Minimizing Nuclear Soil Density and Moisture Content Gage Errors

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The sources of error in the nuclear soil density gages are identified as sensitivity to variations in sample composition, poor calibration technique, and sensitivity to surface heterogeneities. The errors associated with the nuclear moisture content gages are identified as sensitivity to soil composition, sensitivity to soil density, and poor calibration technique. Several approaches are described and evaluated for minimizing these sources of error, including mathematical analyses of the nuclear gaging principles, the calibration model method, and the dual-gage principle for nuclear density gages.

•GAMMA-RAY scatter and neutron moderation gages have been in use for the measurement of soil density and moisture content since about 1950 (1). The immediate obvious advantages of the nuclear methods were nondestructiveness, measurement speed, and good reproducibility. However, when the nuclear gages were put to use in the field and compared to the existing gravimetric methods, discrepancies appeared and the question of the accuracy of these devices arose. To resolve this question, the Highway Research Board formed the Committee on Nuclear Principles and Applications, and NCHRP Project 10-5 (Density and Moisture Content Measurement by Nuclear Methods) was initiated. This paper describes the work that has been done on Project 10-5, the work in progress on Project 10-5A (an extension of Project 10-5), and other pertinent work in this area that has been initiated since the beginning of Project 10-5.

SOURCES OF NUCLEAR GAGE ERROR

The purpose of the initial phase of work on Project 10-5 (2) was to evaluate the nuclear gages for measuring soil density and moisture content in relation to the conventional gravimetric techniques. It was concluded that the nuclear gage results were more reproducible and potentially more accurate if the identified sources of error could be minimized. Sources of error identified for the neutron moisture content gages were sensitivity to soil density, sensitivity to soil composition, and poor calibration techniques.

The primary source of error for the gamma-ray density gage was sensitivity to composition. The calibration problem stems from the composition sensitivity of the gages and is compounded by difficulties in preparing stable, homogeneous samples of soil for laboratory calibration, or by inaccuracies in the gravimetric density measurement techniques when field calibrations are used. Likewise, the primary sources of error for the neutron moisture content gage were sensitivity to density and composition. Therefore, the calibration problem is essentially the same for the neutron moisture content gages as for the gamma-ray density gages.

The Virginia Correlation and Conference (3) sponsored by the HRB Committee on Nuclear Principles and Applications provided valuable quantitative information on the

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sensitivity of the gamma-ray density gages to soil composition. At the conference laboratory, samples of known density, moisture (hydrogen) content, and composition were available for calibrating gages brought by interested users and gage manufacturers. After the gages were calibrated on these laboratory samples, they were used in the field at a prepared test site composed of five typical Virginia construction soils. The average standard error reported for all backscatter-type gamma-ray density gages on the five laboratory samples was ± 11.0 pcf (pounds per cubic foot), whereas the error for transmission-type gages was ± 7.53 pcf. The average standard error reported for all neutron moisture content gages on the four laboratory samples was ± 1.14 pcf water. These standard errors were determined by fitting the gage responses by a least-squares method to straight-line functions of density or moisture content.

MATHEMATICAL ANALYSES OF THE NUCLEAR GAGES

To quantitatively evaluate the extent of the identified sources of error, mathematical analyses have been made of both the gamma-ray and neutron gages. A brief account of these analyses, some analyses made by other workers, and the conclusions that can be made based on these analyses are given here.

Analyses of the Gamma-Ray Soil Density Gages

In the initial phase of Project 10-5, a simple single-scatter mathematical model of the gamma-ray density gages was developed and tested. The simplifying assumptions were (a) that the gage geometry was identical to that of the depth-type gages so that the problem was only two-dimensional, and (b) that the total gage response was directly proportional to the gamma rays emitted by the source and scattered once by the surrounding soil directly into the detector. Using these two assumptions, the response of the gage could be given as the double integral of the product of four separable probabilities:

$$R = \int_{\phi} \int_{r} \frac{dP_1}{d\phi} \frac{dP_2}{dr} P_3 P_4 \, d\phi \, dr \qquad (1)$$

where R is the gage response; ϕ is the angle between the line connecting the source and detector and the direction of the gamma ray being considered; r is the distance from the source to the scattering point; P_1 is the probability that a gamma ray will move in a direction between ϕ and $\phi + d\phi$ and will reach distance r without being scattered or absorbed; P_2 is the probability that a gamma ray described by P_1 will scatter between r and r + dr so that it travels in the direction of the detector; P_3 is the probability that a gamma ray described by P_1 and P_2 reaches the detector from the scattering point without being scattered or absorbed; and P_4 is the detector efficiency for the gamma ray described by P_1, P_2, and P_3.

