Fatigue and Fracture of a Bituminous Paving Mixture

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The fatigue behavior of simply supported sand-asphalt beams is examined by using experimental and analytical methods of fracture mechanics. The fatigue crack growth rates correlate well with the stress-intensity factor in accordance with Paris's law that states that $dc/dN = AK^4$, where dc/dN is the crack per cycle, A is a constant of a material, and K is the stress-intensity factor, which is dependent on the load, geometry, and boundary conditions. It is postulated that in a material such as a sand-asphalt mixture, which is abundantly endowed with flaws, fatigue damage is initiated at the first loading cycle, so that the fatgue life is the number of cycles of repeated loading to propagate a "starter flaw," c_o, into a crack of critical size, c_f. The starter flaw is a material's constant but is subject to statistical variation and is believed to be principally responsible for the statistical variation of fatigue life. The crack reaches the critical stress-intensity factor, K_c, which is a constant for a given material. K_c is the failure criterion for both static fracture and fatigue. A formula for fatigue life, N_f, is given. Methods for determining these constants are presented, and the fatigue lives, determined experimentally are compared with those predicted from the formula and show good agreement.

•MANY PAVEMENT DESIGNERS, in recent years, have expressed concern over the cracking of flexible pavements under repeated load applications. This type of distress, referred to as fatigue failure, appears in the form of alligator or map cracking, which is initially confined to localized zones and spreads at an increasing rate. Other types of pavement failure, such as rutting and shear failure, are also associated with cracking and may occur in conjunction with fatigue cracking. However, fatigue failure differs from other modes of pavement distress by the fact that it usually requires little or no permanent deformation or flow and is often of quasi-brittle type.

The phenomenon of fatigue failure is associated with the concept of damage or those changes in the material that lead to formation of macroscopic cracks and subsequent structural instability. The occurrence of fatigue failure is a result of 2 separate processes: damage initiation and damage growth (1, 2, 3, 4). The occurrence of these 2 processes in a material system results in a gradual weakening of the structural components. In order for the failure state to be reached, however, the damage should approach a critical level. In short, damage initiation and growth are necessary but not sufficient conditions for the occurrence of fatigue failure. In fact, damage growth in a material body can be arrested during the course of repeated loading before reaching the threshold of instability. The arrest of the damage can be attributed either to the inability of the applied load to furnish sufficient energy required for growth or to other changes that occur in the material body and boundary conditions and that alter the state of stress distribution in the structural component.

The processes of damage initiation and growth differ among various materials. Because of the presence of inherent flaws in certain alloys, plastic, polymers, and heterogeneous compositions such as asphaltic materials, damage can reasonably be expected

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to be initiated at the first few cycles of load application. The statistical distribution of such internal discontinuities, in fact, can account for statistical variations in the fatigue life (1, 2).

The process of damage growth has been discussed in various theories (4), but fundamentally it is related to the deformation occurring at the tip of discontinuities and is associated with the energy balance in these regions. The work of external forces in the regions of discontinuity is divided into stored elastic energy, the energy required for irreversible changes, such as viscous or plastic flow, in the material body, and the surface energy required to form a crack. The rate of crack growth then depends entirely on the energy balance, and the path it follows is governed by the minimum energy requirement.

During the cyclic deformation process, the tip of the zone of discontinuity blunts and resharpens resulting in crack growth through the body. This process continues until a crack of critical size has been reached, and the induced state of stress results in structural instability or terminal event of fracture.

The fatigue life, N_f, of a material or structural component can be represented by a parametric equation, such as

$$N_f = F\,[c_o,\ (A,\ n,\ K),\ K_c] \tag{1}$$

This equation expresses the fatigue life in terms of 3 sets of parameters. The parameter, c_o, is associated with the process of crack initiation and may be considered as the starter flaw in the tension face of the beam from which the crack will propagate. Its size is dependent on the material characteristics such as void content, statistical distribution of voids, and surface condition. This parameter cannot be directly measured and should be determined experimentally. It is believed to be principally responsible for the statistical variations in the service life of a mixture.

