

VISCOELASTIC MODEL FOR ELASTOMERIC PAVEMENT JOINT SEALS

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This paper describes the development of a simple viscoelastic model to simulate the time-dependent behavior of elastomeric seals. The model chosen for demonstration is the 3-parameter solid. Standard responses of this model are briefly described and are followed by appropriate analytical expressions. The analytical expressions are given in a general form and refer to three particular cases: (a) response to a fixed level of deformation that has been held for a specified length of time, (b) response on release from such a fixed level of deformation, and (c) load-deflection response in an experiment at a constant rate of displacement. The required parameters of the model are evaluated by using the experimental values obtained for an actual seal. The performance of the seal is then compared with that of the model. The model enables us to make some important acceptable predictions: (a) residual deformations immediately on release from a fixed level of compression, (b) the subsequent recovery of the seal as a function of time, (c) the decay of the effective force exerted by the seal, and (d) the effect of the displacement rate on the load-deflection characteristic. Suggestions for further refinement of the model are also given.

●AN efficient pavement joint seal should at all times be in contact with the sides of the joint opening and should maintain adequate pressure on the seal-pavement interface to prevent the intrusion of water and incompressible materials into the joint opening (6, 7, 8). To function effectively, the seal must be able to expand quickly to bridge any opening created by slab movement. At the same time, it must also maintain effective contact with the sidewalls and exert adequate pressure to meet the requirement previously described.

A variety of solid and liquid pavement sealers are commercially available and have been discussed in adequate detail elsewhere (1, 2, 5, 6, 8). In a recent study, conducted by the author (7, 8), it was observed that elastomeric seals combine flexibility with resilience and seem to be well suited for effectively sealing pavement joints.

The principal items of interest to the highway engineer are the maximum tensile and compressive stresses in a given seal and the effective pressure exerted by the seal on the sidewalls of the joint opening. The maximum stresses in a seal can be evaluated analytically (7, 8) or experimentally by using the photoelastic method (3). The effective pressure exerted by the seal is usually assessed by short-time load-deflection experiments conducted in the laboratory. Although these experiments give a fair idea of the strength of the seal, it is important to recognize that the basic seal material is viscoelastic (1, 2, 7, 8). Consequently, the effective pressure exerted by the seal on the interface will vary with time. Little is known about the correct constitutive relationships that relate the stresses and strains in the basic seal material. It is possible, however, to develop tentative working models that give a better understanding of the product and its performance.

The results of the experimental studies on the recovery of preformed elastomeric seals emphasize two salient features of the seals. When released from a fixed level of deformation that has been held for a specified duration of time, the seals show a definite (but not complete) recovery immediately on release. As a result, all seals tested show a residual deformation that is temporary in nature and that, in most cases, is fully recovered with lapse of time. These findings indicate that the behavior of the product is comparable to that of a viscoelastic solid (8). The simplest possible viscoelastic model that can simulate this behavior is the 3-parameter solid (4) shown in Figure 1.

The main objective of this paper is to simulate the behavior of a seal by means of the viscoelastic model previously mentioned. The necessary mathematical expressions for the 3-parameter solid are developed for three different situations: (a) response to a fixed level of deformation that has been held for a specified duration of time, (b) response on release from such a given level of deformation, and (c) response of the model in a load-deflection experiment conducted at a fixed displacement rate. The mathematical results are presented in a general form suitable for repeated use. The actual response of the model (Fig. 1) will depend on the values of the parameters k_1 , k_2 , and c associated with the springs and the dashpots shown in the figure. In a more general form, these parameters are designated by the symbols β_1 , β_2 , and γ . For practical predictions, the parameters are evaluated from the experimental results obtained for an actual seal. The performance of the corresponding model, so constructed, is then compared with the actual response of the sample. The model is used, subsequently, to predict the effective force exerted by the seal with lapse of time when it is held at a fixed level of deformation. Further, the same model is used to construct the load-deflection curves for a similar seal when tested at different displacement rates.

