Statistical Approximation for Consolidation Settlement

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Statistical procedures are used to describe consolidation data from various types of compressible, fine-grained soils, and an approximate method is developed to estimate consolidation settlements. The suggested technique is presented in graphical form and is illustrated by several example problems.

•WHEN A LOAD is applied to a compressible foundation soil, the magnitude of the resulting settlement depends on both the compressibility of the foundation soil and the magnitude and distribution of the imposed load. The degree of accuracy required in determining such settlements is governed by the nature of the structure and its sensitivity to total and differential settlements. For example, a rigid frame structure will generally not tolerate large differential settlements between adjacent columns, whereas a large-diameter oil tank with a steel bottom may tolerate differential settlements of several inches between the center and the edge. In the case of a highway embankment crossing areas underlain by soft soils, differential settlements may cause dips in the finished road surface, and total settlements may necessitate fill volumes in excess of the original plans. Also, differential settlements in the cross section of a highway embankment may cause serious longitudinal stresses or unacceptable joint separations in a buried culvert, as well as a decrease in its drainage capacity. For this reason, it is usually desirable to install such structures with a camber to compensate for the expected settlements. This work is directed toward the development of an approximate technique whereby consolidation settlements may be estimated for situations that do not require or warrant a more detailed and accurate soils engineering analysis.

SOIL COMPRESSIBILITY AND CONSOLIDATION

The application of a load to a soil will, in general, cause instantaneous and time-dependent deformations to occur. These deformations may consist of two parts, one part resulting primarily from normal stresses and consisting of volume changes and the other part resulting from shear stresses under conditions of constant volume. Depending on the physical properties of the soil, the magnitudes and rates of these deformations may vary considerably. Although all types of deformation may be important in particular situations, the following analysis will be limited to the time-dependent deformations resulting from volume changes or consolidation of the soil.

For a layer of normally consolidated clay of thickness D, natural void ratio e_0, compression index C_c, and natural overburden pressure p_0, the settlement S under an applied load Δp may be expressed as

$$\frac{S}{D} = \frac{C_c}{1 + e_0} \log \frac{p_0 + \Delta p}{p_0} = FL \tag{1}$$

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where

$$F = \frac{C_c}{1 + e_0} \tag{2}$$

and

$$L = \log \frac{p_0 + \Delta p}{p_0} \tag{3}$$

The loading factor L in Eq. 3 includes the loading effects, and its exact values depend on the actual configuration of the applied load. The compressibility factor F expresses the compressibility of the soil.

METHOD OF ESTIMATING SETTLEMENT

Determination of Soil Compressibility

Because of the difficulties, expense, and time delay that normally accompany field sampling and laboratory consolidation tests, it is highly desirable to devise some approximate scheme for determining the soil parameters required to compute consolidation settlements for different types of soils. Rutledge (5) found some correlation between C_c and the natural water content w_n of various soils, and this was later confirmed by Peck and Reed (4) and Osterberg (2). Rutledge also found a correlation between C_c and the natural void ratio e_0 of the different soils tested. Approximately 300 data points

