An Application of Game Theory to Route Selection

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The problem of alternate route selection is analyzed as a 2-person game against nature. By using data on galvanic skin response, a payoff function is formulated in terms of driver tension. Driver tension increases with traffic volume on a freeway but is practically independent of traffic volume on an arterial street. A solution to the problem is given under 4 different criteria for games against nature. If a freeway route and an alternate route on an arterial street are equal in length, the Hurwicz α index indicates that a driver should not select the freeway unless he is at least 62 percent sure of finding good traffic conditions. Application of the minimax-regret principle yields a diversion curve that indicates that all drivers should select the freeway for a distance ratio less than 0.48, 38 percent should select the freeway for a distance ratio of 1.00, and no drivers should take the freeway if the distance ratio exceeds 2.94.

If the driver can estimate a probability distribution for the states of freeway traffic, his optimum strategy can be found by linear programming. A criterion is suggested for evaluating the effectiveness of a driver-information system in terms of the regret associated with this optimum strategy and the minimax regret pertaining to the case of complete uncertainty.

•GAME THEORY PERTAINS to problems of decision-making under risk or uncertainty. Such risk or uncertainty results from a partial lack of control over the results of a given decision because of conflict of interests among a group of other individuals who also have some partial control over the outcome of their collective actions.

The problem of decision-making when the outcomes of one's decisions can be predicted with certainty is a problem of maximizing or minimizing the value of some objective criterion that characterizes the goals sought by the decision-maker. Under the condition of certainty, one is free to manipulate all the parameters of the problem.

Risk is introduced when one must take into account the (possibly) conflicting interests of other groups or individuals. In the risk situation one is aware of what these conflicting interests or objectives may be and must therefore attempt to protect one against the actions of opponents or to maximize one's return in the face of opposition.

Uncertainty is distinguished from risk by the lack of knowledge of the intentions or objectives of one's opponents. Decision-making under uncertainty is often characterized as a 2-person game in which the decision-maker is pitted against a fictitious player called "nature," a player with no known objectives and no discernible strategy.

Game theory is not a descriptive theory in the sense that it attempts to predict the way people behave under given circumstances. It is a conditionally normative theory because it describes the way people should behave if they wish to obtain some stated outcome.

It is a static theory in that the objectives and value systems of the players are considered invariant in time. Dynamic aspects of behavior can be introduced in a game theoretic model by permitting successive plays or iterations with the appropriate changes in the payoff functions of the players introduced at the end of each play.

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APPLICATION OF GAME THEORY TO TRAFFIC OPERATIONS

The principal areas in which game theory has been applied have been the social sciences, military planning, and business. It would seem natural, however, to characterize a traffic stream as a collection of individuals, each of whom possesses objectives and value systems somewhat different from those of others. It seems also that the driving task can be regarded as a process of decision-making under risk or uncertainty. That is, the driver can be thought of as a player in an n-person game who is called on to make frequent decisions that lead to more or less uncertain outcomes if his interests are opposed by other players.

Intersections, freeway ramps, and passing lanes are locations where drivers face a variety of conflict situations in which each driver has only a partial control over the outcome of a particular set of circumstances. It is felt that applications of existing mathematical theory to conflict situations such as unsignalized intersections will yield valuable insights into questions of intersection capacity and accident potential. This paper treats one driver decision problem only: the selection of a route from a pair of alternates in an urban area. The results of formulating the route selection problem in terms of game theory are interesting in themselves, but an important objective of the following presentation is to furnish an illustration of an analytical tool that has not been exploited by highway engineers.

Fundamental principles of the mathematical theory of games are presented in a number of textbooks such as those by Owen (8) and McKinsey (12). No general mathematical treatment will be attempted here, but it will be necessary to introduce a few basic concepts. The notion of utility underlies the entire theory. Each outcome of a game can be ranked by a player in his order of preference from the most to the least desirable. Moreover, there can be constructed a linear utility function that maps each outcome into the real numbers such that a more desirable event is assigned a higher number than a less desirable one, and the difference between utility numbers measures how much more a player prefers one outcome to the other.

It is possible, then, to define a payoff function that assigns an n-vector to each outcome of a game. Each element of the vector is the utility number attributed to the given outcome by one of the n-players.