The four probabilities P_1, P_2, P_3, and P_4 are complicated functions of the original gamma-ray energy, the particular gamma-ray path being considered, and the type of detector (2).

The double integral of Eq. 1 was put into finite difference form and programmed for solution on a digital computer. Solutions of this equation did not reproduce experimental results performed with a prototype depth-type gage on prepared laboratory samples. It was concluded that the second simplifying assumption was not valid, and that the response caused by multiple-scattering events would have to be included in the analysis. This was accomplished by assuming that buildup factors derived from the results reported by Goldstein and Wilkins (4) could be superimposed on each gamma-ray path. The new mathematical model including this buildup factor can be written as

$$R = \int_{\phi} \int_{r} \frac{dP_1}{d\phi} \frac{dP_2}{dr} P_3 P_4 P_5 \, d\phi \, dr \qquad (2)$$

where P_5 is the buildup factor for a particular gamma-ray path.

Solutions to Eq. 2 did reproduce experimental results performed with a prototype depth-type gage on laboratory standards. After this verification of the mathematical model given in Eq. 2, it was used extensively to quantitatively study the effect of certain gage parameters on the sensitivity of the gages to soil composition. Among the conclusions reached were that (a) the composition sensitivity of a given gage was essentially independent of the sample density; and (b) composition sensitivity is affected by the gage housing material and thickness, the detector efficiency vs gamma-ray energy relationship, the source-to-detector distance, and the source collimation angles. Quantitative predictions of these effects are given by Ballard and Gardner (2).

A recent series of papers by Taylor and Kansara (5, 6, 7) reports on a single-scatter model of the gamma-ray gages. These authors were apparently unaware of the previous work just discussed. Unfortunately, they concluded that a single-scatter model was sufficient to describe the gamma-ray scatter technique. Therefore, the subsequent conclusions drawn by these authors are questionable.

The comprehensive multiple-scatter model described in this section served to investigate the practical importance of the possible sources of error that were identified. The model was then used to study the effect of varying all possible gage design parameters on these sources of error. The results of these studies indicated that changes in any single gage design parameter tended to minimize the error caused by variations in soil composition only at the expense of increasing the error caused by surface heterogeneities. For example, suppose that very low angle scattering of gamma rays is accentuated by source collimation so that, on the average, higher gamma-ray energies are detected. Then the effect of soil composition variations is minimized because the photoelectric absorption effect is minimized because of the higher average energy of the detected gamma rays, but the effect of surface heterogeneities is increased because the effective sample depth of the gage is reduced and the average relative path length through the surface heterogeneity is increased. This conflicting effect of varying the design parameters indicated that improvements in gage design would not be able to satisfactorily minimize the two sources of error caused by soil composition and surface heterogeneities. Moreover, even if gage design improvements could satisfactorily minimize these two sources of error, this would not solve the problem of using the many existing gages with acceptable error. For these reasons the calibration model approach described later was pursued.

Analyses of the Neutron Soil Moisture Content Gages

Attempts to perform a detailed mathematical analysis of the neutron soil moisture content gages have not been as successful as those for the gamma-ray soil density gages. This is primarily because neutron transport is a more complex and difficult phenomenon than gamma-ray transport. An excellent early study by Semmler (8) gives several possible mathematical approaches that include the use of various forms of one- and two-group neutron diffusion models. This study served as the primary basis for selection of a two-group neutron diffusion model with a spherical cavity of adjustable radius such as the model used in the studies made in the second phase of Project 10-5 (9). This model is given by

$$R = \frac{K_1}{\Sigma a\ (L_1 + K_2)\ (L_2 + K_2)\ (L_1 + L_2)} + K_3 \qquad (3)$$

where R is the gage response; Σa is the macroscopic thermal absorption probability of the sample; L_1 is the diffusion length of fast neutrons in the sample; L_2 is the diffusion length of thermal neutrons in the sample; K_1 is a factor that depends on source intensity and source-to-detector distance; K_2 is the spherical cavity radius; and K_3 is the background response of the gage. These parameters are described in detail elsewhere (9).