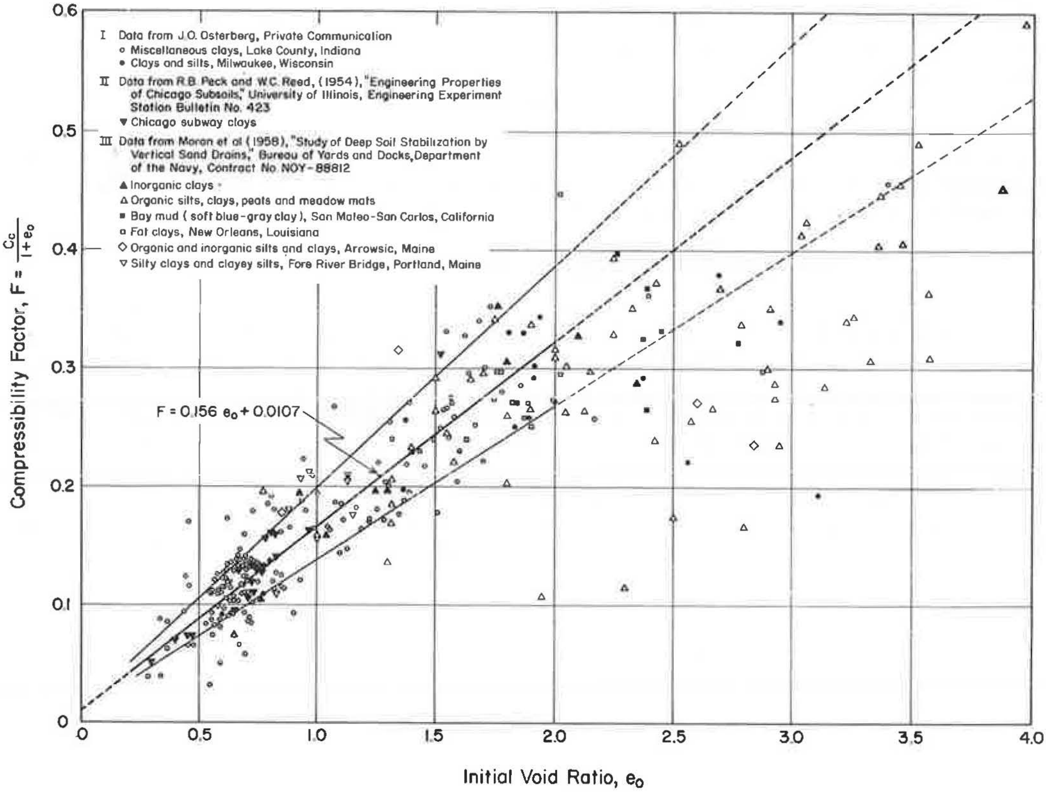

Figure 1. Compressibility factor versus initial void ratio.

representing inorganic and organic clays and silty soils were collected from the literature, and a plot of F versus the initial void ratio e_0 is shown in Figure 1. For values of e_0 less than approximately 2, there appears to be a reasonable correlation between F and e_0. For e_0 greater than 2, parameters other than e_0 obviously influence F to such an extent that any correlation between F and e_0 is indistinguishable. As a result, the following development is restricted to soils with an initial void ratio less than 2. Soils with e_0 greater than approximately 2 are highly compressible and are generally organic in nature, and a considerably greater problem exists.

Approximately 230 of the data points in Figure 1 have an initial void ratio e_0 less than 2, and the method of least squares was used to determine the "best fit" straight line describing these data. With F as the dependent variable and e_0 as the independent variable, the regression line is given by

$$F_{est} = 0.156\, e_0 + 0.0107 \tag{4}$$

To test the "goodness of fit" of Eq. 4, the correlation coefficient r was computed by the equation

$$r = \sqrt{\frac{\sum \left(F_{est} - \overline{F}\right)^2}{\sum (F - \overline{F})}} \tag{5}$$

where F_{est} represents the value of F as estimated from Eq. 4 for a given value of e_0, $\overline{F}$ is the mean value of F for N data points, and F is the observed value. The dimensionless correlation coefficient r measures the goodness of fit achieved by Eq. 4 and it equals unity for perfect correlation. Because the computed value of r is 0.93, a very high linear correlation is implied. The standard error of estimate of F on e_0 is given by

$$S_{F \cdot e_0} = \sqrt{\frac{\sum_{i=1}^{N} \left[F^i - F^i_{est}\right]^2}{N}} \tag{6}$$

For the data points considered, $S_{F \cdot e_0}$ was found to be 0.028, and the standard error of estimate for various subsets of N varied from 0.022 to 0.032. The standard deviation of F is equal to 0.0755, giving a variance of F of 0.0057, or $(0.0755)^2$.

