Spring Analog Model for Flexible Culvert Behavior

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An analog model consisting of pin-jointed rigid segments is used to simulate the soil-culvert interaction of a flexible circular culvert subjected to earth loading. The soil behavior is expressed in terms of the constrained modulus, which some investigators have indicated can be determined from the dry density. The forces due to the soil weight are applied at the joints by means of calibrated springs, where the stiffnesses of the springs are set to correspond with the stress-strain characteristics of the surrounding soil, and the effect of settlement of the adjacent embankment is handled by moving the spring supports through calculated distances. The resulting response of the model is measured, and these data, together with a modification factor to account for ring stiffness, are suggested for possible use in determining directly the ring forces and deformations of the culvert for a wide range of field conditions.

•IN GENERAL, the commonly used methods for flexible culvert design involve establishing a load distribution around the culvert periphery and then using this load distribution to determine the stresses in the culvert wall. If desired, the deformation of the culvert may be computed from an empirical formula; however, this formula requires a knowledge of some rather elusive soil properties. The determination of the load distribution and associated deformations for a soil-culvert system is highly complex because of the interaction between the two, and the situation is further complicated by the fact that the load-deformation behavior of most soils is nonlinear and time-dependent.

As far as major structural considerations are concerned, the designer is interested primarily in the compressive stresses in the culvert wall and the resulting displacements. If it were possible to determine these parameters directly from the geometry of the culvert and the soil properties, a knowledge of the actual load distribution would be unnecessary. The spring analog model discussed here is designed to investigate the response to earth loading of a circular culvert whose walls are unrestricted by bending resistance. Graphical representation of the results in terms of dimensionless parameters and provision for modification to account for wall stiffness facilitate the direct application of model data to actual field conditions.

REVIEW OF RELATED WORKS

The traditional Marston-Spangler approach to the design of positively projecting flexible circular culverts (11, 12, 13) is an extension of earlier work (5) relating to negatively projecting sewer and drain lines laid in excavated trenches. Only two directions of displacement are considered: vertical and horizontal. Vertical displacement is assumed to consist of the displacement of a rectangular prism of soil above the pipe, and horizontal displacement is assumed to have a parabolic distribution with a maximum

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value, Δx, at the horizontal diameter. According to Spangler, Δx can be computed from the formula

$$\Delta x = \frac{D_L K W_C R^3}{EI + 0.061\ E' R^3} \tag{1}$$

where

Δx = horizontal deflection (increase in diameter) of the pipe,
D_L = deflection lag factor,
K = bedding constant,
W_C = vertical load per unit length of pipe,
R = mean radius of pipe,
E = modulus of elasticity of the culvert material,
I = moment of inertia per unit length of the pipe wall cross section, and
E' = modulus of soil reaction.

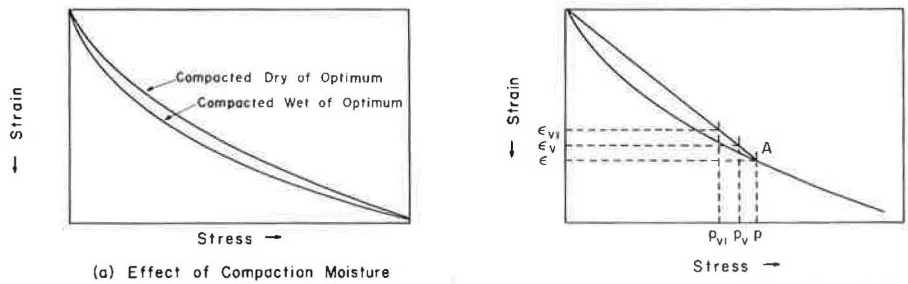
The vertical load on the pipe consists of the uniformly distributed weight of the prism of soil above the pipe less the shear forces acting on the sides of this prism. The reaction to this loading is given by the passive resistance of the soil near the culvert walls and by the vertical forces on the bottom provided by the bedding. The commonly employed design criterion limits Δx to 5 percent of the diameter and does not explicitly consider compressive stresses in the culvert wall, except for seam and lap joint design.