DESCRIPTION OF THE MODEL

The model for the 3-parameter solid (Fig. 1) is composed of a linear spring k_1 in series with a Kelvin solid (4). The latter consists of a linear spring k_2 in parallel with a dashpot whose constitutive constant is c . The symbol c is meant to indicate that the resistance offered by the dashpot is equal to the product of this constant and the time rate of displacement in the dashpot. We may, for the sake of convenience, refer to a dashpot with a high value of c as a stiff dashpot and a dashpot with a low value of c as a weak dashpot. Further, we designate by ℓ the distance between terminals A and B, when the model is completely free of stress. When such a stress-free model is subjected to a load $P(t)$, the spring k_1 and the Kelvin element undergo displacements $\delta_1(t)$ and $\delta_2(t)$, as shown in Figure 1. These displacements are governed by the following relationships:

$$k_1\delta_1(t) = P(t) \quad (1)$$

$$k_2\delta_2(t) + c\dot{\delta}_2(t) = P(t) \quad (2)$$

where

$$(\dot{\quad}) = d/dt(\quad) \quad (3)$$

Equation 3 defines the dot notation for designating the time derivative of a function. The total displacement $\delta(t)$ in the model is given by

$$\delta(t) = \delta_1(t) + \delta_2(t) \quad (4)$$

As implied in Eqs. 1 through 4, forces and displacements are treated as functions of time t . We introduce, next, dimensionless displacements as follows:

$$x_1(t) = \delta_1(t)/\ell \quad (5)$$

Figure 1. The 3-parameter solid.

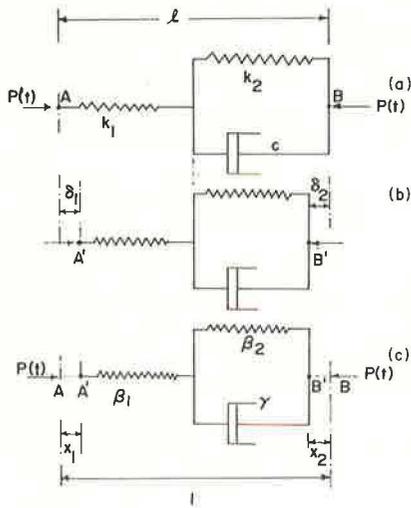


Figure 2. Deformation under a step load of one unit.

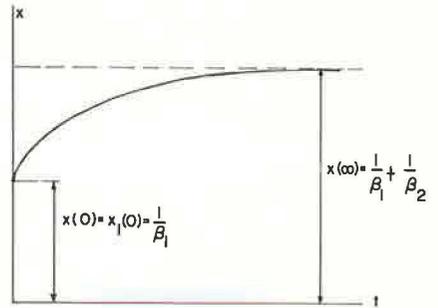


Figure 3. Effective load P(t) when model is subjected to a fixed deformation x0.

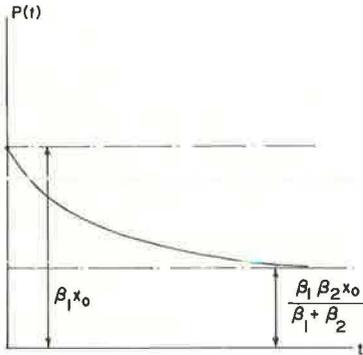


Figure 4. Response of model on release from deformation x0, held for a duration of time tau.

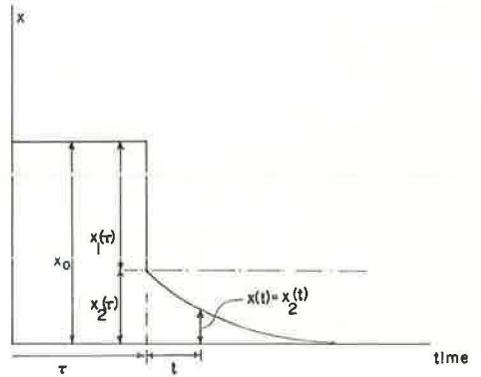


Table 1. Recovery results for a sample at 0 F and 40 percent nominal deformation.