The second parameter set, (A, n, K), expresses the process of crack growth in the material system. A previous paper (4) pointed out that Paris's equation for rate of crack growth can be applied to bituminous mixtures. This equation is

$$dc/dN = AK^n \tag{2}$$

The constant, A, is a material constant and appears to be temperature dependent, and n has been found to be 4.0 as shown by Paris and Erdogan (5, 6). The parameter, K, is the stress-intensity factor at the crack tip affected by the applied load, geometry, support condition, and material stiffness.

The third parameter, K_c, describing the critical limit for damage, is the value of the stress-intensity factor at the start of instability, i.e., at the point where the crack will propagate spontaneously. K_c is determined from fracture tests and has been shown to be a constant of the material (7, 8, 9, 10).

These parameters are considered in this duscussion of the fatigue life of a paving mixture.

MATERIALS AND TESTING PROCEDURES

In this study a sand-asphalt mixture composed of a 60 to 70 penetration asphalt cement and a well-graded Ottawa sand mixture was utilized. The aggregate gradation is given in Table 1, and the characteristics of the asphalt cement and the specimen mixture used are given in Table 2. Six percent asphalt cement by weight of aggregate was used in the preparation of the specimen. Beam specimens of 1 by 1 by 12-in. were prepared by using a drop-hammer compaction assembly. The specimen density and void content are also given in Table 2.

The sand-asphalt beam specimens were subjected to both fatigue and fracture tests. Fracture bending tests were conducted on simply supported beams of 11-in. span loaded at midspan. Both unnotched and notched specimens were used in this investigation. Six notch depths were used ranging from $^5/_{32}$ to $^1/_2$ in. The fracture tests were conducted in stroke control by using an MTS testing machine (Fig. 1). The ramp loading function

TABLE 1

AGGREGATE GRADATION

Sieve Number	Percent Passing (samples)	ASTM Specification D1663-59T
16	100.0	85 to 100
30	88.0	70 to 95
50	64.2	45 to 75
100	30.1	20 to 40
200	15.7	9 to 20

TABLE 2

CHARACTERISTICS OF ASPHALT CEMENT AND SPECIMEN MIXTURE

Characteristics	Amount
Asphalt cement	
Test	60 to 70
Specific gravity	1.010
Softening point, ring and ball, deg F	123
Ductility at 77 F, cm	150+
Penetration	
100 grams, 5 sec, 77 F	63
200 grams, 60 sec, 39.4 F	23.5
Flash point, Cleveland open cup, deg F	455
Specimen mixture	
Asphalt cement, percent by weight of aggregate	6
Average density, grams/cm^3	2.052 ±0.011
Average air voids, percent	17.0 ±0.5

was used. At low temperatues the effect of rate of loading on the fracture test is not significant. Nevertheless, the rate of loading was selected so that it would be comparable to the average rate of loading in fatigue experiments, determined by approximating the sinusoidal curve with a triangular loading function.

The fatigue experiments were conducted at 0.5 cycles per second with a $P = P_0 + P_1 \sin \omega t$ loading function. Nine different combinations of P_0 and P_1 were selected to give a wide spectrum of fatigue loading. These tests were conducted on unnotched specimens. The results of fatigue response of notched specimens have been reported previously (4, 8). Although these experiments are being conducted at various temperatures, only the results of one temperature, 23 F, is presented in this paper.

ANALYSIS OF RESULTS

The fatigue life of a material can be expressed by

$$N_f = F\,[c_0,\ (A,\ n,\ K),\ K_c] \tag{1}$$

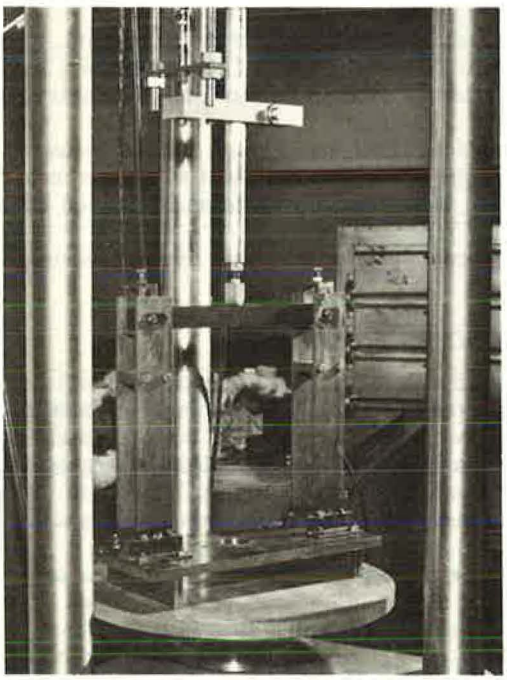

Figure 1. Specimen arrangements in testing frame.

where each parameter set describes different processes involved in the fatigue failure. In the following sections the methods for determining these parameters are discussed.