White and Layer (17) propose that the soil overburden load be assumed to act hydrostatically on the culvert. Based on this assumption, they determine the ring compressive stress, which, in conjunction with a safety factor of either 2 or 4, is used as the structural design criterion for the culvert. Watkins (16) considers the loading of a circular, flexible culvert to be more realistically described by a radial compression phenomenon. Under the free-field compression of the embankment, the soil surrounding the culvert is deformed into an elliptical shape, where the length of the horizontal axis remains unchanged and the length of the vertical axis is reduced. Because the circumference of the culvert remains essentially constant, however, the culvert resists this change and, in so doing, manifests an outward radial displacement relative to the soil mass. The theory of elasticity is used to develop graphs from which the compressive stresses in the culvert wall and the associated deflections may be determined. This approach is used herein as a basis for converting the soil response to an equivalent spring load. When considering the buckling of soil-surrounded tubes, Luscher (4) suggests that the finite elastic continuum immediately surrounding the tube may be approximated by a system of discrete springs; the same approximation is used to analyze the bending of beams that are continuously supported by an elastic subgrade. Further justification for this model will be discussed subsequently.

THEORETICAL CONSIDERATIONS

Transmission of Embankment Forces to the Structural System

As the fill is constructed over a culvert, forces are exerted on the culvert wall. These forces may be considered to have radial and tangential components or, alternatively, vertical and horizontal components. From the viewpoint of the directional distribution of forces and soil properties in an embankment, the latter representation is convenient and will be used here. If the culvert is constructed within a large homogeneous embankment, the vertical component of force at any point on the culvert periphery will depend on (a) the weight of the vertical column of soil above the culvert, (b) the degree of consolidation of the adjacent fill outside the "zone of influence" of the culvert, (c) the "arch action" effect due to the influence of the preceding factor on the "hard spot" provided by the culvert, (d) the vertical component of the modulus of soil reaction, and (e) the vertical component of the culvert deformation. Similarly, the horizontal force component depends on (a) the vertical force component, (b) the coefficient of earth pressure at rest, (c) the horizontal component of the modulus of soil reaction, and (d) the horizontal component of the culvert deformation.



(a) Effect of Compaction Moisture Content on Shape of Curve

(b) Approximation of Nonlinear Soil Compressibility by Linear Spring

Figure 1. Typical stress-strain curves from consolidation test.

Stress-Strain Characteristics of the Soil

The deformation of the embankment outside the zone of influence of the culvert is normally considered to be a uniaxial strain condition; the constrained modulus, M, as measured by the conventional laboratory consolidation test, is commonly used to represent the field stress-strain characteristics. Based on the results of consolidation tests on a variety of laboratory-compacted soils, Osterberg (9) suggested that M for many commonly encountered soils may be uniquely determined from the compacted dry density. Although the use of such a generality must be approached with caution until substantiated by further tests, its application in conjunction with good engineering judgment should not be precluded. If correct, this relationship would permit the compression of the embankment at the culvert level to be determined directly from the dry density of the compacted fill alone.

There are many variables that can affect the compressibility of a compacted soil; among these, Lambe (2) lists temperature, soil composition, characteristics of permeant, void ratio, degree of saturation, and structure. Some, but not necessarily all, of these are taken into account with dry density. As shown in Figure 1a, Lambe suggests that, for clay samples compacted to the same dry density, one above and one below optimum moisture content, the one compacted at the lower moisture content will exhibit a more nearly linear void ratio-pressure relationship. However, the degree of difference is not indicated. Osterberg's work appears to show that this difference would not be sufficient in practice to prohibit the use of some average curve for design purposes. In addition, Lambe's observations apply primarily to clays, and the variation would be expected to be less for soils of lower clay content, such as would probably be used for compacted embankments. The normal use of different load-increment ratios in the laboratory and in the field may have an effect on the consolidation characteristics of the soil, as discussed by Leonards and Altschaeffl (3) and others; however, the few comparisons between laboratory and field settlement characteristics of compacted soils are insufficient to draw any general conclusions, and it seems reasonable to extrapolate laboratory results directly to comparable field situations until a more definite and quantitative evaluation of such load-increment ratio effects is forthcoming.

