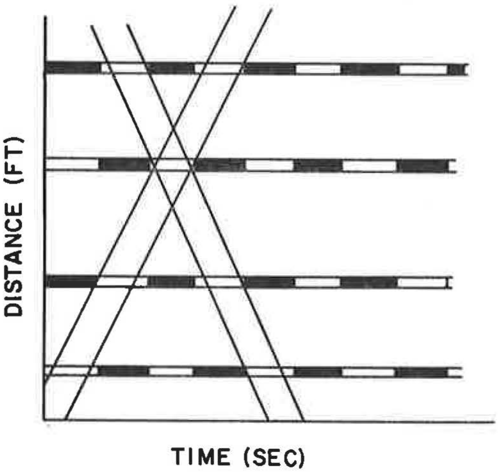

Figure 1. Typical time-space diagram.

that, as a practical expedient, control in the face of random disturbances can be achieved by short-term localized modifications of the progressions.

The demand on an arterial is measured in terms of the number of vehicles per unit of time arriving at the entry points to the system. For the purposes of the discussion, let the points of entry be the two ends of the street. If the flow rate in each direction on the arterial is exactly equal to the demand, then of course no queuing occurs and the progression is said to be operating satisfactorily.

The first question of concern is to determine the flow rate that a given progression will sustain. The classical time-space diagram analysis gives only the bandwidth or time headway between the first and last vehicles in the platoon. The number of vehicles per unit of time that can be served by the street depends not only on the bandwidth of the progression but also on the density of the vehicles in the platoon. The relationship between these quantities can be obtained from the theory of traffic flow.

Definitions for the symbols used in the following discussion are:

Symbols	Definition
B_i	bandwidth—inbound direction
B_o	bandwidth—outbound direction
B	maximum equal bandwidth
G	minimum green time
y	flow rate
y_s	continuous stream flow rate
y_{sm}	maximum continuous stream flow rate
y_{di}	demand—inbound direction
y_{do}	demand—outbound direction
x	lane occupancy
x_j	lane occupancy resulting in jam
v	velocity
v_i	progression speed—inbound direction
v_o	progression speed—outbound direction
v_f	free speed
v_e	progression speed equal in both directions
F	performance index
μ	Lagrange multiplier

Various experimental and theoretical investigations (5, 6, 7) of the flow in a single lane of traffic have established that flow rate is related to the lane occupancy by a parabolic relationship of the form shown in Figure 2. The corresponding speed of the traffic stream varies with lane occupancy (Fig. 3). The behavior indicated in Figure 3 has been shown by car-following models (7) to be the result of the drivers in the traffic stream reducing their speed when confronted with smaller intervehicular spacing (in-

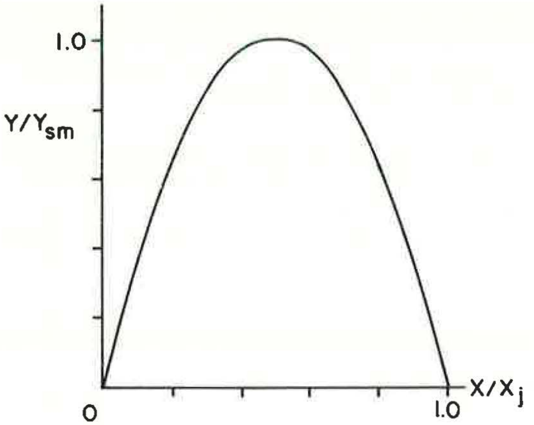

Figure 2. Normalized flow rate (y/y_{sm}) as a function of normalized lane occupancy (x/x_j) for a continuous traffic stream.

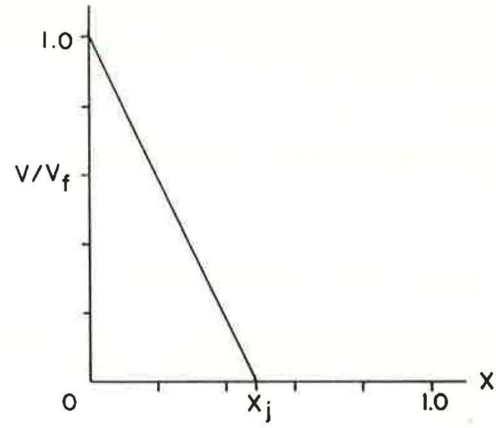

Figure 3. Normalized speed (v/v_f) as a function of lane occupancy (x) for a continuous traffic stream.

creased lane occupancy). Inasmuch as the same basic phenomenon takes place in a platoon as in a continuous stream of vehicles, the maximum flow rate that can be expected without breakdown of the progression is given by the product of the continuous stream flow-occupancy characteristic and the ratio of bandwidth to cycle length. For example, if the continuous stream flow-occupancy characteristic of Figure 2 is given by $y_s = G(x)$ where y_s is the continuous stream flow rate and x is the lane occupancy, the flow-occupancy characteristic for a progression system is given by $y = BG(x)$ where B equals bandwidth per cycle. Thus, the flow-occupancy characteristics in each of the two directions on the arterial are different if the ratios of bandwidth to cycle length are different in the two directions; that is, if $B_i \neq B_o$.