Finally, the concept of a strategy is needed to develop the so-called normal form of a game. A strategy is a complete set of rules for deciding what action to take against all possible moves of an opponent. It is possible to conceive of a strategy, but generally not practical to describe it in detail.

If it were possible to produce a complete description of a strategy, the players could hand their strategies to a referee who, after working out any chance moves, could use the strategies selected by each player to find the final outcome and compute the payoff to each player. Under this condition, the game becomes in the view of an individual player a process of deciding which strategy out of the set available to him will result in the maximum expected payoff to him. It is necessary to consider the mathematical expectation of the payoff function because the outcomes of the chance moves can be described only as a probability distribution.

Now let $\pi(\sigma_1, \sigma_2, \ldots, \sigma_n) = \pi_1(\sigma_1, \ldots, \sigma_n), \pi_2(\sigma_1, \ldots, \sigma_n), \ldots, \pi_n(\sigma_1, \ldots, \sigma_n)$ represent the mathematical expectation of the payoff function, given that player i selects strategy σ_i out of the set of strategies he could choose. It is therefore possible to tabulate all values of $\pi(\sigma_1, \ldots, \sigma_n)$ as an n-dimensional array of n-vectors. Such an array is called the normal form of the game.

For a 2-person game, the normal form is a matrix, the elements of which are 2-dimensional vectors, or pairs of real numbers. In a so-called "zero-sum" game, one person wins exactly what the other loses. It is customary in this case to write the payoff matrix from the standpoint of one player only. The elements of the matrix are therefore numbers rather than vectors. The route-selection game to be discussed will be formulated as a 2-person game in which the driver plays against a fictitious opponent called nature. The driver's payoff matrix will be used exclusively since nature's payoff is the negative of the driver's in every case.

A ROUTE-SELECTION GAME

Michaels (1) has shown that drivers evaluate their experiences on different routes and that they will select an alternate route in order to minimize driver stress. His data indicate further that the objective of minimizing stress is a more significant factor than either direct cost or time considerations in selecting an alternate route.

Michaels' results form the basis of an attempt to formulate in game theoretic terms a model of traffic distribution between 2 alternate routes in an urban area. One such alternate will be an arterial street and the other an expressway with arterial linkages at each end of the trip.

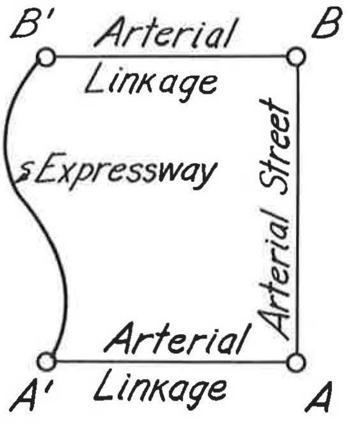

Figure 1. Alternate routes.

The driver's objective is to minimize total stress or tension in a 2-person game against nature. The possible states of nature are represented by the levels of service prevailing on the freeway, each of which generates a different level of driver tension. Using the galvanic skin response (GSR) as a measure of tension (2, 3), Michaels has shown that driver tension on an urban freeway is a function of traffic volume, while arterial street traffic volumes do not greatly affect tension responses. This property will be used, together with Michaels' data on GSR, in constructing a payoff function for a route-selection game.

Consider an urban expressway with arterial linkages, together with an alternate route entirely on the arterial street system, as shown in Figure 1. Let the distance AB by the expressway route be d. For simplicity in presentation, it will be assumed that the distance of this route along the arterial linkages is 0.10d and along the expressway proper 0.90d. The distance ratio for the alternate routes will be equal to d if the arterial route distance is taken as unity.

Let V_a be the average speed on the arterial route and expressway linkages, and let V_{ei} be the average speed on the expressway that prevails under level of service i.

Consider that the driver is faced with a selection of a route home from work during the afternoon peak period.

The duration of the arterial trip on route AB is

$$t_a = \frac{1}{V_a} \tag{1}$$

The corresponding trip duration on the expressway route AB is

$$t_e = t_l + t'_{ei} \tag{2}$$

where

t_l = time on linkages = $0.10d/V_a$, and
t'_{ei} = time on expressway = $0.90d/V_{ei}$.