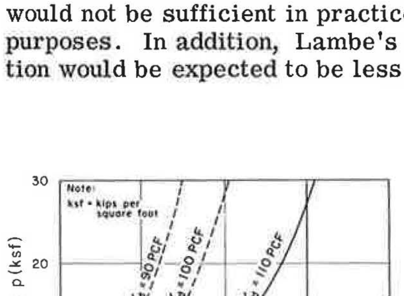


Figure 2. Secant modulus versus stress for compacted soils.

The curves developed from Osterberg's work (Fig. 2) show the secant constrained modulus as a function of stress and compacted dry density. The solid curve is associated with a dry density

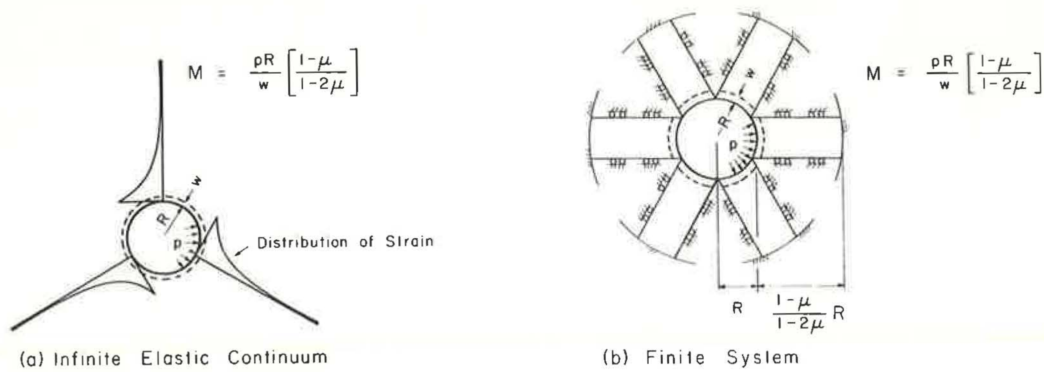


Figure 3. Representation of infinite elastic continuum surrounding a cylindrical hole by equivalent finite system.

of 110 pcf, and it is used as a reference; the dashed curves represent constrained modulus relationships for soils with dry densities of 90, 100, and 120 pcf. Moduli for other dry densities may be determined from

$$M = 0.635^{\left(11 - 0.1\gamma_d\right)} M_{110} \tag{2}$$

where

M = constrained modulus for a soil with a dry density γ_d, and
M_{110} = constrained modulus for a soil with a dry density of 110 pcf.

The compressibility of the soil adjacent to the culvert wall is also of major interest. Watkins (16) suggests that the effect of the downward movement of the fill is to produce a relative radial motion between the culvert circumference and the soil. According to Murphy (7), the radial deformation, w, of a circular hole of radius R in an infinite elastic continuum caused by an internal pressure p is

$$w = \frac{Rp\ (1 + \nu)}{E_s} \tag{3}$$

where

ν = Poisson's ratio, and
E_s = modulus of elasticity of the elastic medium.

Because E_s and M are related by

$$E_s = \frac{M(1 + \nu)\ (1 - 2\nu)}{(1 - \nu)} \tag{4}$$

substitution of Eq. 4 into Eq. 3 and simplification leads to

$$w = \frac{Rp}{M}\ \frac{1 - \nu}{1 - 2\nu} \tag{5}$$

The qualitative distribution of strains for this condition is shown in Figure 3a.

Consider now the system shown in Figure 3b, in which segments of a cylinder are restrained by blocks of elastic material of constrained modulus, M, under the following conditions:

1. The material is of infinite extent parallel to the cylinder axis;
2. The cylinder is restrained from movement in a direction tangential to the circumference; and
3. The radial length of each restraining block is $(1 - \nu) R/(1 - 2\nu)$.