To estimate the average percent error to be expected when using Eq. 4, the following approach is utilized. Consider a straight line passing through the point $F_{est} = 0$ and making an angle of $\pm\theta$ with the regression line. The percent error in using the regression line will be the same for all data points lying on this line. More generally, the data points lying in the angular sectors $\Delta\theta$ between two pairs of straight lines radiating from the point $F_{est} = 0$ at $\pm(\theta - \frac{1}{2}\Delta\theta)$ and $\pm(\theta + \frac{1}{2}\Delta\theta)$ to the regression line will have approximately the same percent deviation from the regression line. For any given angle θ, the number n of data points in the particular angular sector $\Delta\theta$ under consideration was counted and normalized with respect to the total number N of data points to give a parameter f equal to n/N. Values of f were plotted versus θ in Figure 2, and from these data a probability density function can be defined; that is, the curve that, for a large sample of size N, approximates the frequency curve. A normal distribution curve was found to describe these data very closely, as verified by a goodness-of-fit test. Approximately 69 percent of the data points lie between ±5 deg of the regression line, and 94 percent lie between ±10 deg. As seen in the inset to Figure 2, this means that there is a 69 percent chance that the computed value for F will be within 20 percent of the actual value, and a 94 percent chance that the computed value will be within 44 percent of the actual value.

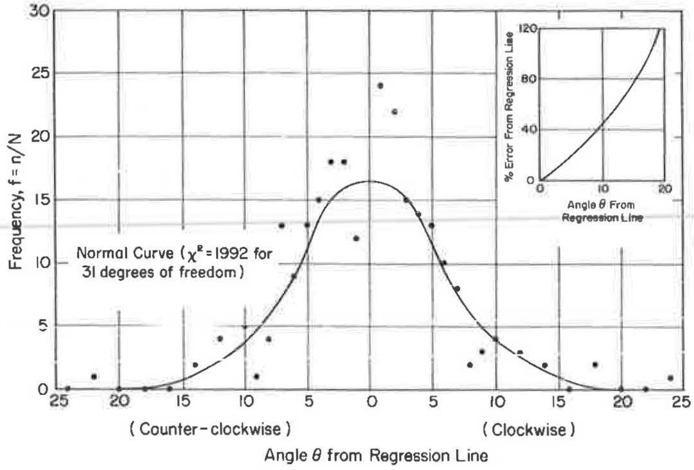

Figure 2. Statistical distribution of compressibility factors.

The preceding correlation between F and e_0 indicates that soils with the same in-situ void ratio would consolidate approximately the same magnitude. Such a correlation, however, should not be used indiscriminately; rather, it must be applied in conjunction with sound engineering judgment in order to benefit from its full potential. In particular, it should be emphasized that the foregoing concept is only valid statistically, and its results when applied to an individual situation may be misleading. For example, as can be seen in Figure 1, the compressibility factor may vary from the regression line by a factor of 2 or ½ for particular cases. In order to provide some suggested alternatives in applying the principles outlined, two additional lines at ±5 deg from the regression line are shown in Figure 1. These lines may be regarded as upper and lower median estimates, F_{um} and F_{lm} respectively. Under certain conditions, the use of one or the other of these lines will serve to place lower bounds on the expected error. Which of these lines best represents the characteristics of a given soil is left to engineering judgment. In the absence of any positive reason for using the F_{um} or the F_{lm} line, use of the F_m line is recommended. Extreme caution is advised against the over-enthusiastic or indiscriminate use of this approach for determining expected settlements. The procedure is a very approximate one and intended for use only in situations where it is not necessary to determine precisely the magnitude of anticipated consolidation settlements. For structures that are sensitive to slight settlements, a different approach should be used, and consolidation tests should be performed on good undisturbed samples to evaluate the consolidation characteristics of the soils underlying the specific structure.

The procedure suggested herein involves the determination of the natural void ratio of the soil. One convenient way of obtaining e_0 is by use of the expression

$$e_0 = G_s \frac{\gamma_w}{\gamma_d} - 1 \qquad (7)$$

where γ_d is the dry density, γ_w is the density of water, and G_s is the specific gravity of the soil particles. Figure 3 shows curves for γ_d versus e_0 for different values of G_s as well as the same series of lines shown in Figure 1 and discussed previously. The use of the chart in Figure 3 is demonstrated by the following example.