The important assumptions made in arriving at this model are that (a) the gage geometry consists of a point source of monoenergetic fast neutrons surrounded by a spherical cavity of radius K_2 that is, in turn, surrounded by an infinite, homogeneous sample; and (b) the neutrons emitted by the source diffuse like gas molecules until they are removed from each energy group. It is not likely that this model describes very accurately the behavior of surface-type neutron gages except over very limited ranges of gage design parameters and sample compositions and densities. The gage geometry chosen is very artificial when applied to surface-type gages.

A more rigorous approach is possible with the depth-type neutron moisture content gages. Olgaard and Haahr (10) devised and tested a three-group neutron diffusion model for the depth-type gages that appears to be quite accurate for the gages and samples tested to date. A Monte Carlo model is presently being developed for surface-type gages in NCHRP Project 10-5A. This approach should prove to be quite accurate, but generally requires a large amount of digital computer time for each model prediction. It is possible that a simple model can be devised and tested based on the results of the Monte Carlo model.

THE CALIBRATION MODEL METHOD

To minimize the identified sources of error to acceptable levels, the calibration model approach was formulated in the initial phase of work on Project 10-5 (2). This approach consists essentially of developing a simple mathematical or calibration model for both nuclear gages that includes all the soil parameters that affect the gage response. This model contains constants that must be determined for each gage by a least-squares analysis of gage responses taken on samples with known characteristics. A set of calibration samples consisting of pure materials, such as aluminum and magnesium of known density and composition, is used with these models. The advantages of using such samples for calibration are that they can be chosen to be stable, homogeneous, and representative of typical soils.

The calibration model method is no longer required for the gamma-ray density gages because the dual-gage principle described later will be able to minimize satisfactorily the identified sources of error for those gages. However, a similar principle probably does not exist for the neutron moisture content gages, so that the calibration model approach should prove important in minimizing the sources of error for those gages. Therefore, the rest of the discussion of the calibration model approach is confined to the neutron moisture content gages.

The least sophisticated use of this method would be to calculate one calibration curve with the calibration model appropriate to one average soil composition and density. Even this relatively simple method of use should represent a considerable improvement over other previous methods of calibration. A slightly more sophisticated method would be to use the calibration model to calculate calibration curves representative of various densities of several soil types that can be visually identified, such as sand and clay. This method requires a knowledge of the soil composition as a function of the soil classification, and a knowledge of the sample density that can be obtained from a measurement with a gamma-ray density gage. This second method should be quite accurate, but it depends on the gage user's ability to identify visually the soil type and also on each soil type having a relatively constant composition. The most sophisticated use of the method would be to obtain the composition of the soil sample of interest and calculate a calibration curve from the calibration model specifically for that soil at various densities. This method would only be practical if the same soil is to be encountered for an extended period of time. This method still requires a measured value of the sample density. One shortcut method that would alleviate most of the work involved in compensating the gages for variable composition and density would be first to obtain the gage response to a soil sample of known density. Then the entire calibration curve for that soil would be back-calculated from the single point and the calibration model for the gage. This technique has not been tried yet, but should prove valuable.

Gamma-Ray Density Calibration Models

Although the calibration model approach is no longer necessary for the gamma-ray density gages, the calibration model developed for this type of gage may prove useful in the orderly optimum design and use of the dual-gage techniques that will probably replace the calibration model approach. Therefore, a discussion of the model that was developed in the second phase of work on Project 10-5 (9) and other possible models is given here.

The calibration model developed in Project 10-5 is given by

$$R = C \exp_{10} (a + bC + cP) \tag{4}$$

where R is the gage response; C is the Compton scattering probability; P is the photoelectric absorption probability; and a, b, and c are constants for a given gage that are determined by a least-squares analysis of gage responses taken on samples of known density and composition.