Time t (min)	Observed Values of $x(t) = x_2(t)$ on Release From Deformation x		
	$\tau^a = 24$ hours $x^b = 0.424$	$\tau = 48$ hours $x = 0.398^c$	$\tau = 168$ hours $x = 0.398^c$
0	0.154	0.156 ^c	0.193
1	0.124	0.097	0.150
2	0.092	0.087	0.139
3	0.088	0.077	0.130
4	0.082	0.069	0.115
5	0.082	0.066	0.110
10	0.076	0.053	0.099

^aDuration τ in hours.

^bFixed deformation x in the duration τ .

^cValues were used in evaluating the required parameters of the viscoelastic model.

$$x_2(t) = \delta_2(t)/\ell \quad (6)$$

$$x(t) = \delta(t)/\ell \quad (7)$$

Further, let

$$\beta_1 = \ell k_1 \quad (8)$$

$$\beta_2 = \ell k_2 \quad (9)$$

$$\gamma = \ell c \quad (10)$$

By using Eqs. 5 through 10, Eqs. 1, 2, and 4 may be rewritten as

$$\beta_1 x_1(t) = P(t) \quad (11)$$

$$\beta_2 x_2(t) + \gamma \dot{x}_2(t) = P(t) \quad (12)$$

$$x(t) = x_1(t) + x_2(t) \quad (13)$$

In Eqs. 11 through 13, x is related to the total percentage of compression in the model; x_1 and x_2 are related to the contribution from the two components of the basic model. We may look on these expressions as providing a description of the performance of the normalized model shown in Figure 1c.

The 3-parameter solid is a basic viscoelastic model and is very adequately described elsewhere (4). Some salient features of the model are as follows. When the model is subjected to a unit step load, the linear spring β_1 responds immediately with an initial displacement $x = 1/\beta_1$, which is followed by viscous creep (Fig. 2). If, on the other hand, the model is subjected to a fixed displacement x_0 , the force required to maintain this displacement diminishes exponentially as shown in Figure 3. The response of the model on release from such a fixed displacement x_0 , held for a duration of time τ , is shown in Figure 4. As this figure shows, the elastic component $x_1(\tau)$ is recovered immediately, whereas the Kelvin component $x_2(\tau)$ is recovered exponentially with lapse of time from release. The eventual recovery of this Kelvin component is an important feature of the response and is responsible for the designation of the model as a viscoelastic solid.

ANALYTICAL EXPRESSIONS FOR THE THREE CASES

The behavior described in the preceding section compares favorably with that observed experimentally for elastomeric seals. The expressions given in this section will be helpful in finding a working 3-parameter model for a given seal. All of the expressions are derived from Eqs. 11 through 13 by using appropriate conditions for each particular case.

Case One

Displacement x_0 held for time τ .

Conditions:

$$x(0) = x_1(0) = x_0 \quad (14)$$

$$x_2(0) = 0 \quad (15)$$

$$t \leq \tau \quad (16)$$

Solutions:

$$x_2(t) = [\beta_1/(\beta_1 + \beta_2)] x_0 [1 - e^{-at}] \quad (17)$$

$$x_1(t) = [\beta_1/(\beta_1 + \beta_2)] x_0 [(\beta_2/\beta_1) + e^{-at}] \quad (18)$$

$$P(t) = [\beta_1/(\beta_1 + \beta_2)] x_0 [\beta_2 + \beta_1 e^{-at}] \quad (19)$$

where

$$a = (\beta_1 + \beta_2)/\gamma \quad (20)$$

Case Two

Response on release from a displacement x_0 held for time τ . In case two we have to proceed from the information obtained from case one. From the basic character of the model, we recognize that as soon as the displacement x_0 is released, after it has been held for duration τ , there will be an immediate recovery equal in magnitude to $x_1(\tau)$. The displacement $x_2(\tau)$ will appear as a residual deformation, which will be recovered eventually with lapse of time. Then, for this case, we have the following situation.

Conditions:

$$t > 0_+ \text{ (measured from the instant of release)} \quad (21)$$

$$P(t) = 0 \quad (22)$$

$$x_2(0) = x_2(\tau) \quad (23)$$

where $x_2(\tau)$ is obtained from Eq. 17.