For a given distance ratio d, the time ratio T can be shown to be

$$T = d/10 \left[1 + \frac{9}{(V_{ei}/V_a)}\right] \tag{3}$$

Let driver stress per unit time on arterial route AB and on linkages AA′ and BB′ be S_a. Consider S_a a constant (independent of traffic volumes). Similarly, let driver stress on expressway A′B′ be S_{ei}, a function of the level of service as characterized by traffic volume.

Total driver stress on any trip is treated by Michaels as a linear function of time. Let A be the total driver tension on the arterial route, and let the corresponding value on the expressway route be E_i. Then

$$\frac{E_i}{A} = d/10 \left[\frac{9(S_{ei}/S_a)}{(V_{ei}/V_a)} + 1\right] \tag{4}$$

The Highway Capacity Manual (4) defines 6 levels of service on freeways, designated A through F. For the afternoon peak period it is considered that levels A and B can be assigned probability zero. This leaves a total of 4 possible states of nature corresponding to levels of service C through F. The payoff function is therefore a 2 by 4 matrix as follows:

Level of service	C	D	E	F
State of nature, i =	1	2	3	4
Driver strategy on expressway, α_1	E_1/A	E_2/A	E_3/A	E_4/A
Driver strategy on arterial, α_2	1	1	1	1

The values E_i are such that $E_1 < E_2 < E_3 < E_4$. Therefore, if $E_4 < A$, the driver will be assured of less total tension on the expressway, no matter what level of service may prevail. In this case, the strategy of taking the expressway, α_1, is said to be a strictly dominant strategy. This situation will exist provided that

$$d < \frac{10(V_{e4}/V_a)}{9(S_{e4}/S_a) + (V_{e4}/V_a)} \tag{5}$$

On the other hand, if $E_1 > A$, the strategy α_2 strictly dominates and the driver should always take the arterial route. In this case

$$d > \frac{10(V_{e1}/V_a)}{9(S_{e1}/S_a) + (V_{e1}/V_a)} \tag{6}$$

For values of distance ratio d lying between those given by Eqs. 5 and 6, there will exist optimal mixed strategies. That is, the driver can minimize his total tension by taking the expressway sometimes and selecting the arterial route at other times. In mathematical terms, the driver should select α_1 with probability p and choose α_2 with probability (1 - p). Determination of a value for p constitutes a solution to the problem in terms of game theory.

If it is assumed that the driver has some knowledge of the probability of each state of nature, the set of these probabilities can be regarded as nature's strategy. Determination of the driver's optimum strategy in this case can be reduced to a problem in linear programming. But if the driver is entirely without knowledge of nature's probable behavior, he is faced with a decision to be made under a condition of uncertainty.

There are a number of possible ways to proceed under uncertainty. One way is to assume that nature is "out to get you" and to select a strategy that will make the worst damage nature could do as small as possible. This is the maximim-utility criterion, which in the present case leads to the conclusion that the driver should never select the expressway.

Only a little less conservative is the Laplace principle of insufficient reason that assumes that, if nothing is known about the probabilities of the various states of nature, they should be considered equally likely. This latter assumption makes possible a linear programming solution that again indicates that the driver should always select the arterial route.

There are 2 other criteria that yield more interesting results. One is the Hurwicz α or pessimism-optimism index (10). The α index is intended to provide a basis for a decision that is not so ultraconservative as the maximin-utility method. To apply the

Hurwicz method, one selects a number α ($o \le \alpha \le 1$) which expresses one's degree of pessimism or optimism in the following way:

Find for each act (row in the payoff matrix) the smallest and the largest values of the payoff. Then associate with each act an index.

$$\alpha m_i + (1 - \alpha)M_i \qquad (7)$$

where m_i is the smallest number in row i and M_i is the largest. Then select as one's decision the act with the largest α index. Note that, for $\alpha = 1$, this procedure reduces to the maximin-utility criterion illustrated earlier. If $\alpha = o$, the decision-maker is completely optimistic. He will seek the largest payoff possible.

The other approach to games against nature is the so-called minimax-regret criterion. The regret is defined as the difference between the maximum payoff possible under a given strategy selected by one's opponent and the payoff associated with some strategy of one's own.