In the analysis which follows, Greenshield's traffic model (6)

$$y = 4y_{sm} \left(\frac{x}{x_j}\right) \left(1 - \frac{x}{x_j}\right) \tag{1}$$

$$v = v_f \left(1 - \frac{x}{x_j}\right) \tag{2}$$

is used, where y_{sm} is the peak flow rate, x_j is the jam lane occupancy, and v_f is the free speed. Comparable results can be obtained using other models which have been proposed.

In a continuous steady-state traffic stream, the vehicles adjust their speeds so that the flow rate just satisfies the demand, provided the capacity of the street is not exceeded. Up to the point of saturation, increased demand is accommodated by higher lane occupancy and a lower individual vehicle speed. If the demand exceeds the capacity of the street, congestion very quickly results upstream. In an analogous fashion, the service flow rate of a two-way progression system should be adjusted to the demand. More specifically, the service flow rate in each direction is given by the product of the bandwidth-to-cycle ratio and the continuous stream flow rate that would prevail on that street at the progression speed. When this service flow rate is just matched to demand in both directions, the bandwidths can be expected to be filled with vehicles. Setting the progression speed higher than the value for which demand and service rate are balanced invites queuing and congestion, while setting it lower implies a lower quality of service than necessary.

If x is eliminated from Eqs. 1 and 2, the result is

$$y = 4y_{sm}\left(\frac{v}{v_f}\right)\left(1 - \frac{v}{v_f}\right) \tag{3}$$

Applying the principle that bandwidths and progression speeds should be selected so that service rates equal demand requires

$$y_{di} = 4B_i y_{sm}\left(\frac{v_i}{v_f}\right)\left(1 - \frac{v_i}{v_f}\right) \tag{4}$$

and

$$y_{do} = 4B_o y_{sm}\left(\frac{v_o}{v_f}\right)\left(1 - \frac{v_o}{v_f}\right) \tag{5}$$

where y_{di} and y_{do} are the inbound and outbound demands and v_i and v_o are the progression speeds.

A reasonable method of apportioning the potential service between the traffic demands in the two directions is to select the progression which minimizes total vehicle hours of travel time in the system. A measure of total vehicle hours of travel time is

$$F = \frac{y_{di}}{v_i} + \frac{y_{do}}{v_o} \tag{6}$$

Note that F is directly proportional to total vehicle hours under steady-state operation, since y is a measure of the number of vehicles served in any interval of time, and the individual trip time is proportional to the reciprocal of the steady-state velocity. F must be minimized while taking into account the constraints of realizability of the bandwidths and progression speeds selected.

Before proceeding with the optimization process, a few remarks about progression speeds are necessary. It is possible to vary the progression speeds within the constraint

$$\frac{1}{v_i} + \frac{1}{v_o} = \frac{2}{v_e} \tag{7}$$

without changing the bandwidths, where v_e is the progression speed that would prevail for these bandwidths if v_i and v_o are equal. A change in the offset of each signal will force drivers in one direction to reduce their transit time between intersections by the same amount that drivers in the other direction must increase their transit time if bandwidths are to be preserved. Since distance between any pair of intersections remains constant, the sum of the reciprocals of progression speeds must remain constant. Inspection of Eq. 7 indicates that allowing v_e to increase without disturbing bandwidths results in higher permissible values of v_i and v_o.

If Eqs. 4 and 5 are substituted into Eq. 6, the result is

$$F = \frac{4B_i y_{sm}}{v_f}\left(1 - \frac{v_i}{v_f}\right) + \frac{4B_o y_{sm}}{v_f}\left(1 - \frac{v_o}{v_f}\right) \tag{8}$$

If F is minimized by the method of Lagrange multipliers subject to the constraint of Eq. 7, the optimum will occur when

$$\frac{\partial}{\partial v_i}\left(F + \mu\left[\frac{1}{v_i} + \frac{1}{v_o} - \frac{2}{v_e}\right]\right) = 0 \tag{9}$$

and

$$\frac{\partial}{\partial v_o}\left(F + \mu\left[\frac{1}{v_i} + \frac{1}{v_o} - \frac{2}{v_e}\right]\right) = 0 \tag{10}$$

where μ is the Lagrange multiplier.