The Critical Stress Intensity Factor, K_c

At the terminal event of fracture, when the internal flaw has reached a critical size, the stress field in the vicinity of the crack tip is represented by a parameter, K_c, the critical stress-intensity factor. K_c is proportional to the critical strain energy release rate and Young's modulus. It is a material constant and is independent of the crack-to-beam depth ratio (c/d ratio) (7, 8, 9, 10). However, this parameter is affected by temperature and rate of loading.

K_c can be determined experimentally from fracture tests by using Winne and Wundt's equation (7):

$$K_c^2 = \sigma_n^2\, hf\,(cf/d) \tag{2}$$

where

σ_n = nominal stress at the root of the notch = $M/[b(d-c)^2]$;
M = bending moment = P1/4;
P = load at fracture;
b = width of the beam;
d = depth of the beam;
c_f = crack depth at failure and is equal to the initial notch in the beam assuming that the crack just starts to propagate at the instant of failure;
h = d - c; and
f(c/d) = a function given by Winne and Wundt (7 Fig. 16).

The Parameters, A, n, and K, Describing the Crack Growth-Stress-Intensity Factor Relation

To determine the rate of crack growth requires that a direct measurement of the actual length of the crack be made. This is rather difficult to do in asphaltic materials and requires expensive equipment such as X-ray units. Other techniques such as the successive breaking of electrical circuits or ink staining give only qualitative or partial information (11). An indirect method, however, can be used to obtain the crack length from measurements of the compliance of the beam. The compliance, L, is defined as the inverse of the slope of the load-deflection diagram (Fig. 2). This method for obtaining the crack length is a recognized practice in experimental fracture mechanics (11, 12). The method is based on the fact that a beam will be weakened by the introduction of a crack or by the increase in length of an existing crack. The presence of the crack will cause a stress concentration in its vicinity resulting in an increase in the compliance of the beam. This increase in compliance is uniquely related to the increase in the crack depth, and this relation remains true regardless of the cause of the increase in the crack length, provided that the material behaves elastically and the changes in the material properties are negligible. In fact the relationship between the increase in compliance and the crack length is given by the equation (12)

$$\delta L/\delta c = (2/E)\,[(K/P)^2] \qquad (3)$$

where L is compliance, c is crack depth, P is load, E is Young's modulus, and K is a stress-intensity factor. In order to apply this technique for obtaining the crack growth, beams with notches of predetermined lengths were fabricated and tested so that a master curve of compliance versus crack length could be plotted. The artificial notch had the same effect on the compliance as a real crack of the same length. The results of the normalized compliance L/L_o versus crack length is shown in Figure 3. L_o is the compliance of the unnotched beam. The accuracy of this graph was checked

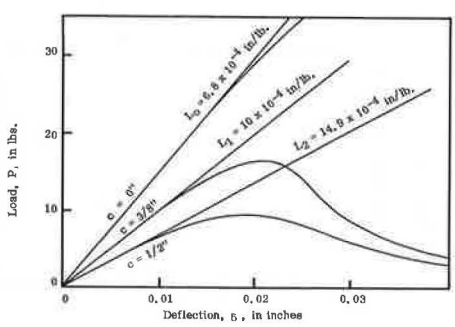

Figure 2. Load versus deflection and values of compliance.

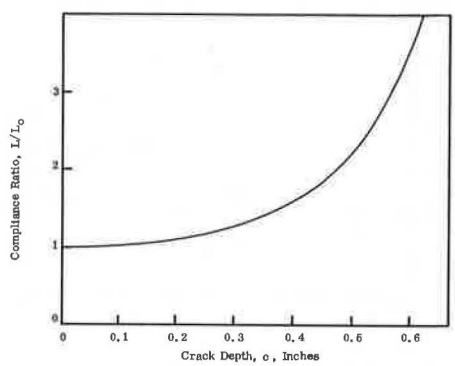

Figure 3. Normalized compliance versus crack depth from fracture tests.