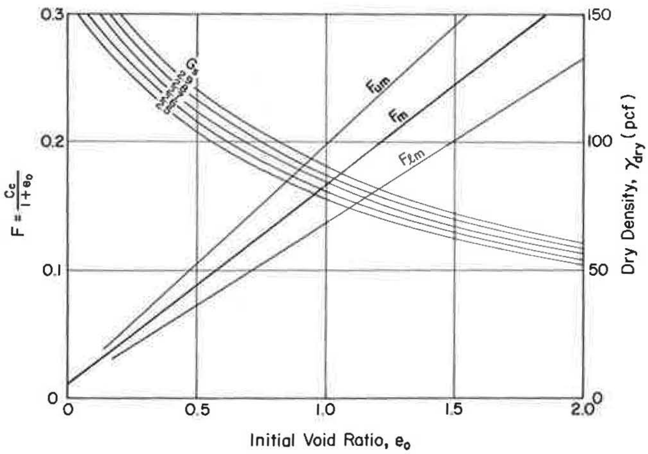

Figure 3. Nomograph for compressibility factor.

Example 1

Given: γ_d = 90 pcf; G_s = 2.63

Determine:

(a) Under normal conditions, what is the mean value of the compressibility factor?
(b) If visual inspection of the soil samples indicates some organic matter, which is evident by the dark color and the odor of the soil, what is the probable value for the compressibility factor?
(c) What is the percentage difference in F if the mean value, case (a), is used for the relatively more compressible soil, case (b)?

Solution: Referring to Figure 3, and using the given values for γ_d and G_s, we obtain $e_o = 0.82$.

(a) for $e_o = 0.82$, $F_m = 0.138$.
(b) The soil in this case will probably be more compressible than average because of its content of organic matter; therefore, for $e = 0.82$, $F_{um} = 0.165$.
(c) The percent difference in F is

$$\frac{0.165 - 0.138}{0.165} \times 100 = \frac{0.027}{0.165} \times 100$$

$$= 16.4 \text{ percent}$$

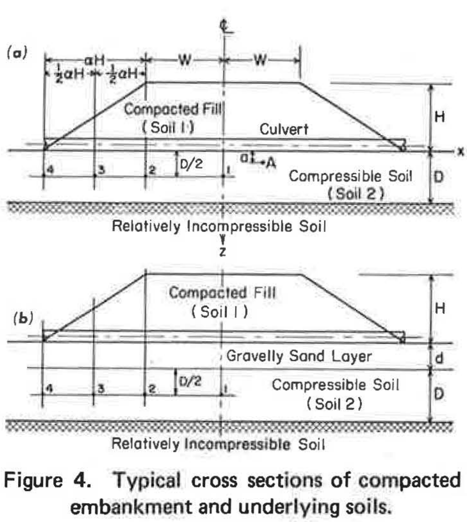

Figure 4. Typical cross sections of compacted embankment and underlying soils.

Determination of Settlement

In order to develop this technique in a reasonably general manner, a trapezoidal load distribution (such as encountered in a highway embankment) will be assumed. Uniformly distributed loads may then be treated as special cases. Figure 4a shows the cross section of a compacted fill embankment (soil 1)

of top width 2W, height H, and side slopes α to 1, underlain by a compressible normally consolidated soil layer (soil 2) of depth D resting on a relatively incompressible foundation soil. This general cross section is, for example, typical of situations in which culvert camber should be determined. If we assume that the groundwater surface is approximately the same as the surface of the compressible layer, the submerged unit weight γ_2' of the soil in the latter layer is $\gamma_2 - \gamma_W$, where γ_2 is the total unit weight of the compressible soil and γ_W is the unit weight of water. In addition, the total unit weight of the compacted fill may be designated as γ_1.