Then, as a result of the conditions specified, the relationship among w, p, and M is the same as that in Eq. 5 for the cylindrical hole in a body of infinite extent. Hence, the artificial system shown in Figure 3b may be used to simulate the infinite case shown in Figure 3a, provided the indicated relationships exist. For ν values of 0.2, 0.3, and 0.4, the factor $(1 - \nu)/(1 - 2\nu)$ is equal to 1.33, 1.75, and 3.00 respectively. If a value of 0.33 is arbitrarily assumed for ν, Eq. 5 reduces to

$$M = \frac{2Rp}{w} \tag{6}$$

The use of 2 for the factor $(1 - \nu)/(1 - 2\nu)$ may warrant subsequent refinement after further study; in particular, a variation of ν with M would indicate that this factor should vary. Alternatively, an evaluation of this factor from field measurements may provide compensation for errors introduced in the theory by the various approximations.

The preceding development involves the major assumptions that the material is elastic and that small strains occur. Although it is possible that neither of these conditions strictly applies for the case of a soil-surrounded flexible culvert, it is probable that they will be acceptable approximations for many situations. For a well-compacted fill the assumption of an elastic material seems reasonable, and for strains up to 5 to 10 percent the associated error should be tolerable. Note that, for the particular case where ν equals 0.33, strains of 5 to 10 percent in the soil correspond to changes in the culvert diameter of 10 to 20 percent. A further assumption concerns the use of Eq. 3 to determine radial deformations in accordance with the concept proposed by Watkins (16) and explained previously herein. Compression in the vicinity of the culvert wall involves both the vertical and horizontal soil moduli; although these moduli probably differ to some extent, few data are available on the relationship between them. For purposes of this work, they will be assumed equal, and the soil modulus relationships shown in Figure 2 will be used.

ANALOG MODEL

If the bending resistance of a circular, flexible culvert is neglected, Eq. 1 (Iowa formula) reduces to

$$\frac{\Delta x}{2R} = C \frac{p_v}{E'} \tag{7}$$

where p_v is the weight of a unit column of soil above the culvert and C is a constant of proportionality. In fact, some investigators (6) consider such a form sufficiently accurate for flexible culvert design. The object of this study is to investigate the response of a flexible culvert having no bending resistance, so that the behavior reflects purely the influence of the soil. It is recognized that a real culvert having a continuous circular wall must exhibit some bending strength in order to resist local buckling. However, the buckling mode of failure, which virtually never occurs in smaller diameter culverts, is specifically precluded in this work. It is important to note that a prismatic culvert having essentially rigid flat faces joined by hinges (similar to the model suggested) can be realistically constructed, and, in fact, Peck (10) discussed a London tunnel that recently was lined in this manner. Whereas such a structure would be unstable if surrounded by a fluid having no shear strength, the shear strength of the surrounding soil normally provides ample stability except in cases of low cover height or excessive deformation. If the number of segments in such a structure is large, the response to loading would closely represent that of a circular culvert with little or no bending resistance.

Because the segments are considered rigid, the continuous force system may be replaced by equivalent concentrated forces at the hinges without affecting the behavior. In addition, the use of concentrated tensile forces, instead of compressive forces, will result in an equivalent, but reverse, response. Such an analysis, whereby "stable" geometry under compression forces is determined by means of a tension system, has been used for surface structures in the form of the "linear arch" (14). In the case of a concrete arch subjected to surface loads, the geometry producing equilibrium of a flexible system in tension is equivalent to that producing unstable equilibrium in compression. For design, of course, some bending resistance is required to account for load variations and design inaccuracies. The difference in applying the "linear arch" concept to the deformation of a flexible culvert is that the resulting shape is not one of unstable equilibrium under the compression system, but one of stability owing to the shear strength of the soil.

Based on this reasoning, an analog model consisting of rigid strip segments connected at pin joints and loaded in tension at these joints by helical springs will be used to describe the response of a flexible culvert to earth loads. The tests using this model are based on the following assumptions, some of which may be removed by further refinement of the technique: (a) the culvert is perfectly flexible; (b) the response of the culvert can be determined from tests on one quadrant only; (c) the quadrant can be represented by a series of six rigid chords connected at flexible pivot points; (d) forces acting on the culvert can be represented by a combination of horizontal and vertical forces acting at the pivot points; (e) the horizontal pressure distribution over the height of the culvert due to the weight of the fill is uniform and equal to the value at the horizontal diameter of the culvert; and (f) no consideration is given to buckling.