Solutions:

$$x(t) = x_2(t) = x_2(\tau)e^{-(\beta_2/\gamma)t} \quad (24)$$

Case Three

Load-deflection response in a test at constant rate of displacement α .

Conditions:

$$\alpha = \text{imposed displacement rate (units of length/units of time)} \quad (25)$$

$$\dot{x} = \alpha/\ell \quad (26)$$

$$x(0) = x_1(0) = x_2(0) = \dot{x}(0) = 0 \quad (27)$$

Solutions:

$$x_1(x) = (\beta_1\alpha/\gamma a^2\ell) \left[1 + (\beta_2/\beta_1) (a\ell/\alpha) x - e^{(a\ell/\alpha)x} \right] \quad (28)$$

$$x_2(x) = (\beta_1\alpha/\gamma a^2\ell) \left[e^{-(a\ell/\alpha)x} + (a\ell/\alpha) x - 1 \right] \quad (29)$$

$$P(x) = (\beta_1^2 \alpha / a^2 \gamma \ell) \left[1 + (\beta_2 / \beta_1) (a / \alpha) x - e^{-(a / \alpha) x} \right] \quad (30)$$

or

$$u(x) = \alpha \left[1 + (\beta_2 / \beta_1) (a / \alpha) x - e^{-(a / \alpha) x} \right] \quad (31)$$

where

$$u(x) = \left[P(x) a^2 \gamma \ell \right] / \beta_1^2 \quad (32)$$

APPLICATION TO ELASTOMERIC SEALS

The expressions derived in the preceding section are useful in predicting the time-dependent response of a unit length of an actual seal. For the purpose of such a prediction, however, we need the appropriate numerical values of the constants β_1 , β_2 , and γ or the ratios β_2 / β_1 , $\beta_1 / (\beta_1 + \beta_2)$, $(\beta_1 + \beta_2) / \gamma$, and β_2 / γ . These ratios can be easily evaluated from the results of recovery experiments for an actual seal.

In a recent study conducted by the author (8), a number of seals were tested at 0, 75, and 120 F for their recovery as a function of time on release from fixed levels of deformation that had been held for specified durations of time. The recovery of all the seals tested was found to be weakest at the low temperature of 0 F. Some results for one of the seals, tested at 0 F, are given in Table 1. These results are used in the numerical example that follows to evaluate the ratios mentioned previously. It should be noted that the corresponding model applies to a rather stringent practical situation.

When the sample is held at 39.8 percent deformation (Table 1) for 48 hours, or 2,880 minutes, and then released, it has a temporary residual deformation of 15.6 percent. By using this information in Eq. 17, we get

$$\left[\beta_1 / (\beta_1 + \beta_2) \right] \left[1 - e^{-2880 a} \right] = 0.156 / 0.398 = 0.392 \quad (33)$$

Similarly, from the results of the 168-hour (1-week) test

$$\left[\beta_1 / (\beta_1 + \beta_2) \right] \left[1 - e^{-10080 a} \right] = 0.193 / 0.398 = 0.485 \quad (34)$$

In Eqs. 33 and 34, we have two equations and two unknown quantities, $\beta_1 / (\beta_1 + \beta_2)$ and a . These equations can be solved quickly by the following iterative scheme. We first solve Eq. 34 for $\beta_1 / (\beta_1 + \beta_2)$, assuming that the contribution from the exponential term is negligible. Then, we use this value in Eq. 33 to evaluate a . Next, we substitute the value of a in Eq. 34 to reevaluate $\beta_1 / (\beta_1 + \beta_2)$ for the second iteration. This iterative process is continued until satisfactory convergent values are obtained for the two unknowns. In the present example, we converge to the following values at the end of two cycles

$$\left[(\beta_1 + \beta_2) / \beta_1 \right] = 2.06, \beta_2 / \beta_1 = 1.06, a = 0.571 \times 10^{-3} \quad (35)$$

Next, we consider the response of the seal on release from 39.8 percent compression (held for 48 hours). From the value at $t = 0$ (Table 1, column 3) and Eq. 24, we get

$$x_2(t) = x(t) = 0.156 e^{-(\beta_2 / \gamma)t} \quad (36)$$

where t = time in minutes from release. From these results (Table 1, column 3), we obtain the following average value for β_2 / γ :

$$\beta_2 / \gamma = 0.202 \quad (37)$$

It may be pointed out here that the model parameters are assumed to be temperature-dependent. The numerical values obtained in this example refer to a model that may be used to represent the particular seal considered at a temperature of 0 F.