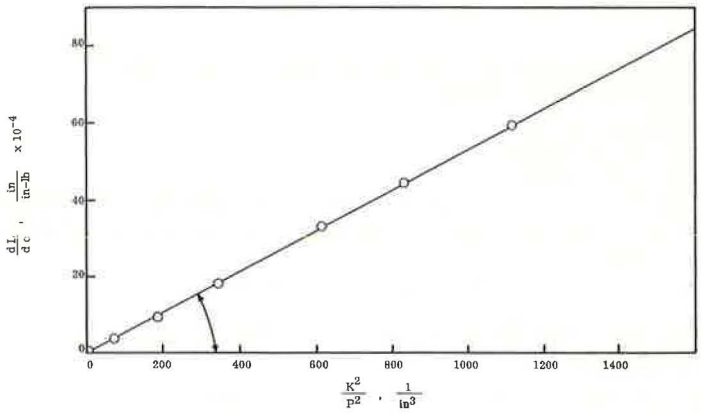

Figure 4. Relationship between dL/dc from fracture tests and theoretical K^2/P^2.

by differentiating it to obtain $\delta L/\delta c$ as a function of c. The values of $\delta L \delta c$ were then plotted versus the calculated values of K^2/P^2 from Winne and Wundt's formula (Fig. 4) and show a proportionality relation as indicated by the equation.

The fatigue tests were conducted on unnotched, simply supported beams loaded at midspan; the testing temperature was 23 F. During the test the load was maintained constant using the MTS in load control. The deflection increased as the test progressed, very slowly at first but with an exponential type of increase with the number of cycles. Figure 5 shows the normalized compliance versus the number of cycles. The only phenomenon that could account for the increase in the compliance was the increase in the crack depth at midspan.

Because the compliance and the crack depth are uniquely related, the crack depth at any stage of the fatigue process can be obtained as shown in Figure 5, where the crack depth after N cycles in the fatigue test corresponds to the crack depth at the point on the master curve where the compliance is the same as that on the fatigue curve. In this way it is possible to obtain the c-N curve for each fatigue test. Figure 6 shows a typical c-N curve.

By knowing the crack depth, c, at any cycle, N, one can calculate the stress-intensity factor, K, at any stage of the fatigue test. Similarly, K^2/P^2, which is expected to be proportional to the rate of

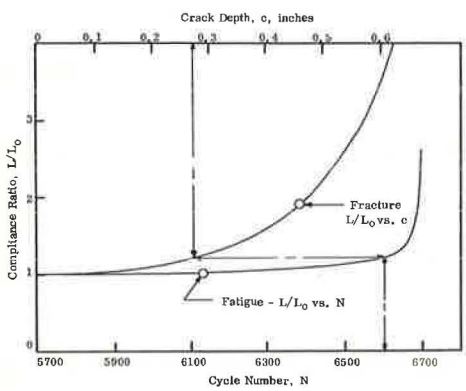

Figure 5. Normalized compliance versus number of cycles for fatigue test and normalized compliance versus crack depth for fracture test.

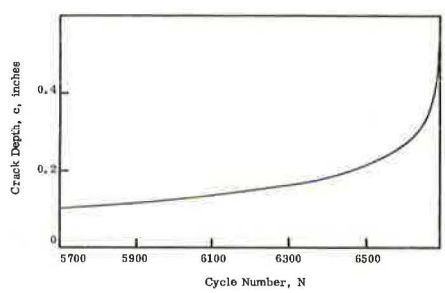

Figure 6. Crack depth versus number of cycles for fatigue test.