Function of the Model

The field conditions in the model are simulated by means of a superposition process. The effect of the difference in vertical and horizontal pressures is first allowed to act on the model. Then, superimposed on this force system is the effect of the free-field compression of the soil mass. The validity of such a process depends on an estimate of the final vertical and horizontal components of stress in the soil and on setting the spring stiffnesses accordingly. Because a constant secant constrained modulus is used instead of the actual stress-dependent modulus, the spring stiffnesses are set accordingly, and, once the spring stiffnesses are set for a given load, any partial loading condition for the model will be meaningless. If, for example, the curve shown in Figure 1b is the constrained stress-strain relationship for the soil and p is the estimated final stress acting on the culvert, then a stiffness corresponding to the slope of line OA is set in one of the associated springs while the entire model is held fixed. If this spring represents a vertical force and if the initial load setting, p_v, represents the weight of the overburden, the corresponding strain in the spring will be represented by ϵ_v. However, this value of strain is not correct, inasmuch as the point (ϵ_v, p_v) does not lie on the stress-strain curve for the soil. If the loads in the horizontal springs are set in accordance with $K_o p$ (where $K_o < 1$) and the system is released, the load in the spring under consideration will decrease to p_{v1}; this condition will correspond to a strain of ϵ_{v1}, which again will be unrealistic for the soil. If vertical deformations due to the free-field compression (time-dependent consolidation plus instantaneous deformations) of the fill adjacent to the culvert are then introduced into the system, the spring deformation will correspond with point A, provided the estimate for p was correct, and the corresponding strain in the soil will be realistic.

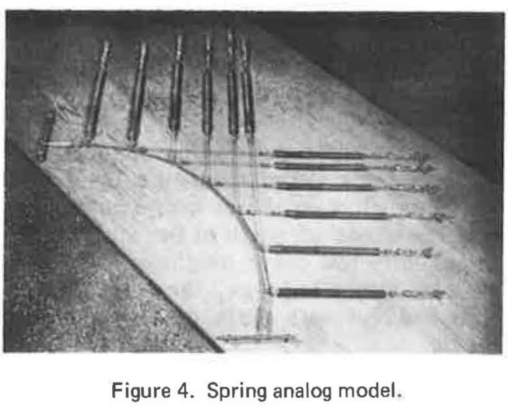


Figure 4. Spring analog model.

The culvert quadrant, shown in Figure 4, is represented by six equal $\frac{1}{16}$- by $\frac{3}{8}$-in. brass bars joined by hooks to $\frac{5}{16}$-in. diameter brass washers. The centers of the washers represent points on the circumference of a flexible culvert with a radius of 2 ft. Except for the end washers, each washer has two springs attached, one vertical and one horizontal. The resultant of the forces applied by these springs represents the load acting on the culvert wall over an arc distance of 7.5 deg to either side of the pivot point. The end points of the quadrant are each supported by only one spring, and the motion at these points is restricted to one direction.

For loading the model, an arbitrary modulus, M_c, is selected to represent that of the soil near the culvert wall, and the modulus is then converted to an equivalent spring stiffness. According to Eq. 6, the soil deformation, w, may be approximated by $2Rp/M_c$. The soil stress, p, acting on the component, L, of the 15-deg arc produces a load, P, given by pL, and P is the load applied to the spring. The spring stiffness, k, is then given by

$$k = \frac{P}{w} = \frac{pL}{w} = \frac{M_c L}{2R} \tag{8}$$

Each spring stiffness is set separately by allowing a required number of coils to be active, depending on the premeasured calibration of the particular spring. The quadrant of the model is clamped rigidly in a circular position while the springs are tensioned. As previously discussed, the vertical springs are loaded in accordance with the unit load of the overlying soil and the component of the culvert area on which it acts ($P_v = \gamma HL$), while the horizontal springs are loaded similarly except for an adjustment for the coefficient of earth pressure at rest ($P_h = K_o \gamma HL$).

For the work presented herein, the value of the coefficient of earth pressure at rest, K_o, is taken as 0.5. Tschebotarioff (15) discusses early tests by Terzaghi on remolded soils, for which values of 0.42 for sand and 0.70 to 0.75 for clays were reported. However, because the latter values for clay are considered by many to be too high, 0.5 appears to be a reasonable average value. Furthermore, from the theory of elasticity, $K_o = \nu/(1 - \nu)$, which yields K_o equal to 0.5 for ν equal to 0.33, this latter value for ν is consistent with an earlier assumption.