The regret is computed by determining the number that must be added to an entry in a payoff matrix to make it equal the maximum entry in its column. A player's objective is to minimize his regret, or the difference between his maximum possible reward and the payoff he actually receives. The proper mixed strategy to attain this objective results from transforming the problem from one in game theory to linear programming.

NUMERICAL STATEMENT OF THE PAYOFF MATRIX

The Hurwicz and minimax-regret solutions to the route-selection game will be illustrated, and numerical data developed by Michaels and others will be used to compute the payoff matrix. An average speed on an arterial street during a peak traffic period will be taken as V_a = 20 mph (5). Other traffic parameters have been taken from the Highway Capacity Manual (4) as given in Table 1.

Michaels reports an average magnitude per minute for GSR on arterial streets as 18.7 (2). His data are deviations from a basal or "rest" value of GSR for a series of test drivers and are referred to a GSR resulting from an arbitrary shock stimulus. The scale units, therefore, have no physical meaning in themselves (2, 3). Figure 1 in Michaels' paper (2) shows the variation of GSR magnitude per minute with traffic volumes on urban freeways. Readings for S_{e1} and S_{e2} can be read directly from this figure, while S_{e3} can be extrapolated. Michaels shows that GSR on a freeway increases linearly with traffic volume up to a volume of about 2,800 vehicles per hour in one direction (2). For larger traffic volume, GSR seems to increase exponentially (2).

It is not practicable to associate a traffic volume with level of service F. In order to complete the payoff matrix it is necessary to hypothesize that high driver tension is induced by delays and by possible physical discomfort or fatigue caused by stop-and-go operation and by long vehicle queues. The value of S_{e4} as given in Table 2 has no empirical support but is regarded as a reasonable estimate of driver tension under unstable flow conditions, in view of the results reported by Michaels. The values of (S_{ei}/S_a) have been rounded to simplify calculations.

TABLE 1
FREEWAY TRAFFIC DATA

Level of Service	State of Nature (i =)	Service Volume (vehicles/hr in one direction)	V_{ei} = Minimum Speed (mph)	V_{ei}/V_a
C	1	2,700	50	5/2
D	2	3,200	40	2
E	3	3,600	30	3/2
F	4	—[a]	15[b]	3/4

[a]Indefinite. [b]Hypothesized.

TABLE 2
DRIVER TENSION

Level of Service	State of Nature (i =)	S_{ei}	S_{ei}/S_a
C	1	12	2/3
D	2	17	9/10
E	3	22[a]	7/6
F	4	30[b]	5/3

[a]Extrapolated from Michaels. [b]Hypothesized.

It is possible to determine time and distance ratios that form the upper and lower bounds of the range of d for which mixed strategies are optimal. These limiting values are as follows:

$$0.476 \le d \le 2.94$$
$$0.62 \le T \le 1.35 \qquad (8)$$

For a distance ratio equal to one, the payoff matrix is as follows:

$$P_1 = \begin{bmatrix} 0.34 & 0.50 & 0.80 & 2.10 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad (9)$$

SOLUTIONS FOR CASE OF COMPLETE UNCERTAINTY

Consider the Hurwicz α criterion. First find a value of the Hurwicz α that makes the driver's 2 possible strategies indifferent.

$$\alpha(E_1/A) + (1 - \alpha)E_4/A = \alpha + (1 - \alpha) \qquad (10)$$

$$\alpha = \frac{A - E_4}{E_1 - E_4} \qquad (11)$$

The arterial route is the more attractive alternative if the driver's degree of optimism is such that his α index is less than the value given by Eq. 11. The Hurwicz α index can be interpreted as the probability of good freeway traffic conditions. For a distance ratio of 0.50, Figure 2 shows that a driver would be well advised to take the freeway even if he considers that he has only a 6 percent probability of finding good freeway conditions. If the freeway route and the arterial alternate are equal in length, the driver needs a 62 percent probability of finding good traffic conditions in order to make the freeway route attractive. The required probability exceeds 90 percent when