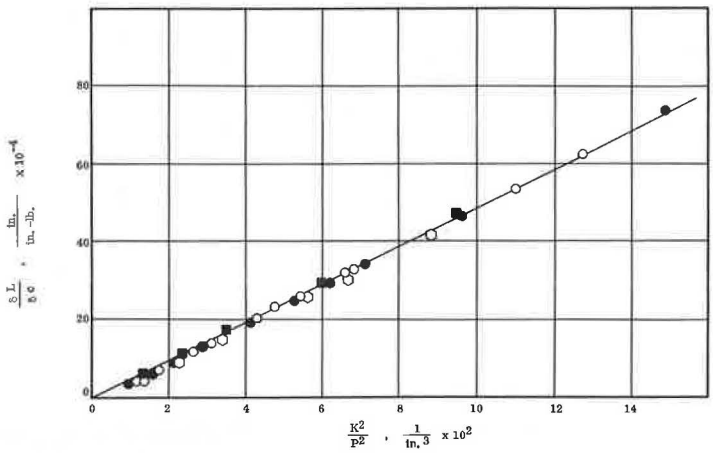

Figure 7. Relationship between $\delta L/\delta c$ from fracture tests and K^2/P^2 from fatigue tests.

change of compliance with crack depth, can be determined. The experimental values of $(\delta L/\delta c)$ versus calculated K^2/P^2 were plotted for a number of fatigue experiments, and a linear relationship between the two (Fig. 7) was indicated.

To determine the rate of crack growth-stress-intensity factor relation requires that the curve shown in Figure 6 be differentiated to obtain dc/dN versus c. Knowing c and the load P, one can calculate the stress-intensity, K. The (dc/dN) - K relationship is then plotted on a log-log scale in the search for a relationship of the form

$$dc/dN = AK^n \tag{4}$$

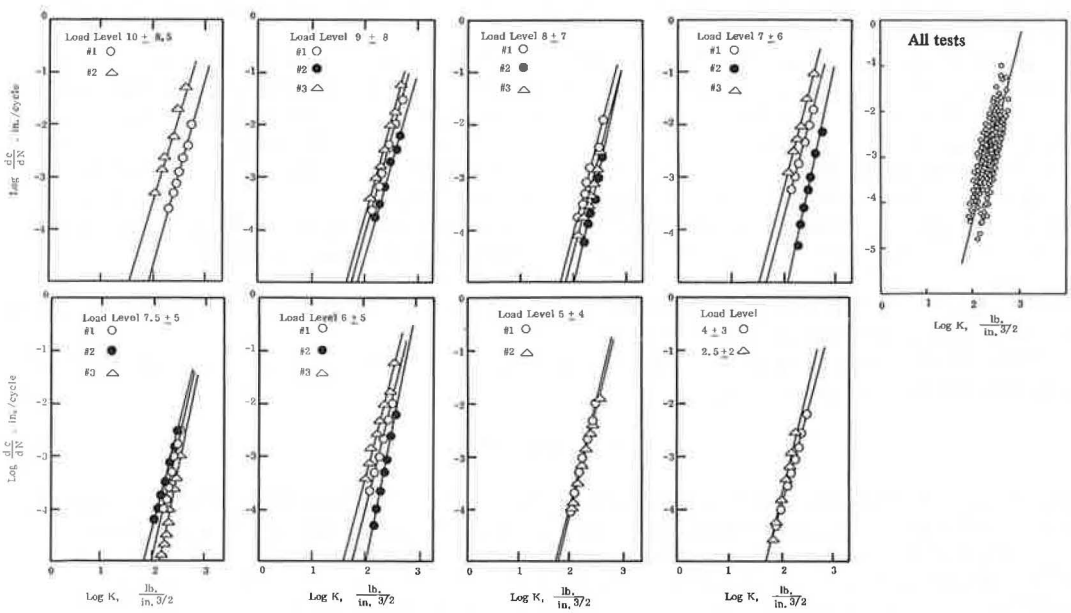

Figure 8. K versus dc/dN for different load levels and for all tests.