DISCUSSION OF RESULTS

It is important to verify whether the model we have constructed is satisfactory. For this purpose we compare the response of the model with that observed for the seal. The comparison is shown in Figure 5 and Table 2. The plots in Figure 5 were constructed by using the model parameters already evaluated in Eqs. 35 and 37 and the observed values given in Table 1. From the plots, it is evident that the 3-parameter solid satisfactorily predicts the performance of the actual seal. The model predicts a more favorable recovery than that observed, but the differences are not large.

An important value from a practical standpoint is the residual deformation that occurs immediately on release. A comparison of these values for the model and the actual seal is given in Table 2. The last row in this table gives the largest deviation between the observed and the predicted values. The observed residual deformation in this case is 36.8 percent, whereas that predicted by the model is 29.3 percent, a difference of 7.5 percent. All the other values in this table show a deviation of less than ± 4 percent. In view of the large deformations involved, the prediction made by the model is quite satisfactory.

As mentioned in the introduction, if the seal is to function effectively, it must maintain adequate force on the sidewalls of the joint. In this context, the results of the short-time load-deflection experiment and the use of the viscoelastic model yield some important information. From the load-deflection curve obtained by laboratory testing, the force $P(0)$ exerted by the seal at a given percent deformation x can be readily obtained. When the seal is compressed to this same level of deformation in the pavement joint, it may be assumed that it exerts the same force $P(0)$ on the interface at the time of installment. If the compression is maintained at all times, the seal, according to the model, will relax. Correspondingly, it will show a loss in the effective force exerted at the interface. From Eq. 19, the effective force $P(t)$, after a lapse of time t from installment, is obtained as:

$$P(t) = P(0) \left[(\beta_2/\beta_1) + e^{-at} \right] / \left[(\beta_2/\beta_1) + 1 \right] \quad (38)$$

The foregoing relationship is shown in Figure 6. The effective force decays exponentially and reaches 51.5 percent of its initial value. This limiting value of the effective force is obtained by using β_2/β_1 (from Eq. 35) in Eq. 38 and proceeding to the limit as $t \rightarrow \infty$. It is interesting to note that the expression in Eq. 38 is independent of the imposed step displacement. We also notice that, for this seal, at the end of 3 days the effective force reduces to 52 percent of its initial value, which is very close to the limiting value. The result, in this particular case, supports the ASTM recommendation of a 72-hour recovery test at low temperature. However, the ASTM recommendation may be confirmed or revised only after the model parameters are carefully evaluated for a large variety of seals.

An important test used in assessing the strength of a given seal is the short-time load-deflection experiment. Most testing machines that are used in experimental work load the specimen at a constant rate of displacement. As indicated by Eq. 31, the displacement rate at which the specimen is tested does affect the load-deflection response. The seal for which we have evaluated the model parameters seems to have a very stiff dashpot. For such a model, the load-deflection response is not appreciably affected by a practical rate of loading. However, this characteristic curve will be significantly influenced by the rate of loading if we use a very weak dashpot in the model (Fig. 7). The curves in this figure are plotted for a model with $a = 0.0571$ (very weak dashpot). Other model parameters are assumed to be the same as those evaluated in the preceding section. When the model is tested at a displacement rate of $\alpha = 0.5$ in./min, the viscous effects are almost imperceptible. However, as shown in Figure 7, the load values de-

Figure 5. Predicted and observed recovery of a seal at 0 F on release from 40 percent nominal deformation.