The Compton scattering probability is taken as

$$C = \rho \sum_{i=1}^{n} \frac{w_i Z_i}{A_i} \tag{5}$$

where ρ is the sample density; w_i is the weight fraction of element i; Z_i is the atomic number of element i; A_i is the atomic weight of element i; and n is the total number of elements in the sample. The photoelectric absorption probability is taken as

$$P = \rho \sum_{i=1}^{n} \frac{w_i Z_i^5}{A_i} \tag{6}$$

This sample model inherently assumes that one average gamma-ray path can be established for a given gage that is essentially constant over the range of sample compositions and densities that are to be encountered. In spite of the simplicity of the model, it has been found that it is quite accurate for a wide range of gage designs and sample compositions and densities. It has the additional advantage of being able to fit the boundary conditions $R = 0$, $\rho = 0$, and $R = 0$, $\rho \to \infty$. It has the disadvantage that it does not explicitly give the role of the gage design parameters, such as source-to-detector distance, source energy, collimation angles, and the detector efficiency, as a function of gamma-ray energy. These parameters are implicitly contained within the a, b, and c constants.

Prior to this study, very similar models to that of Eq. 4 were proposed by Irick (11) and Semmler et al (12), but they did not separate the Compton scattering probability from the photoelectric absorption probability. A recent paper by Czubek (13) describes a model similar to that given by Eq. 4 that attempts to extract explicitly the gage design parameters from the a, b, and c constants. This model may prove quite valuable in the orderly optimum design of dual-gage techniques.

The simple calibration model of Eq. 4 has proved to be quite valuable in leading to the discovery of the dual-gage principle and in optimizing the air-gap dual-gage method developed by Kühn (14). Gardner et al (15) applied the calibration model to air-gap responses and showed that the model gave a basis for a system of using the air-gap method that gave improved accuracy. A simple nomograph method of use was developed based on the proper application of the calibration model to air-gap responses. The details of this treatment and a method for determining the necessary nomograph are given by Gardner and Roberts (9) and by Gardner et al (15).

Neutron Moisture Content Gage Calibration Models

The neutron moisture content gage calibration model developed and used in NCHRP Project 10-5 is that given as Eq. 3, which also served as the detailed mathematical analysis model. For use as a calibration model, the K_1, K_2, and K_3 parameters in this model are determined for a particular gage by a trial-and-error analysis of gage responses taken on prepared samples. This model does not offer sufficient accuracy for use as a detailed mathematical analysis model, and has too much freedom of gage response shape to be used without additional information as a calibration model. The model might be useful as a calibration model if the gage response shape for a particular type of gage is established, and the factor K_2 in the model is restricted to values that will give rise to the correct gage response shape. It has been found that gage responses vs moisture content shapes can be concave, straight, or convex, depending on the value of K_2 that is used. Additional experimental work or results from the Monte Carlo model studies will be used to establish the gage response shape for particular values of the gage design parameters such as source-to-detector distance and the amount of moderator surrounding the source and detector.

DUAL-GAGE PRINCIPLE FOR GAMMA-RAY SCATTER GAGES

The dual-gage principle was discovered when gage responses to laboratory calibration samples from several different gages were being fitted to the calibration model given by Eq. 4. It became obvious that different gages had different relative sensitivities to the Compton scattering and photoelectric absorption probabilities. Because the primary effect of variations in soil composition is manifested in the photoelectric absorption probability, the possibility existed to use two gages simultaneously to determine density while eliminating the effect of soil composition by eliminating the photoelectric absorption probability. This is accomplished by obtaining the specific calibration model given by Eq. 4 for each of two different gages. If the calibration models are denoted for each of the two gages by the subscripts 1 and 2, then one obtains

$$R_1 = C \exp_{10}(a_1 + b_1C + c_1P) \qquad (7)$$

and

$$R_2 = C \exp_{10}(a_2 + b_2C + c_2P) \qquad (8)$$

From the definition of C, the density ρ can be extracted from these equations if it is assumed that

$$\sum_{i=1}^{n} \frac{w_i Z_i}{A_i} = 0.05 \qquad (9)$$

Unfortunately, the simultaneous solution of Eqs. 7 and 8 is not straightforward, but several techniques are given by Gardner et al (15). A quadratic solution can be obtained if two terms of a series expansion of log ρ are used:

$$\rho = \frac{-B - (B^2 - 4AC)^{1/2}}{2A} \qquad (10)$$

$$A = 0.05\,(c_1b_2 - c_2b_1) \qquad (11)$$

$$B = c_2 \log R_1 - c_1 \log R_2 - 1.6815(c_2 - c_1) + c_1c_2 - c_2a_1 + 6.5(c_1b_2 - c_2b_1) \qquad (12)$$

$$C = 130c_2 \log R_1 - 130c_1 \log R_2 + 7.2384(c_2 - c_1) + 130(c_1a_2 - c_2a_1) \qquad (13)$$

The density ρ in Eq. 10 is given in pounds per cubic foot. A solution can also be obtained from a nomograph and this procedure is outlined by Gardner and Roberts (9).