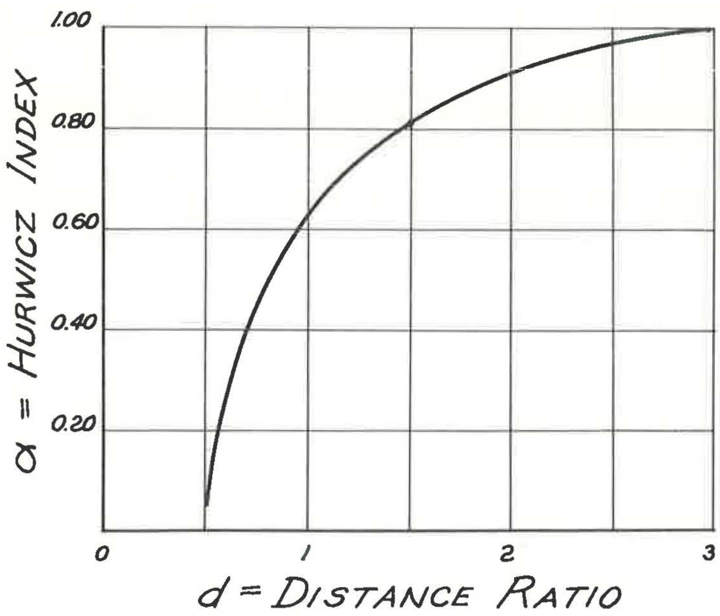

Figure 2. Minimum Hurwicz α to make freeway attractive.

the distance ratio increases to 2.00. It is also clear that the limiting distance ratios computed earlier are in agreement with the results of the Hurwicz α analysis. The freeway is attractive for a distance ratio less than 0.48 regardless of probable traffic conditions. Similarly, the arterial alternate is optimal for a distance ratio larger than 2.94, even if good freeway driving is a virtual certainty.

The minimax-regret criterion can be used to develop a traffic diversion curve. The regret matrix can be written as follows for a distance ratio of one:

$$R_1 = \begin{bmatrix} 0 & 0 & 0 & 1.10 \\ 0.66 & 0.50 & 0.20 & 0 \end{bmatrix} \quad (12)$$

Equation 12 shows that nature should select either its strategy 1 or 4 if it wishes to inflict the most damage on the driver. Nature's strategies 2 and 3 are said to be "dominated" by strategy 1 and can be deleted from the matrix without changing the solution of the game. All regret matrices in the present problem can be reduced to the form

$$R = \begin{bmatrix} 0 & a_{12} \\ a_{21} & 0 \end{bmatrix} \quad (13)$$

The problem is now to find a mixed strategy that will minimize the maximum possible regret. Let p be the probability of selecting row 1 and q be the probability of selecting row 2. Then

$$p \geq o, \; q \geq o, \; p + q = 1 \quad (14)$$

If m is the maximum regret, then

$$a_{12}p \leq m$$
$$a_{21}q \leq m \quad (15)$$

Writing $r = p/m$, $s = q/m$, $k = 1/m$ yields a linear programming problem.

Maximize $k = r + s$, subject to

$$r \geq 0, \; s \geq 0, \; a_{12}\, r \leq 1, \; a_{21}\, s \leq 1 \quad (16)$$

It turns out that

$$p = \frac{a_{21}}{a_{12} + a_{21}} \quad (17)$$

Plotting p against the distance ratio d yields the diversion curve shown in Figure 3, because p can be interpreted as the percentage of drivers who should select the freeway route for a given distance ratio.

PARTIAL KNOWLEDGE OF STATE OF FREEWAY TRAFFIC

Drivers who must return home from work every day over one of a group of alternate routes can be expected to acquire some knowledge of traffic conditions over these routes as a result of repeated use. It is therefore of interest to consider the case of decision-making under risk wherein it is assumed that the driver can estimate a probability distribution over the various possible states of freeway level of service. Once the state probabilities are estimated, this case can be solved by transforming it to a linear programming problem.