The simultaneous solution of Eqs. 9 and 10 after substitution of Eq. 8 results in

$$\left(\frac{v_i}{v_o}\right)^2 = \frac{B_o}{B_i} \tag{11}$$

Taking the ratio of Eqs. 4 and 5 results in

$$\frac{y_{di}}{y_{do}} = \frac{B_i}{B_o}\frac{v_i}{v_o}\frac{(v_f - v_i)}{(v_f - v_o)} \tag{12}$$

Substituting Eq. 11 into Eq. 12 yields

$$\frac{y_{di}}{y_{do}} = \frac{v_o}{v_i}\frac{(v_f - v_i)}{(v_f - v_o)} = \frac{\left(\frac{v_f}{v_i} - 1\right)}{\left(\frac{v_f}{v_o} - 1\right)} \tag{13}$$

Simultaneous solution of Eqs. 7 and 13 results in

$$\frac{v_i}{v_f} = \frac{\left(1 + \frac{y_{do}}{y_{di}}\right)}{\frac{y_{do}}{y_{di}} + \frac{2v_f}{v_e} - 1} \tag{14}$$

$$\frac{v_o}{v_f} = \frac{\left(1 + \frac{y_{di}}{y_{do}}\right)}{\frac{y_{di}}{y_{do}} + \frac{2v_f}{v_e} - 1} \tag{15}$$

Thus, for a given level of demand, the progression speeds will be determined by the largest value of v_e for which the values of B_i and B_o, satisfying Eqs. 4 and 5, can be realized.

It has been shown (8) that the progression obtained for maximum equal bandwidths with equal progression speeds can be modified to allow B_i to be increased up to min [2B, G], where B is the maximum equal bandwidth for the given progression speed and G is the minimum green. With B_i set in this range, the corresponding value for B_o is

$$B_o = \max\left[0, 2B - B_i\right]$$

Solving Eq. 4 for B_i and eliminating v_i using Eq. 14 results in

$$B_i = \frac{y_{di}}{4y_{sm}} \frac{\left[\frac{y_{do}}{y_{di}} + \frac{2v_f}{v_e} - 1\right]^2}{\left[1 + \frac{y_{di}}{y_{do}}\right]\left[\frac{2v_f}{v_e} - 2\right]} \tag{16}$$

A similar procedure using Eqs. 5 and 15 yields

$$B_o = \frac{y_{do}}{4y_{sm}} \frac{\left[\frac{y_{di}}{y_{do}} + \frac{2v_f}{v_e} - 1\right]^2}{\left[1 + \frac{y_{di}}{y_{do}}\right]\left[\frac{2v_f}{v_e} - 2\right]} \tag{17}$$

By using the method of Little et al (8), one can determine values of 2B as a function of v_e for the arterial geometry and specified traffic signal splits. The optimum value

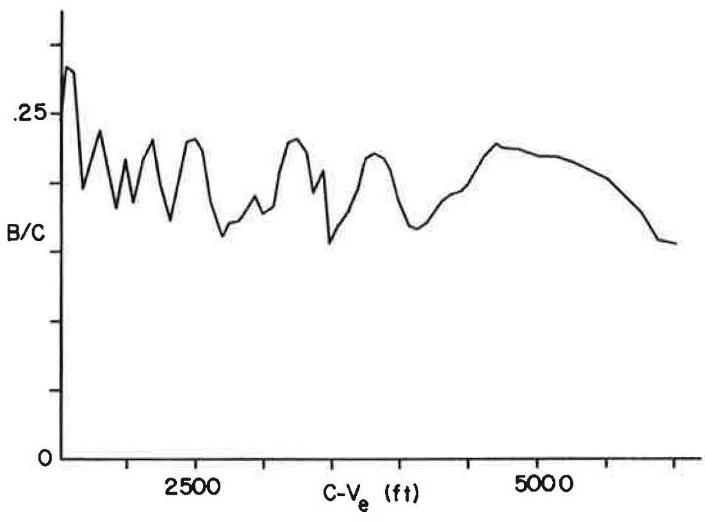

Figure 4. Realizable equal bandwidth as a function of progression speed for a section of Euclid Avenue in Cleveland, Ohio.

TABLE 1

OPTIMAL PROGRESSION CHARACTERISTICS FOR 10 INTERSECTION SECTIONS WITH VARIOUS DEMAND CONDITIONS

$\frac{y_{di}}{y_{sm}}$	$\frac{y_{do}}{y_{sm}}$	$\frac{v_e}{v_f}$	B_i	B_o	$\frac{v_i}{v_f}$	$\frac{v_o}{v_f}$
0.23	0.23	0.5	0.23	0.23	0.5	0.5
0.23	0.115	0.75	0.27	0.19	0.69	0.82
0.23	0.0767	0.75	0.26	0.15	0.67	0.86
0.23	0.0575	0.8	0.28	0.17	0.71	0.91
0.23	0.046	0.8	0.27	0.16	0.71	0.92

of v_e is the largest value of v_e for which this procedure indicates that 2B is greater than the sum of the values for B_i and B_o which are computed using Eqs. 16 and 17.

A plot of B as a function of the product of cycle length and v_e is shown in Figure 4 for a section of Euclid Avenue in Cleveland, Ohio. It can be seen that there is little bandwidth advantage obtained by using low values of v_e. Some examples of progression speeds and bandwidths which would be selected by the procedure outlined in this paper are given in Table 1.

Existing traffic responsive systems adjust the progression by modifying cycle length only. Changing cycle length reduces or increases the progression speeds in both directions by the same factor. This unduly penalizes the traffic in the low demand directions. The results (Table 1) indicate it is possible to give better service to the heavy demand direction in terms of a higher progression speed by taking account of the light demand.

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