The effect of the free-field compression of the fill is represented in the model by displacing the outer supports for the vertical springs by amounts proportional to the distances, y, between the corresponding pivot points and the horizontal axis of the model. These displacements, δ, are computed from the equation

$$\delta = \frac{p_v y}{M} \tag{9}$$

in which M is the modulus of the soil mass. Although a value of 0.9 for M/M_c was used for these tests, final results indicate that a ratio of unity is more nearly correct. However, the influence of this factor on the test results is considered negligible.

After these adjustments have been made, the spring deformations are measured to determine the forces at the joint; in addition, the new locations of the pivot points are plotted. With a knowledge of the vertical and horizontal forces at each joint, the two unknown forces, T, in each of the chords at a given joint are determined by a graphical procedure. Because each chord extends between the two joints, the load, T, in each chord is determined twice with the independent solution of the force system at each joint. By varying the relative values of p_v and M, various values for T, Δx, and Δy can be found. Presentation of the results in dimensionless form facilitates application of the resulting curves to actual field conditions.

Discussion of Results

Typical test results obtained from this model are shown in Figure 5. Note from Figure 5a that, for a deflection of 5 percent, the parameter p_v/M equals 0.056. The height of fill corresponding to this value may vary from about 17 ft for a material having a compacted dry density of 90 pcf to about 200 ft for a material having a compacted dry density of 120 pcf. Culvert deformation, and thus structural performance, is highly sensitive to compacted dry density. It is interesting to compare the model test results shown in Figure 5c for the compressive load in the culvert wall with those obtainable by assuming a hydrostatic load on the culvert. The parameter T/Rp_v for the model tests varies from about 0.8 at the top and bottom of the culvert to about 1.2 at the side points. Were it not for shear stresses tangent to the culvert wall (Fig. 6a), a constant value of about unity would be obtained, which, coincidentally, is the value for T/Rp determined under the hydrostatic load conditions assumed in the ring compression theory (17). Apparently, the reduction in horizontal pressure due to the use of K_0 approximately compensates for the arching effect due to compression of the adjacent fill. In addition, a comparison may be made between these results and those calculated from Eq. 1 (the Iowa

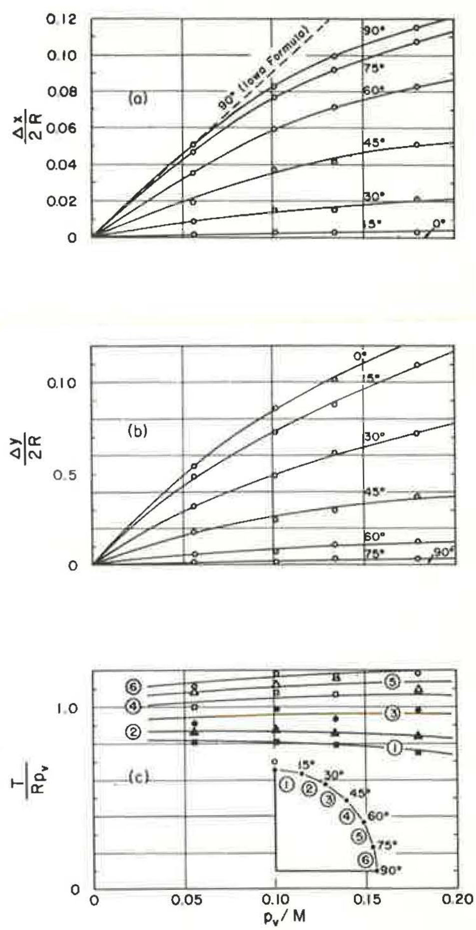


Figure 5. Deformation and ring compression load characteristics determined from test results.