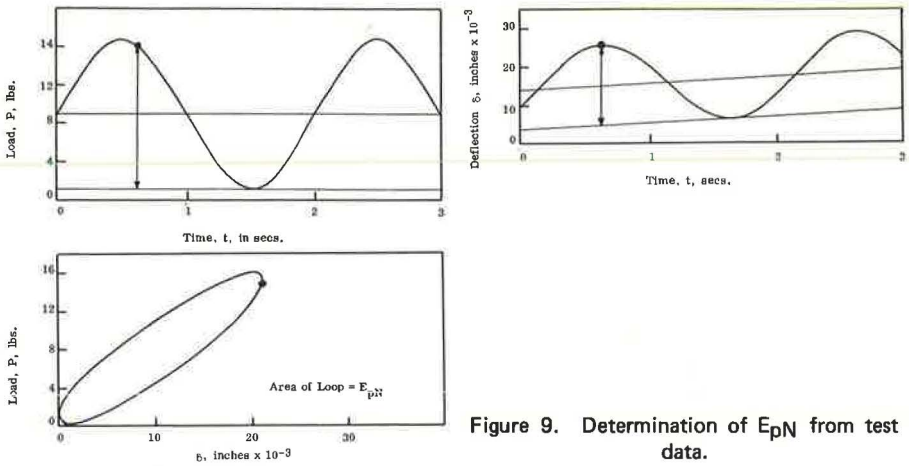

Figure 9. Determination of E_{pN} from test data.

The results for 9 stress levels and all levels combined are shown in Figure 8. From these graphs, the crack-growth relation is

$$dc/dN = 5 \times 10^{-3} K^4 \tag{5}$$

which is in accordance with Paris's law.

It is also possible to correlate the rate of crack growth with the hysteresis energy loops obtained from the dynamic load-deflection diagrams at various stages of the fatigue process. The hysteresis loop in the Nth cycle, E_{pN}, measures the total energy dissipated in the Nth cycle by viscous, plastic, and surface work. This energy may be conveniently divided into 2 parts: one part is concerned with the energy dissipated in viscous and plastic deformation when no crack is present and the other is concerned with the viscous and plastic deformation in the zone of influence of the crack and the energy consumed in forming any new surfaces of crack that may develop during the particular cycle of loading.

During the first cycle of loading in an unnotched beam, it may be assumed that no increment of crack length occurs (because the rate of crack propagation at this stage is negligible) so that the hysteresis loop is a measure of the energy loss in the usual viscous and plastic deformation process. This initial loop will be called E_{p1}. If during the subsequent loading the crack does not propagate, the area of the hysteresis loop will remain constant. In most cases, however, the area of the hysteresis loop will increase over the initial value because of the formation of new crack surfaces and the intensification of the local stress field in the zone of influence of the crack. This increase in the hysteresis energy loop will be called E_p, and it is equal to $E_{pN} - E_{p1}$, where E_{pN}, the hysteresis loop in the Nthecycle, can be determined as shown in Figure 9, and E_{p1} is the hysteresis loop at N = 1.

The relationship between the part of the hysteresis loop, E_p, and the rate of crack growth, dc/dN, is given as (6)

$$E_p = T(dc/dN) \tag{6}$$

where T is a constant. This relationship holds only if the size of the plastic zones at the crack tip is small in comparison to crack length and the specimen dimensions.

A plot of E_p versus dc/dN shows a linear relationship for all stress levels (Fig. 10). Thus the hysteresis loop is an indication of the rate at which the crack is propagating or the rate at which damage is progressing during fatigue.

In this study of the rate of crack growth, the indirect method of deducing the crack length from the compliance of the beam was used. Although it is considered to be a reliable technique in fracture mechanics, it is desirable that the rate of crack growth

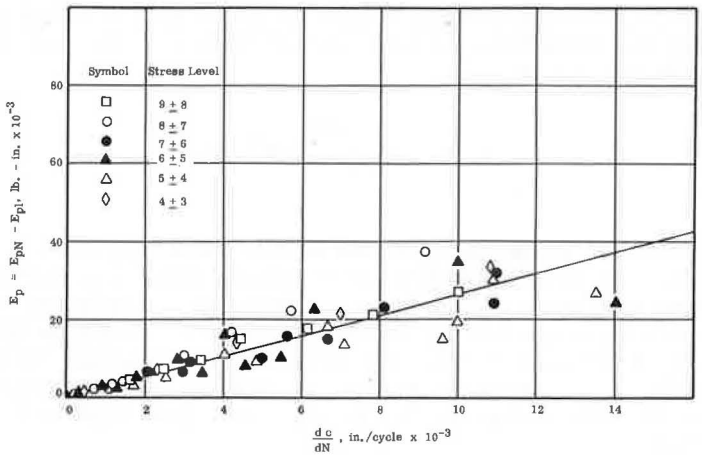

Figure 10. Relationship between hysteresis loops and dc/dN from fatigue tests.

be determined by direct experimental methods. Work is in progress at the present time to utilize direct observational techniques to measure crack depth during fatigue tests.

