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Stress Distribution in Rectangular Concrete Slabs Under Wheel Loads

Y. H. HUANG, Department of Civil Engineering, University of Kentucky

•ENGINEERS have long used the theory of elasticity for determining the stresses in concrete pavements due to wheel loads. One of the best-known and most frequently used methods is Westergaard's analysis (1) for determining the maximum stress in an infinite large slab loaded at a corner, near an edge, or at the interior of a slab far from any edge. In order to check the validity of the theory, experimental pavements were constructed (2) and the maximum stress under a given wheel load was determined and compared with the theoretical solution. These comparisons based on the maximum stress generally could not give a definite indication on the applicability of the theory because the theoretical stress depends strongly on the modulus of elasticity and the Poisson's ratio of the concrete as well as the modulus of subgrade reaction, the exact values of which are quite difficult to ascertain. It is believed that a comparison between theoretical solutions and experimental measurements should be made on the distribution of stresses rather than just on the maximum stress. Although the distribution of stresses in concrete pavements was measured in the AASHO Road Test and contours of major and minor principal stresses were presented (3), because of the mathematical difficulty involved no effort has been made to compare these contours with those obtained by the elastic theory.

The purposes herein are twofold: (a) to introduce an approximate method for determining the stress distribution in rectangular concrete slabs, based on the theory of thin plates on elastic foundations; and (b) to determine theoretically the contours of principal stresses and compare them with those obtained experimentally from the AASHO Road Test.

The theoretical method employed in this study was first developed by Vint and Elgood (4) as early as 1935 for determining the deflections in a rectangular plate. In 1937 Murphy (5) applied the method to obtain stresses and deflections in a rectangular plate with four free edges. He showed that the method could easily be extended to the case where part of the plate is not in contact with the subgrade. Because the method is quite cumbersome and requires the solution of a large number of simultaneous equations, it has not received the attention it merits. However, this difficulty has been completely overcome with the advent of high-speed computers. The inclusion of partial contact between pavement and subgrade is an outstanding feature not considered in Westergaard's analysis.

In this method, the deflection function is represented by a double series in the form of two orthogonal functions. Under a given wheel load, the coefficients of the series can be determined by minimizing the total energy. Because of the particular deflection function selected, it is necessary to neglect the effect of Poisson's ratio when determining the moment and shear from the deflection so that the boundary condition that no moment exists at the free edge can be satisfied. This surely will lead to error, and the method may be considered as only approximate. However, the error is believed to be small, especially when the point at which the stresses are to be sought is not too far from the edge. Details of the method can be found elsewhere (4, 5).

Another difficulty in comparing theoretical solutions with the experimental data from the AASHO Road Test is that the former are based on free edges while dowel and tie

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bars were used in the latter. Although the effect of dowel and tie bars on stress distribution is very significant when the load is close to the joints, their effect becomes smaller as the load moves farther away from the joints. For this reason, only the case when the load is at a distance of 6 ft from the transverse joint (Fig. 1) was used for comparison.

Figure 1 shows the contours of major and minor principal stresses in a 5-in. pavement over a 36-sq ft region bounded by the pavement edge and a transverse joint. A total load of 6 kips was applied to the pavement through 2 wooden pads, each having 11 × 14-in. area and spaced on 6-ft centers. The stress is considered positive when the top of slab is in tension and negative when in compression. Figure 1a shows the experimental stresses determined in the AASHO Road Test (3). The stresses were measured at the 15 points indicated by the black dots. Figure 1b shows the theoretical stresses based on the assumption that the slab and subgrade are in full contact. Figure 1c shows the theoretical stresses when a 0.6-ft strip adjacent to the outside edge is not in contact. In the theoretical calculations, the following data employed or determined in the Road Test were used: length of slab = 180 in., width of slab = 144 in., thickness of slab = 5 in., modulus of elasticity of concrete = 6.25×10^6 psi, Poisson's ratio of concrete = 0.28, and modulus of subgrade reaction = 80 pci. The deflection function was approximated by 144 terms. The stresses were determined at 7 × 7 or 49 points, each 6 in. apart, and the contours of equal stresses were then interpolated. The method was programmed for the IBM 360 high-speed computer available at the University of Kentucky.