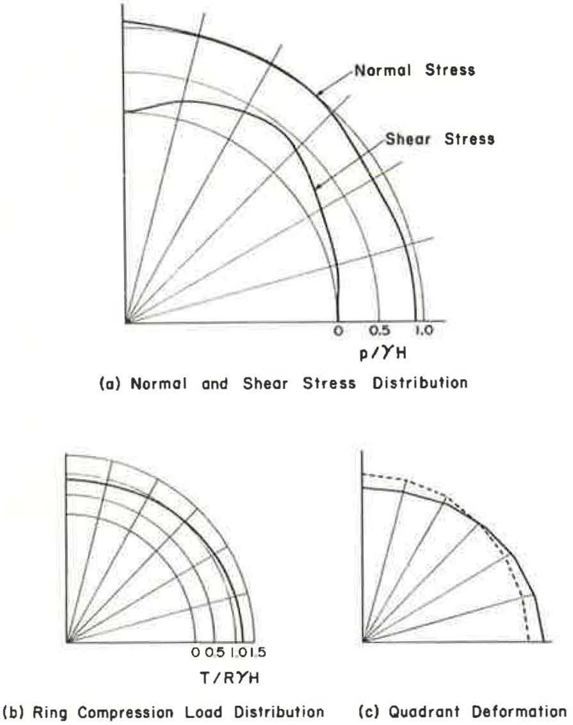


Figure 6. Stress and deformation distribution for model test (p_v/M_0 = 0.10).

formula). If E' is taken as 1.5 M, as shown by Nielson (8), if D_L, K, and W_c are taken as 1.0, 0.083 (corresponding to the maximum bedding angle), and $2Rp_v$ respectively, and if the stiffness parameter, EI, is taken as zero, Eq. 1 becomes

$$\frac{\Delta x}{2R} = 0.91 \frac{p_v}{M} \qquad (10)$$

As shown by the plot of Eq. 10 on Figure 5a, very close agreement is obtained for deflections less than 5 percent.

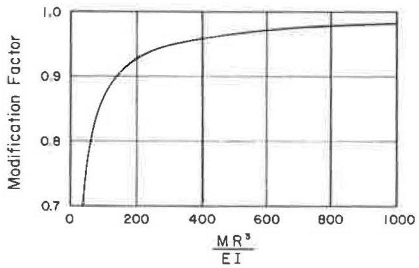


Figure 7. Modification factor for ring stiffness.

Detailed stress and deformation data from a typical model test are shown in Figure 6. The distribution of normal and shear stresses as a function of the overburden pressure, p_v, is shown in Figure 6a. Although the shear stress approaches a maximum of about 0.45 times the normal stress, it is possible that shear failure will occur before this stress is reached, so that some redistribution of stresses will occur. Figures 6b and 6c show respectively the ring compression load distribution and the ring deflection pattern for the same test. The effect of varying degrees of rigidity for the culvert wall may be considered by referring to Figure 7; this curve is based on data presented by Burns and Richard (1). An increase in ring stiffness will result in a reduction in the deformation, and data from the spring analog model may be multiplied by the modification factor to take culvert stiffness into account.

Suggested Extension

The suggested spring analog model, although developed herein for the somewhat restricted conditions in which a circular culvert is embedded in a homogeneous fill of infinite extent, is a versatile technique for solving the complex soil-structure interaction problem. (See the Appendix for an example problem.) With certain minor modifications, it is applicable to cases where the pipe geometry or the soil properties or both of these vary. Generally, however, these cases will require that one-half instead of one-fourth of the culvert be represented. When the soil cover is shallow in relation to the culvert diameter, the assumption of a uniformly distributed horizontal pressure on the culvert may not be valid, and it may be necessary to conduct tests in which the horizontal force resulting from the weight of the overlying soil varies. Also, if the material underlying the culvert has a compressibility substantially different from that of the embankment material, it is possible to represent this difference by adjusting the vertical springs near the bottom of the model. Similarly, the effect of the "imperfect ditch" on a culvert can be studied by adjusting the vertical springs near the top of the culvert.