The advantage of the dual-gage principle is that it shows promise of being able to eliminate the effect of composition while not accentuating the effect of surface heterogeneities. It also has the capability of being implemented with existing gages or more efficiently with gages designed specifically and optimally for the dual-gage principle.

There are many possible methods for obtaining practical dual-gage systems. Any combination of two different source energies, source collimations, source-to-detector separations, detector efficiencies including the use of energy filters, and gage positions above the sample are possible. Some of these combinations can be used with existing gages and some can only be incorporated in new gage designs. Both types of dual-gage systems are discussed in the following. The optimum dual-gage technique is the one that will minimize the total error composed of those resulting from composition sensitivity, sensitivity to surface heterogeneities, and normal source emission fluctuations.

Use of Existing Gages

It is important that the dual-gage principle be capable of use with existing gages so that they can be used in an optimum fashion until a new generation reaches the market. Of the possible dual-gage techniques capable of use with existing gages, the air-gap method introduced by Kühn (14) is most promising. This technique consists of taking a gage response in the usual manner and then raising the gage to a fixed height above the sample surface where a second response is taken. A nomograph can be obtained that gives density independent of the sample composition as a function of the normal flush response and gap response. This technique is described in detail by Gardner and Roberts (9) and a sample application is described by Gardner et al (15).

To implement the air-gap method with the existing commercial gages, one only needs a jig for raising the gage to a predetermined height above the sample surface. Several of the gage manufacturers are now supplying such a jig with the gages. The gages can be easily calibrated with four laboratory samples. Suitable sample materials for calibration are described by Gardner and Roberts (9).

Other possible techniques are the use of two separate gages that inherently have different characteristics, and the use of energy filters, such as the placement of a thin lead sheet under an existing gage by a shutter mechanism of some sort. It is possible that one of these techniques would minimize the total error discussed in the previous subsection better than the air-gap method if it is found to be less sensitive to surface heterogeneities. This possibility is being studied in NCHRP Project 10-5A. To date only the feasibility of these other dual-gage techniques has been established.

Design of New Gages

The next generation of gamma-ray density gages will probably include optimally designed dual-gage systems. A major portion of the work in NCHRP Project 10-5A now under way is devoted to determining optimally designed dual-gage systems that will give minimum total error. The gage design parameters presently being studied for dual-gage implementation include source energy, source-to-detector separation, detector efficiency spectra, and source and detector collimation.

FUTURE WORK

Work is presently in progress on NCHRP Project 10-5A on several aspects of improving the design and use of nuclear gages. These include the optimum design of a dual-gage gamma-ray density system, a study of the energy discrimination technique for minimizing the composition effect of gamma-ray density gages, a study of possible methods of minimizing the surface heterogeneity effect on gamma-ray density gages, and the improvement of the analysis and calibration model for the neutron moisture content gages.

Two related programs of interest to nuclear gage users have recently been initiated. The International Atomic Energy Authority recently sponsored the writing of a guidebook on neutron moisture gages that should be published very soon. This guidebook will describe how the gages work, what the advantages and disadvantages of the gages are, state-of-the-art development of the gages, the role and present status of theortical analyses of the gages, and suggested methods of calibration and use of the gages. The other program, also endorsed by the International Atomic Energy Authority, is an extensive evaluation of the nuclear gages. This program is being carried out at Brno, Czechoslovakia, by the Czechoslovak National Association of the International Union of Testing and Research Laboratories for Materials and Structures. It consists of a series of tests to be performed on all commercially available nuclear gages including stability, temperature dependence, effective sample volume, and composition sensitivity. A report of the results of these tests will be published.

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