Under the assumption that levels of service E and F are the most probable states of freeway traffic during peak periods, a complete strategy for route selection can be formulated from a knowledge of the probability of encountering level of service F. In

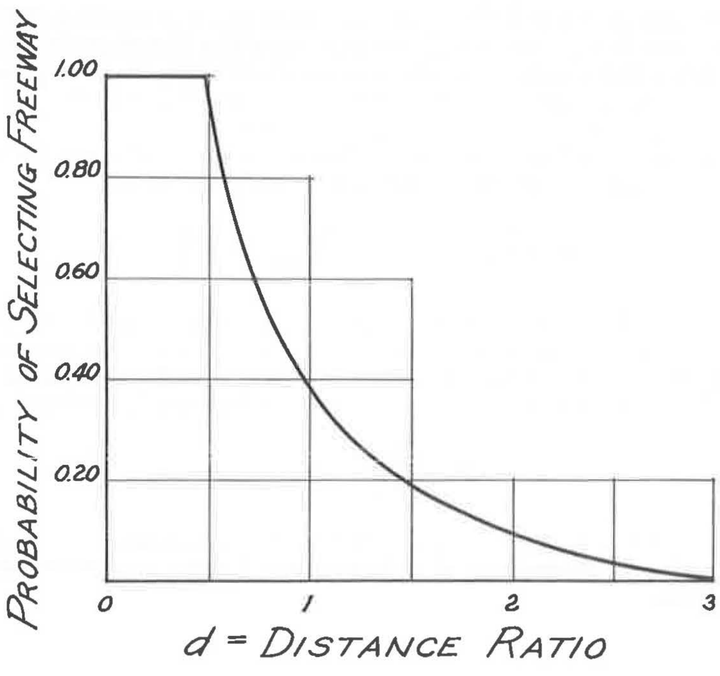

Figure 3. Diversion curve based on minimax regret.

any event, a knowledge of the vector of traffic state probabilities for the freeway would permit a driver to make an optimal route selection.

If a driver is advised that at time 0 the state of freeway traffic is i (i = 1, . . ., 4), the entries in the vector of state probabilities at time 0 would be

$$\pi_k = 1 \quad (k = i)$$
$$\pi_k = 0 \quad (k \neq i) \tag{18}$$

Suppose that the state of freeway traffic is determined at discrete intervals Δt by a surveillance system and that this information is displayed on signs or broadcast over commercial radio at such intervals or both. If a driver could enter the freeway at time 0 (and could expect the state of traffic to remain substantially constant for the duration of his trip), his route selection strategy would be a simple problem.

But the driver will probably not be able to enter the freeway until t units of time after receiving a status report. At time t, the vector of freeway traffic state probabilities will be given by

$$\pi(t) = \pi(o)e^{At} \tag{19}$$

where A is the matrix of transition rates of a continuous time Markov process.

If the interval Δt between successive announcements of the state of freeway traffic is relatively long, the driver will be forced to assume that $\pi(t)$ is equal to the limiting value of π either that he has learned to estimate by repeated use of the system or that he must estimate under conditions of uncertainty.

On the other hand, if Δt is made relatively short, the driver can estimate $\pi(t)$ by applying his knowledge of $\pi(o)$. That is, frequent announcements of the state of freeway traffic will tend to make the time interval t comparatively short between a driver's route selection and the actual beginning of his freeway trip. $\pi(t)$ is clearly a continuous function of time. Consequently, if one element of $\pi(o)$ is unity and the others are zero, it is reasonable to expect that there will be a significant interval of time during which one element of $\pi(t)$ will be numerically dominant over all the others. It is during this time interval that a driver can benefit from information on the state of freeway traffic. As the time following a freeway report increases, the state probabilities will approach their limiting values and the driver's information will become stale.

A driver with accurate knowledge of the state of freeway traffic can reduce his regret to zero in the route-selection game because he can always select the optimum strategy if all but one column in the payoff matrix are known to have probability zero. On the other hand, as the reliability of his traffic information deteriorates with the passage of time, his minimax regret will approach the values computed for the case of uncertainty.

If a dollar value could be assigned to a driver's regret, it would be possible to design an information system by selecting Δt so that the cost of providing data on the state of freeway traffic was equal to the value of the total regret of potential freeway users. The efficiency of an information system could also be defined as

$$E = 100 \left(1 - \frac{R}{R^*}\right) \tag{20}$$

where R is the maximum regret associated with the optimum strategy resulting from the state probability vector $\pi(t)$ and R^* is the minimax regret associated with route selection under uncertainty.

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