An approximation of the stress distribution in the compressible soil under and adjacent to the embankment may be obtained from Figure 5, which was originally presented by Osterberg (3). With a knowledge of this stress distribution and the compressibility characteristics of the compressible soil, the settlement at any point A may be calculated from the relationship

$$S_A = \frac{C_c}{1 + e_0} \int_a^D \log \frac{p_0(z) + \Delta p(z)}{p_0(z)} \, dz \tag{8}$$

where $p_0(z)$ is the overburden stress distribution under point A and $\Delta p(z)$ is the stress distribution under point A resulting from the embankment load. Although the application of Eq. 8 is rather straightforward and is susceptible to the development of a family of curves, this degree of refinement is probably not justified for the type of problems considered here. Alternatively, if we make the simplifying assumption that the vertical stresses existing at the midpoints of the compressible layer represent the average values of the vertical stresses in a vertical strip, the settlement at the top of the compressible layer may be written as

$$\frac{S}{D} = \frac{C_c}{1 + e_0} \log \frac{p_0(D/2) + \Delta p(D/2)}{p_0(D/2)} \tag{9}$$

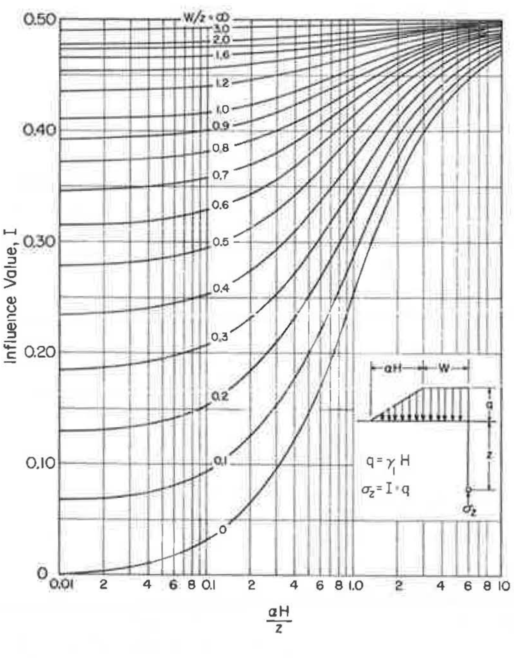

Figure 5. Influence chart for vertical stresses under a trapezoidally distributed strip load.

where the overburden effective stress $p_0(D/2)$ is given by $0.5D\gamma_2$.

To obtain values for the induced vertical stresses $\Delta p(D/2)$ and the associated settlements S, we consider four points 1, 2, 3, and 4, as shown in Figure 4. For point 1, which is under the center of the embankment, the weight of the embankment may be assumed as fully acting; hence, Δp at this point is $\gamma_1 H$. If the further approximations are made that γ_2' equals 60 pcf and γ_1 equals 120 pcf, we have from Eq. 3

$$L = \log \left\{ 1 + 4 \frac{H}{D} \right\} \tag{10}$$

A plot of L versus H/D is shown in Figure 6. For convenience, the scale of H/D was enlarged, as shown by the curve labeled $(H/D) \times 10^{-1}$, and reduced, as shown by the curve labeled $(H/D) \times 10$. To determine settlement at point 1 in Figure 4, the solution of Eq. 1 is shown in Figure 6, where the settlement ratio S/D

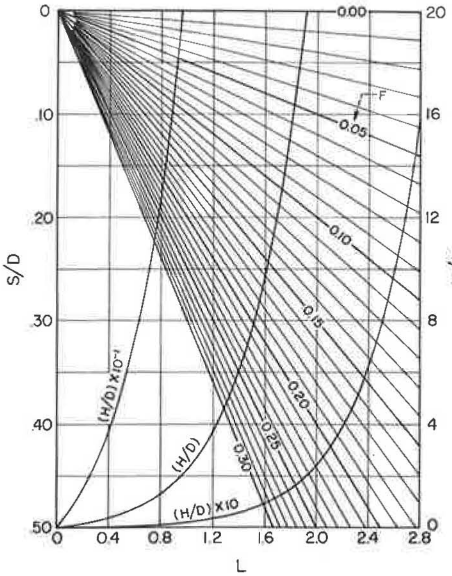

Figure 6. Nomograph for determination of consolidation settlements.