The Parameter, c_o, the Starter Flaw

From the crack-growth law, $dc/dN = AK^4$, the crack, c, at any stage is given by

$$c = c_f - \int_{N}^{N_f} AK^n \, dN \tag{7}$$

Because of the limits of experimental accuracy and the relative insensitivity of the L/L_o versus c curve from which the c-N curve is obtained, the crack-growth law could

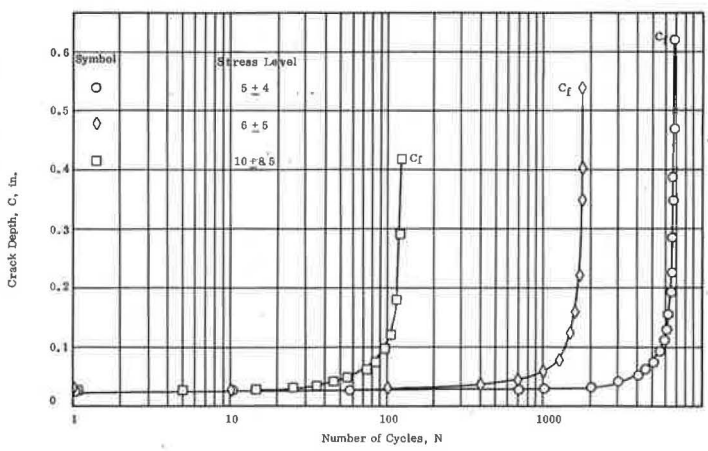

Figure 11. Crack depth versus number of cycles for 23 F from a computer solution of crack-growth law.

not be obtained experimentally for crack depths smaller than about 0.2 in. Assuming, however, that the law holds from the first loading cycle, then the "crack" depth at the first loading cycle is given by

$$c_o = c_f - \int_1^{N_f} AK^4 \, dN \tag{8}$$

where all the quantities on the right side are known. However, because K is a function of c, the solution of the integral is difficult; therefore, numerical methods utilizing computer programming were used. The crack depth, c_o, is referred to as the equivalent size of the most critical flaw or discontinuity that will propagate into a crack of engineering size. In this paper it is referred to as the starter flaw or the equivalent initial crack depth, c_o. Typical c-N curves extrapolated analytically to N = 1 are shown in Figure 11. The values obtained for c_o ranged from 0.007 to 0.13 in. with an average of 0.05 in.

PREDICTION OF FATIGUE LIFE

After the constants of the material, c_o, (A, n, K), and K_c, are determined, the fatigue life of a simply supported beam can be predicted for any stress level. In accordance with the foregoing concepts, the fatigue life is obviously the number of cycles for the starter flaw, c_o, to propagate into a crack of critical length, c_f. Thus the fatigue life, N_f, is given by

$$N_f = \int_{c_o}^{c_f} (1/AK^n) \, dc \tag{9}$$

where c_f can be obtained from

$$K_c^2 = \sigma_n^2 h \, f(c_f/d) \tag{10}$$

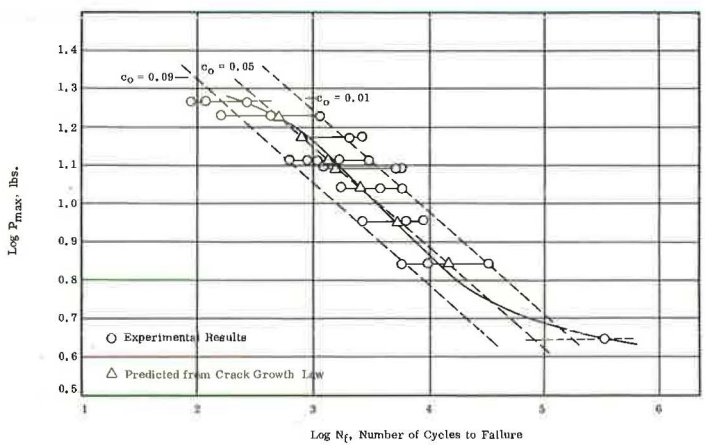

Figure 12. Relationship between P_{max} and N_f for 23 F.