CONCLUSIONS

A direct, rational, approximate solution to the complex soil-structure interaction problem involved in the design and analysis of flexible, circular culverts is possible by the use of a spring analog model. The validity of the technique depends on the accuracy with which the weight of the overburden, the coefficient of earth pressure at rest, the constrained soil modulus, and the initial structure geometry can be represented in the model. The independent treatment of stresses due to soil weight and stresses due to free-field settlement of the adjacent embankment produces a more logical and realistic representation of stresses acting on the culvert; the compressibility of the infinite soil mass can be reasonably well approximated by the compressibility of an equivalent system of springs; and the representation of the culvert ring by a system of rigid chords connected by pin joints is a sufficiently close approximation to conditions encountered in many field situations. The existence of a general relationship between the compressibility and compacted dry density of a variety of embankment soils obviates the need for individual compressibility testing. In addition to deflection data, the model provides a guide to the distribution of normal and shear stresses on the culvert and the compressive stresses in the culvert wall. Although the normal stress distributions obtained from the model are close to the hydrostatic pressure condition assumed in the ring compression theory, it appears that shear stresses between the culvert and the surrounding soil may cause a maximum increase of about 20 percent in the compressive stress in the culvert wall.

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Appendix

ILLUSTRATIVE EXAMPLE

To illustrate the applicability and simplicity of a culvert design procedure based on the spring analog model, the following problem, previously used by Spangler (13), is solved.

Problem

A 60-inch, 10-gage corrugated metal pipe (standard corrugations ½ in. deep at 2⅔ in. centers) is to be installed as a projecting conduit with a 60-deg bedding (bedding angle equals 30 deg) and covered with an embankment 20 ft high. Assume the projection ratio equals 0.7, the settlement ratio equals 0, the unit weight of soil equals 120 pcf, and the value of E′ equals 700 psi. Determine the long-time deflection of the pipe with the deflection lag factor of 1.25.

Solution

Because $H = 20 + 0.5\ (5) = 22.5$ ft, we have $p_v = \gamma H = 120\ (22.5) = 2{,}700$ psf. For a soil having a total unit weight of 120 pcf, a dry density of 100 pcf is a reasonable estimate; therefore, from Figure 3 and Eq. 2, for p_v equal to 2,700 psf and γ_d of 100 pcf, M is found to be 79,000 psf, which gives

$$\frac{p_v}{M} = \frac{2{,}700}{79{,}000} = 0.035$$

From Figures 6a and 6b, we obtain $\Delta x/2R$ and $\Delta y/2R$ equal to 0.034 and 0.037, giving horizontal and vertical deflections of 2.0 and 2.2 in. respectively. An adjustment for the wall stiffness of the culvert may be made by computing the soil-culvert flexibility parameter,

$$\frac{MR^3}{EI} = \frac{79{,}000 \times 2.5^3 \times 12}{30 \times 10^6 \times 0.0045} = 110$$

and entering Figure 7 to obtain a modification factor of 0.875. Although the computed deflections are due to primary consolidation only, the stated problem includes an additional allowance of 25 percent ($D_L = 1.25$) to account for long-term deformation. Hence, if the originally computed deflections are multiplied by the product of 0.875 and 1.25 to account for both ring stiffness and deflection lag, the final horizontal and vertical deflections will be 2.2 and 2.4 in. respectively.

The circumferential load per lineal foot, T, can be determined from Figure 5c by noting that the maximum value of T/Rp_v for $p_v/M = 0.035$ is 1.11; accordingly, the maximum circumferential thrust in the culvert ring is

$$T = 2.5 \times 2{,}700 \times 1.11 = 6{,}920 \text{ lb/ft}$$

In computing a horizontal culvert deformation of 2.68 in., Spangler (13) selected the following values for the parameters used in his equations: (a) the settlement ratio, r_{sd}, is 0; (b) the modulus of soil reaction, E′, is 700 psi; (c) the bedding factor, K, is 0.102; (d) the load coefficient, C_c, is 4; and (e) the deflection lag factor, D_L, is 1.25. By comparison, the use of the spring analog model involves the following assumptions: (a) the soil in the vicinity of the culvert and that in the adjacent embankment is reasonably homogeneous; (b) a good correlation exists between compacted dry density and confined modulus of the soil; (c) the laboratory consolidation test is representative of the field of compression characteristics; (d) the coefficient of earth pressure at rest is 0.5; and (e) the analogy between the culvert in the ground and the spring model is valid.