is plotted as a function of L and F for a given H/D ratio. Equation 10 applies to situations where the embankment width W is large relative to the depth of the compressible stratum. In order to use the equation for general cases of imposed loads, divide the load intensity by 120 pcf to obtain an equivalent height H. The use of this chart will be illustrated later. The settlements at points 2, 3, and 4 in Figure 4 may be determined from Figure 6 by multiplying H/D by a factor β. For points 2 and 3, β is on the order of 0.97 and 0.50 respectively. Values for β at point 4 are shown in Figure 7 as a function of the embankment height H and the side slope α.

For a case where the compressible layer of soil is covered by a relatively incompressible soil layer of thickness d, as shown in Figure 4b, the H/D values for any point under the embankment should be further adjusted by multiplying by λ, where

$$\lambda = \frac{D}{D + 2d} \tag{11}$$

For conditions other than those discussed here, Figure 6 may still be used for settlement determinations provided that H/D is multiplied by the appropriate factor. Suggested approximations are not given in such cases, and engineering judgment will play a large role in this choice. The following examples illustrate the proposed method.

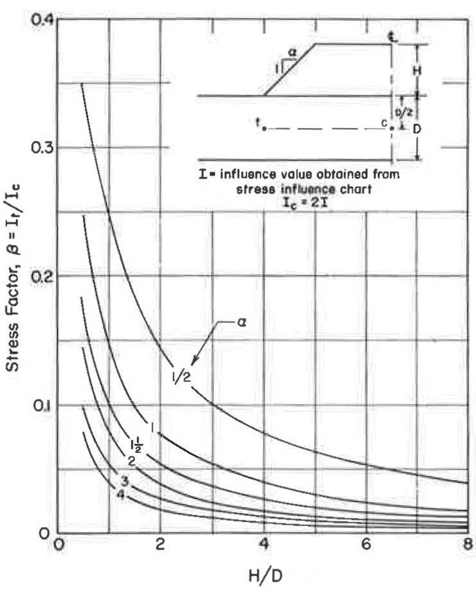

Figure 7. Dependence of stress factor on side slope and height of compacted embankment.

Example 2

Given: The embankment shown in Figure 4 has a height H of 35 ft and a width 2W of 40 ft, with side slopes of 2:1. The foundation soils below the original ground surface are silty clay to a depth of 15 ft underlain by a thick stratum of gravelly sand.

Determine: What is the settlement profile of the original ground surface for the soils described in cases (a) and (b) of Example 1?

Solution: For point 1 in Figure 4, we have

$$\frac{H}{D} = \frac{35}{15} = 2.33$$

which for case (a) yields F = 0.138. Use of Figure 6 gives

$$\frac{S}{D} = 0.140$$

or

$$S = 0.14 \times 15 \times 12$$
$$= 25.2 \text{ in.}$$

For point 2

$$\frac{H}{D} = 0.97 \times 2.33 = 2.26$$

which gives

$$\frac{S}{D} = 0.137$$

or

$$S = 0.137 \times 15 \times 12 = 24.7 \text{ in.}$$

In similar manner, for point 3

$$\frac{H}{D} = 0.5 \times 2.33 = 1.17$$

which gives

$$\frac{S}{D} = 0.104$$

or

$$S = 0.104 \times 15 \times 12 = 18.7 \text{ in.}$$

Finally, at the toe, or at point 4, for $\alpha = 2$ and $H/D = 2.33$, we have from Figure 7 that $\beta = 0.032$; hence, we get