Using the values

$c_o = 0.05$ in.,
$A = 5 \times 10^{-3}$,
$n = 4$, and
$K_c = 400$,

we calculated the fatigue lives from this equation by using numercial methods for evaluating the integral. Figure 12 shows the close agreement of the calculated values of the fatigue lives with those determined experimentally. Further research is being done to determine the variation of the constants of the materials at different temperatures and to study the applicability of the method in designing pavement systems.

SUMMARY AND CONCLUSIONS

In this paper the fatigue life of a bituminous paving mixture is expressed in terms of 3 parameter sets. These are K_c, the critical stress-intensity factor, which is the failure criterion; (A, n, K) that express the rate of crack growth; and c_o, the initial flaw size or starter flaw from which damage is initiated. The results presented indicate the following:

1. The rate of change of compliance (inverse slope of the load-deflection curve) with crack depth is in accordance with Irwin's equation, which is

$$dL/dc \propto K^2/P^2$$

where K is calculated from Winne and Wundt's equation.

2. The rate of crack growth follows Paris's law, $dc/dN = AK^n$, where A and n are material constants and $n = 4$.

3. The crack-growth law is used to analytically extrapolate the c-N curve to the point where $N = 1$ to obtain the starter flaw, c_o. This parameter is shown to be a constant of the material and is believed to be responsible for statistical variations in the fatigue life.

4. The fatigue life of bituminous paving mixtures may be predicted when all the constants of the material, such as c_o, (A, n, K), and K_c, and the variables load, geometry, and boundary conditions, which determine the stress-intensity factor, K, are known. The fatigue life may be expressed as

$$N_f = \int_{c_o}^{c_f} (1/AK^n)\,dc$$

where c_f is given by

$$K_c^2 = \sigma_n^2\, h\, f(c/d)$$

The comparison of experimentally observed fatigue lives and calculated values for simply supported beams is presented.

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REFERENCES

1. McEvily, A. J., Jr., Boettner, R. C., and Johnston, T. L. On the Formation and Growth of Fracture Cracks in Polymers. In Fatigue, an Interdisciplinary Approach, Proc., 10th Sagamore Army Mat. Res. Conf., Syracuse Univ. Press, N. Y., 1964.

2. McEvily, A.J., and Boettner, R.C. On Fatigue Crack Propagation in F.C.C. Metals. Acta Metallurgica, Vol. 11, 1963, p. 725.
3. Frost, N.E. The Growth of Fatigue Cracks. Proc., First Internat. Conf. on Fracture, Sendai, Japan, Vol. 3, 1965, p 1433.
4. Majidzadeh, K., Ramsamooj, D.V., and Fletcher, T.A. Analysis of Fatigue of Sand-Asphalt Mixtures. Paper presented at the annual meeting of Assn. of Asphalt Paving Technologists, 1969.
5. Paris, P., and Erdogan, F.J. A Critical Analysis of Crack Propagation Laws. Jour. Basic Engineering, Trans. ASME, Series D, Vol. 85, 1963, p. 528.
6. Paris, P.C. The Fracture Mechanic Approach to Fatigue. In Fatigue, an Interdisciplinary Approach, Proc., 10th Sagamore Army Mat. Res. Conf., Syracuse Univ. Press, N.Y., 1964.
7. Winne, D.H., and Wundt, B.M. Application of the Griffith-Irwin Theory of Crack Propagation to the Bursting Behavior of Discs, Including Analytical and Experimental Studies. Trans. ASME, Vol. 80, 1958, pp. 1643-1655.
8. Kauffmann, E.M. Analysis of Fracture Mechanics in Bituminous Materials. Ohio State Univ., Columbus, MS thesis, Dec. 1968.
9. Moavenzadeh, F. Asphalt Fracture. Proc., Assn. of Asphalt Paving Technologists, Vol. 36, 1967.
10. Bahgat, A.G., and Herrin, M. Brittle Fracture of Asphaltic Mixes. Paper presented at the annual meeting of the Assn. of Asphalt Paving Technologists, Feb. 1968.
11. Glucklich, J. Static and Fatigue Fractures of Portland Cement Mortar in Flexure. Proc., First Internat. Conf. on Fracture, Sendai, Japan, 1965.
12. Irwin, G.R. Fracture. In Encyclopedia of Physics, Vol. 6.