A comparison between the experimental stresses and the theoretical stresses based on full contact indicates that the general pattern of stress distribution is quite similar, although the magnitudes of the computed stresses are somewhat smaller than those of the measured stresses. This result is reasonable because the stresses were measured when the corners and edges of the slab were curled upward, whereas the theoretical

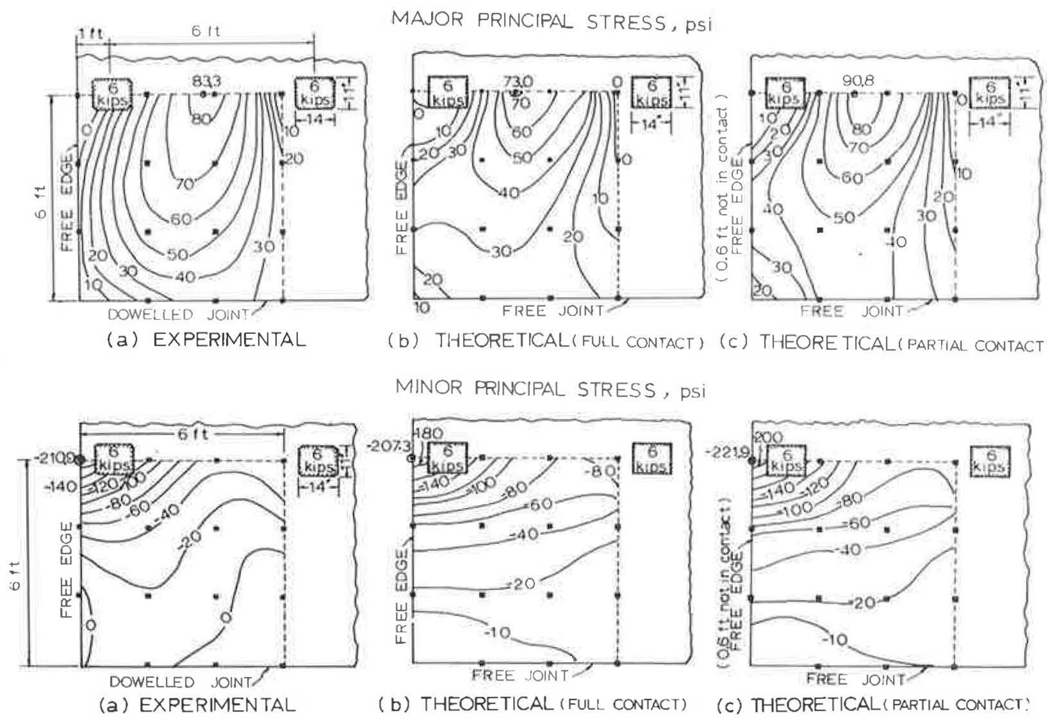

Figure 1. Contours of major and minor principal stresses, experimental vs. theoretical.

analysis is based on full subgrade contact. As can be seen from the figure, the assumption of partial contact gives stresses that check more closely with actual measurements than the assumption of full contact.

A note should be made on the case of partial contact. To compute the stresses based on partial contact, it is necessary to know the area over which the slab and subgrade are in contact. This area can be estimated by a method of successive approximations if the curling of the slab at various points is known. Because reliable information on the curling of the slab is not available, it is arbitrarily assumed that a 0.6-ft strip, or 5 percent of the slab width, adjacent to the outside edge is not in contact. The inside edge is at the longitudinal joint and is assumed in full contact. Although the area adjacent to the transverse joints may also be curled, its effect on stress distribution for the given loading position is comparatively small and can therefore be neglected.

The agreement between the theoretical and the experimental stress distribution for the 9.5-in. and 12.5-in. slabs is quite similar to that for the 5-in. slab. Because of space limitations, they are not presented here.

The close agreement between the theoretical solutions and the experimental measurements indicates that this approximate method, based on the theory of thin plates on elastic foundations, can be used to determine the stress distribution in concrete pavements when the load is far from the joints. Fortunately, this is also the most critical position as revealed by the AASHO Road Test.

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