$$\frac{H}{D} = 0.032 \times 2.33 = 0.075$$

which corresponds to

$$\frac{S}{D} = 0.015$$

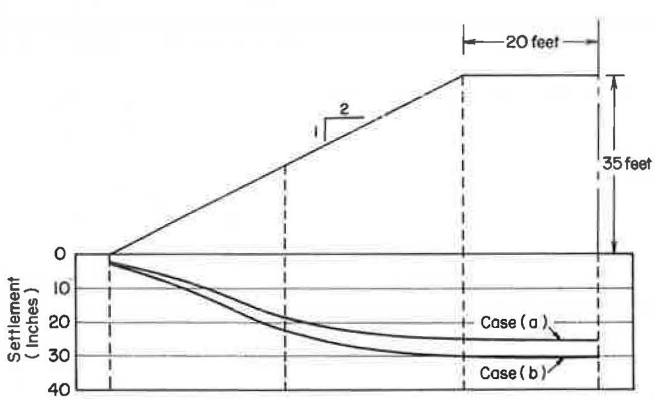

Figure 8. Example of consolidation settlements under a culvert.

or

$$S = 0.015 \times 15 \times 12 = 2.7 \text{ in.}$$

For case (b), F = 0.165, and the settlements at points 1,2, 3, and 4 are, at point 1, S = 0.168 × 15 × 12 = 30.2 in.; at point 2, S = 0.166 × 15 × 12 = 29.9 in.; at point 3, S = 0.123 × 15 × 12 = 22.1 in.; and at point 4, S = 0.018 × 15 × 12 = 3.2 in. The predicted settlement profiles for these cases are shown in Figure 8.

Example 3

What are the predicted settlements at points 1 and 4 for case (a) of Example 2 if the silty clay layer were covered by 5 ft of dense sand? From Eq. 11 we calculate

$$\lambda = \frac{15}{15 + 2 \times 5} = \frac{15}{25} = 0.6$$

At point 1, we have

$$\frac{H}{D} = 2.33 \times 0.6 = 1.40$$

which for F = 0.138 yields

$$\frac{S}{D} = 0.113$$

or

$$S = 0.113 \times 15 \times 12 = 20.3 \text{ in.}$$

At point 4, we have

$$\frac{H}{D} = (0.032 \times 2.33) \times 0.6 = 0.045$$

which gives

$$\frac{S}{D} = 0.010$$

or

$$S = 0.10 \times 15 \times 12 = 1.8 \text{ in.}$$

OVERCONSOLIDATED SOILS

If soils are highly overconsolidated, it is unlikely that settlement problems will arise for the type of situations under consideration. When the weight of embankment and other loads, plus the existing overburden stress, is less than the preconsolidation stress of the soil at a given point, no appreciable settlement may be expected. On the other hand, if the stresses resulting from the proposed loads are higher than the preconsolidation stress, the soil will consolidate essentially under the stress in excess of the preconsolidation stress. Therefore, an evaluation of the preconsolidation stress is essential for reasonably accurate settlement predictions, and laboratory consolidation tests

may be required. Although the method presented herein is not directly applicable to overconsolidated soils, it may be used with modification. If the stress in excess of the preconsolidation stress is denoted by p_c, an equivalent height of compacted fill H_e may be determined from the relationship

$$H_e = \frac{p_c}{\gamma_1} \tag{12}$$

If H_e is equal to or greater than H, the height of the embankment, settlement may be considered negligible. On the other hand, if H_e is equal to nH, where n is less than unity, the method presented herein may be used with engineering judgment to predict settlements if H/D is reduced by a factor of (1 - n). Such a procedure is particularly advantageous if p_c is known without the necessity of conducting laboratory tests.

SUMMARY AND CONCLUSIONS

Statistical procedures are used in conjunction with reported consolidation test data on various types of compressible, fine-grained soils to develop a simple graphical procedure whereby approximate consolidation settlements might be quickly and inexpensively estimated. The technique developed is illustrated for several typical example problems. It must be emphasized that this method yields only approximate settlements in the statistical sense, and it must be used in conjunction with good engineering judgment. For cases where a more accurate settlement determination is required, a comprehensive soils investigation is necessary.

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