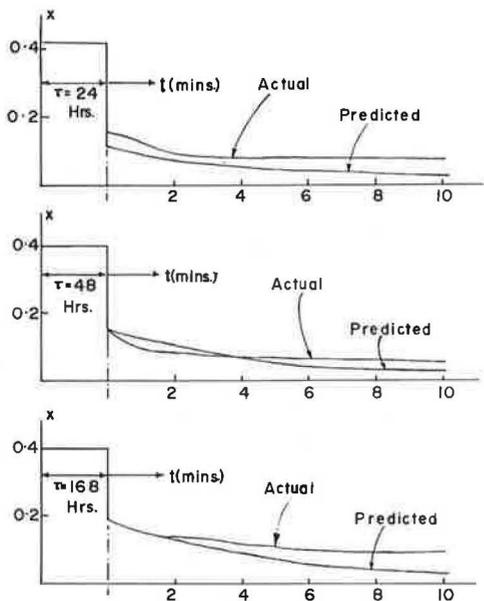


Figure 6. Predicted effective force P(t) at 0 F when the seal is held at a constant deformation.

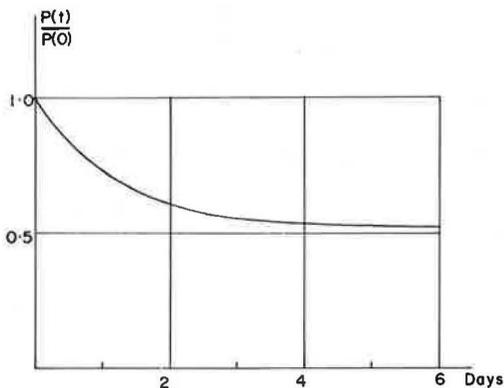
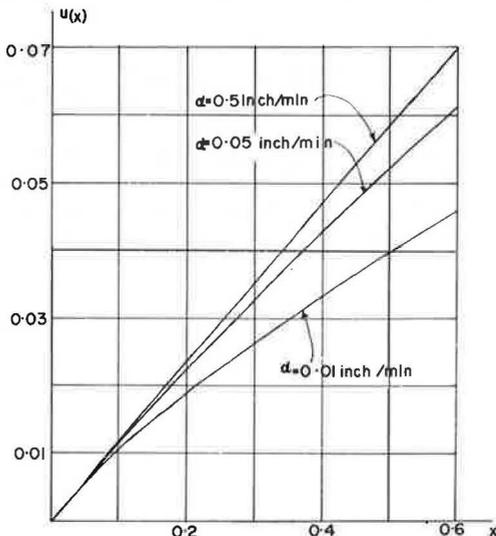


Table 2. Comparison of the actual immediate recovery of a seal and the recovery predicted by the model.

Test Duration τ (hour)	Nominal Deformation x Before Release (percent)	Values of x Immediately on Release	
		Actual Seal	Model Prediction
24	20	0.079	0.061
	40	0.154	0.116
	60	0.206	0.167
48	20	0.101	0.073
	40	0.156	0.156
	60	0.210	0.232
168	20	0.102	0.096
	40	0.193	0.193
	60	0.368	0.293

Figure 7. Load-deflection response in tests at different displacement rates.



crease with a decrease in the loading rate. For example, for $\alpha = 0.05$ in./min, the load at 40 percent compression will be 91 percent of its value when tested at $\alpha = 0.5$ in./min. If the loading rate is reduced to 0.01 in./min, the corresponding load value will be reduced to 71 percent.

CONCLUDING REMARKS

Viscoelastic models can be effectively used to represent elastomeric seals. The 3-parameter solid described in this paper gives a good prediction of the time-dependent response of the product considered. It is worthwhile to note that we have used one of the simplest possible models to simulate the actual behavior of a given seal. Further refinements can be incorporated in the analysis by adding more Kelvin elements to the 3-parameter solid. With such complicated models it may be possible to simulate the time-dependent behavior of elastomeric seals much more accurately. A considerable additional amount of well-planned experimental work is necessary for the development of these more sophisticated models.

It has been pointed out elsewhere (7) that the short-time response of the seals can be predicted by a nonlinear analysis of the seal section as a structure. A corresponding modification can be incorporated in the viscoelastic model by using nonlinear springs. The analysis of the resulting model may become much more complicated; however, such a model may lead eventually to an optimum design of the product.

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